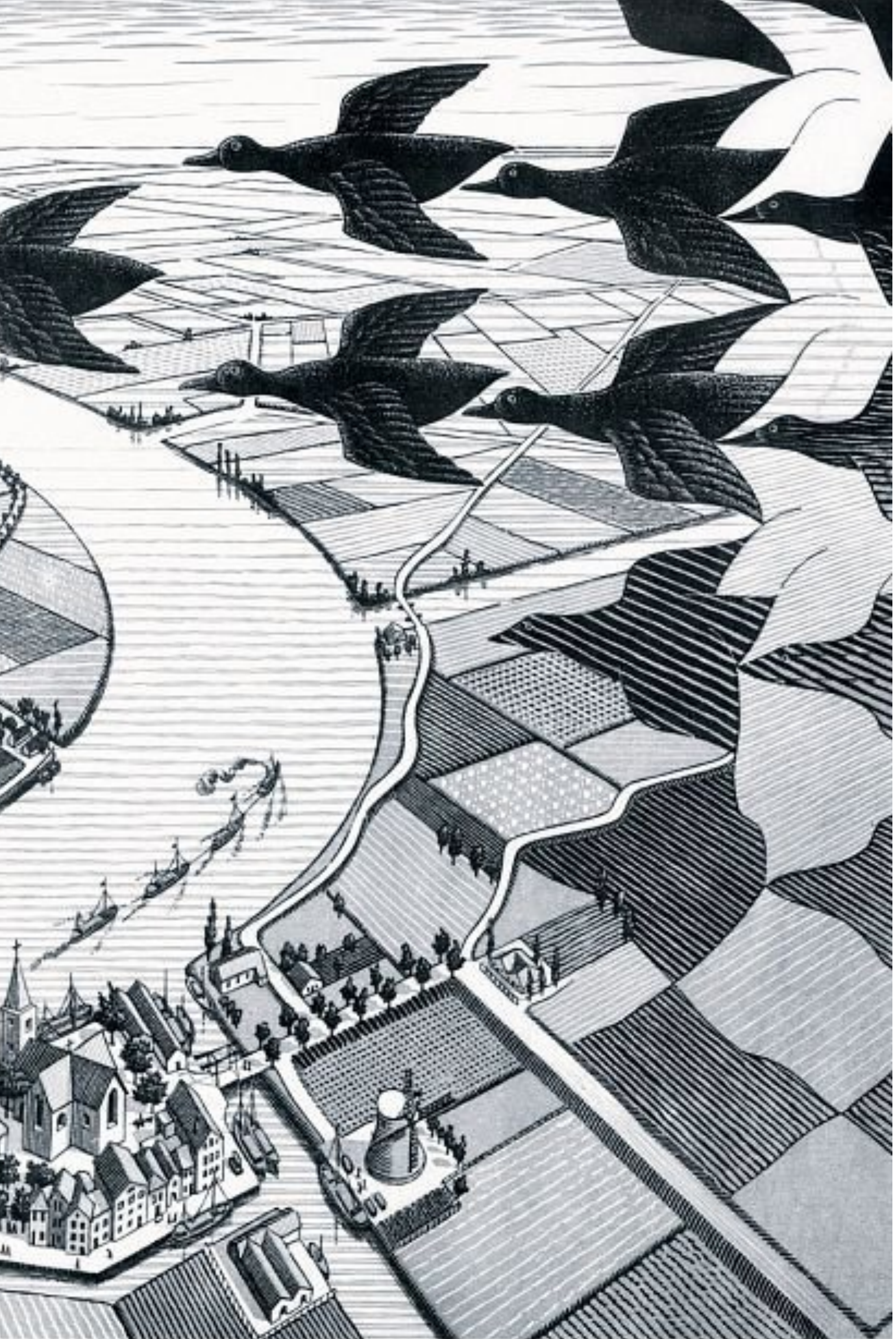
HEAVY QUARK PRODUCTION AT NNLO

HP2 2018, 02.10.18

Simone Devoto - University of Zürich

In collaboration with: Stefano Catani, Massimiliano Grazzini, Javier Mazzitelli



CONTENTS

- Motivations
- q_T subtraction formalism
- Top pair production within q_T subtraction formalism
- Conclusions

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- **Production** → Pair production;
  top pair production is the main source of top quark events in the SM;

- **At NNLO** → Next to Next to Leading Order;
  leading order only give us an order of magnitude prediction, higher orders are necessary to obtain a reliable prediction.

WHY TOP QUARK?

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- Heaviest elementary particle known so far ($m_t \simeq 173$ GeV);
  Strong coupling with the Higgs Boson;
  Study of $t\bar{t}$ production can shed light on electroweak symmetry breaking mechanism.

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- Top quarks are abundantly produced at the LHC;
  Its production is an important background both for NP model and SM precision measurements;
  Experimental measurements require reliable predictions of $t\bar{t}$ production.

QCD CORRECTIONS FOR TOP PAIR PRODUCTION

- **NLO**:
  - Total Cross Section:
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  - Differential distribution:
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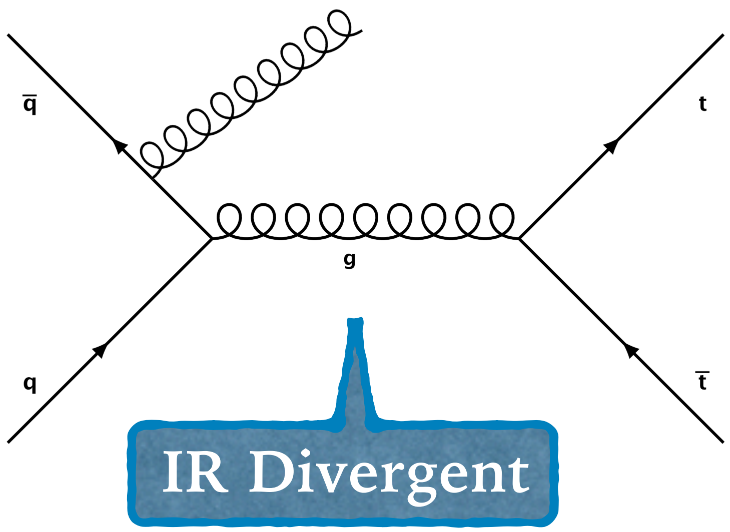
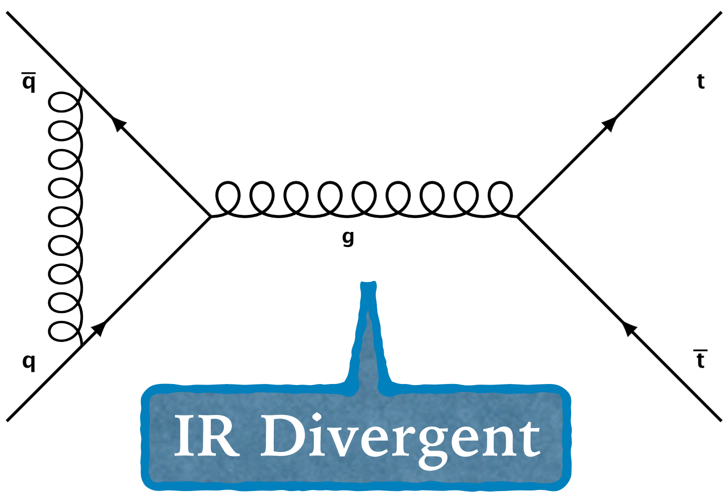
Independent result for NNLO computation would be a good check of the complex calculation performed.

WHY ARE QCD CORRECTIONS CHALLENGING?

Real (NLO)



Virtual (NLO)



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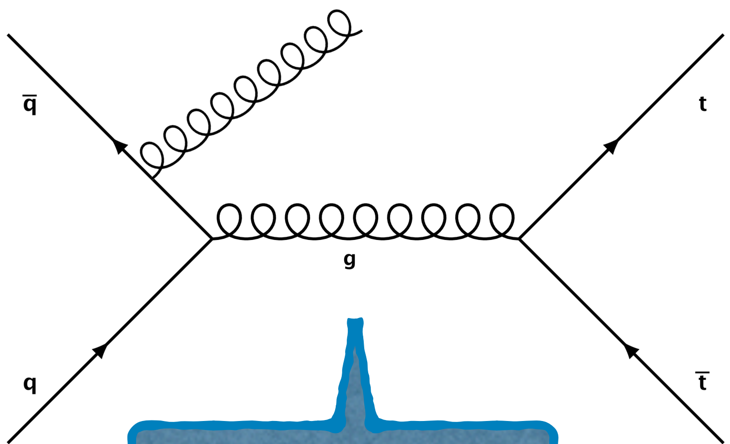

IR Divergent

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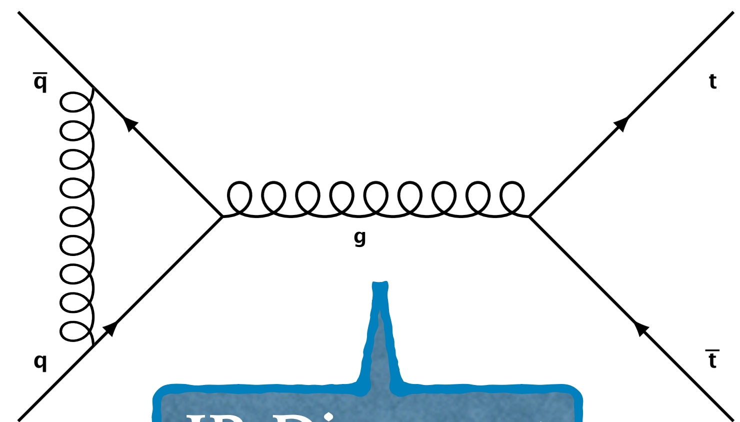

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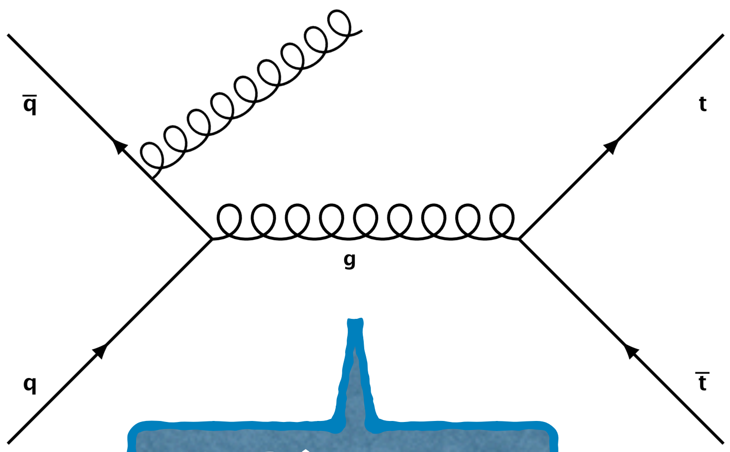

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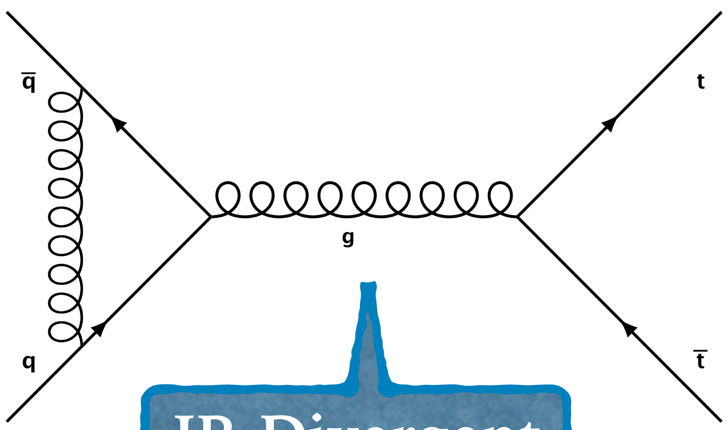

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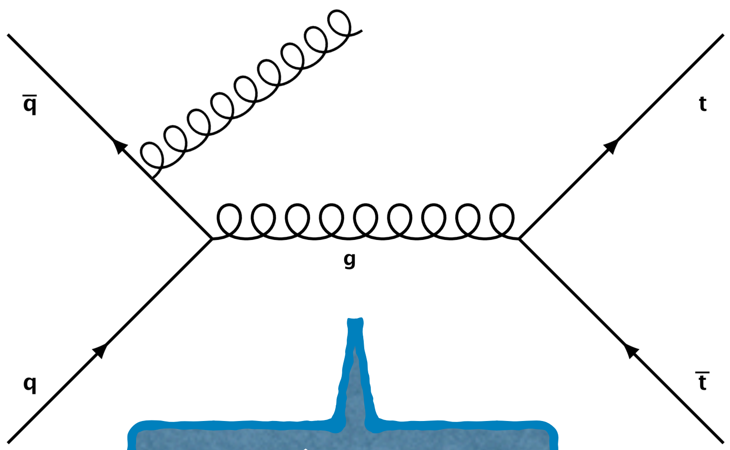

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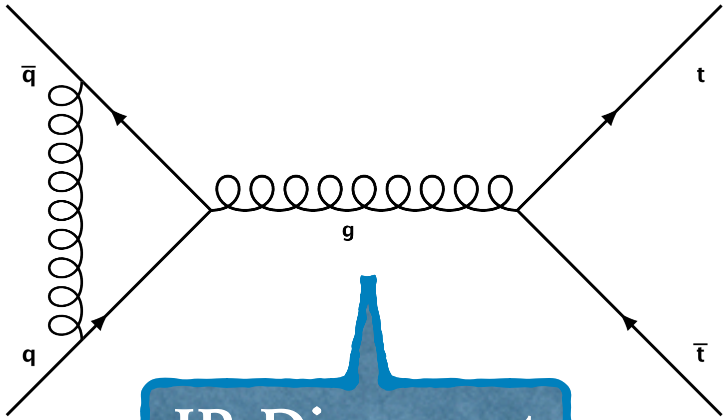

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Presence of IR divergences at intermediate steps of the computation of QCD higher order corrections does not allow a straightforward implementation of numerical techniques.

We need methods to overcome this difficulties!

WHY ARE QCD CORRECTIONS CHALLENGING?

- **NLO methods:**
  - *Catani-Seymour dipole subtraction* *[S. Catani, M. Seymour (1996)]*
  - *FKS subtraction* *[S. Frixione, Z. Kunszt, A. Signer (1996)]*
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  - q_T *subtraction formalism* *[S. Catani, M. Grazzini (2007)]*
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$\mathbf{Q_T}$ SUBTRACTION FORMALISM

[S. Catani, M. Grazzini: arXiv 0703012]

$$d\sigma^F_{(N)NLO} = d\sigma^F_{(N)NLO}\Big|_{q_T=0} + d\sigma^F_{(N)NLO}\Big|_{q_T\neq 0}$$

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Extra singularities of NNLO type associated to the $q_t \to 0$ limit need additional subtraction.
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HARD COLLINEAR COEFFICIENT
Contains information on virtual corrections to the process.

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```
[devoto:/mnt/runs2/devoto/MATRIX_v1.0.0] ./matrix
```

MATRIX

[M.Grazzini, S. Kallweit, M. Wiesemann: arXiv 1711.06631]

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Computational framework which, implementing q_T subtraction, allows us to evaluate fully differential cross sections for a wide class of processes at hadron colliders where the final state is a colour singlet in next-to-next-to-leading order (NNLO) QCD.

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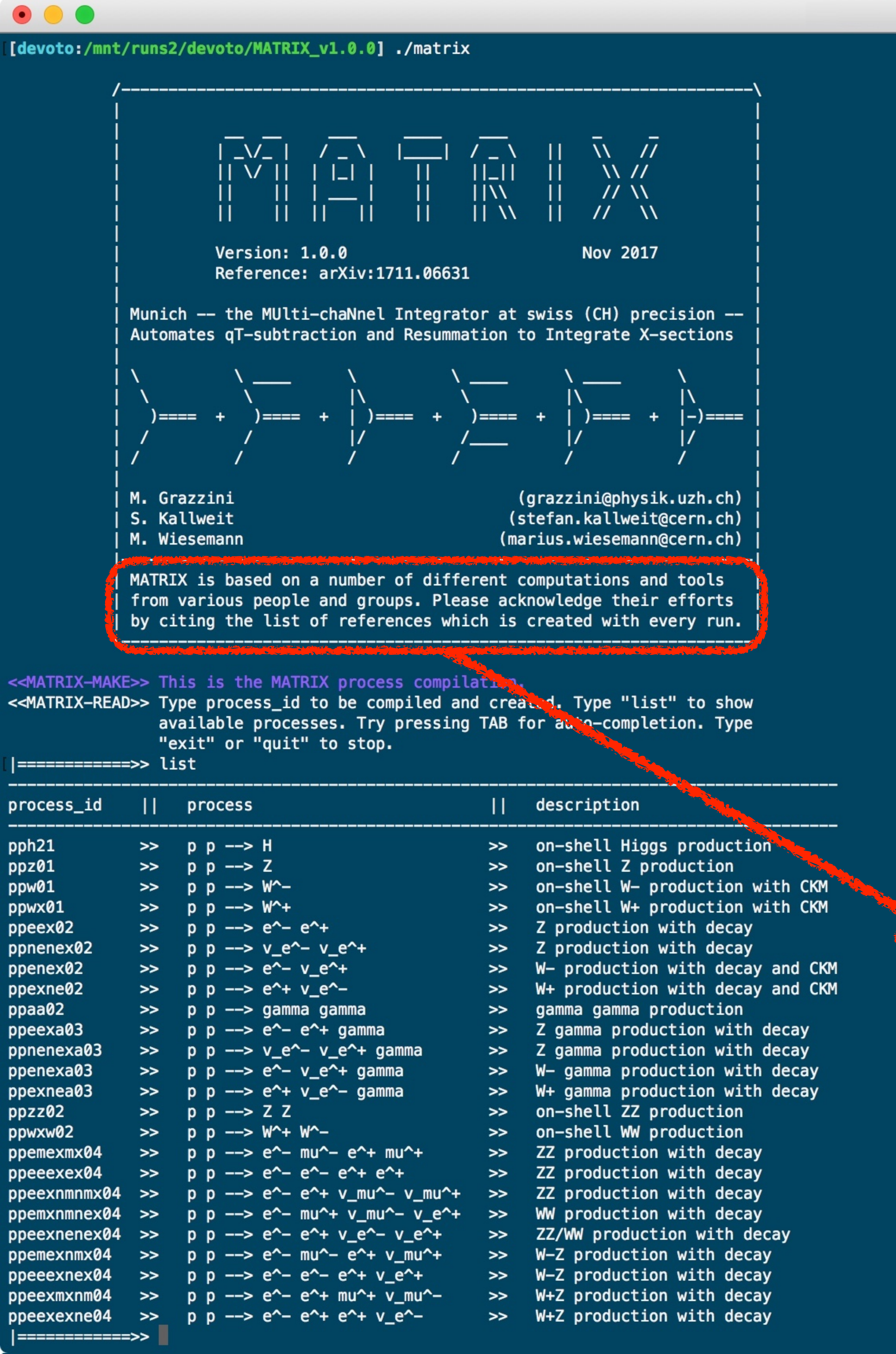

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```
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           |                                                                   |
           |         Version: 1.0.0                              Nov 2017      |
           |         Reference: arXiv:1711.06631                               |
           |                                                                   |
           | Munich -- the MUlti-chaNnel Integrator at swiss (CH) precision -- |
           | Automates qT-subtraction and Resummation to Integrate X-sections  |
           |                                                                   |
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           | /          /         /          /          /           /          |
           |                                                                   |
           | M. Grazzini                               (grazzini@physik.uzh.ch) |
           | S. Kallweit                              (stefan.kallweit@cern.ch) |
           | M. Wiesemann                            (marius.wiesemann@cern.ch) |
           |                                                                   |
           | MATRIX is based on a number of different computations and tools   |
           | from various people and groups. Please acknowledge their efforts   |
           | by citing the list of references which is created with every run.  |
           \-------------------------------------------------------------------/

<<MATRIX-MAKE>> This is the MATRIX process compilation.
<<MATRIX-READ>> Type process_id to be compiled and created. Type "list" to show
                available processes. Try pressing TAB for auto-completion. Type
                "exit" or "quit" to stop.
|============>> list
--------------------------------------------------------------------------------------
process_id     ||  process                          ||  description
--------------------------------------------------------------------------------------
pph21          >>  p p --> H                        >>  on-shell Higgs production
ppz01          >>  p p --> Z                        >>  on-shell Z production
ppw01          >>  p p --> W^-                      >>  on-shell W- production with CKM
ppwx01         >>  p p --> W^+                      >>  on-shell W+ production with CKM
ppeex02        >>  p p --> e^- e^+                  >>  Z production with decay
ppnenex02      >>  p p --> v_e^- v_e^+              >>  Z production with decay
ppenex02       >>  p p --> e^- v_e^+                >>  W- production with decay and CKM
ppexne02       >>  p p --> e^+ v_e^-                >>  W+ production with decay and CKM
ppaa02         >>  p p --> gamma gamma              >>  gamma gamma production
ppeexa03       >>  p p --> e^- e^+ gamma            >>  Z gamma production with decay
ppnenexa03     >>  p p --> v_e^- v_e^+ gamma        >>  Z gamma production with decay
ppenexa03      >>  p p --> e^- v_e^+ gamma          >>  W- gamma production with decay
ppexnea03      >>  p p --> e^+ v_e^- gamma          >>  W+ gamma production with decay
ppzz02         >>  p p --> Z Z                      >>  on-shell ZZ production
ppwxw02        >>  p p --> W^+ W^-                  >>  on-shell WW production
ppemexmx04     >>  p p --> e^- mu^- e^+ mu^+        >>  ZZ production with decay
ppeeexex04     >>  p p --> e^- e^- e^+ e^+          >>  ZZ production with decay
ppeexnmnmx04   >>  p p --> e^- e^+ v_mu^- v_mu^+    >>  ZZ production with decay
ppemxnmnex04   >>  p p --> e^- mu^+ v_mu^- v_e^+    >>  WW production with decay
ppeexnenex04   >>  p p --> e^- e^+ v_e^- v_e^+      >>  ZZ/WW production with decay
ppemexnmx04    >>  p p --> e^- mu^- e^+ v_mu^+      >>  W-Z production with decay
ppeeexnex04    >>  p p --> e^- e^- e^+ v_e^+        >>  W-Z production with decay
ppeexmxnm04    >>  p p --> e^- e^+ mu^+ v_mu^-      >>  W+Z production with decay
ppeexexne04    >>  p p --> e^- e^+ e^+ v_e^-        >>  W+Z production with decay
|============>>
```

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q_t subtraction formalism has been successfully applied to top pair productional NLO and at NNLO considering the off-diagonal channels.

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Cross section [pb]	NLO	$\mathcal{O}(\alpha_S^4)\vert_{qg}$	$\mathcal{O}(\alpha_S^4)\vert_{q(\bar{q})q'}$
q_T subtraction	226.2(1)	−2.25(5)	0.151(3)
Top++	226.3	−2.253	0.148

Table 1: Total cross sections for $t\bar{t}$ production. NLO and (partial) NNLO results from q_T subtraction compared with the corresponding results from Top++ for pp collisions at $\sqrt{s} = 8$ TeV.

Good agreement with TOP++!

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To compute the missing ingredients to complete NNLO q_t subtraction for top pair production, also for diagonal channels.

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IR behaviour known from studies in q_t resummation
[*A. Ferroglia, M. Neubert, B. D. Pecjak, L. L. Yang: arXiv:0908:3676;*
Hai Tao Li, Chong Sheng Li, Ding Yu Shao, Li Lin Yang, Hua Xing Zu: arXiv:1307:2464
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Colourless final state

$$\mathscr{H} \propto \left\langle \tilde{M} \middle| \tilde{M} \right\rangle$$

$$\left| \tilde{M} \right\rangle = \left(1 - I_C\right) \left| M \right\rangle$$

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All loop renormalised virtual amplitude

HARD COLLINEAR COEFFICIENT

Colourless final state

Colourless subtraction operator

Specified up to order α_s^2 in [S.Catani, L. Cieri, D. De Florian, G. Ferrera, M. Grazzini: arXiv:1311.1654].

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$$\mathscr{H} \propto \langle \tilde{M} | \Delta | \tilde{M} \rangle$$

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For top pair production, the finite part of the 2 loop renormalised virtual amplitude can be found in [P. Baernreuther, M. Czakon, P. Fiedler: arXiv:1312.6279].

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All loop renormalised virtual amplitude

Colourful final state

Additional soft radiative factor

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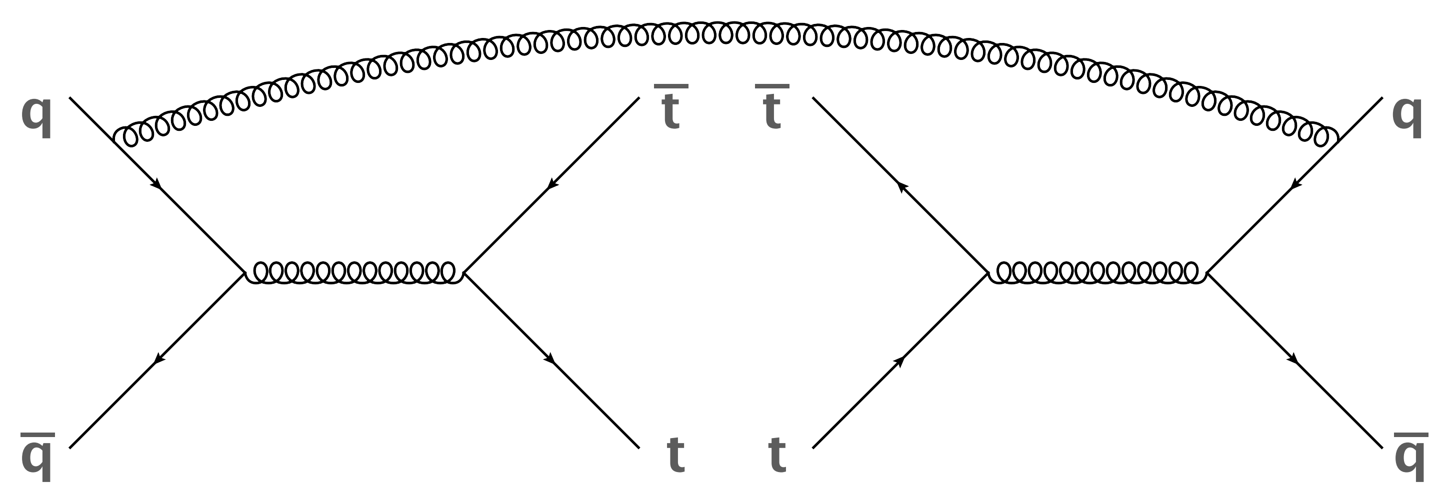

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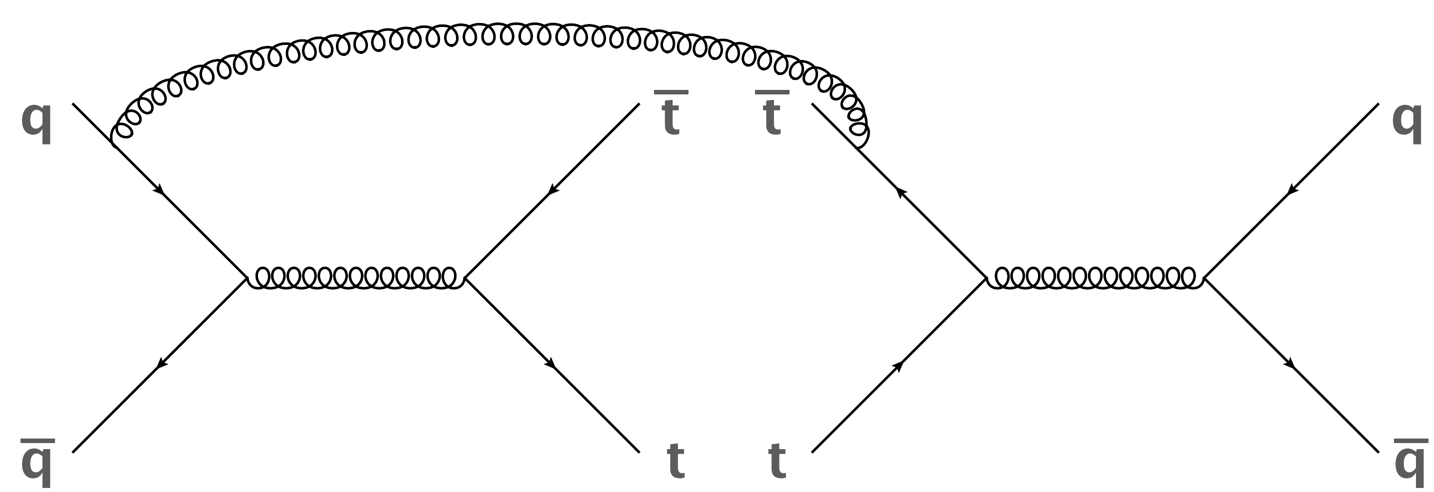

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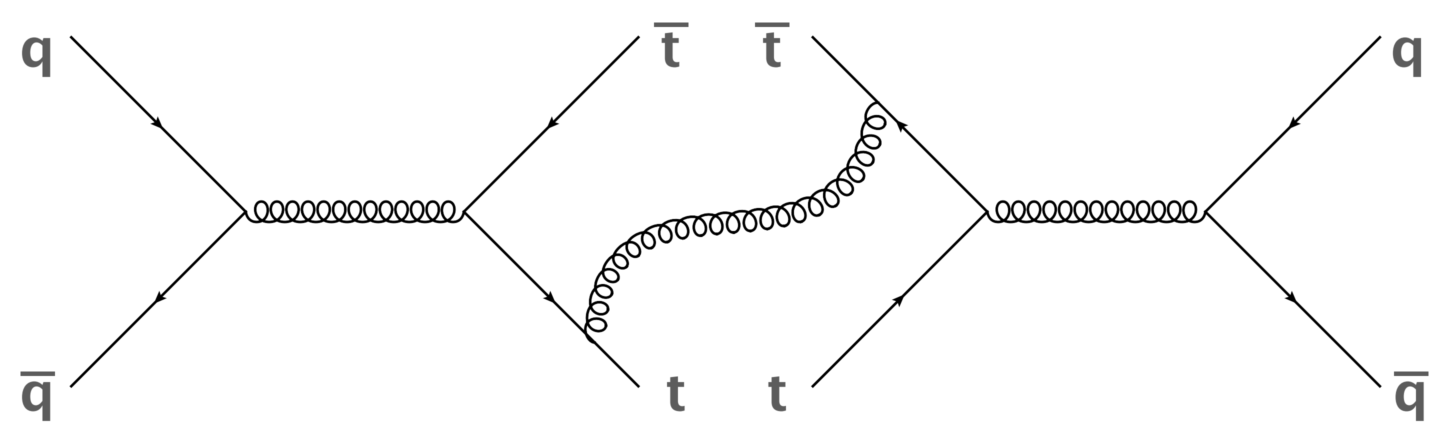

COLOURFUL FINAL STATE - NLO

[S. Catani, M. Grazzini, A. Torre: arXiv 1408:4564]

Computation of the soft contribution → *Integration of a suitably subtracted soft current.*

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- J_{sub} is the soft current after a proper subtraction of the (known) colourless contribution:

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Integration of NNLO soft current recently fully carried out in SCET formalism by using a combined analytical-numerical approach.

[R. Ángles-Martínez, M. Czakon, S. Sapeta: arXiv: 1809.01459]

ONE GLUON EMISSION AT 1 LOOP

*[I. Bierenbaum, M.Czakon, A. Mitov: arXiv:1107.4384;
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The expansion in ε of R_{ij}, I_{ij} can be found in *[I. Bierenbaum, M.Czakon, A. Mitov: arXiv:1107.4384]*.
Simplified expressions recently published in *[M. Czakon, A.Mitov: arXiv:1804.02069]*.

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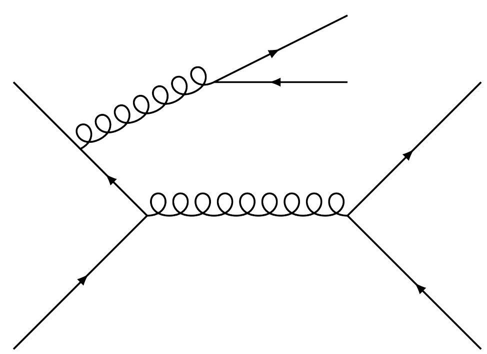

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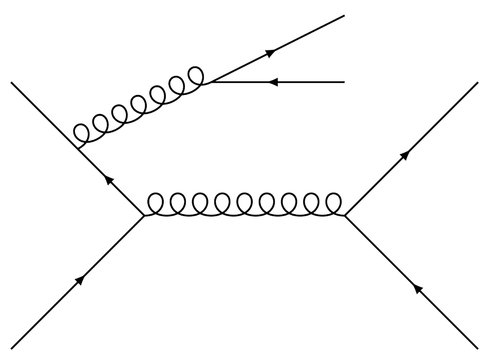

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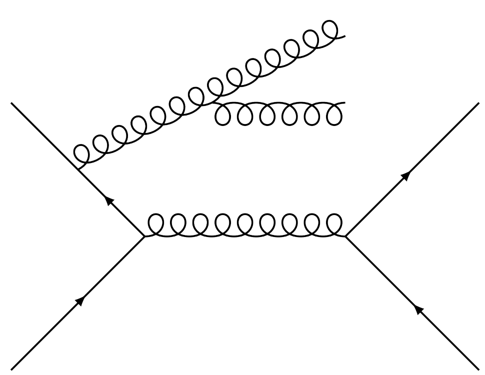

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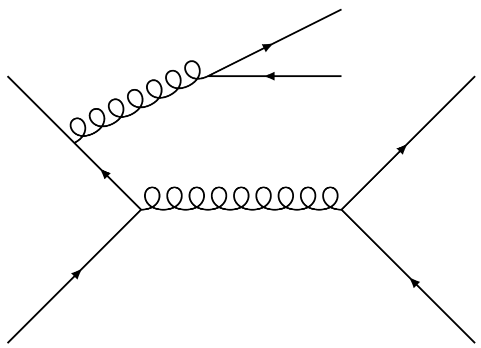

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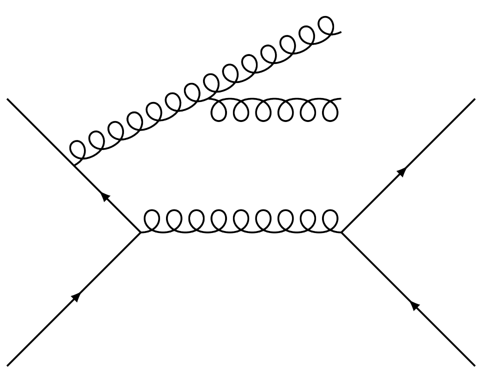

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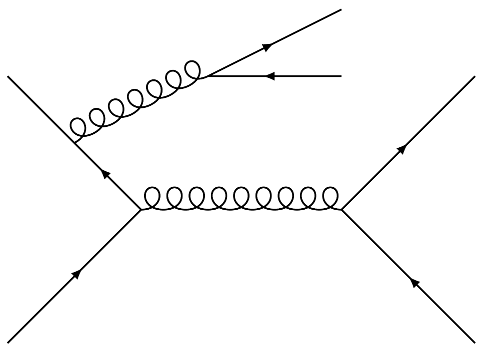

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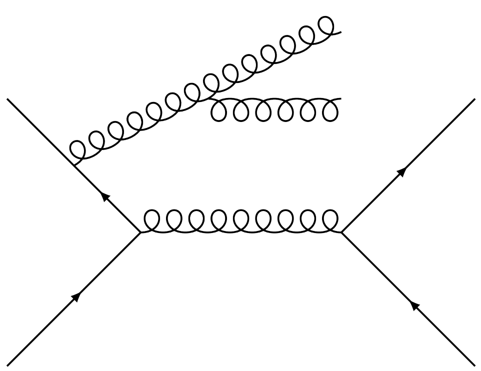

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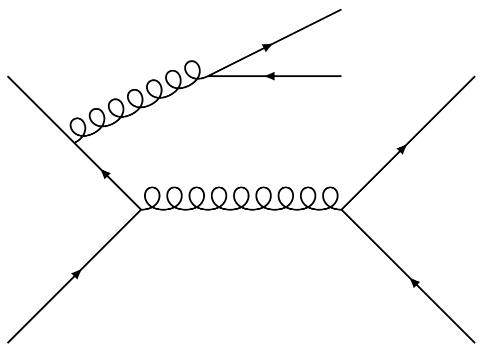

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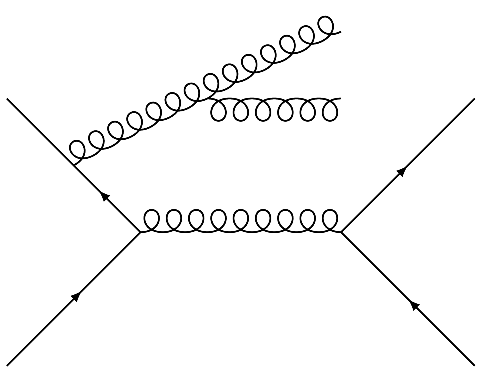

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NLO integral with off-shell emitters:
$$\delta_+\left(k^2\right) \quad \rightarrow \quad \left(k^2\right)^{-1-\epsilon} \theta\left(k^2\right)$$

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Process: $a_1(p_1^\mu)\, a_2(p_2^\mu) \quad \to \quad Q(p_3^\mu)\, \bar{Q}(p_4^\mu)\, g(k_1^\mu)\, g(k_2^\mu)$

Need to compute: $$\boxed{\int \frac{d^n k_1}{2\pi^{n-1}} \frac{d^n k_2}{2\pi^{n-1}} \delta_+(k_1)\delta_+(k_2) \left| J_{sub}^{NNLO(gg)}(k_1,k_2) \right|^2 e^{i\vec{b}\cdot\left(\vec{k}_{T1}+\vec{k}_{T2}\right)}}$$

$$\left| J_{sub}^{NNLO(gg)}(k_1,k_2) \right|^2 = \frac{1}{2}\left\{ \mathbf{J}^2(k_1), \mathbf{J}^2(k_2) \right\} - C_a \sum_{i,j=1}^{n} \mathbf{T}_i \cdot \mathbf{T}_j\, \mathcal{S}_{ij}(k_1,k_2)$$

Iteration of NLO results

DOUBLE REAL EMISSION

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Iteration of NLO results

New part

$$\mathcal{S}_{ij}(k_1,k_2) = \mathcal{S}_{ij}^{m=0}(k_1,k_2) + \left(m_i^2 \mathcal{S}_{ij}^{m\neq 0}(k_1,k_2) + m_j^2 \mathcal{S}_{ji}^{m\neq 0}(k_1,k_2)\right)$$

DOUBLE REAL EMISSION

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$$\mathcal{S}_{ij}^{m=0}(q_1,q_2) = \frac{(1-\epsilon)}{(q_1\cdot q_2)^2}\frac{p_i\cdot q_1\, p_j\cdot q_2 + p_i\cdot q_2\, p_j\cdot q_1}{p_i\cdot(q_1+q_2)\, p_j\cdot(q_1+q_2)}$$
$$-\frac{(p_i\cdot p_j)^2}{2\, p_i\cdot q_1\, p_j\cdot q_2\, p_i\cdot q_2\, p_j\cdot q_1}\left[2-\frac{p_i\cdot q_1\, p_j\cdot q_2 + p_i\cdot q_2\, p_j\cdot q_1}{p_i\cdot(q_1+q_2)\, p_j\cdot(q_1+q_2)}\right]$$
$$+\frac{p_i\cdot p_j}{2\, q_1\cdot q_2}\left[\frac{2}{p_i\cdot q_1\, p_j\cdot q_2}+\frac{2}{p_j\cdot q_1\, p_i\cdot q_2}-\frac{1}{p_i\cdot(q_1+q_2)\, p_j\cdot(q_1+q_2)}\right.$$
$$\left.\times\left(4+\frac{(p_i\cdot q_1\, p_j\cdot q_2 + p_i\cdot q_2\, p_j\cdot q_1)^2}{p_i\cdot q_1\, p_j\cdot q_2\, p_i\cdot q_2\, p_j\cdot q_1}\right)\right]$$

$$\mathcal{S}_{ij}^{m\neq 0}(q_1,q_2) = -\frac{1}{4\, q_1\cdot q_2\, p_i\cdot q_1\, p_i\cdot q_2}+\frac{p_i\cdot p_j\, p_j\cdot(q_1+q_2)}{2\, p_i\cdot q_1\, p_j\cdot q_2\, p_i\cdot q_2\, p_j\cdot q_1\, p_i\cdot(q_1+q_2)}$$
$$-\frac{1}{2\, q_1\cdot q_2\, p_i\cdot(q_1+q_2)\, p_j\cdot(q_1+q_2)}\left(\frac{(p_j\cdot q_1)^2}{p_i\cdot q_1\, p_j\cdot q_2}+\frac{(p_j\cdot q_2)^2}{p_i\cdot q_2\, p_j\cdot q_1}\right)$$

DOUBLE REAL EMISSION

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$$\mathcal{S}_{ij}^{m=0}(q_1,q_2) = \frac{(1-\epsilon)}{(q_1\cdot q_2)^2}\frac{p_i\cdot q_1\; p_j\cdot q_2 + p_i\cdot q_2\; p_j\cdot q_1}{p_i\cdot(q_1+q_2)\; p_j\cdot(q_1+q_2)}$$
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$$\mathcal{S}_{ij}^{m\neq0}(q_1,q_2) = -\frac{1}{4\, q_1\cdot q_2\; p_i\cdot q_1\; p_i\cdot q_2}+\frac{p_i\cdot p_j\; p_j\cdot(q_1+q_2)}{2\, p_i\cdot q_1\; p_j\cdot q_2\; p_i\cdot q_2\; p_j\cdot q_1\; p_i\cdot(q_1+q_2)}$$
$$-\frac{1}{2\, q_1\cdot q_2\; p_i\cdot(q_1+q_2)\; p_j\cdot(q_1+q_2)}\left(\frac{(p_j\cdot q_1)^2}{p_i\cdot q_1\; p_j\cdot q_2}+\frac{(p_j\cdot q_2)^2}{p_i\cdot q_2\; p_j\cdot q_1}\right)$$

DOUBLE REAL EMISSION

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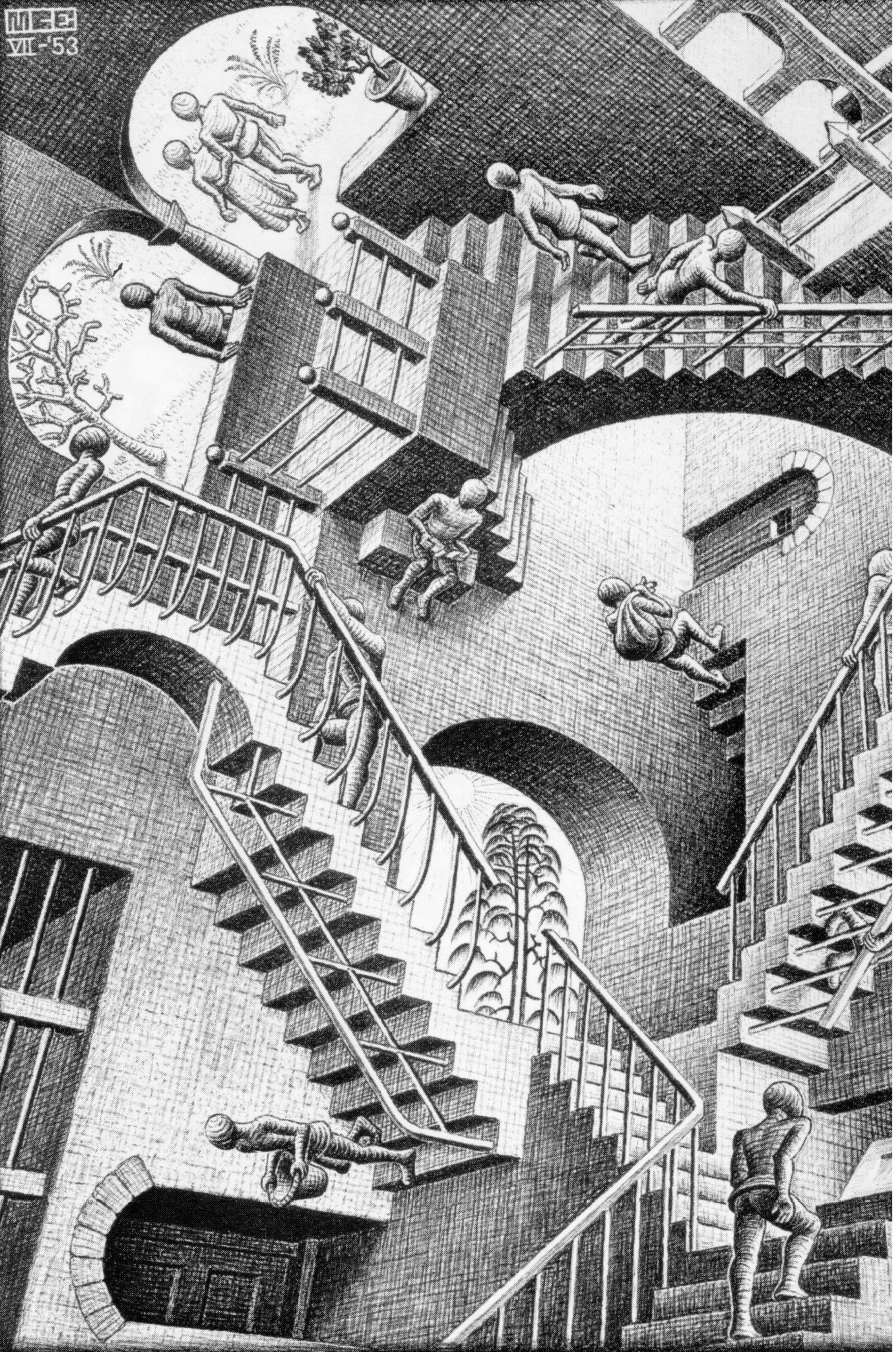
$$\tilde{\mathcal{S}}_{ij}^{m=0}(q_1,q_2) = -\frac{(p_i\cdot p_j)^2}{2(p_i\cdot k)(p_j\cdot k)}\left(\frac{2}{(p_i\cdot q_1)(p_j\cdot q_1)}+\frac{1}{(p_i\cdot q_1)(p_j\cdot q_2)}\right)+\frac{(p_i\cdot p_j)}{k^2}\frac{2}{(p_i\cdot q_1)(p_j\cdot q_2)}$$
$$-\frac{(p_i\cdot p_j)}{2k^2(p_i\cdot k)(p_j\cdot k)}\frac{((p_i\cdot q_1)(p_j\cdot q_2)-(p_i\cdot q_2)(p_j\cdot q_1))^2}{(p_i\cdot q_1)(p_j\cdot q_2)(p_i\cdot q_2)(p_j\cdot q_1)}+(1\leftrightarrow 2)$$

$$\tilde{\mathcal{S}}_{ij}^{m\neq0}(q_1,q_2) = \frac{(p_i\cdot p_j)}{2(p_i\cdot k)^2}\left(\frac{1}{(p_i\cdot q_1)(p_j\cdot q_1)}+\frac{1}{(p_i\cdot q_1)(p_j\cdot q_2)}\right)$$
$$-\frac{1}{k^2(p_i\cdot k)}\frac{1}{(p_i\cdot q_1)}\left(\frac{(p_j\cdot q_1)^2}{(p_j\cdot k)(p_j\cdot q_2)}-\frac{(p_i\cdot q_1)^2}{(p_i\cdot k)(p_i\cdot q_2)}\right)+(1\leftrightarrow 2)$$

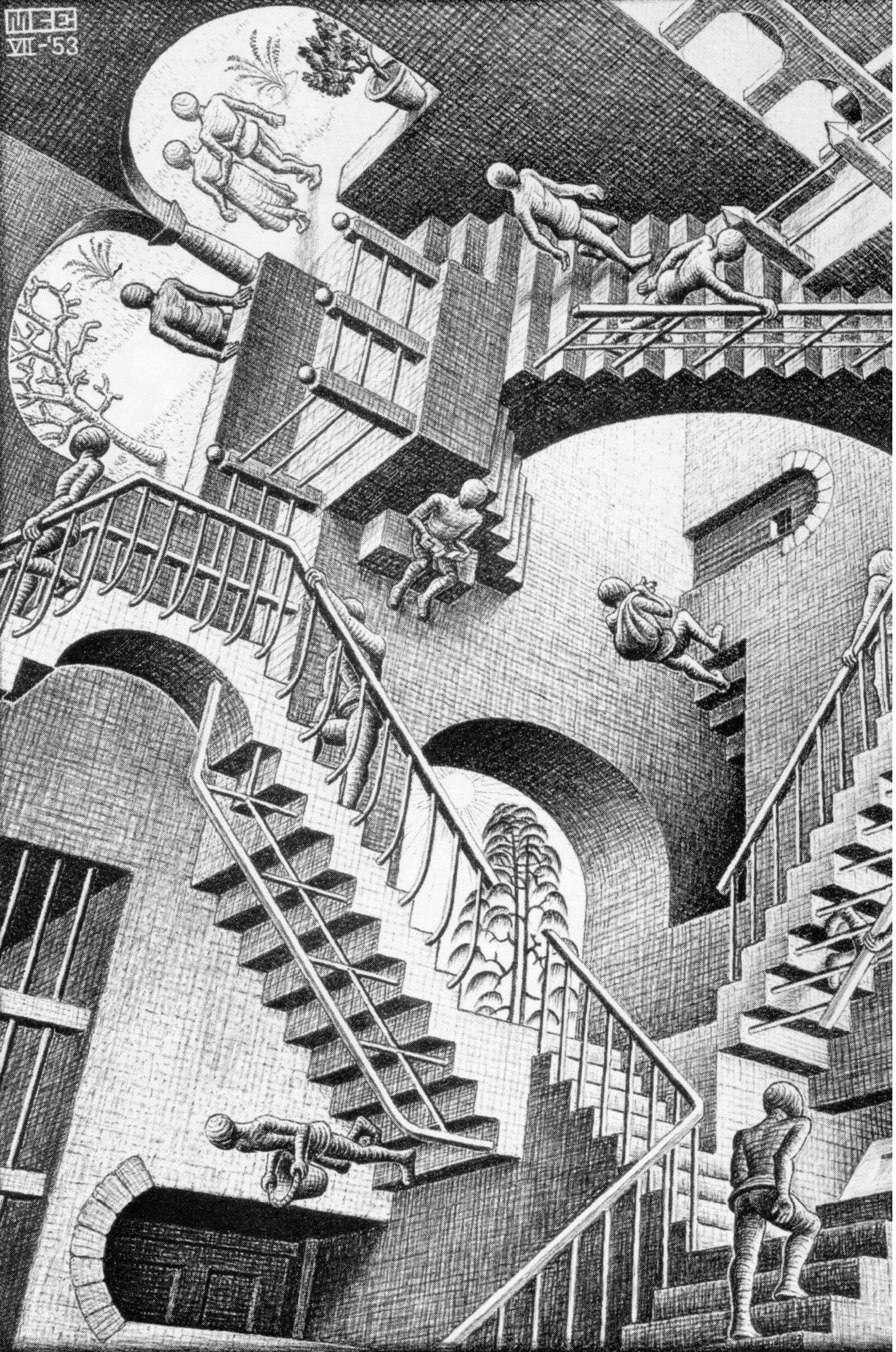
$$\boxed{k = q_1+q_2}$$

STATUS OF THE PROJECT

Contribution				Computed?
n_f -dependent part	Single gluon emission			✔
	$q\bar{q}$ production			✔
[1,2,i → massless 3,4,j → massive]	Single gluon emission at 1 loop		$\propto \mathbf{T}_i \cdot \mathbf{T}_j$	✔
			$\propto \mathbf{T}_3 \cdot \mathbf{T}_4$	✘
	Double gluon emission	$\mathcal{S}^{m=0}$	$\propto \mathbf{T}_i \cdot \mathbf{T}_j$	✔
			$\propto \mathbf{T}_j \cdot \mathbf{T}_j$	✔
			$\propto \mathbf{T}_3 \cdot \mathbf{T}_4$	✘
		$\mathcal{S}^{m\neq 0}$	$\propto \mathbf{T}_i \cdot \mathbf{T}_j$	✔
			$\propto \mathbf{T}_j \cdot \mathbf{T}_j$	✔
			$\propto \mathbf{T}_3 \cdot \mathbf{T}_4$	✘



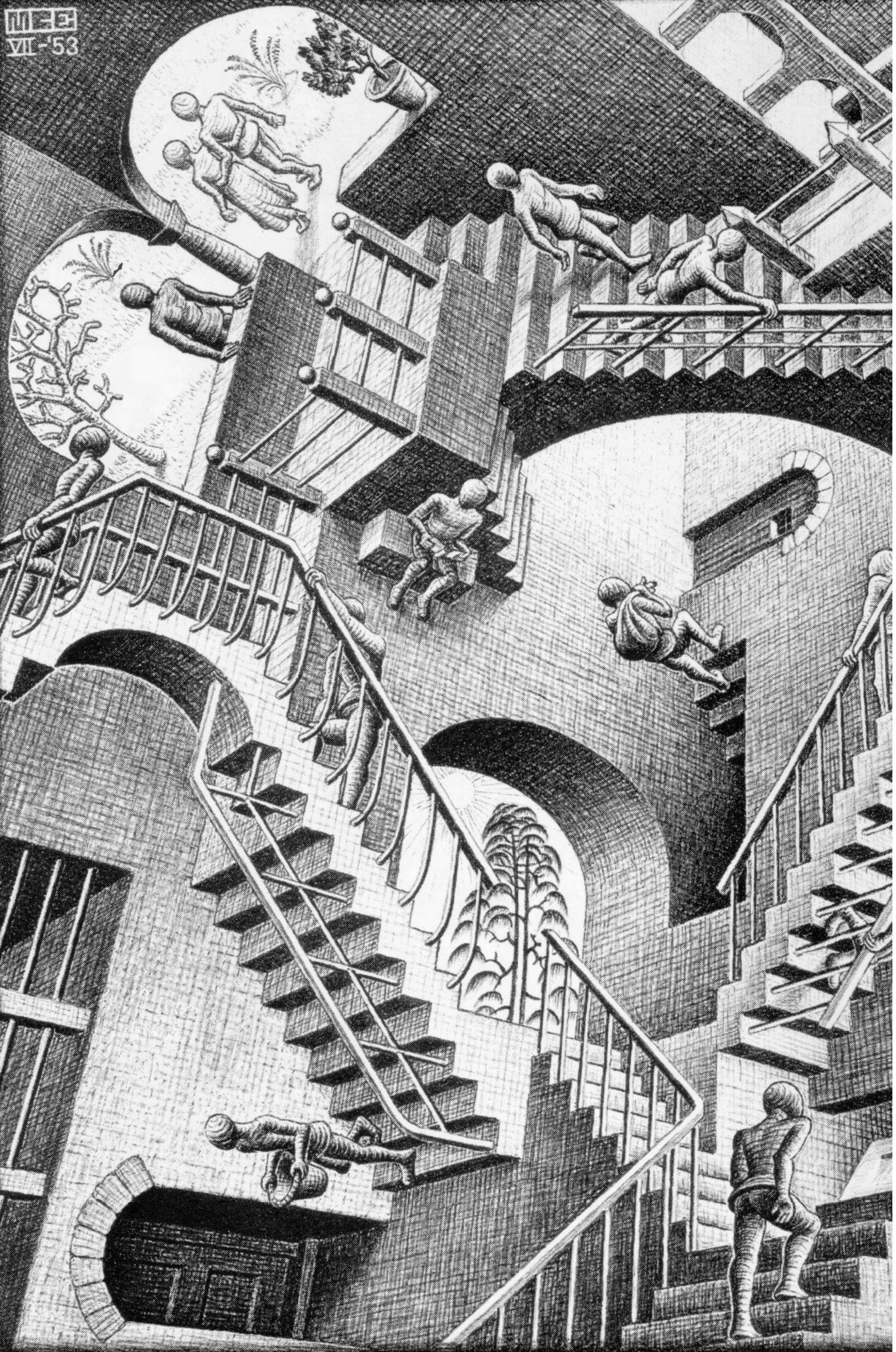
CONCLUSIONS



CONCLUSIONS

- What?

  Computation of the hard collinear coefficient for top pair production.



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- What?

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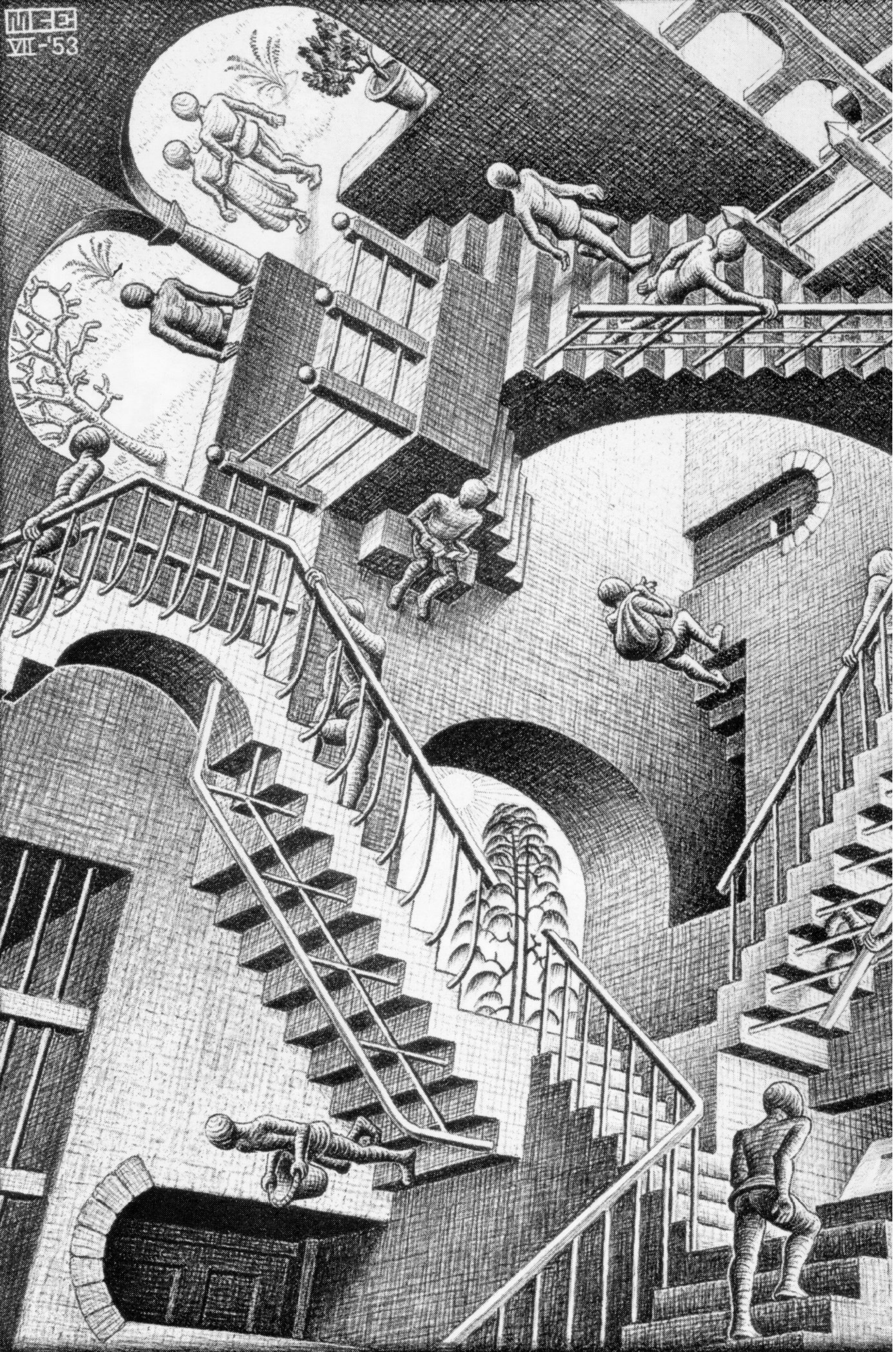
  To implement q_t subtraction for coloured final state.



CONCLUSIONS

- What?

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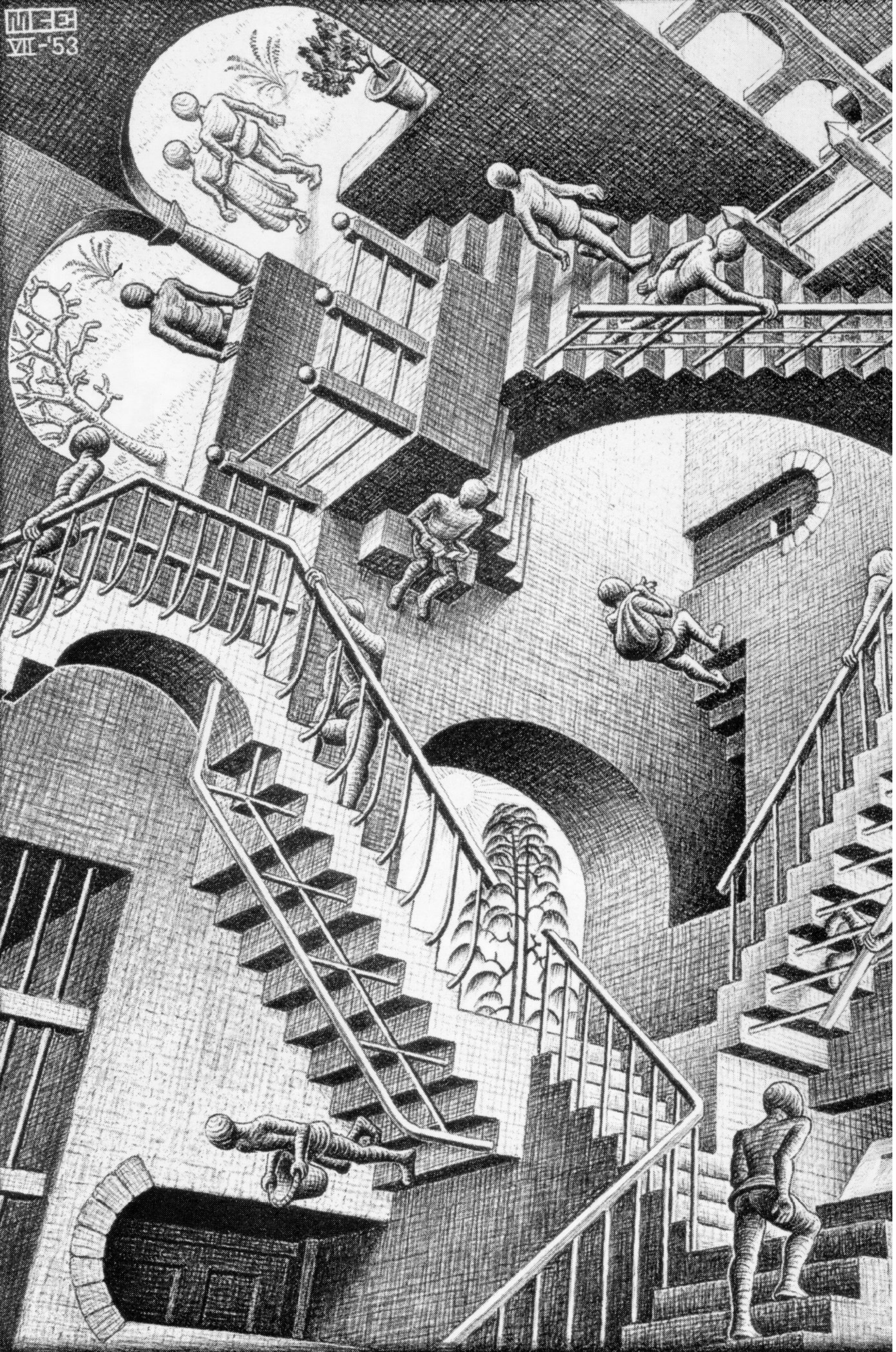
- Done:

  Analytic computation of factorised integrals (NLO type) and of part of the remaining ones.



CONCLUSIONS

- What?

  Computation of the hard collinear coefficient for top pair production.

- Why?

  To implement q_t subtraction for coloured final state.

- Done:

  Analytic computation of factorised integrals (NLO type) and of part of the remaining ones.

- To do:

  Complete the computation of the last integrals.