On the recent progress of computing non-planar pentaboxes

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HP2 2018

Freiburg, 01-03 October 2018

Some (in-)famous five-point integrals

- Interested in two-loop, five-point MI with ***<u>one</u>*** external mass and massless propagators
- Relevant e.g. for virtual-virtual contribution to $2 \to 3$ LHC processes such as $\boldsymbol{H+2j}, \boldsymbol{V+2j}$ at NNLO QCD
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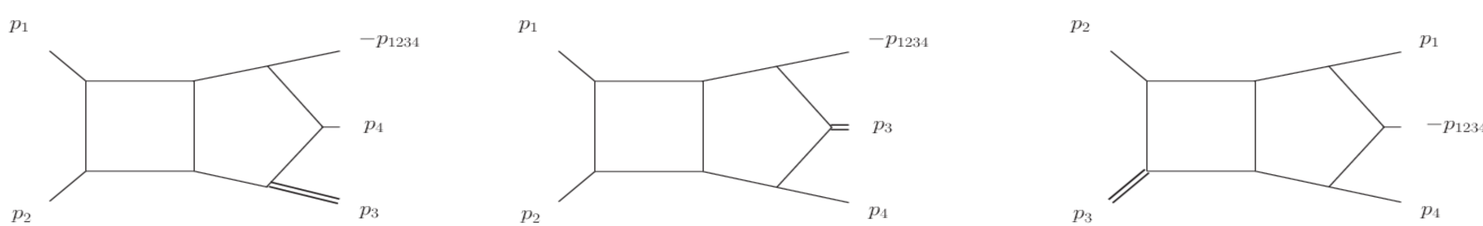

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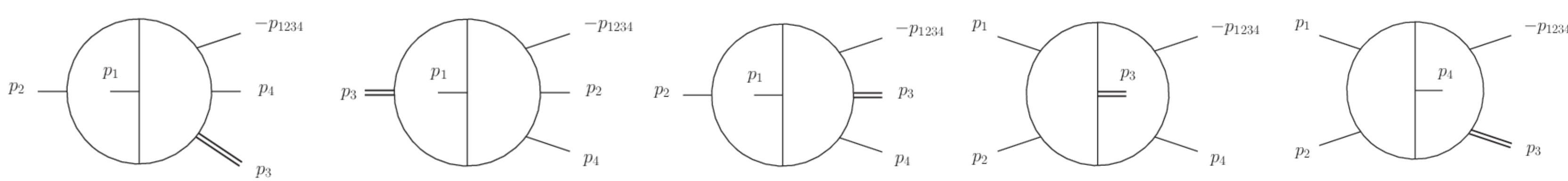

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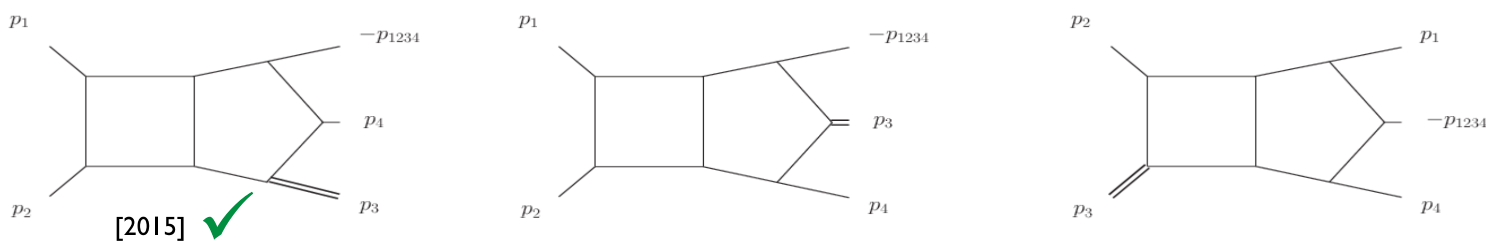

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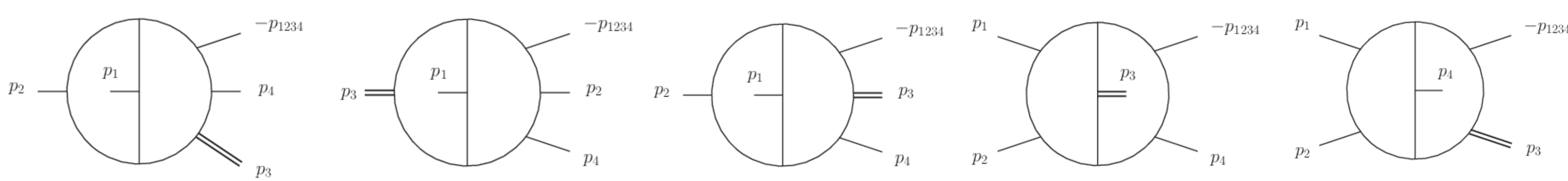

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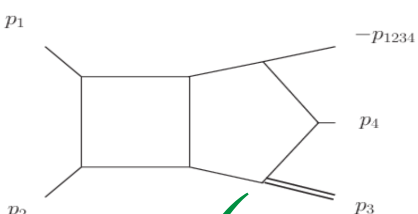
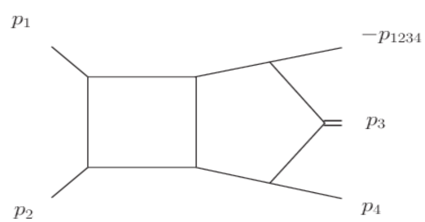
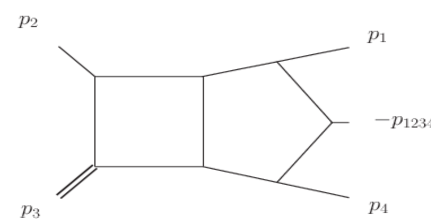


[2015] ✓





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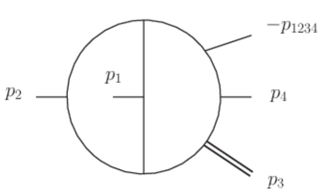

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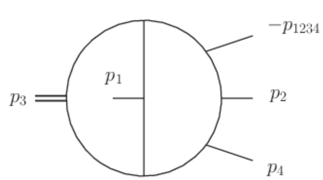
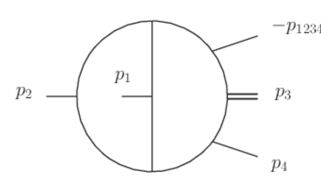
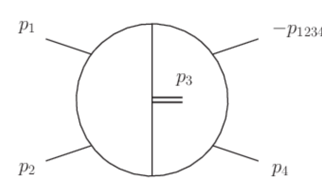
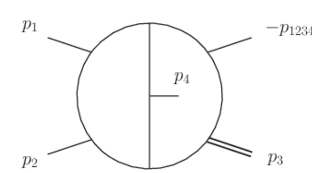
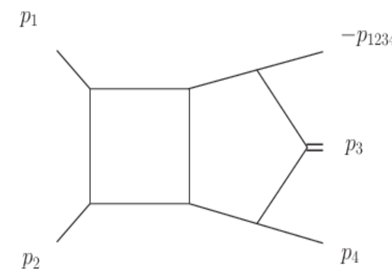






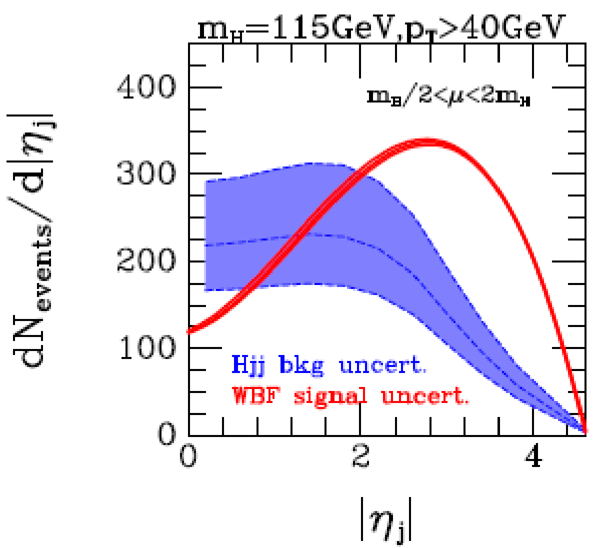


Why the massive pentabox?



$gg \to Hj_1j_2$

- Pentabox contribution at NNLO QCD for $gg \to H + 2j$ (infinite top-mass)
- Current inclusive NLO QCD error ~ 5-10 %



[LO gluon fusion, NLO VBF, Berger & Campbell '04]

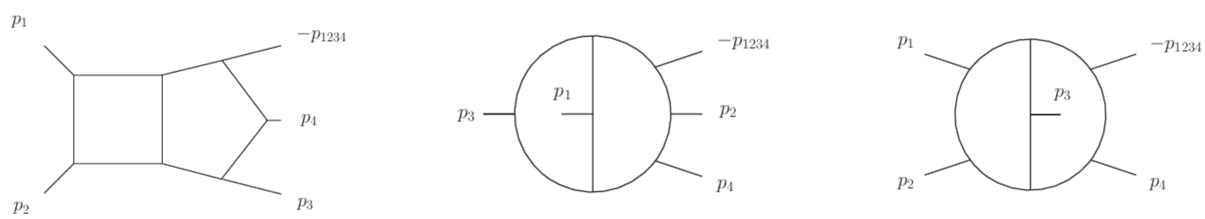
- *H+2j*: require NNLO to measure HWW coupling to 5% at 300 fb^-1
- Serves as bkg to VBF signal production

Further applications:

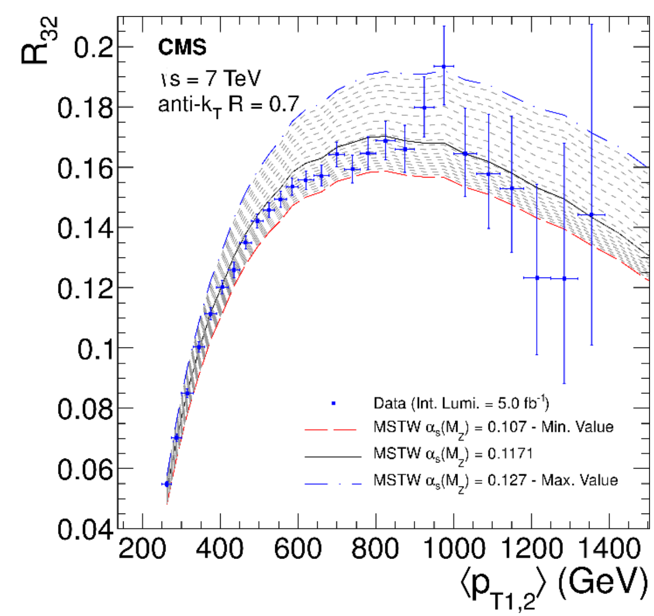
- $(Z \to b\bar{b})$+2j, $(Z \to \tau\bar{\tau})$+2j : background to $(H \to b\bar{b})$+2j, $(H \to \tau\bar{\tau})$+2j
- $Vb\bar{b}$: large background to $pp \to V(H \to b\bar{b})$. NLO QCD error ~ experimental error ~ 20%

Why the massless pentabox?

- Massless pentabox contribution at NNLO QCD for three-jet production $pp' \to j_1 j_2 j_3$



- Measure α_s running: ratio of 3j to 2j production cross sections
- Cancellation of systematic experimental errors (such as the luminosity) and also removes much of the sensitivity to gluon distribution
- Current NLO QCD: 5% scale dependence at high pT
- NNLO expected to reduce errors to order of few %



Progress on Five-point amplitude

- Difficulty: during the reduction to Master Integrals (MI), large intermediate expressions for coefficients encountered
- Most of the recent progress gone into trying to understand how to go beyond simple Laporta at two-loops for the reduction to MI
- Many methods: integrand and integral reduction, numerical unitarity, reduction on unitarity cuts, polynomial division and syzygys, direct solution of IBP systems, projection onto masters,… [Abreu, Badger, Chawdhry, Febres Cordero, Frellesvig, Hansen, Hartanto, Ita, Kosower, Larsen, Manteuffel, Mastrolia, Mitov, Ossola, Page, Peraro, Schabinger, Zeng, Zhang,…]

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- One judicious and efficient method is reduction with finite fields, in particular when combined with (numerical) unitarity-inspired integrand reduction [Manteuffel, Peraro, Schabinger]

- Massless two-loop planar 5-gluon all-plus-helicity amplitude [Gehrmann et al ’15]

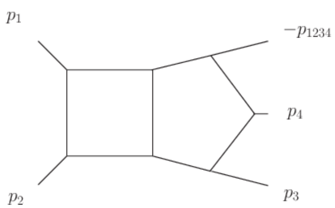
- Remaining helicity configurations for planar on-shell case done [Febres Cordero, Ita et al., Badger et al. ‘17]

⟶ Master integrals also needed…

Progress on Five-point masters

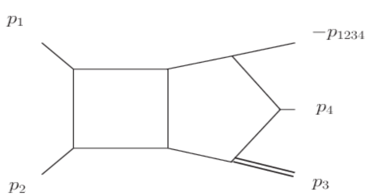
- Pentagon functions for massless five-point amplitudes classified with bootstrapping and conformal method [Henn at al. '18]

- On-shell planar pentaboxes computed



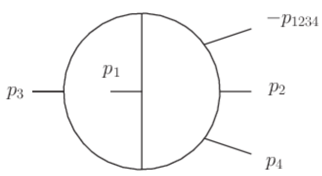
[Gehrmann, Henn, Lo Presti '15,'18]

- One family of massive planar pentaboxes computed



[Papadopoulos, Tommasini, CW '15]

- One family of on-shell non-planar pentaboxes (hexabox) computed



[Gehrmann, Henn, Lo Presti '18]

- Computations done with DE method [Kotikov '91, Remiddi '97, Gehrmann & Remiddi '00, Henn et al '13]

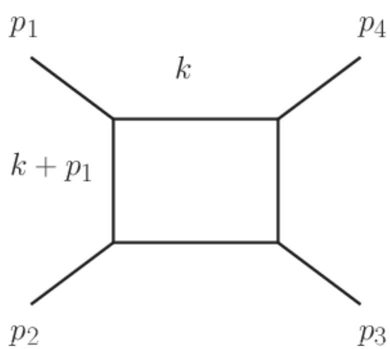
- Relies on IBP reducing five-point integrals to get DE, also not trivial

Feynman plus IBP (1/3)

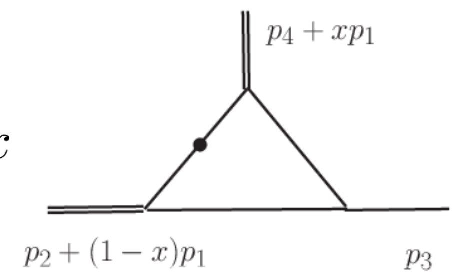
- Observation 1: IBP reduction of integrals with many scales a bottleneck
- Observation 2: IBP reduction of an integral I_2 that has one propagator less but one dot extra than I_1 is typically easier (in particular if scale removed)

  $\longrightarrow$ use Feynman parameters to achieve this

- Example 1



$$= \int_0^1 dx$$

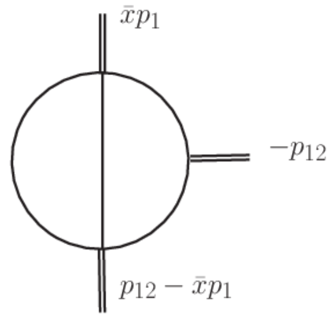


$$= \int_0^1 dx \left(-\frac{2\,(d-3)}{S_3\,(S_2-S_3)} \mathrm{Bub}_1 + \frac{2\,(d-3)}{S_2\,(S_2-S_3)} \mathrm{Bub}_2 \right) = \frac{2}{\varepsilon} \left[\frac{(-s)^{-1-\varepsilon}(1-x)^{-1-\varepsilon}}{s\,(1-x)-tx} - \frac{(-t)^{-1-\varepsilon} x^{-1-\varepsilon}}{s\,(1-x)-tx} \right]$$

$$= \frac{2}{\varepsilon^2} \frac{1}{st} \left[(-s)^{-\varepsilon} \, {}_2F_1 \left(1, -\varepsilon; 1-\varepsilon; \frac{s+t}{t} \right) + (-t)^{-\varepsilon} \, {}_2F_1 \left(1, -\varepsilon; 1-\varepsilon; \frac{s+t}{s} \right) \right]$$

Feynman plus IBP (2/3)

- Example 2



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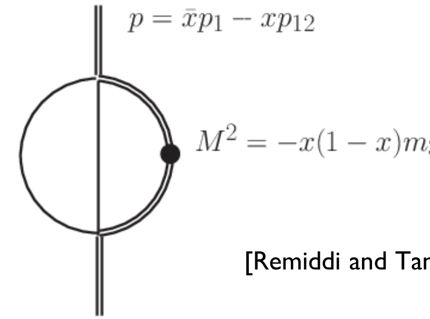
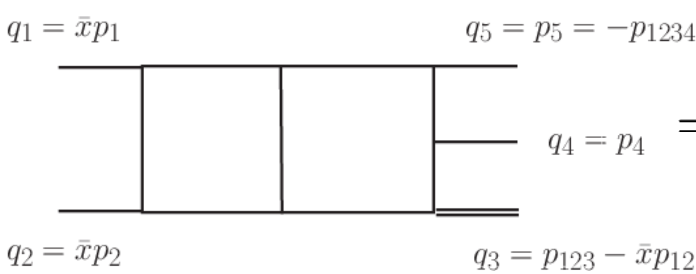


[Remiddi and Tancredi '16]

- Integrand is known expanded in epsilon in GPLs: integration in x can be performed analytically and been checked to agree with left hand side

- <u>Non-trivial example</u>: planar massive pentabox

[On-shell: Gehrmann et al '18]



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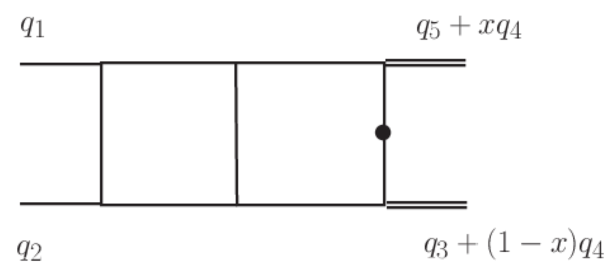


- Right hand side computed by many groups

[Henn et al.' 14, Papadopoulos et al ' 15, Tancredi et al. '15]

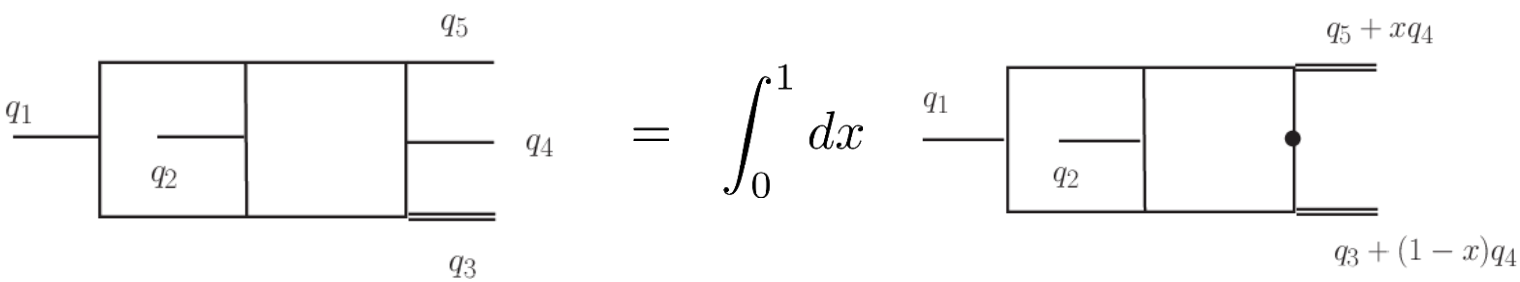
- Use Henn et al. results to for example evaluate in a physical point

- Agreement found with previous results in physical point

[Papadopoulos et al ' 15]

Feynman plus IBP (3/3)

- Non-trivial example 2: nonplanar massive pentabox (or hexabox)



- Right hand side known in terms of GPLs [Henn et al.' 14, Papadopoulos et al ' 15, Tancredi et al. '15]

- We used [Henn et al.' 14] for physical and [Papadopoulos et al.' 15] for Euclidean point

- The integrand contains one square root in x

$$\sqrt{4s_{45}(-1+x)(m_3 - m_3 x + s_{34}x) + (m_3 - s_{12} + s_{45} - m_3 x + s_{34}x - s_{45}x)^2}$$

- Since it is only power two in the integration variable we can rationalize it

- The integrand is then GPLs multiplied with rational functions of integration variable and integrates again to GPLs

Dealing with end point singularities

- Integrand typically contains singularities at x=1 and/or x=0

$$I_5(s_{12}, s_{23}, s_{34}, s_{45}, s_{51}, m_3) = \underbrace{\int_0^1 dx \{I_4(x) - I_4(x \to 1)\}}_{\text{perform numerically or analytically}} + \int_0^1 dx I_4(x \to 1)$$

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- E.g. in the case of x=1 singularity the resummed version looks like

$$I_4(x \to 1) = \sum_{j=1,2,4} (1-x)^{-1-j\epsilon} I_4^{(j)}(s_{12}, s_{23}, s_{34}, s_{45}, s_{51}, m_3)$$

- Its integral in x can be performed exactly in epsilon

$$\int_0^1 dx I_4(x \to 1) = \sum_{j=1,2,4} \frac{-1}{j\epsilon} I_4^{(j)}(s_{12}, s_{23}, s_{34}, s_{45}, s_{51}, m_3)$$

- Compute $I_4(x \to 1)$ for complete double box basis from DE around x=1

$$\partial_x \mathbf{I_4}(x \to 1) = \frac{\mathbf{H}(\epsilon)}{(x-1)} \mathbf{I_4}(x \to 1)$$

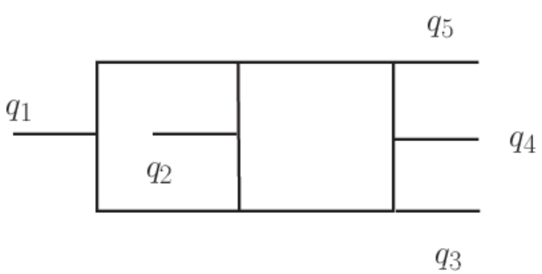
- Easiest to solve the DE for the canonical double box I_4 basis

[Henn et al.' 14]

Preliminary results and massless case

Massless case:

- The same method can be applied for the completely on-shell case [Gehrmann, Henn, Lo Presti '18]
- In on-shell case the integrand contains singularities at both x=0 and x=1



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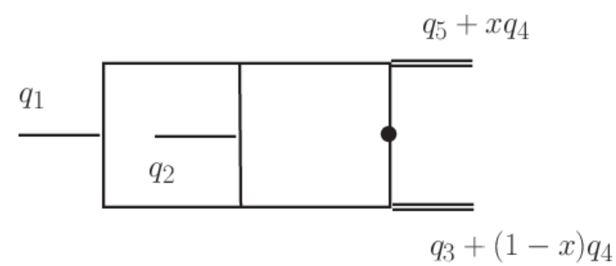

Preliminary results:

- Naïve numerical Mathematica NIntegrate takes about 5 minutes per phase space point
- Analytic result: about ~ 8000 Generalized Polylogs
- Agreement with Secdec in a Euclidean region for <u>massless and massive case</u> [Borowka, Heinrich et al '11-'18]
- Could not extract reliable results with current Secdec version in a physical region

Summary

- Many phenomenologically relevant $2 \to 2$ two-loop integrals have been computed
- However only a few five-point integrals computed
- No complete (including non-planar) five-point two-loop SM amplitude computed yet

- I presented some ongoing work on the computation of a non-planar five-point two-loop family with one external mass
- Computation performed with a combined Feynman representation and IBP reduction approach (Internal Reduction)
- Approach is still in the playground phase
- Used to confirm previously computed planar massive pentabox solutions in physical point

Backup slides

Coupled DE of 5-point integrals

[Papadopoulos et al. '15]