# off-shell $t\bar{t}j$ production

## Top quark mass studies at the LHC

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# Introduction

LHC is a top quark factory: $t\bar{t}$ pairs are abundantly produced enabling high precision measurements of top quark properties

The study of (associated) $t\bar{t}$ production has a wide range of applications...

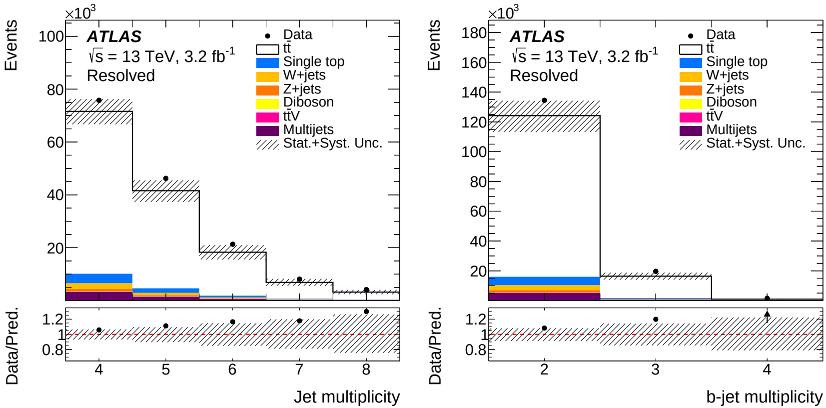
- SM benchmarks (e.g. $t\bar{t}$ cross section)
- precision measurements of SM parameters (e.g. $m_t$)
- probing Higgs-Yukawa sector (e.g. $t\bar{t}H$)
- constraining PDFs (especially gluon at large $x$)
- searches for BSM physics (e.g. heavy resonances decaying to tops)

... and many more

We study the associated $t\bar{t}$ production with one additional hard jet ($pp \to t\bar{t}j$) and its impact on the precise determination of $m_t$ at the LHC

# Why study $pp \to t\bar{t}j$?

- Large fraction of $t\bar{t}$ events accompanied by hard jets [ATLAS, arXiv:1708.00727]



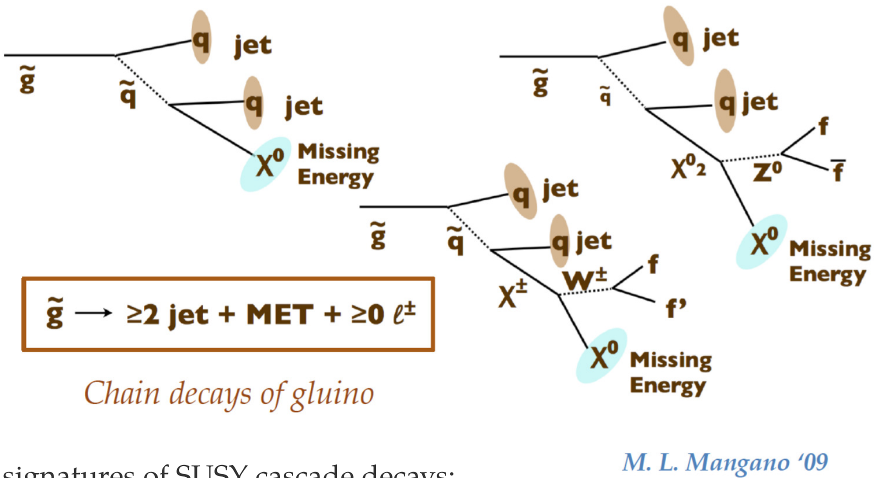

- Compare NLO $t\bar{t}j$ to inclusive $t\bar{t}$ cross section [Dittmaier et al.]

| $p_T^{cut}$ [GeV] | $\sigma_{t\bar{t}j}$ [pb] | $\sigma_{t\bar{t}j}/\sigma_{t\bar{t}}$ [%] |
|---|---|---|
| 40 | $296.97 \pm 0.29$ | 41 |
| 60 | $207.88 \pm 0.19$ | 29 |
| 80 | $152.89 \pm 0.13$ | 21 |
| 100 | $115.60 \pm 0.14$ | 16 |
| 120 | $89.05 \pm 0.10$ | 12 |

- Background for SM Higgs production in VBF: $qq \to Hqq \to W^+W^-qq$

[Englert et. al]

- Background for SUSY searches




Typical signatures of SUSY cascade decays:

$$\text{hadronic jets} + \text{charged leptons} + \text{missing } p_T$$

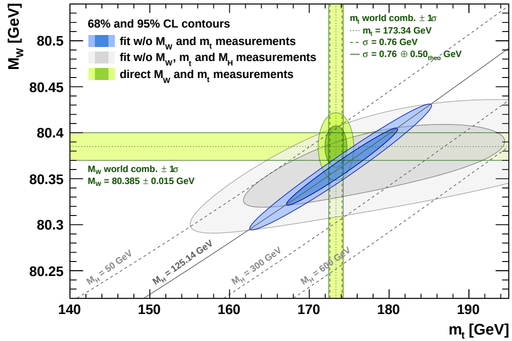
# Top quark mass: precision matters

Precision tests of the Standard Model:

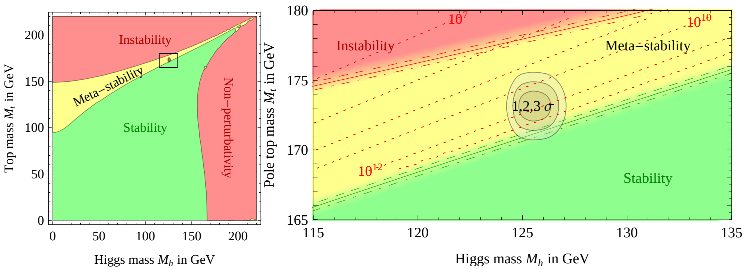
global EW fit: [Riemann et al., Baak et al., ...]

→ check self-consistency through

$m_t, m_W, m_H$ correlations




[Gfitter, Eur. Phys. J C74 (2014) 3046]




Stability of EW vacuum:

stable or meta-stable?

[Degrassi et al., JHEP 1208 (2012) 098]

# Theoretical predictions for $t\bar{t}j$

**NLO QCD Fixed-order**

- on-shell $t\bar{t}j$ production

  [Dittmaier, Uwer, Weinzierl '07'09]

- on-shell $t\bar{t}j$ production with LO decays

  [Melnikov, Schulze '10]

- on-shell $t\bar{t}j$ production with NLO decays

  [Melnikov, Scharf, Schulze '12]

- off-shell $t\bar{t}j$ production with leptonic decays

  [Bevilacqua, Hartanto, MK, Worek '15'16]

**Matched to parton shower**

- Powheg + Pythia → no spin correlations

  [Kardos, Papadopoulos, Trocsanyi '11]

- Powheg + Pythia/Herwig → spin correlations at LO

  [Alioli, Moch, Uwer '12]

- MC@NLO + Deductor → without decays

  [Czakon, Hartanto, MK, Worek '15]

# Narrow Width Approximation

$$\lim_{\Gamma_t/m_t \to 0} \frac{1}{(p_t^2 - m_t^2)^2 + m_t^2\Gamma_t^2} = \frac{\pi}{m_t\Gamma_t}\delta(p_t^2 - m_t^2)$$

NWA allows to factorize complicated matrix elements

$$\begin{aligned}\lim_{\Gamma_t/m_t \to 0} \left|M^{WWb\bar{b}j}\right|^2 = &\left|M^{t\bar{t}\mathbf{j}}\right|^2 \otimes \mathcal{B}r(t \to Wb) \otimes \mathcal{B}r(\bar{t} \to W\bar{b}) \\ &+ \left|M^{t\bar{t}}\right|^2 \otimes \mathcal{B}r(t \to Wb\mathbf{j}) \otimes \mathcal{B}r(\bar{t} \to W\bar{b}) \\ &+ \left|M^{t\bar{t}}\right|^2 \otimes \mathcal{B}r(t \to Wb) \otimes \mathcal{B}r(\bar{t} \to W\bar{b}\mathbf{j}) + \underbrace{\mathcal{O}\left(\frac{\Gamma_t}{m_t}\right)}_{\approx 1\%}\end{aligned}$$

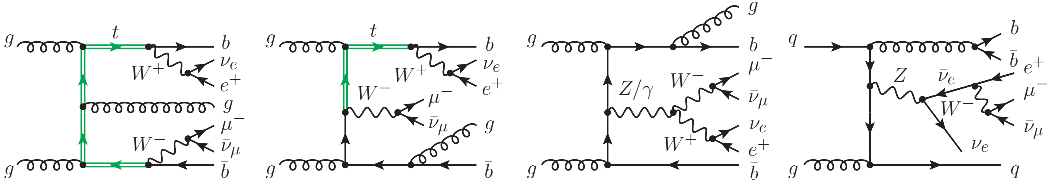
- on-shell tops and $W$
- hard jet radiation in decay
- Calculation provided by M. Schulze

[Melnikov, Scharf, Schulze '12]

NWA is best suited for inclusive observables. At the differential level, off-shell effects can reach tens of percent.

# Full off-shell calculation

$$pp \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}j + X$$

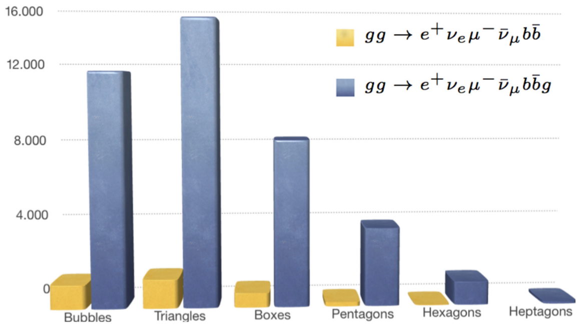



- Double-, single- and Non-resonant contributions

$$\left|M^{WWb\bar{b}j}\right|^2 = \left|M^{t\bar{t}j} + M^B\right|^2 = \underbrace{\left|M^{t\bar{t}j}\right|^2}_{2\times\ Resonant} + \underbrace{\left|M^B\right|^2}_{0,1\times\ Resonant} + \underbrace{2Re\left[M^{t\bar{t}j}M^{B\dagger}\right]}_{Interference}$$

- QCD corrections to $2 \to 5$ process ($WWb\bar{b}j$)
- $t\bar{t}j$ production at $\mathcal{O}(\alpha_s^4\alpha^4)$ in the dilepton decay channel

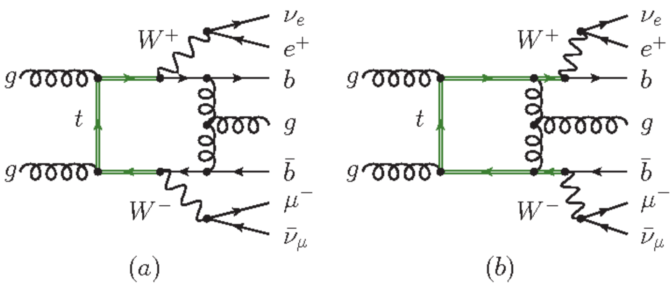
Number of 1-Loop Feynman diagram split by topology: $t\bar{t}$ vs. $t\bar{t}j$




[source: QGRAF (Nogueira '13). Special thanks to A. Kardos]

- up to 1155 hexagons and 120 heptagons
- Scalar integrals with complex masses: $m_t^2 \to m_t^2 - i m_t \Gamma_t$

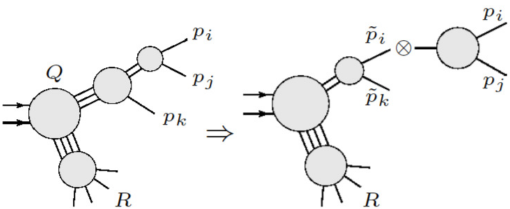
- NWA for $t\bar{t}j$ only up to pentagons!



## Number of subtracted terms for representative subprocesses

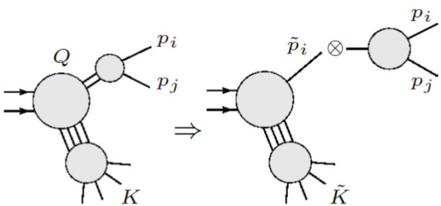
| Subprocess | # Diags | # CS dipoles | # NS dipoles |
|---|---|---|---|
| $gg \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}gg$ | 4447 | 56 | 14 |
| $gg \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}b\bar{b}$ | 3904 | 48 | 12 |
| $gg \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}q\bar{q}$ | 1952 | 40 | 10 |
| $qq \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}qq$ | 930 | 20 | 5 |
| $qq' \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}qq'$ | 501 | 12 | 3 |

### Catani-Seymour



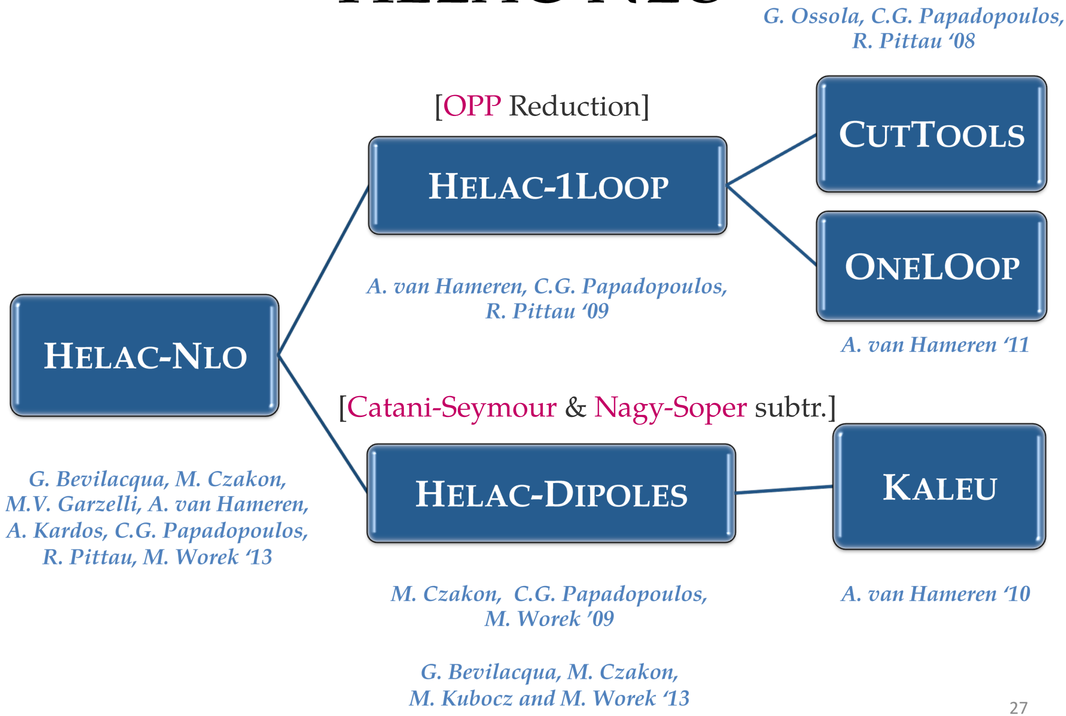

$$\{p\}_{m+1} \to \{\tilde{p}\}_m^{(ijk)}$$

### Nagy-Soper




$$\{p\}_{m+1} \to \{\tilde{p}\}_m^{(ij)}$$

# HELAC-NLO




- NEW: Ntuple generation + re-weighting for different $\mu_{R,F}$ and PDFs

# Numerical results for $t\bar{t}j$ at the LHC

# Setup for LHC 13 TeV

## Final state and parameters

- Fully leptonic decays: $pp \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}j + X$
- Light quarks (also bottom) and leptons are massless → 5 FS
- Top quark mass: $m_t = 173.2$ GeV
- Top quark width with off-shell $W$: [Jesabek,Kühn,'99, Denner et al. '12]

  $\Gamma_t^{\text{LO}} = 1.47834$ GeV $\qquad \Gamma_t^{\text{NLO}} = 1.35146$ GeV
- Complex Mass Scheme: $m_t^2 \to m_t^2 - i m_t \Gamma_t$ [Denner et al. '99, '05]

## Kinematics

- exactly 2 b-jets, $\geq 1$ light jet, 2 charged leptons, missing $p_T$
- partons with $|\eta| < 5$, anti-$k_T$, $\Delta R = 0.5$
- cuts:

$$p_{T\,\ell} > 30 \text{ GeV}\,, \qquad p_{T\,j} > 40 \text{ GeV}\,, \qquad \not{p}_T > 40 \text{ GeV}\,,$$
$$\Delta R_{jj} > 0.5\,, \qquad \Delta R_{\ell\ell} > 0.4\,, \qquad \Delta R_{\ell j} > 0.4\,,$$
$$|y_\ell| < 2.5\,, \qquad |y_j| < 2.5\,,$$

# Inclusive cross sections

PDF: CT14

Fixed scale: $\mu_0 = m_t$

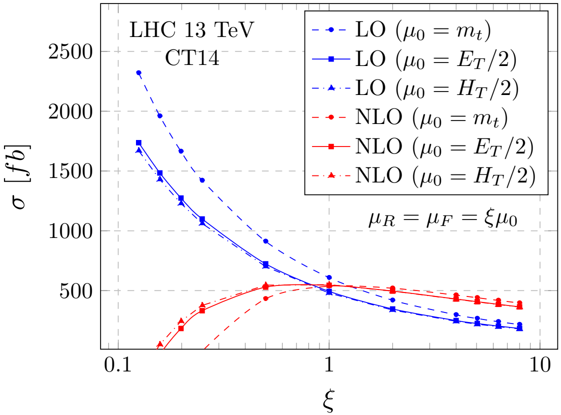
Dynamical scale: $\mu_0 = E_T/2$ and $H_T/2$

$$E_T = \sqrt{m_t^2 + p_T^2(t)} + \sqrt{m_t^2 + p_T^2(\bar{t})}$$

$$H_T = \sum_i p_{T,i} + \not{p}_T$$

Scale uncertainties:

$$\frac{1}{2} \leq \left(\frac{\mu_R}{\mu_F}\right) \leq 2\,, \quad \frac{1}{2} \leq \mu_R, \mu_F \leq 2$$

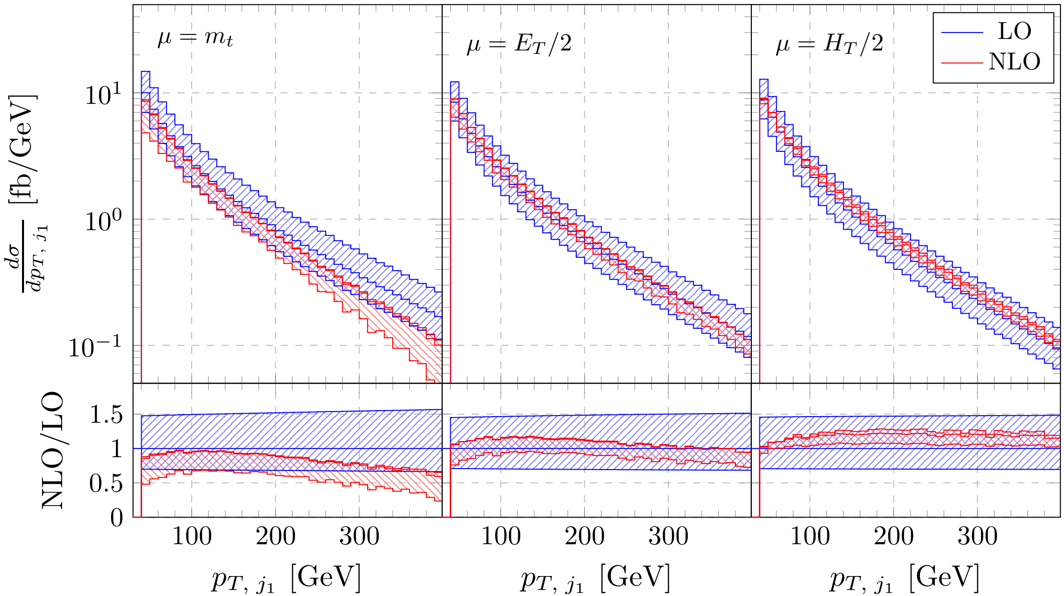



| $\mu_0$ | $\sigma_{LO}[fb]$ | $\sigma_{NLO}[fb]$ | $K$ |
|---|---|---|---|
| $m_t$ | $608.09\ ^{+50\%}_{-31\%}$ (scale) | $537.24\ ^{+2\%}_{-35\%}$(scale) $^{+3\%}_{-3\%}$(pdf) | 0.88 |
| $E_T/2$ | $493.54\ ^{+47\%}_{-30\%}$ (scale) | $544.64\ ^{+1\%}_{-22\%}$(scale) $^{+3\%}_{-3\%}$(pdf) | 1.10 |
| $H_T/2$ | $479.38\ ^{+46\%}_{-30\%}$ (scale) | $549.65\ ^{+2\%}_{-10\%}$(scale) $^{+3\%}_{-3\%}$(pdf) | 1.15 |

[Bevilacqua, Hartanto, MK, Worek, JHEP 1611 (2016) 098]

# Differential distribution - I

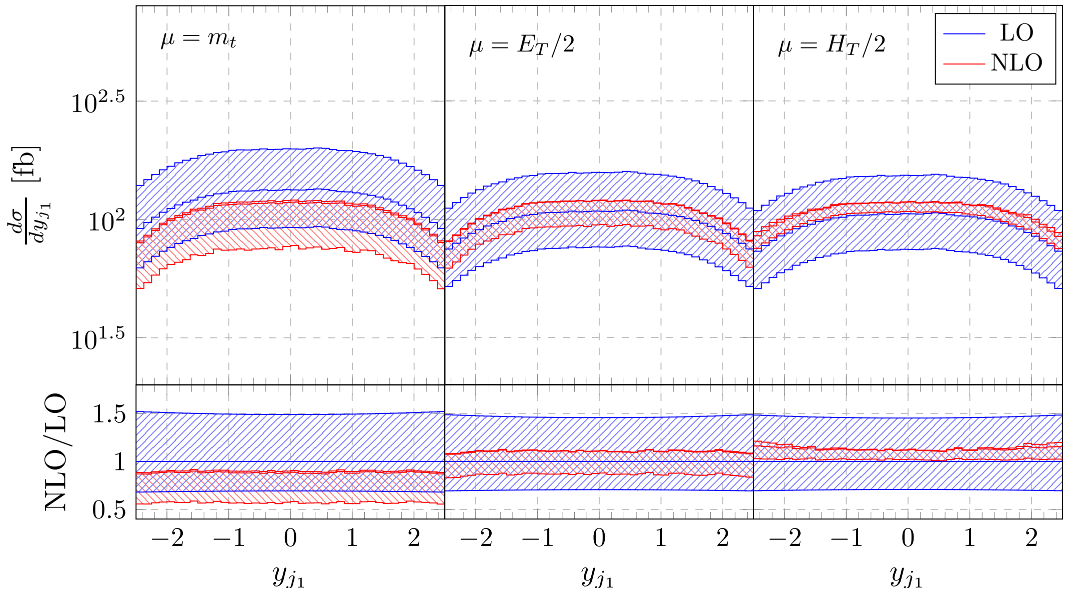
Transverse momentum of the hardest jet




[Bevilacqua, Hartanto, MK, Worek, JHEP 1611 (2016) 098]

# Differential distribution - II

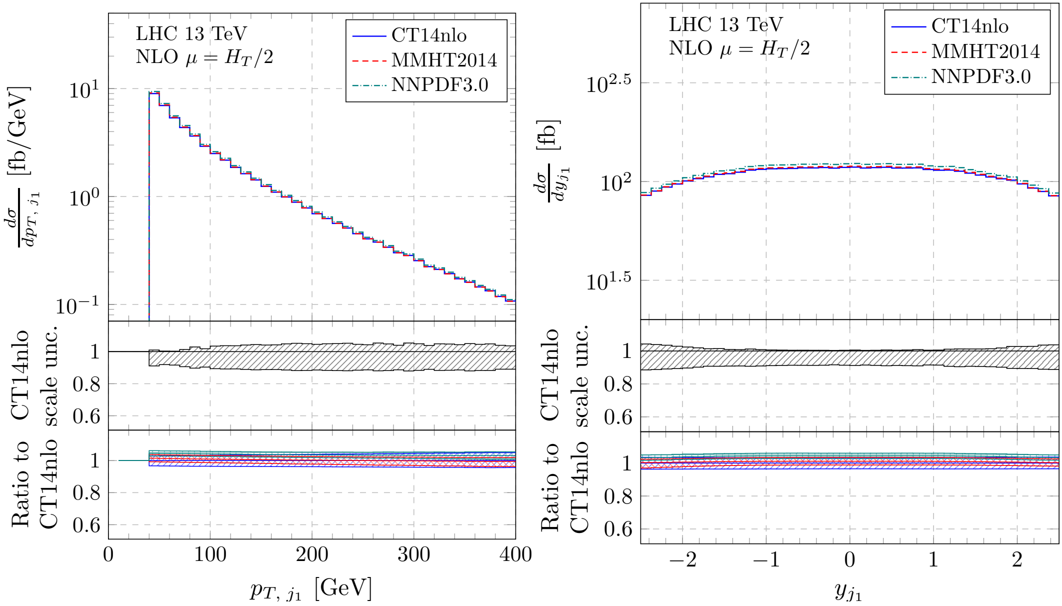
Rapidity of the hardest jet




[Bevilacqua, Hartanto, MK, Worek, JHEP 1611 (2016) 098]

# Differential distribution - III

PDF vs. Scale uncertainties



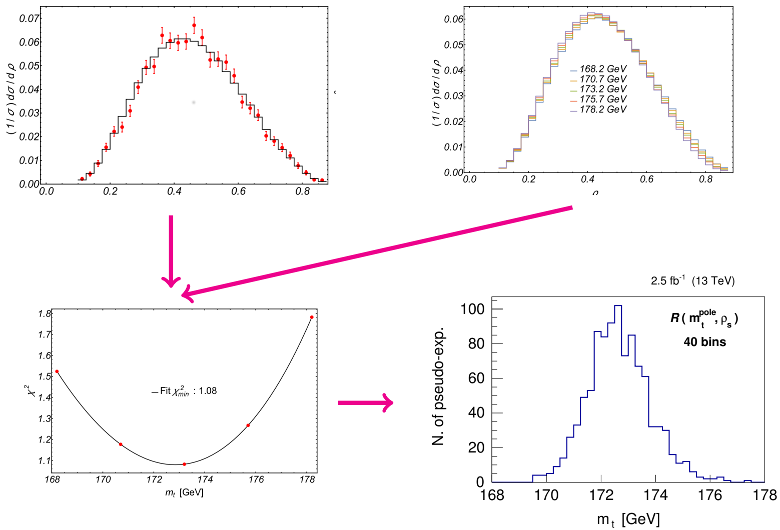

Scales ~ 10%, PDF CT14 ~ 5%

[Bevilacqua, Hartanto, MK, Worek, JHEP 1611 (2016) 098]

# Top quark mass studies with $t\bar{t}j$

## Basic idea

- generate pseudo-data for a given luminosity, distributed according to the best prediction ($\rightarrow$ NLO with off-shell effects, $\mu_0 = H_T/2$)
- fit pseudo-data with templates for different $m_t$
- compare results obtained with templates of different accuracy (NWA vs full)



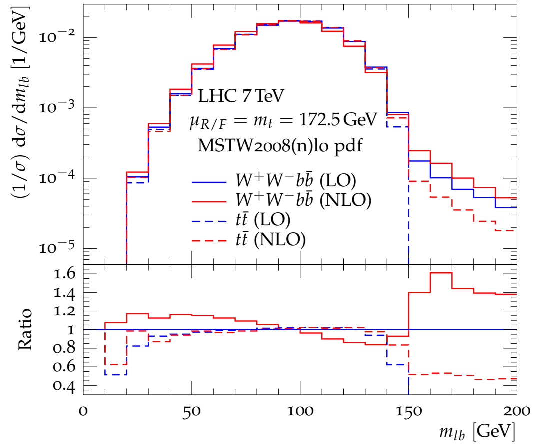
Case 1: min. invariant mass distribution of lepton and b-jets ($M_{b\ell}$)

- Assuming on-shell top and W decays, $M_{b\ell}$ has a sharp kinematical endpoint:

$$M_{b\ell}^{max} \approx \sqrt{m_t^2 - m_W^2} \approx 153 \text{ GeV}$$

- Off-shell and radiative effects smear the kinematical endpoint.
  Studied in detail for $t\bar{t}$ [Denner et al. '12, Heinrich et al. '13,...]
- Study on impact of off-shell effects on $m_t$ measurement by

[Heinrich et al, arXiv:1212.6659, arXiv:1709.08615]

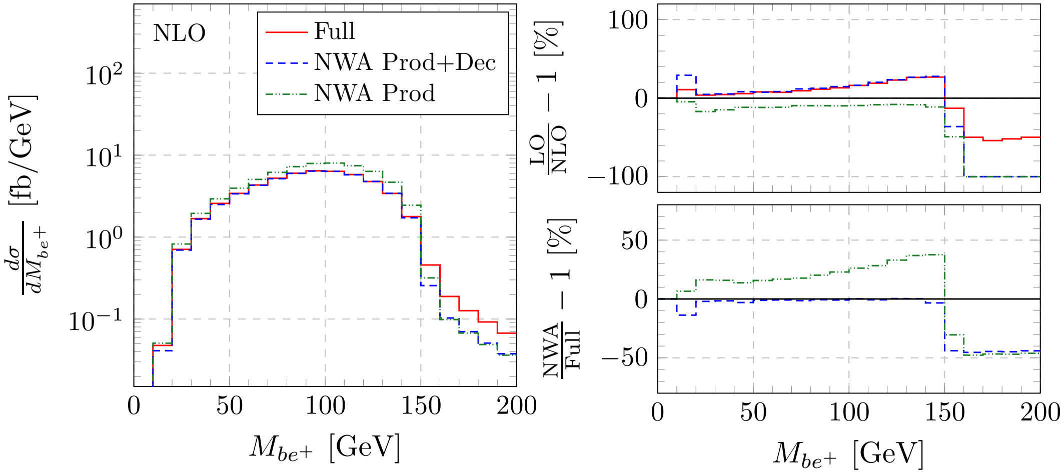


# Comparison to NWA

[Bevilacqua, Hartanto, MK, Schulze, Worek, JHEP 1803 (2018) 169]

PDF: CT14

Scale: $\mu_R = \mu_F = m_t$



# Fit results for $M_{be^+}$

[Bevilacqua, Hartanto, MK, Schulze, Worek, JHEP 1803 (2018) 169]

| Theory, NLO QCD CT14 PDF | $m_t^{out} \pm \delta m_t^{out}$ [GeV] | Averaged $\chi^2$/d.o.f. | Probability *p-value* | $m_t^{in} - m_t^{out}$ [GeV] |
|---|---|---|---|---|
| | | 2.5 $\text{fb}^{-1}$ | | |
| *Full*, $\mu_0 = H_T/2$ | $173.09 \pm 0.48$ | 1.05 | 0.38 $(0.9\sigma)$ | +0.11 |
| *Full*, $\mu_0 = E_T/2$ | $173.01 \pm 0.50$ | 1.06 | 0.37 $(0.9\sigma)$ | +0.19 |
| *Full*, $\mu_0 = m_t$ | $173.07 \pm 0.49$ | 1.22 | 0.18 $(1.3\sigma)$ | +0.13 |
| *NWA*, $\mu_0 = m_t$ | $173.90 \pm 0.50$ | 1.11 | 0.30 $(1.0\sigma)$ | −0.70 |
| $NWA_{\text{Prod.}}$, $\mu_0 = m_t$ | $172.56 \pm 0.54$ | 1.64 | 0.01 $(2.6\sigma)$ | +0.64 |
| | | 25 $\text{fb}^{-1}$ | | |
| *Full*, $\mu_0 = H_T/2$ | $173.18 \pm 0.15$ | 1.02 | 0.42 $(0.8\sigma)$ | +0.02 |
| *Full*, $\mu_0 = E_T/2$ | $173.23 \pm 0.15$ | 1.03 | 0.41 $(0.8\sigma)$ | −0.03 |
| *Full*, $\mu_0 = m_t$ | $173.22 \pm 0.16$ | 1.78 | 0.005 $(2.8\sigma)$ | −0.02 |
| *NWA*, $\mu_0 = m_t$ | $173.98 \pm 0.16$ | 2.56 | $5 \cdot 10^{-6}$ $(4.6\sigma)$ | −0.78 |
| $NWA_{\text{Prod.}}$, $\mu_0 = m_t$ | $172.62 \pm 0.17$ | 8.23 | 0 $(\gg 5\sigma)$ | +0.58 |

- NWA vs off-shell: shift of $\mathcal{O}(800)$ MeV
- Scale uncertainties: dyn. $\rightarrow \mathcal{O}(50)$ MeV, fix. $\rightarrow \mathcal{O}(1)$ GeV
- PDF uncertainties: $\mathcal{O}(30)$ MeV

# Top quark mass dependence of $pp \to t\bar{t}j$

Case 2: normalized inverse $t\bar{t}j$ mass ($\mathcal{R}(m_t, \rho_s)$)

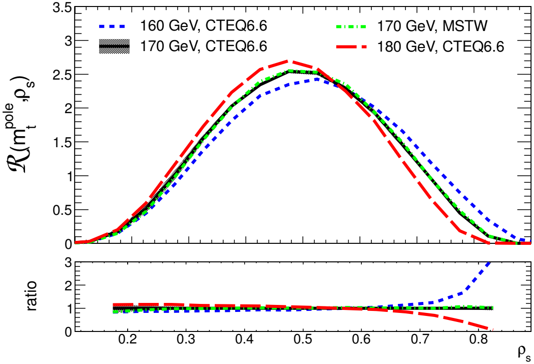
[Alioli, Fernandez, Fuster, Irles, Moch, Uwer and Vos '13]

$$\mathcal{R}(m_t, \rho_s) = \frac{1}{\sigma_{t\bar{t}j}} \frac{d\sigma_{t\bar{t}j}}{d\rho_s}(m_t, \rho_s)$$

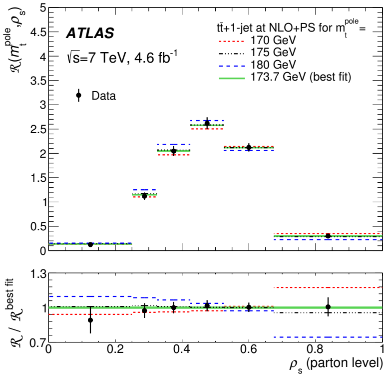
$$\rho_s = \frac{2m_0}{\sqrt{s_{t\bar{t}j}}}$$

- $\rho_s$ shape is sensitive to $m_t$
- $\rho_s \approx 1 \to t\bar{t}$ threshold
- $t\bar{t}j$ has higher sensitivity than $t\bar{t}$

[Alioli et al., arXiv:1303.6415]




[ATLAS, arXiv:1507.01769]




What is the impact of off-shell effects on $\rho_s$ distribution?

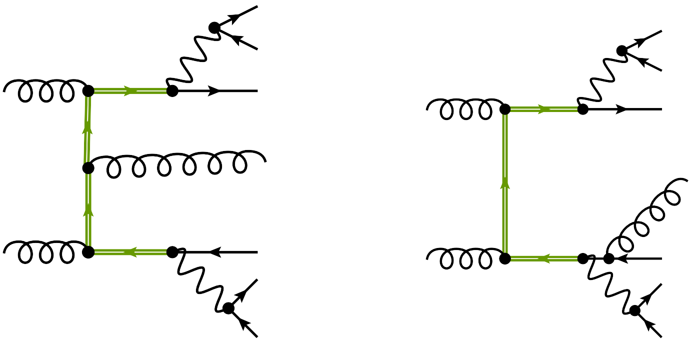
# Top quark mass dependence of $pp \to t\bar{t}j$

What is the impact of off-shell effects on $\rho_s$ distribution?

$$\rho_s = \frac{2m_0}{\sqrt{s_{t\bar{t}j}}} \to \rho_s = \frac{2m_0}{\sqrt{s_{WWb\bar{b}j}}}$$

Including (radiative) top-quark decays:

$$s_{WWb\bar{b}j} \sim a\, s_{t\bar{t}j} + b\, s_{t\bar{t}} + c\, s_X + \cdots$$



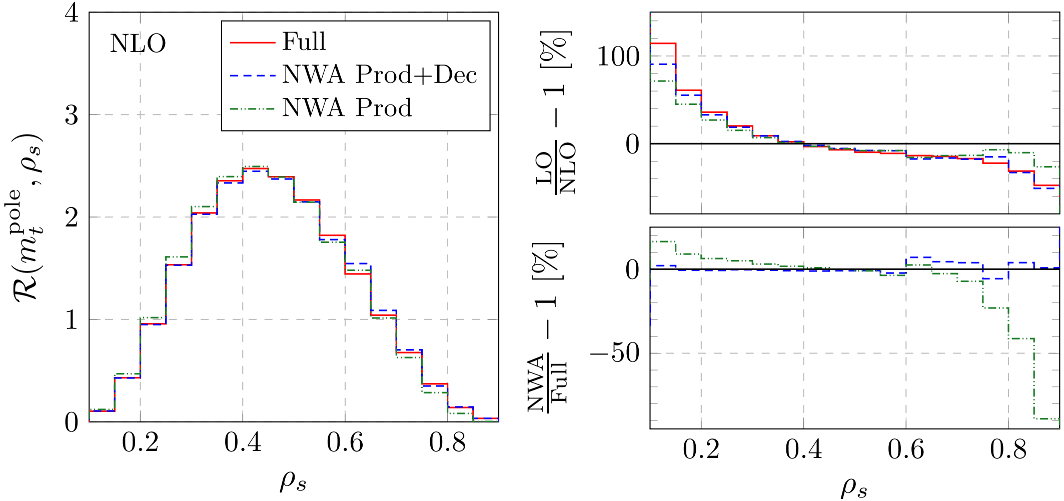
**Disclaimer:**

Not directly comparable with measurements!

[Bevilacqua, Hartanto, MK, Schulze, Worek, JHEP 1803 (2018) 169]

PDF: CT14

Scale: $\mu_R = \mu_F = m_t$



# Fit results for $\mathscr{R}(m_t, \rho_s)$

[Bevilacqua, Hartanto, MK, Schulze, Worek, JHEP 1803 (2018) 169]

| Theory, NLO QCD CT14 PDF | $m_t^{out} \pm \delta m_t^{out}$ [GeV] | Averaged $\chi^2$/d.o.f. | Probability *p-value* | $m_t^{in} - m_t^{out}$ [GeV] |
|---|---|---|---|---|
| | | $2.5\ \text{fb}^{-1}$ | | |
| *Full*, $\mu_0 = H_T/2$ | $173.05 \pm 1.31$ | 0.99 | $0.42\ (0.8\sigma)$ | $+0.15$ |
| *Full*, $\mu_0 = E_T/2$ | $172.19 \pm 1.34$ | 1.05 | $0.39\ (0.9\sigma)$ | $+1.01$ |
| *Full*, $\mu_0 = m_t$ | $173.86 \pm 1.39$ | 1.42 | $0.21\ (1.2\sigma)$ | $-0.66$ |
| *NWA*, $\mu_0 = m_t$ | $175.22 \pm 1.15$ | 1.38 | $0.23\ (1.2\sigma)$ | $-2.02$ |
| $NWA_{\rm Prod.}$, $\mu_0 = m_t$ | $169.39 \pm 1.46$ | 1.12 | $0.35\ (0.9\sigma)$ | $+3.81$ |
| | | $25\ \text{fb}^{-1}$ | | |
| *Full*, $\mu_0 = H_T/2$ | $173.06 \pm 0.44$ | 0.97 | $0.44\ (0.8\sigma)$ | $+0.14$ |
| *Full*, $\mu_0 = E_T/2$ | $172.36 \pm 0.44$ | 1.38 | $0.23\ (1.2\sigma)$ | $+0.84$ |
| *Full*, $\mu_0 = m_t$ | $173.84 \pm 0.42$ | 5.12 | $1 \cdot 10^{-4}\ (3.9\sigma)$ | $-0.64$ |
| *NWA*, $\mu_0 = m_t$ | $175.23 \pm 0.37$ | 5.28 | $7 \cdot 10^{-5}\ (4.0\sigma)$ | $-2.03$ |
| $NWA_{\rm Prod.}$, $\mu_0 = m_t$ | $169.43 \pm 0.50$ | 2.61 | $0.02\ (2.3\sigma)$ | $+3.77$ |

- NWA vs off-shell: shift of $\mathcal{O}(1.4)$ GeV
- Scale uncertainties: dyn. $\rightarrow$ 0.6 – 1.2 GeV, fix. $\rightarrow$ 2.1 – 2.8 GeV
- PDF uncertainties: 0.4 – 0.7 GeV

# Conclusions

**Full calculation of $pp \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}j + X$ at NLO QCD** [JHEP 1611 (2016) 098]

- Full description of $t\bar{t}j$ with all resonant and non-resonant contributions
- Comparison of fixed and dynamic scales: $m_t$, $E_T/2$, $H_T/2$
- Scale and PDF uncertainties for $\sigma$ and various $d\sigma/dX$

**Comparison with NWA**

- off-shell effects on inclusive cross section $\sim 2\%$, non negliable on $d\sigma/dX$ far away from the threshold
- NLO corrections to top decays are important!

**First application: top quark mass extraction** [JHEP 1803 (2018) 169]

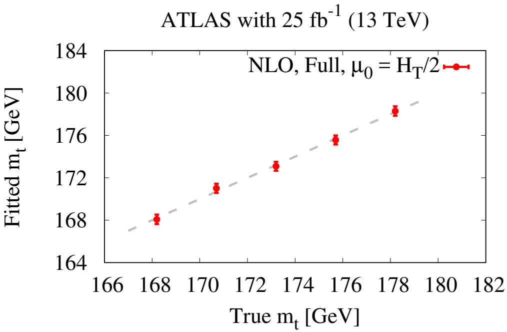
- Presented analysis for $M_{be^+}$ and $\mathcal{R}(m_t, \rho)$
- fixed scale $\to$ larger uncertainties, better: dynamical scales
- Complete study for: $M_{be^+}$, $\mathcal{R}(m_t, \rho)$, $M_{t\bar{t}}$, $H_T$ $\mathcal{R}(m_t, \rho')$
- off-shell effects can have an impact on parameter extraction

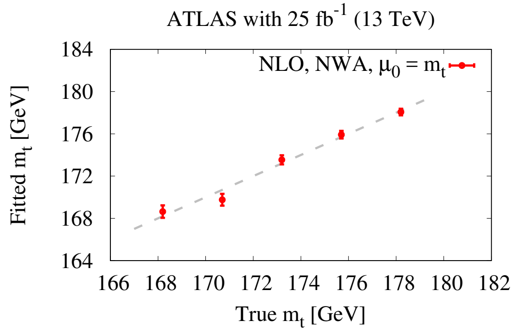
# Backup

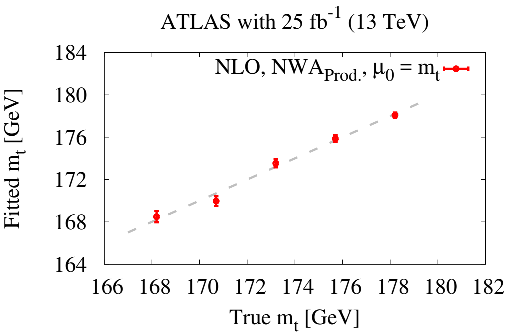
Event samples for $pp \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}j + X$ for $m_t = 173.2$ GeV used for reweighting

| Contribution | Nr. of Events | Nr. of files | (avg) events/file | Size |
|---|---|---|---|---|
| Born | $21 \cdot 10^6$ | 60 | $350 \cdot 10^3$ | 38 GB |
| Born + Virtual | $33 \cdot 10^6$ | 380 | $87 \cdot 10^3$ | 72 GB |
| Int. dipoles | $80 \cdot 10^6$ | 450 | $178 \cdot 10^3$ | 160 GB |
| Real subtracted | $626 \cdot 10^6$ | 18000 | $35 \cdot 10^3$ | 1250 GB |
| Total: | $760 \cdot 10^6$ | 18890 | $40 \cdot 10^3$ | 1520 GB |

# Consistency check



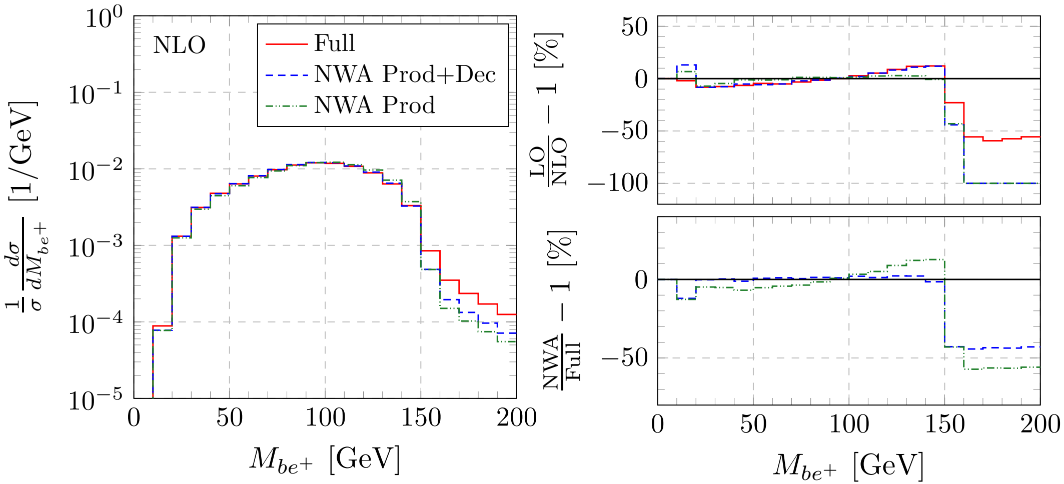




Consider $\mathcal{R}(m_t, \rho_s)$ (using ATLAS binning)

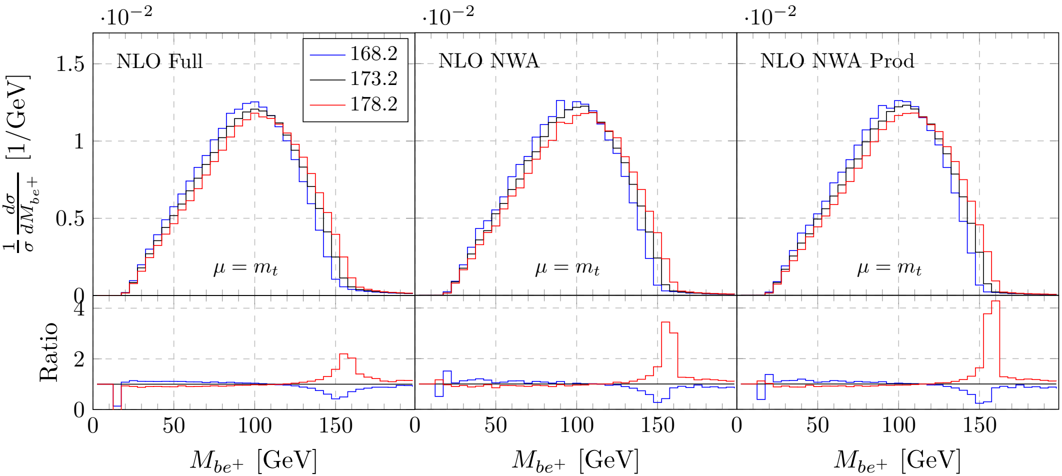
Same theoretical prediction for templates and pseudo-data

True and fitted $m_t$ agree within errors
$\rightarrow$ no bias observed

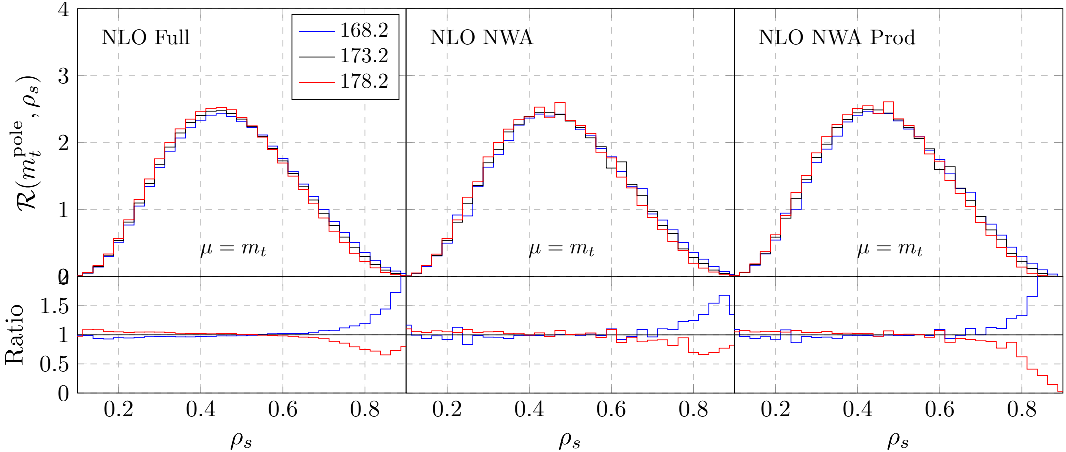
# Shape differences in $M_{be^+}$



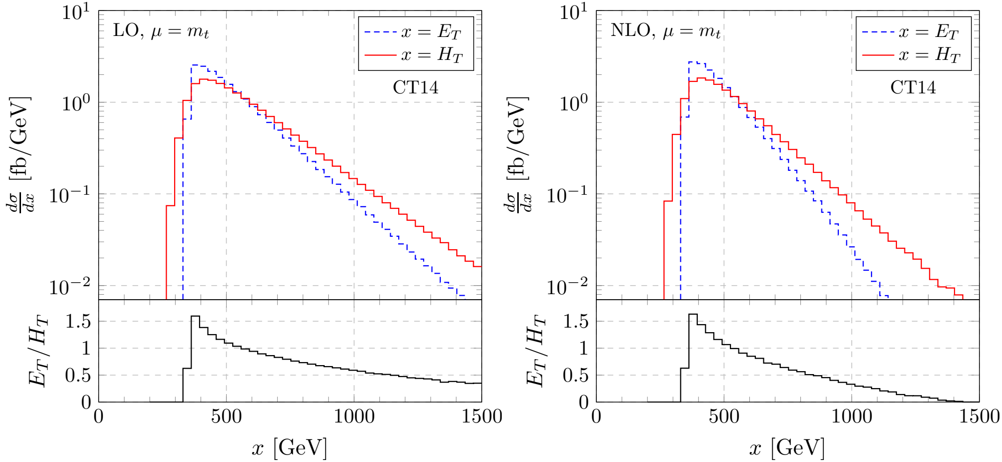
# Top quark mass dependence in $M_{be^+}$



# Top quark mass dependence in $\mathcal{R}(m_t, \rho_s)$



# Comparison of the dynamical scales



# Transverse momentum of hardest light jet