The matrix element method at next-to-leading order QCD

Generating un-weighted jet events according to NLO cross section predictions

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based on [JHEP 1509 (2015) 083, JHEP 1805 (2018) 141 and arxiv:1018.xxx]

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Motivation

So far no "smoking gun" type signal for new physics has been observed at the LHC

New physics may manifest itself in small deviations from the SM predictions

⇒ Need for precise theory predictions

How do we observe small deviations ?

Motivation

- Traditional approach:

Study various distributions of different cross sections and search for deviations

Drawback:

Which distribution should we use ?
Different distributions may have different sensitivities to different SM extensions

- Alternative approach:

Matrix **E**lement **M**ethod (MEM)

→ Given a theory model, method allows to calculate likelihood to observe an event sample

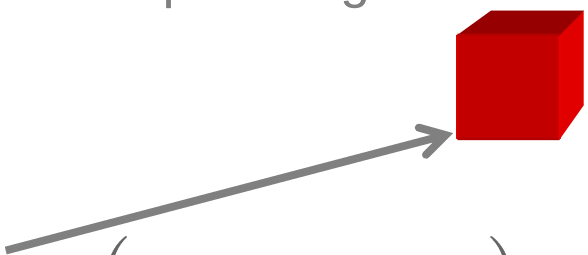
[Kondo 88,91]

The differential cross section

Model parameter $(\alpha_s, m_t \ldots)$

$$d\sigma(\vec{\omega}) \propto \frac{1}{2s} \left| \mathcal{M}(\vec{\omega}, p_1, \ldots, p_n) \right|^2 dR_n$$

is a measure for the probability to observe an event in the infinitesimal phase space region



dR_n

$(p_1, \ldots, p_n)$

$$\mathcal{P}(\vec{x}|\vec{\omega}) = \frac{1}{\sigma_t(\vec{\Omega})} \frac{d\sigma(\vec{\omega})}{dx_1 \ldots dx_r}$$

Given an event sample $E = \{\{p_1^{(i)}, \ldots, p_n^{(i)}\}_{i=1\ldots N}\}$ it is possible to calculate the model dependent likelihood to observe this sample:

$$\mathcal{L}(\vec{\omega}, E) \propto \prod_i \left| \mathcal{M}(\vec{\omega}, p_1^{(i)}, \ldots, p_n^{(i)}) \right|^2$$

The matrix element method in a nutshell

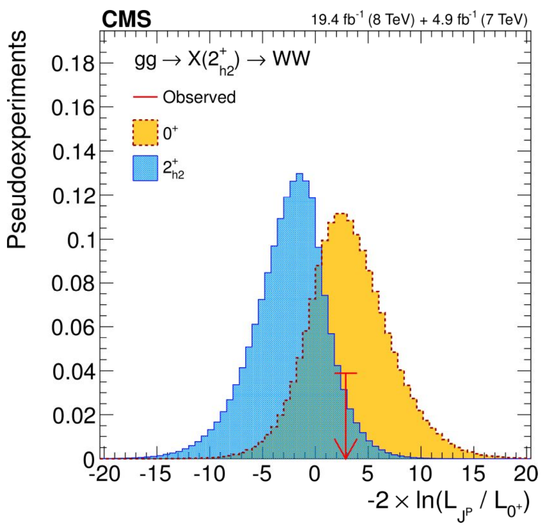
→ Multivariate method with a solid foundation in statistics

Applications:

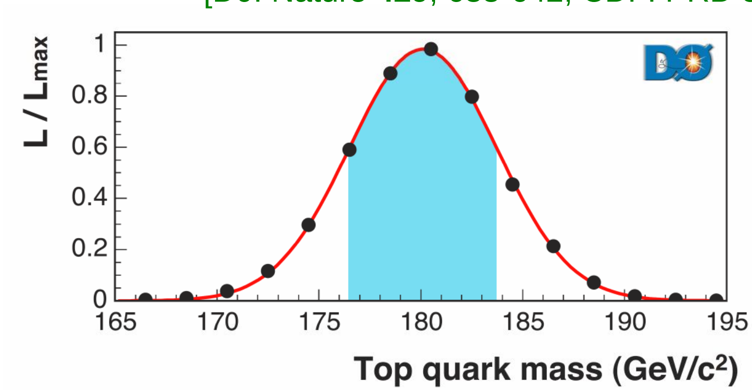
- Signal-background discrimination
- New physics searches
- Maximum likelihood parameter extraction

Unbiased estimator if dσ is “the true probability distribution”

[CMS, Phys. Rev. D92 (2015) 012004]



[D0: Nature **429**, 638-642, CDF: PRD 50, 2966]



e.g. top-quark mass measurements at Tevatron based on O(70) events!

“optimal use of information contained in event sample”

Matrix element method — transfer functions

In real life situation slightly more complicate:

- We observe hadrons and not partons
- No perfect detector

} modeled with transfer functions

→ Need cross section differential in hadronic variables including detector effects

$$\frac{d\sigma}{dx_1 \ldots x_r} = \int dy_1 \ldots dy_\ell \frac{d\sigma}{dy_1 \ldots y_\ell} W(\vec{x}, \vec{y})$$

Transfer function, probability to observe a partonic event x as hadronic event y in the detector

→narrow gaussians for leptonic variables and angles, ….

not a conceptual problem, but a potential limitation because of finite computer resources

for simplicity assume δ-functions in the following

Summary MEM

Powerful multivariate method with a clean statistical interpretation

However:

So far most applications rely on LO matrix elements

→ Estimator for model parameter is in general biased, calibration required which leads to additional uncertainties

→Not sufficient to search for small deviations from the SM (calibration is not an option…)

Outline

- Motivation and Notation ✓
- MEM@NLO accuracy
- Validation
- Applications
- Summary/Conclusion

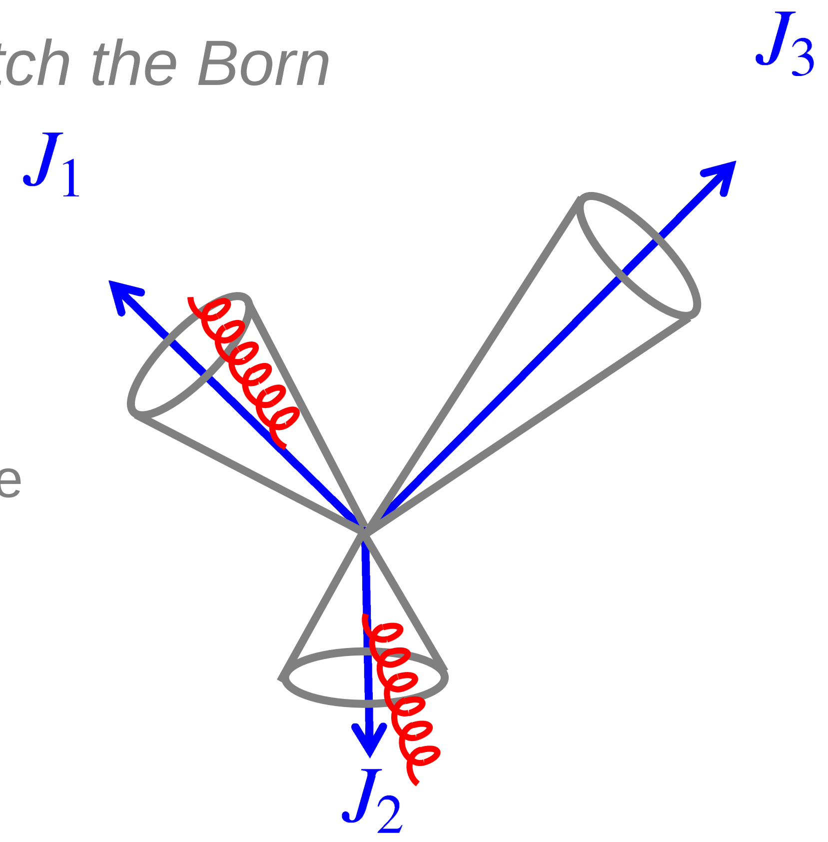
MEM@NLO

Steps towards MEM@NLO:

- Effect of real radiation [Alwall, Freitas, Mattelaer ’11]
- Final states without strongly interacting particles [Campbell,Giele, Williams ’12] [Campbell, Ellis, Giele, Williams ’13]
- First step towards final states with strongly interacting particles [Campbell,Giele, Williams ’13]
- General algorithm for arbitrary processes using a modified jet algorithm [Martini,PU ’15]
- Formal solution for arbitrary jet algorithms no proof of concept [Baumeister, Weinzierl ‘17]
- Approach for arbitrary jet algorithms with proof of concept [Martini, PU, to appear] (related work: [Figy, Giele 18])

What needs to be done:

- Integrate out in an efficient way unresolved real emission
- Left over variables *must match the Born kinematics*



Strictly speaking not much more than choosing the right variables, calculate the jacobian and some combinatorics to identify the relevant phase space regions…

Fully differential event weight in NLO:

$$d\sigma_{\rm NLO}(J_1, J_2, \ldots, J_n) =$$

$$\mathcal{V}_n dR_n(J_1^{(n)}, \ldots, J_1^{(n)}) + \int_1 \mathcal{R}_{n+1} dR_{n+1}(p_1, \ldots, p_n)$$

Need factorization of the form

$$dR_{n+1}(p_1, \ldots, p_{n+1}) = dR_n(J_1^{(n+1)}, \ldots, J_n^{(n+1)}) \times d\Phi$$

to allow integration over all unresolved phase space regions

However:

$$J_i^{(n+1)} = J_i^{(n+1)}(p_1, \ldots, p_{n+1}) \leftrightarrow J_i^{(n)} = p_i$$

$$(J_i^{(n+1)}) \neq (J_i^{(n)})^2$$

momenta $J_i^{(n)}$ and $J_i^{(n+1)}$ satisfy different kinematics, pointwise combination non-trivial

!

Two different approaches:

❶ Modify recombination procedure such that the clustered jets satisfy the Born kinematics

(use for example a 3→2 recombination procedure inspired by Catani-Seymour subtraction procedure)
Universal approach, allows to use the "Born variables" as a subset of the variables used to describe the real corrections, requires however that the new recombination procedure is also applied in the experimental analysis

❷ To describe the differential cross section use variables which do not distinguish between real and virtual corrections eg. do not allow to reconstruct the jet masses

e.g. $d^4 p_i \delta(p_i^2 - m_i^2) = \frac{p_i^{\perp}}{2\cosh(\eta_i)} dp_i^2 d\eta_i d\phi_i dE_i \delta(p_i^2 - m_i^2)$

MEM@NLO

Both approaches allow a factorization of the phase space:

$$dR_n(p_1,\ldots,p_n) = c\,dx_1\ldots dx_r$$

$$dR_{n+1}(p_1,\ldots,p_{n+1}) = dx_1\ldots dx_r \otimes d\Phi_{\mathrm{un}}$$

The differential cross section is than given by

$$\frac{d\sigma_{\mathrm{NLO}}}{dx_1\ldots dx_r} = c\,\mathcal{V}_n + \sum_i \int_{G_i} d\Phi_i \mathcal{R}_{n+1}$$

(an appropriate regulator to regulate IR divergences is required)

Sum runs over all unresolved regions G_i, phase space parameterization depends on region

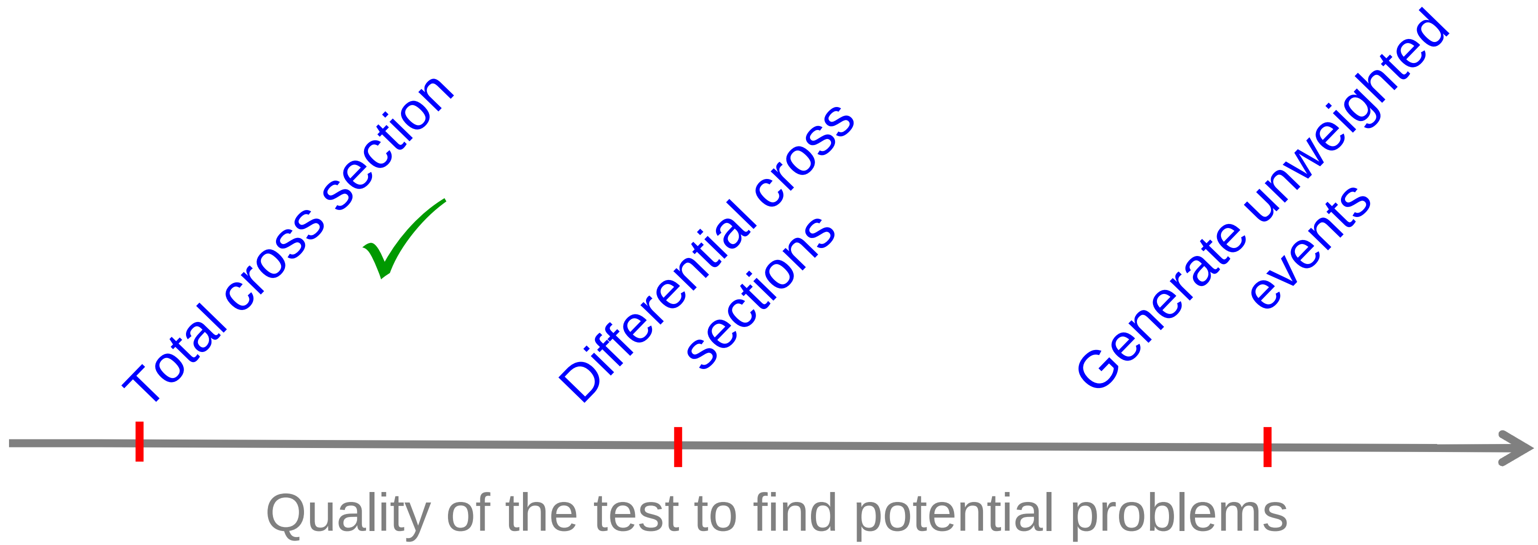
MEM@NLO

Remarks:

- The fact that the variables must be insensitive to the jet masses can also be understood as a consequence of IR safety
- Approach allows *point wise* combination of real and virtual corrections (different from conventional parton level MC's)
- Point-wise combination may simplify numerical integration since problems at bin boundaries are avoided [Figy, Giele '18]
- As long as perturbation theory works: positive weights!
- Possibility to generate un-weighted events according to the NLO cross section

Validation

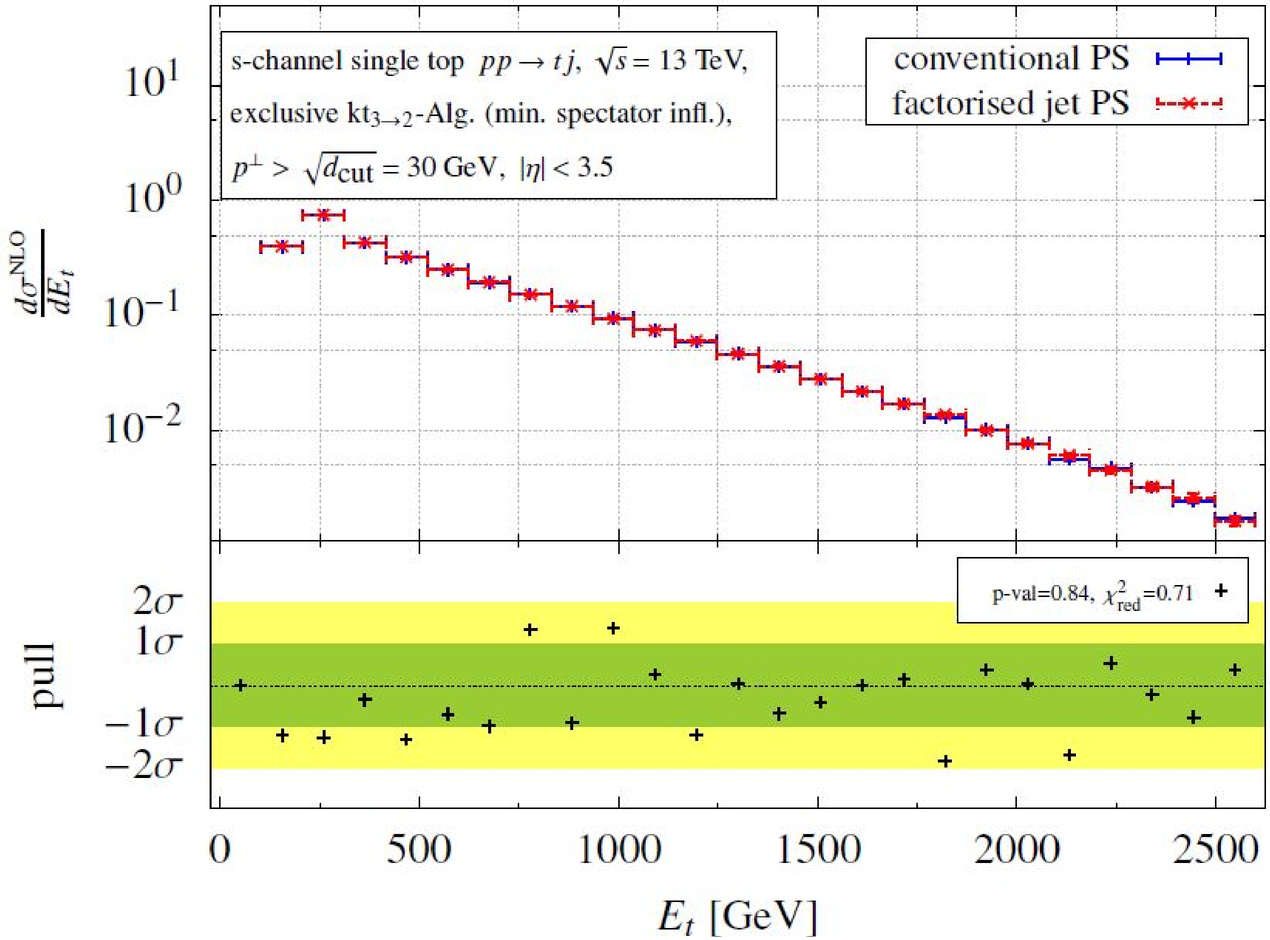
Use aforementioned phase space parametrisation and compare with results obtained from conventional Monte Carlo program



Use as test case single top-quark production where the MEM has been used recently to observe s-channel production [ATLAS '17]

Allows to test initial state as well as final state radiation with colored partons in the final state

Conventional phase space parameterization vs. factorized parameterization



[Martini, PU '17]

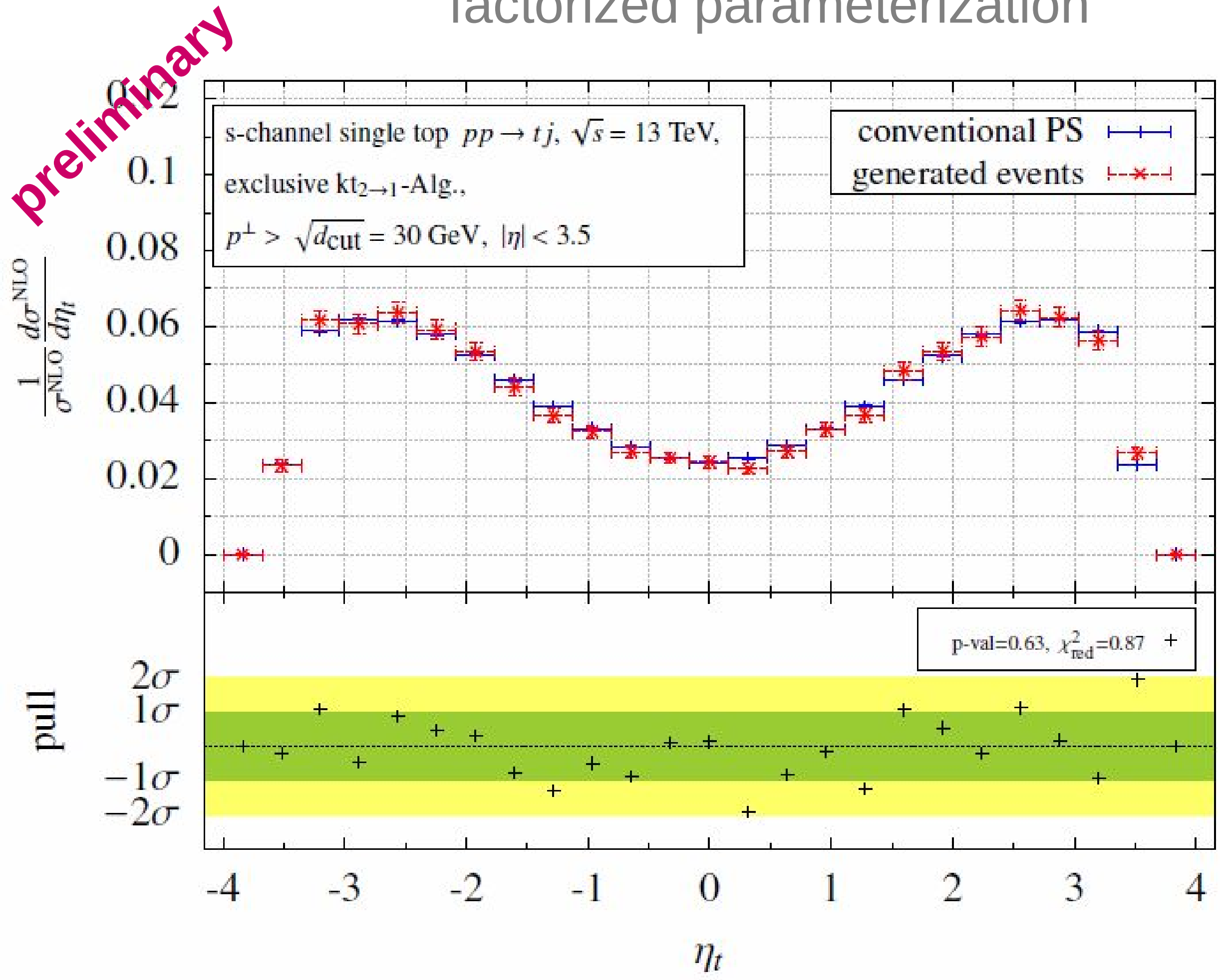
(modified jet algorithm, 3→2 recombination)

Similar plots for t-channel and other distributions

→ Perfect agreement within statistical uncertainties

Validation: Phase space parameterization

Conventional phase space parameterization vs. factorized parameterization



[Martini,PU]

traditional 2→1 recombination

Similar plots for t-channel and other distributions

→ Perfect agreement within statistical uncertainties

Validation: Generation of un-weighted jet events

Differential weight

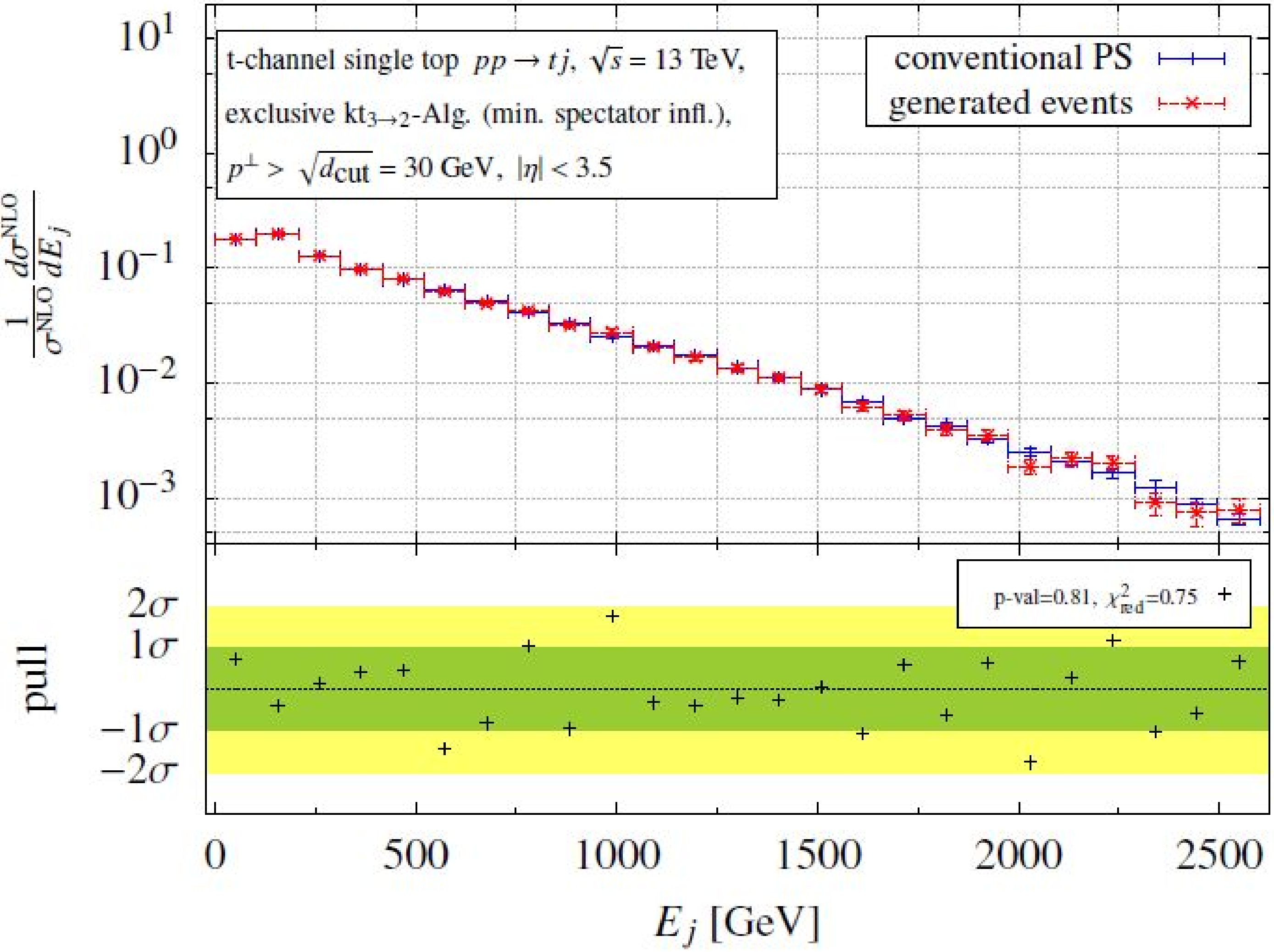
$$\frac{d\sigma_{\mathrm{NLO}}}{dx_1 \ldots dx_r} = c\mathcal{V}_n + \sum_i \int_{G_i} d\Phi_i \mathcal{R}_{n+1}$$

can be used to generate un-weighted jet events parameterized using variables $x_1, \ldots, x_r$

To generate events use von Neumann "acceptance and rejection" approach

Validation: Generation of un-weighted events

Normalized distributions from un-weighted events **[Martini, PU '17]**



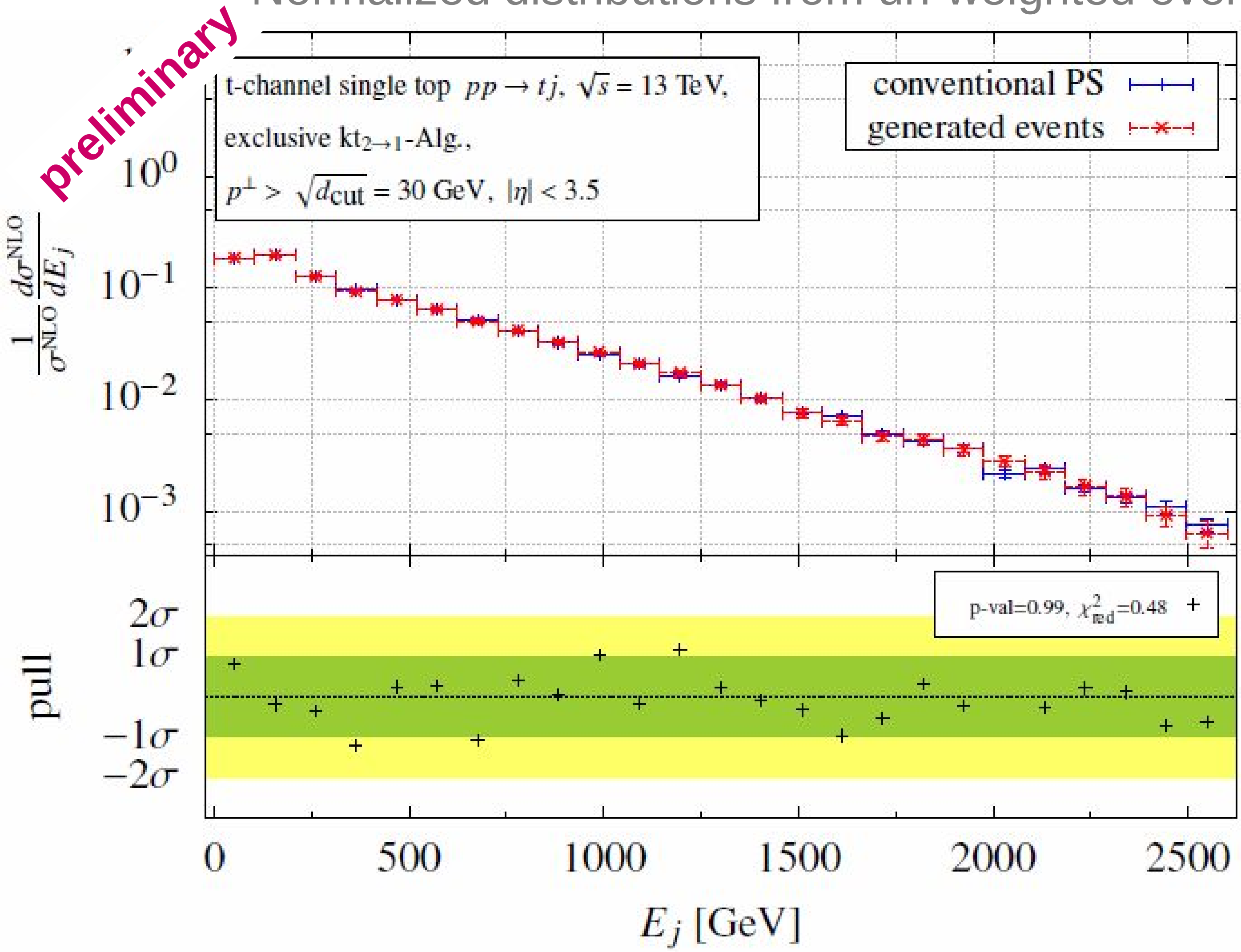
(modified jet algorithm, 3→2 recombination)

→ Perfect agreement with conventional MC

Validation: Generation of un-weighted events

Normalized distributions from un-weighted events

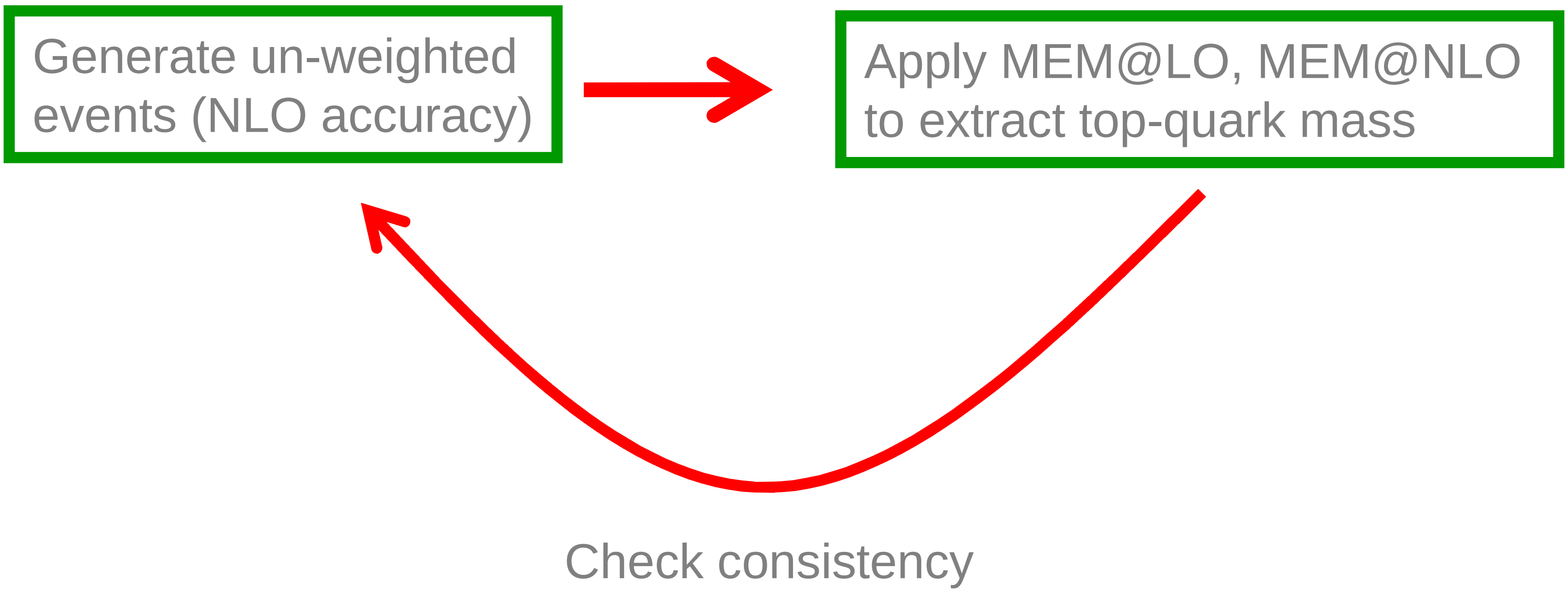
[Martini,PU]



traditional 2→1 recombination

→ Perfect agreement with conventional MC

To analyze method simulate measurement:

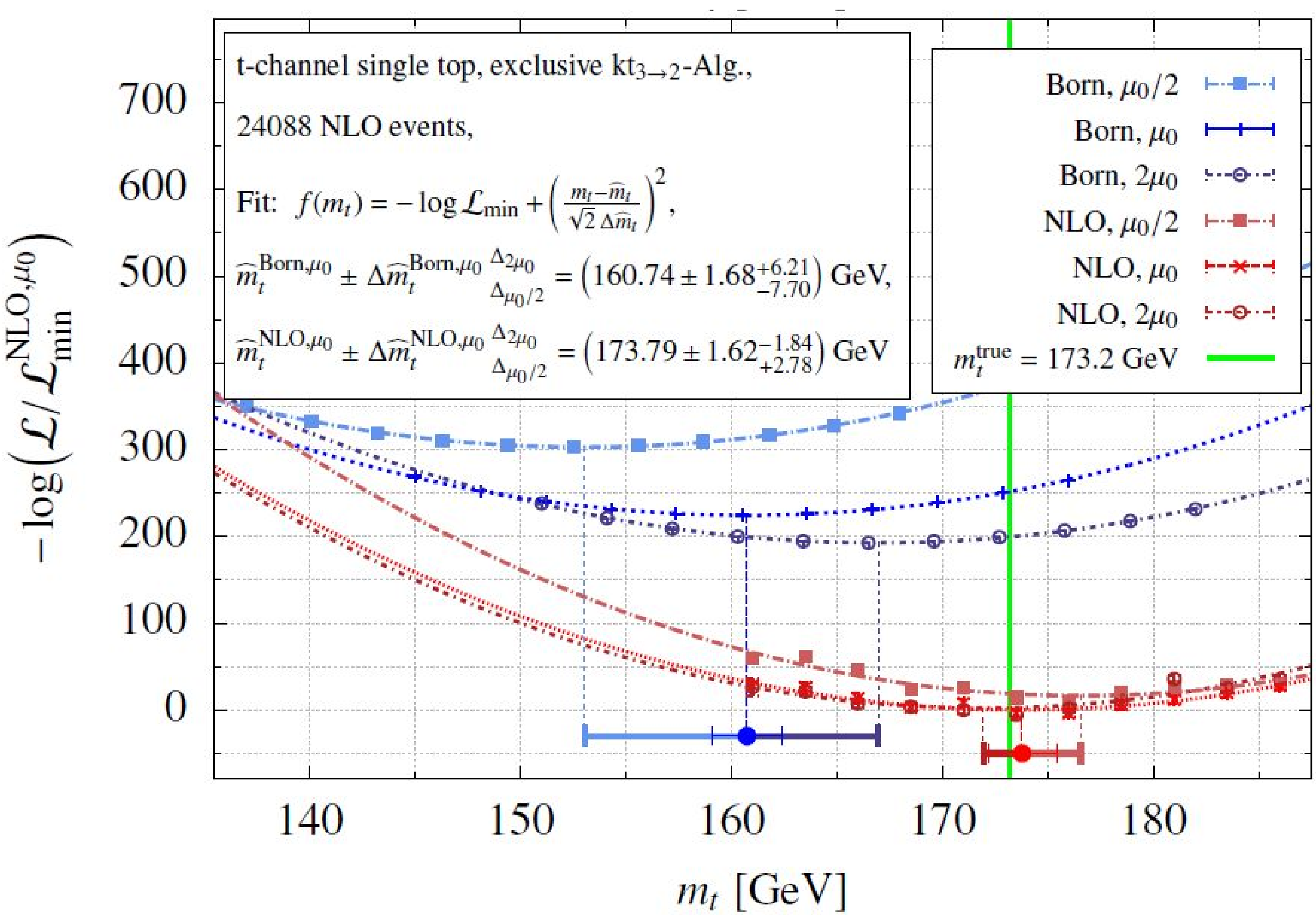


$$\mathcal{L}^{\mathrm{NLO}}(\omega) = \prod_{i=1}^{N} \mathcal{L}^{\mathrm{NLO}}(\omega|\vec{x}_i) = \left(\frac{1}{\sigma^{\mathrm{NLO}}(\omega)}\right)^N \prod_{i=1}^{N} \frac{d\sigma^{\mathrm{NLO}}(\omega)}{d\vec{x}_i}$$

Application: Top-quark mass from single top events

t-channel, 3→2 recombination

[Martini, PU ’17]

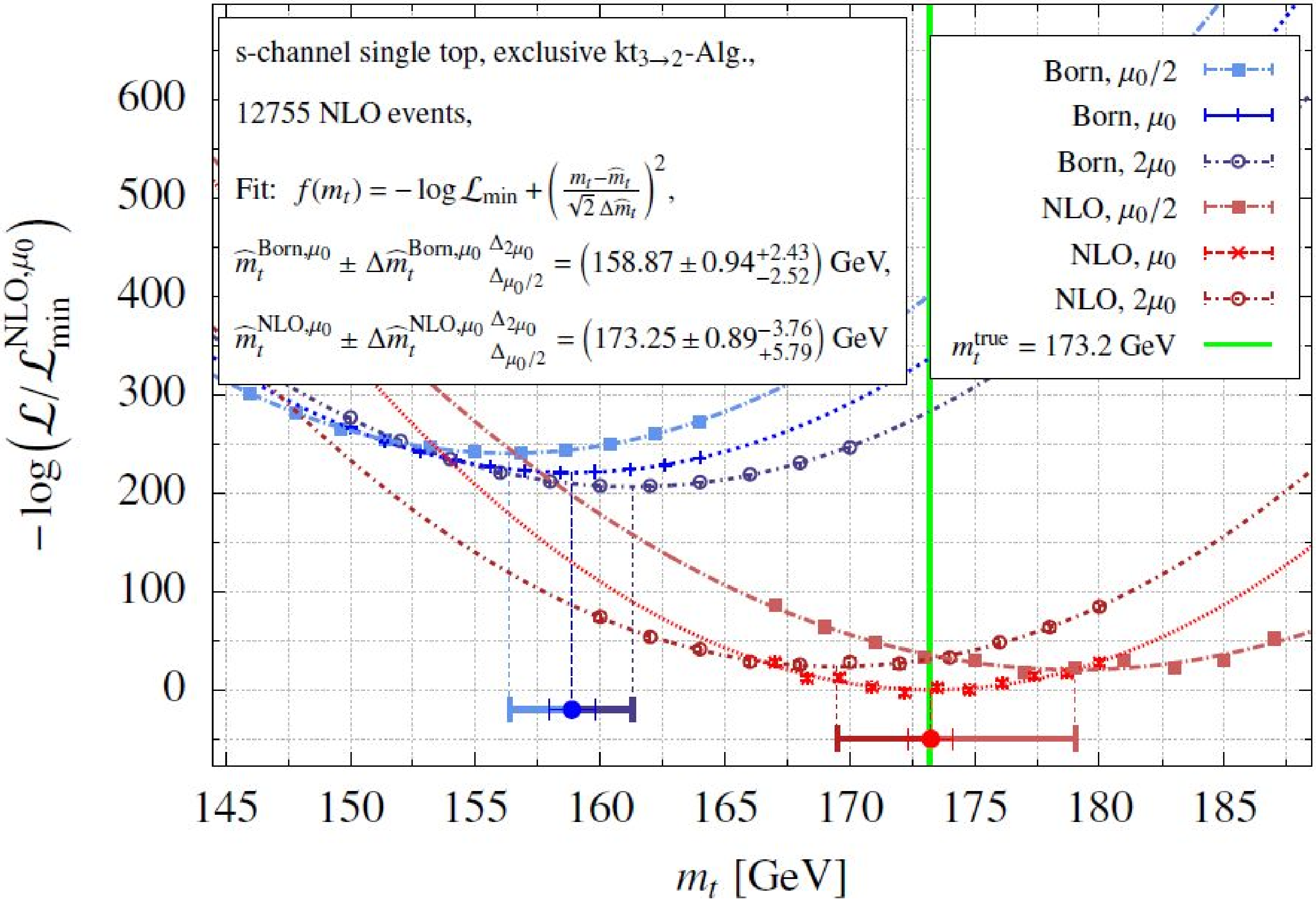


- “closure test” at NLO works
- Scale uncertainty improved
- Significant bias using MEM@LO

Large scale uncertainties remain in NLO…

s-channel, 3→2 recombination

[Martini, PU ’17]

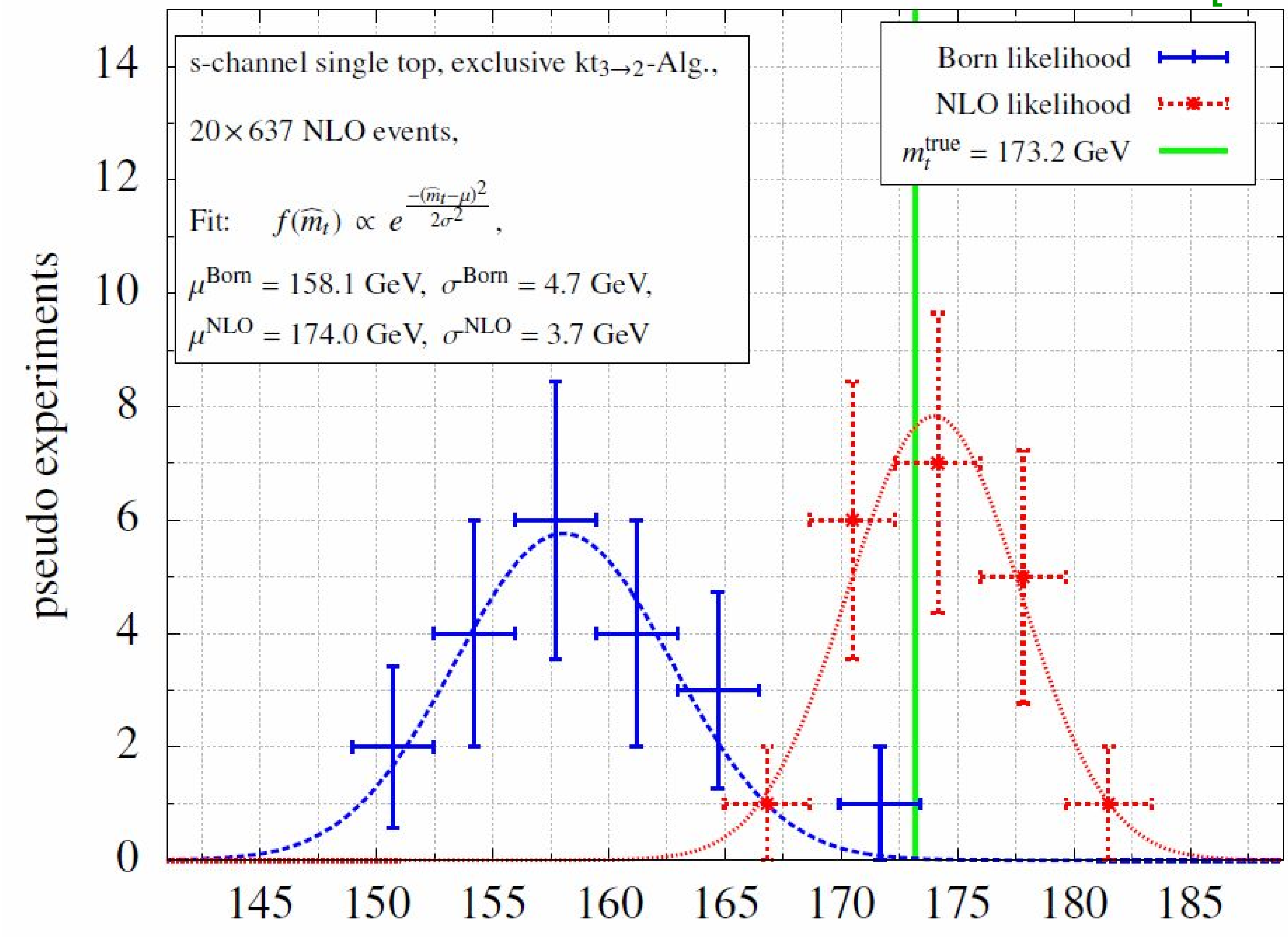


- No improvement in scale dependence
- NLO required to obtain reliable uncertainty estimate
- Again significant bias using MEM@LO

Application: Top-quark mass from single top events

s-channel, 3→2 recombination

[Martini, PU '17]

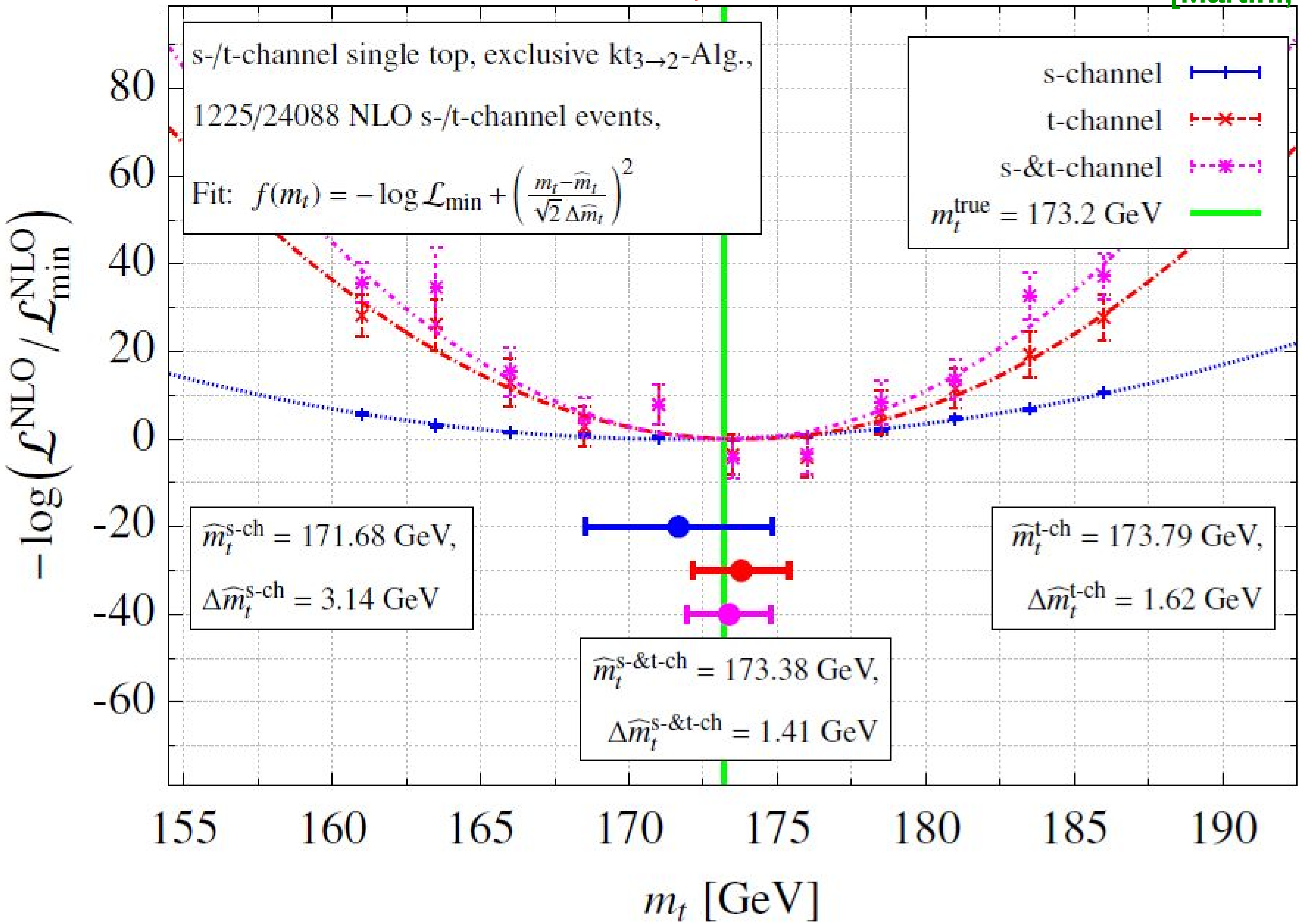


→ Bias is **not** an artefact due to some statistical outliers!

Application: Top-quark mass from single top events

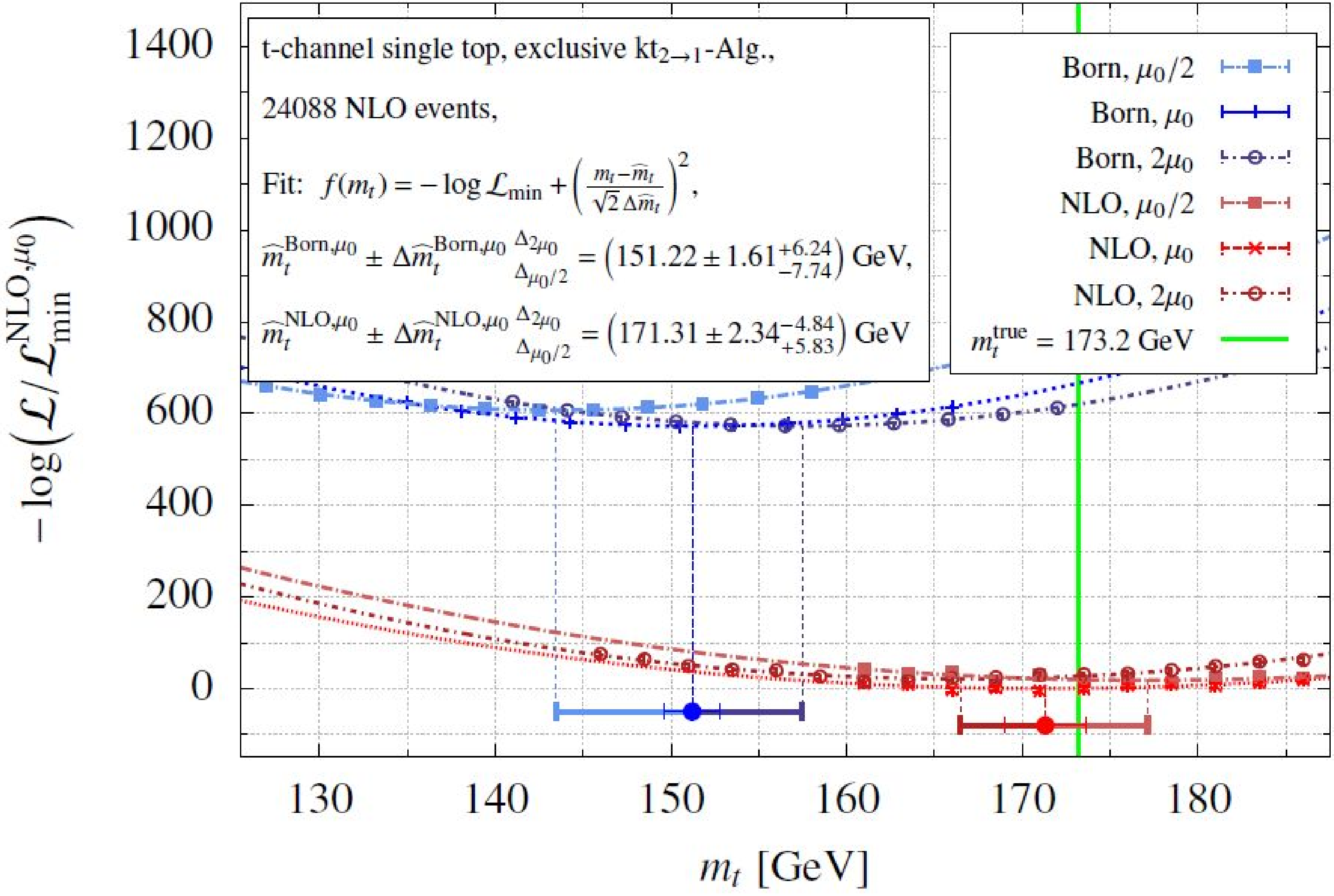
s+t-channel, 3→2 recombination

[Martini, PU '17]



t-channel, 2→1 recombination

[Martini, PU ’17]



Summary and Outlook

Summary:

- Two methods for MEM@NLO
- Allows generating of un-weighted jet events according to NLO cross sections
- Sizeable shifts in extracted parameters using NLO instead of LO
- MEM@NLO required to obtain reliable estimates of uncertainties
- Parton shower does not change the picture

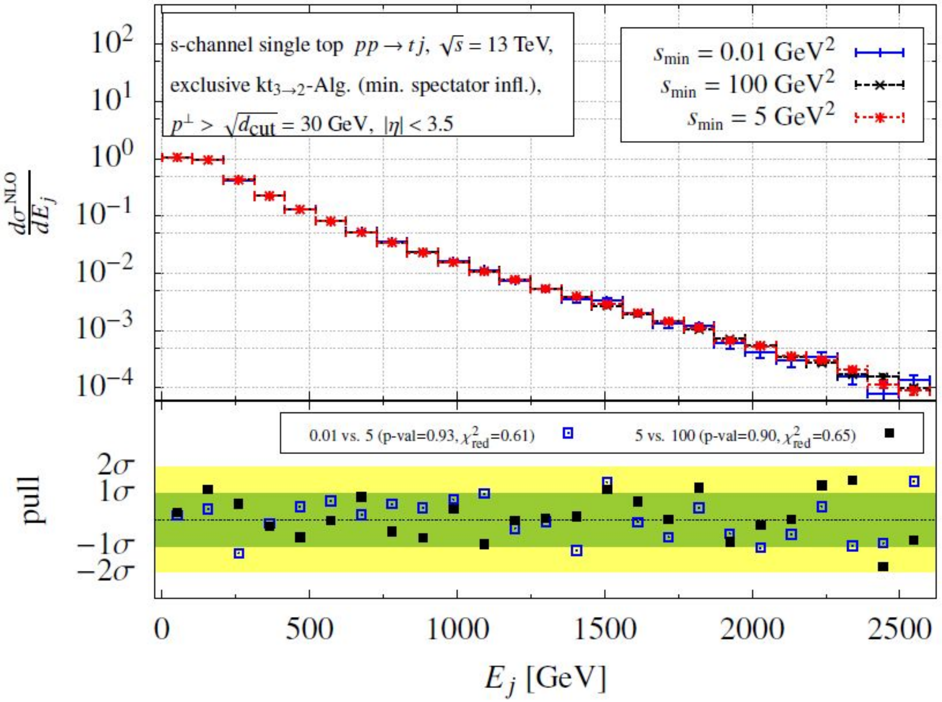
Outlook:

- Improve handling of IR divergences (slicing → FKS)
- Apply to other interesting signal processes

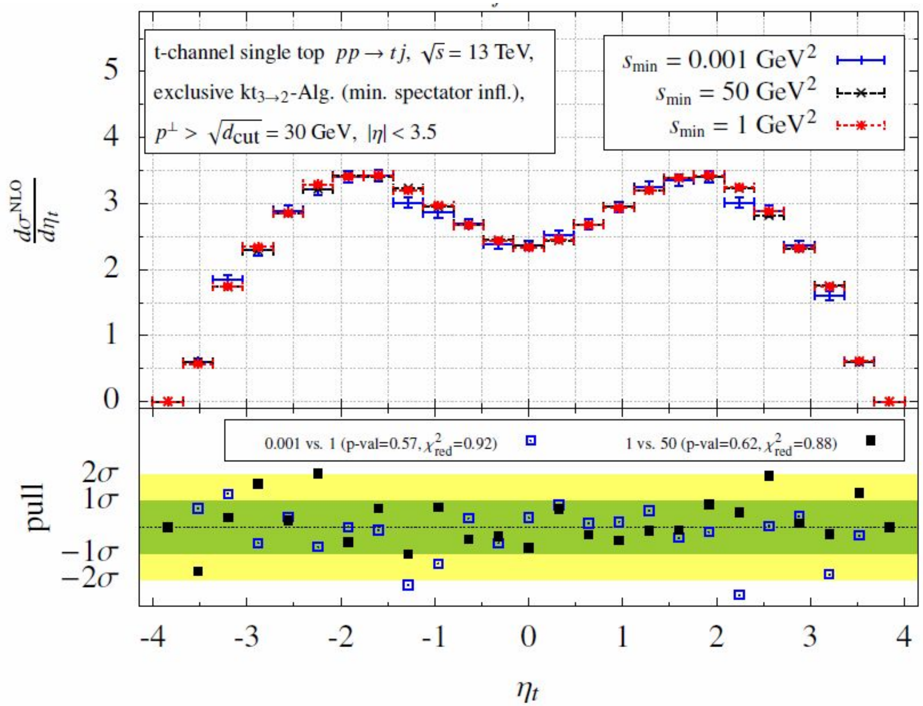
Backup

Backup: smin-(in)dependence

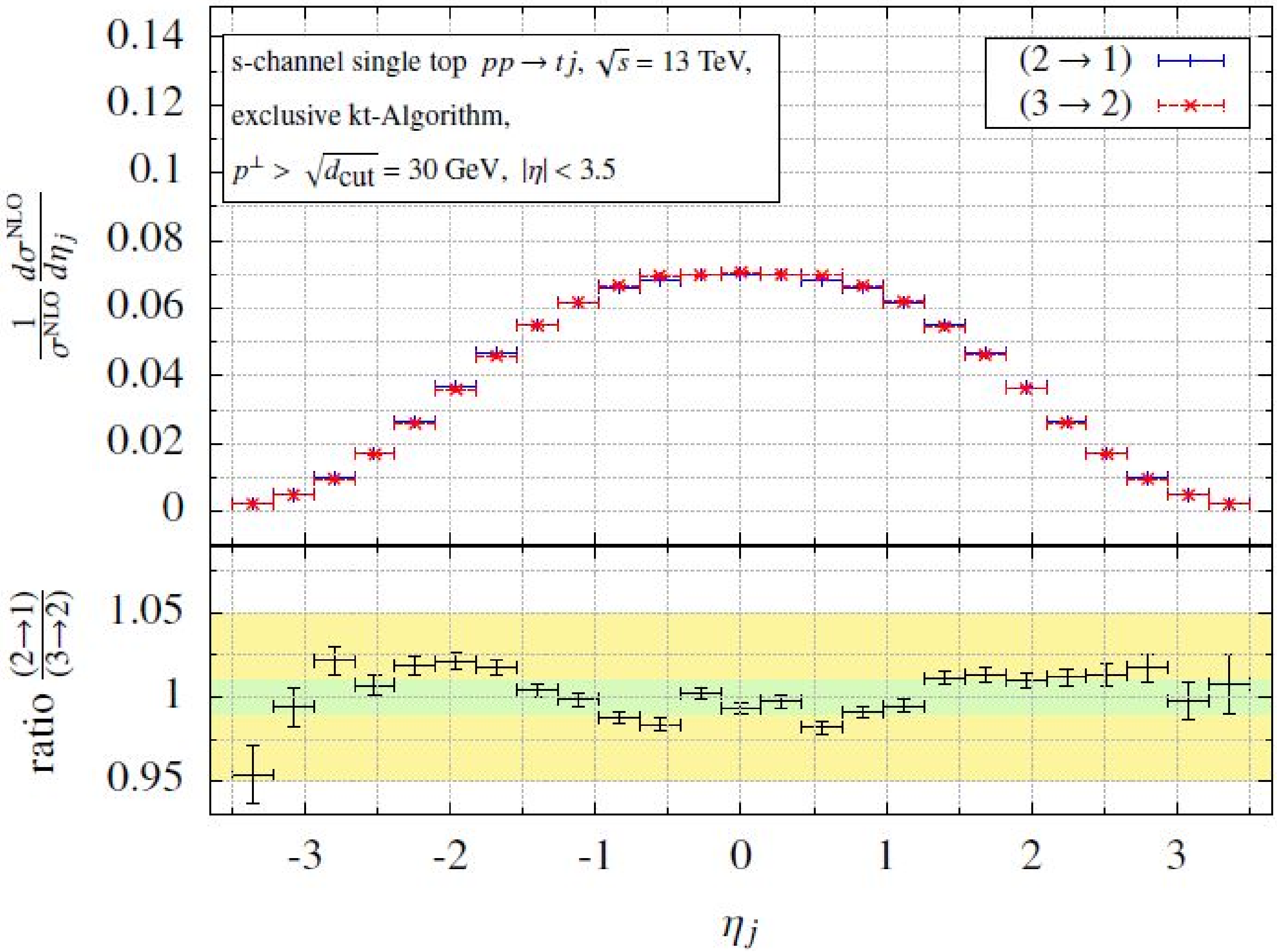
s-channel



t-channel

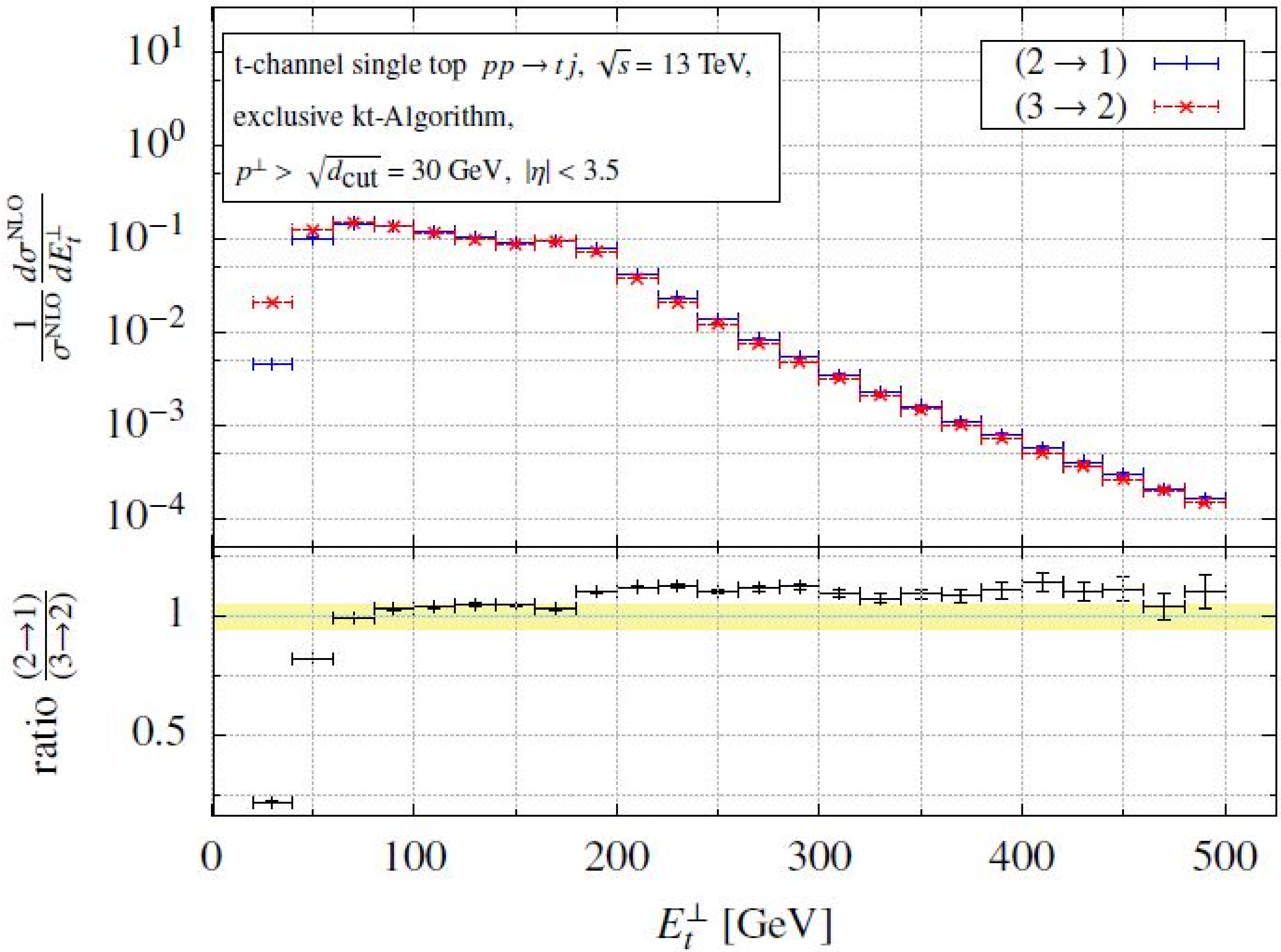


Comparison: 2→1 vs 3→2 recombination



Small difference for distributions relying on angles

Comparison: 2→1 vs 3→2 recombination



Sizeable differences in distribution sensitive to the jet mass

Jet mass for different recombinations