Five-loop UV behavior of $\mathcal{N} = 8$ supergravity

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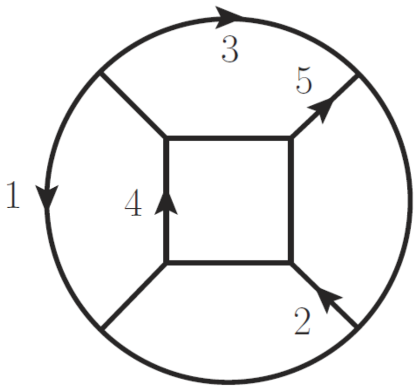
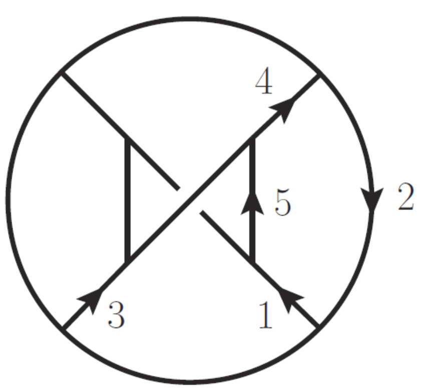




With *Zvi Bern, John Joseph Carrasco, Wei-Ming Chen, Alex Edision, Henrik Johansson, Julio Parra-Martinez, Radu Roiban*

arXiv:1708.06807, Phys. Rev. D. 96, 126012
arXiv:1804.09311, accepted by Phys. Rev. D.

Outline

- Background
- Five-loop results for $\mathcal{N} = 8$ supergravity
- Maximal-cut calculation
- Full calculation
- All-loop patterns & future outlook

Is $\mathcal{N} = 8$ SUGRA a finite theory of gravity?

- Previous calculations: finite up to 4 loops, $D_c = 4 + 6/L$

  Deser, Kay, Stelle, 1977; Tomboulis, 1977; Bern, Carrasco, Dixon, Johansson, Kosower, Roiban 2007; Bern, Carrasco, Dixon, Johansson, Roiban 2009, 2012

- Symmetry arguments: divergent at 7 loops, or 5 loops at $D_c = 24/5$, due to counterterm $\sim D^8 R^4$.

  Green, Russo, Vanhove 2010; Bossard, Howe, Stelle 2011; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger 2010; Vanhove 2010; Bjornsson, Green 2010; Bjornsson 2010

- "Enhanced cancellations" beyond symmetry arguments

  $N = 4$ finite in $D = 5$ at 2 loops: Bern, Davies, Dennen, Huang 2012
  $N = 4$ finite in $D = 4$ at 3 loops: Bern, Davies, Dennen, Huang 2012
  $N = 5$ finite in $D = 4$ at 4 loops: Bern, Davies, Dennen 2014

- This talk: the 5-loop calculation & paths to higher loops

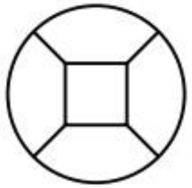
The five-loop results

[Bern, Carrasco, Chen, Edison, Johansson, Parra-Martinez, Roiban, MZ, 2018]

- $\mathcal{N} = 8$ SUGRA is UV finite in $D = 22/5$, confirming symmetry predictions.

- $\mathcal{N} = 8$ SUGRA diverges in $D = 24/5$, as positive-definite vacuum integrals. $\Longrightarrow$ Nonzero coefficient of $D^8 R^4$.

$$\mathcal{M}_4^{(5)}\Big|_{\text{leading}} = -\frac{16 \times 629}{25}\left(\frac{\kappa}{2}\right)^{12}(s^2+t^2+u^2)^2 stu M_4^{\text{tree}}\left(\frac{1}{48}\right.$$



$$+\ \frac{1}{16}$$

$$\left.\right)$$

$$= -17.9\left(\frac{\kappa}{2}\right)^{12}\frac{1}{(4\pi)^{12}}(s^2+t^2+u^2)^2 stu M_4^{\text{tree}}\frac{1}{\epsilon}.$$

FIESTA: Smirnov, Smirnov, Tentyukov

Challenges in a 5-loop calculation

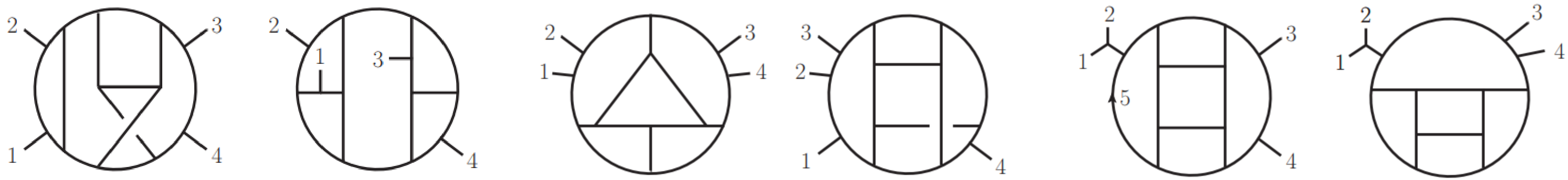
1) The loop integrand:

Explosion of terms in Feynman diagrams, ~ 10^{31} terms at 5-loop 4-point

Solution: (Generalized) double copy

$\mathcal{N}_{GR} \sim \mathcal{N}_{YM} \tilde{\mathcal{N}}_{YM} + J\tilde{J}$ [Bern, Carrasco, Chen, Johansson, Roiban 2017]

$J \sim \text{BCJ discrepancy function}$ gives $N^{\geq 2}$ Max contact terms



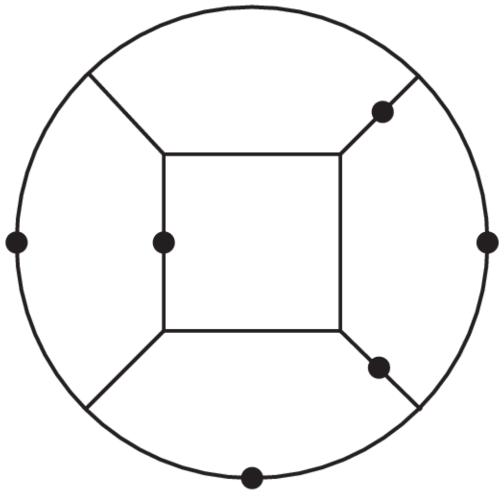
(0: 33) (0: 36) (0: 414) (0: 424) (0: 457) (0: 547)

Challenges in a 5-loop calculation

2) Integration in UV region:

   Large number of vacuum integrals, high-degree numerators

   Solutions: finding better integrand, Unitarity cuts and IBP for vacuum integrals



$\times$ (degree-6 numerator) $=$?

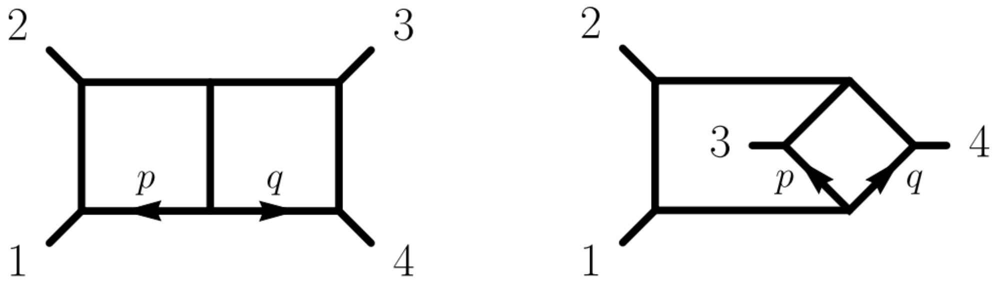
Warmup: 2-loop $\mathcal{N}=4$ SUGRA in $D=5$

[Bern, Enciso, Parra-Martinez, MZ, 2018]

$$\mathcal{N}=4\ \text{SUGRA} \cong (\mathcal{N}=4)\otimes(\mathcal{N}=0)$$

$$\mathcal{A}_{\text{SUGRA}} = \sum_i \frac{n_i^{\text{SYM}} \cdot n_i^{\text{YM}}}{\text{propagators}}$$

Enhanced cancellation from double copy
[Bern, Davies, Dennen, Huang 2012]

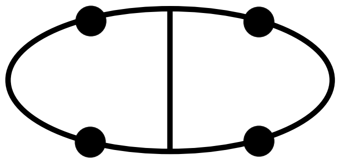


(++++) numerators (both diagrams)

$$\lambda_p^2\lambda_q^2 + \lambda_p^2\lambda_{p+q}^2 + \lambda_q^2\lambda_{p+q}^2$$

1[st] step: vacuum expansion

$$\int d^5p\, d^5q\, \frac{\lambda_p^2\lambda_q^2 + \lambda_p^2\lambda_{p+q}^2 + \lambda_q^2\lambda_{p+q}^2}{(p^2)^A(q^2)^B[(p+q)^2]^C}$$

 $\equiv I_{3,1,3}$

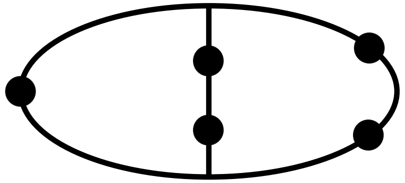
2[nd] step: Lorentz invariance

$$\frac{3}{70}\int d^5p\, d^5q\, \frac{p^2+q^2+(p+q)^2}{(p^2)^A(q^2)^B[(p+q)^2]^C}$$

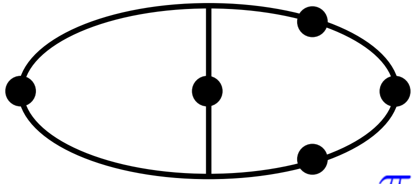
(Planar) + (Nonplanar) $\propto I_{1,2,2} + 2I_{1,1,3} \sim 0\,?$

3[rd] step: integration …

Maximal-cut vacuum integrals



$+\ 2$



$$= I_{1,2,2} + 2I_{1,1,3} = \text{UV finite?}$$

$$= -\frac{\pi}{192\epsilon} + \mathcal{O}(\epsilon^0) \qquad = \frac{\pi}{96\epsilon} + \mathcal{O}(\epsilon^0)$$

No one-loop divergence in 5D, UV from max. cut! $\frac{1}{l^2 - m^2} \to \delta(l^2 - m^2)$

Classic use of cuts: *cut integrand* = product of trees

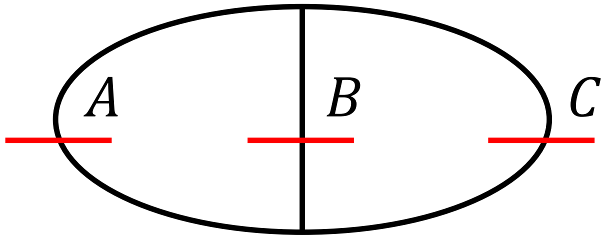
Also consider *cut integrals*, defined on contours preserving integral relations

Kosower, Larsen, 2012; Caron-Huot, Larsen, 2012; Sogaard, 2013;
Johansson, Kosower, Larsen, 2013; Sogaard, Zhang, 2013; Sogaard, Zhang, 2014;
Abreu, Britto, Durh, Gardi, 2017; Bosma, Sogaard, Zhang 2017; Schabinger 2017;
Primo, Tancredi 2016, 2017

Maximal-cut vacuum integrals

Baikov representation of Feynman integrals [Baikov, 1996]

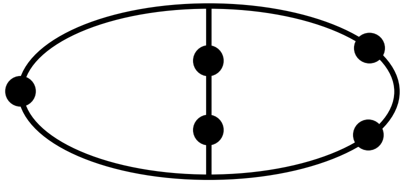
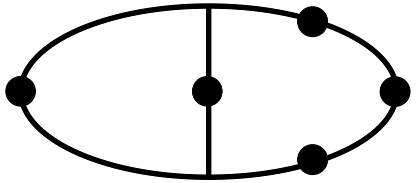
$$I_{A,B,C} = \int d^5p \, d^5q \, \frac{1}{(p^2)^A (q^2)^B [(p+q)^2]^C}$$



$$\propto \int \frac{dz_1}{z_1^A} \frac{dz_2}{z_2^B} \frac{dz_3}{z_3^C} P(z_1, z_2, z_3),$$

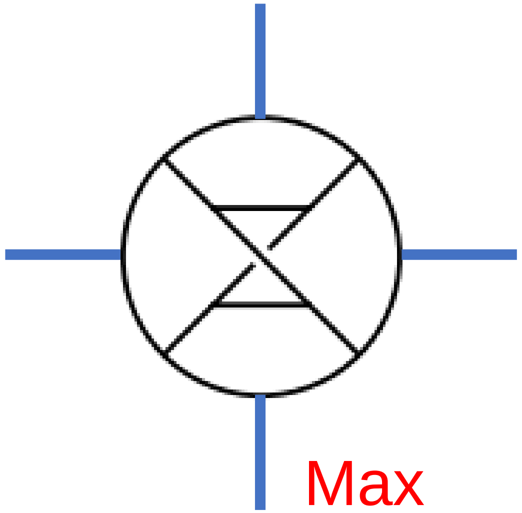
where $P(z_1, z_2, z_3) = 2z_1z_2 + 2z_2z_3 + 2z_3z_1 - z_1^2 - z_2^2 - z_3^2$

contour prescription: $\int \frac{dz}{z^A} \to \oint_{\Gamma_\epsilon(0)} \frac{dz}{z^A}.$ $\quad I_{A,B,C} = \text{Coeff of } z_1^{A-1} z_2^{B-1} z_3^{C-1} \text{ in } P$

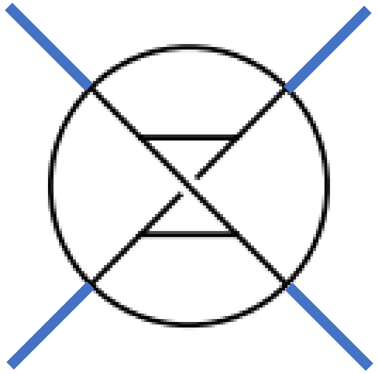
 $+\,2$  $\cong 2 + 2 \cdot (-1) = 0$ ✓ UV finite

Max. cut calculation at 5 loops

1) Vacuum expansion, only from diagrams containing top-level vacuums



Max



up to N^4Max

2) Apply Lorentz invariance (transverse-space integrand reduction)

e.g. talk by Christian Bronnum-Hansen

3) Integration of vacuums on maximal cut

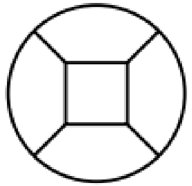
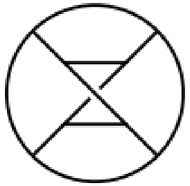
Max. cut calculation at 5 loops

- Analytic integration possible in some cases, e.g. crossed cube topology
  [Bern, Carrasco, Chen, Johansson, Roiban, MZ, 2017]
- Generic cases: *unitarity-compatible integration-by-parts (IBP) reduction*.
  [Gluza, Kajda, Kosower, 2010; Ita 2015; Larsen, Zhang 2015 …] Talks by David Koswer, Vasiliy Sotnikov
- Linear relations between max.-cut vacuum integrals. Solve small linear system of size ~ 500.

$$D = \frac{22}{5}: \ \mathcal{M}_4^{(5)}\Big|_{\text{leading}} \propto \ 0$$

Surprisingly, both are the full results!

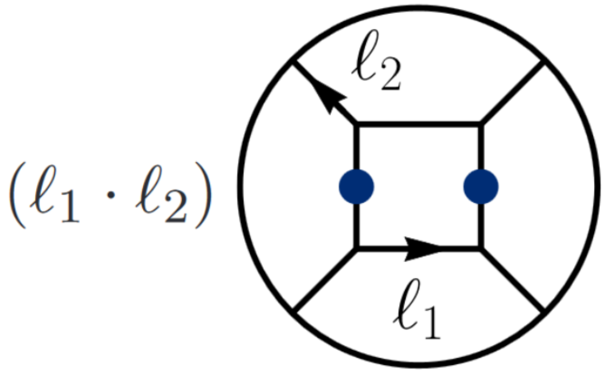
$$D = \frac{24}{5}: \ \mathcal{M}_4^{(5)}\Big|_{\text{leading}} \propto$$

($\frac{1}{48}$  $+ \frac{1}{16}$ )

Full calculation at 5 loops

- Want full results without assumptions.

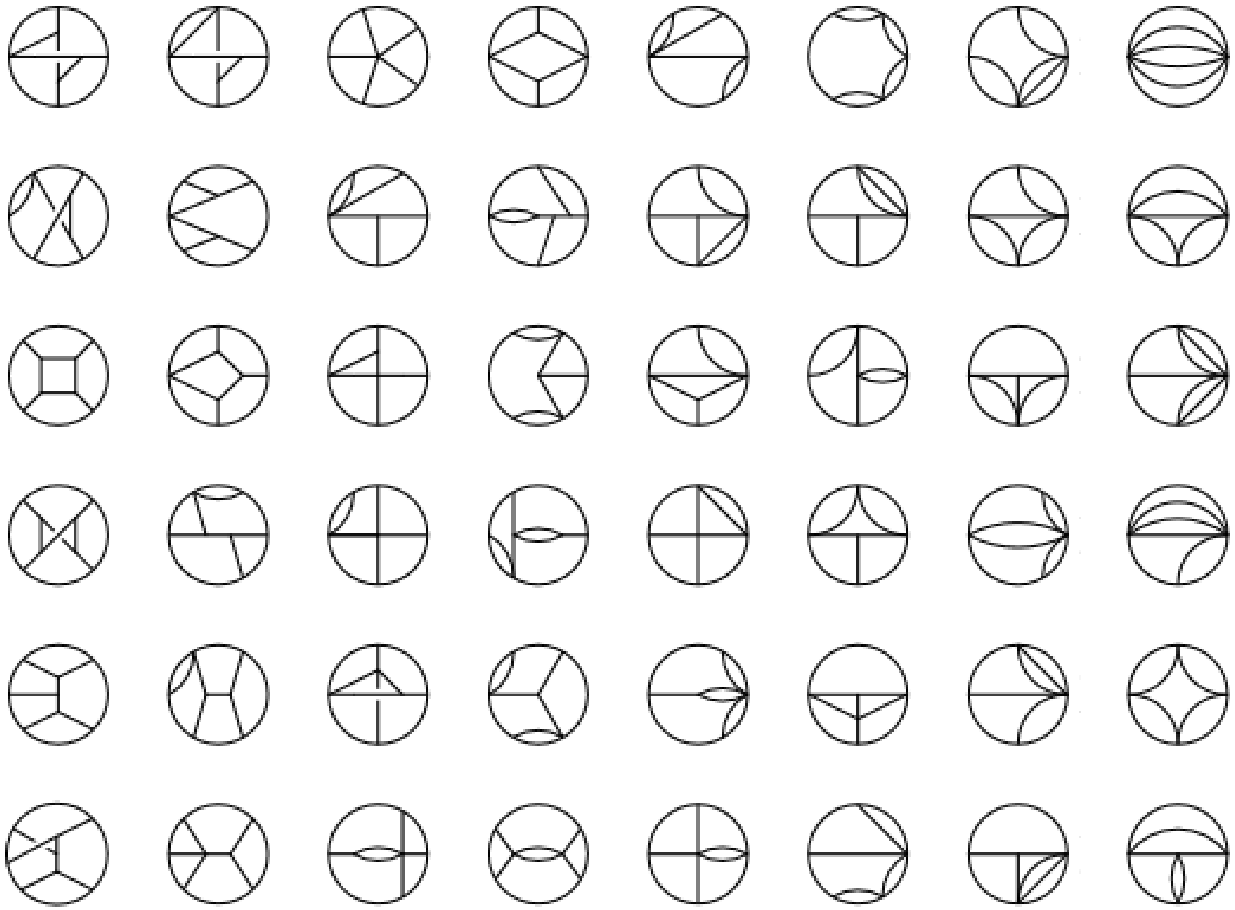
- Old integrand: spurious quartic divergence in $D = 24/5$
  ~ 17 million distinct vacuum integrals (up to **6 dots**).



- Found Improved integrand: log. divergent at top level,
  ~ 140 thousand distinct vacuum integrals (up to **4 dots**).

Five-loop vacuum topologies

[Thomas Luthe 2015 thesis]



QCD beta function:

Baikov, Chetyrkin, Kuhn, 2016

Herzog, Ruijl, Ueda, Vermaseren, Vogt, 2017

Luthe, Maier, Marquard, Schroder, 2017

More propagators ⟷ **Fewer propagators**

Relations from SL(L) relabeling symmetry

- Feynman integrals have linear relations from *integration by parts*

  [Chetyrkin, Tkachev, 1981]

- IR regulators (e.g. internal mass) would cause huge redundancy.
  Solution: no regulator, *d*-indep. IBP relations w/o mixing IR & UV

  [Bern, Carrasco, Dixon, Johansson, Kosower, Roiban 2007
  Bern, Carrasco, Dixon, Johansson, Roiban 2009, 2012]

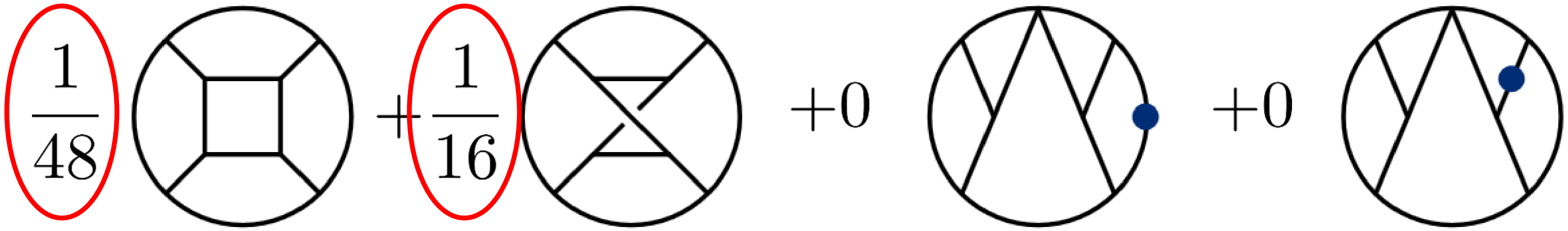
- Simple systematic construction: [Bern, Enciso, Parra-Martinez, MZ, 2017]
  Infinitesimal SL(L) relabeling symmetry $\Delta l_i^\mu = \omega_{ij} l_j^\mu$
  Directly relating log. divergent vacuums

Full calculation results

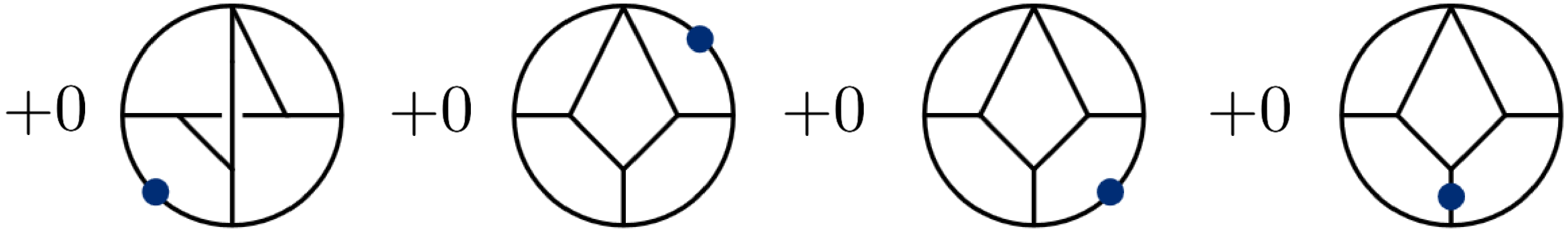
- In $D = 22/5$: UV finite, as expected.

[Integral identification: A. Pak, 2011; J. Hoff, 2016]

- In $D = 24/5$: 2.8 million relations between 0.85 million *canonical* integrals.
  8 master integrals. Linear algebra over finite fields. Software: Finred, Linbox
  [Schabinger, von Manteuffel, 2014; Peraro, 2016; Maierhofer, Usovitsch, Uwer, 2017]
- Summing up diagrams… Cancellation of lower-level coefficients! No triangles.

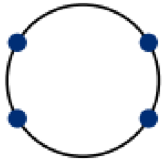


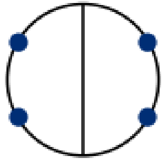
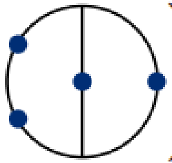
=diagram symmetry factors! $|S_4 \times S_2|$, $|D_8|$

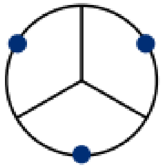
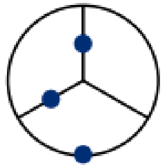


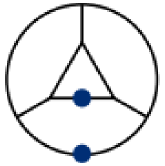
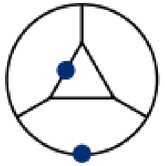
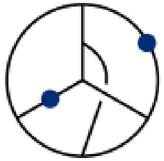
All-loop patterns: $\mathcal{N} = 8$ SUGRA

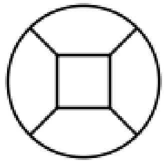
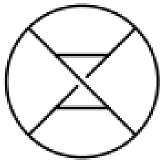
Cross-order relations from removing propagators

$$\mathcal{M}_4^{(1)}\Big|_{\text{leading}} = -3\,\mathcal{K}_{\text{G}}\left(\frac{\kappa}{2}\right)^4$$  ,

$$\mathcal{M}_4^{(2)}\Big|_{\text{leading}} = -8\,\mathcal{K}_{\text{G}}\left(\frac{\kappa}{2}\right)^6 (s^2+t^2+u^2)\left(\frac{1}{4}\right.$$  $$+\frac{1}{4}$$  $\Big)$,

$$\mathcal{M}_4^{(3)}\Big|_{\text{leading}} = -60\,\mathcal{K}_{\text{G}}\left(\frac{\kappa}{2}\right)^8 stu\left(\frac{1}{6}\right.$$  $$+\frac{1}{2}$$  $\Big)$,

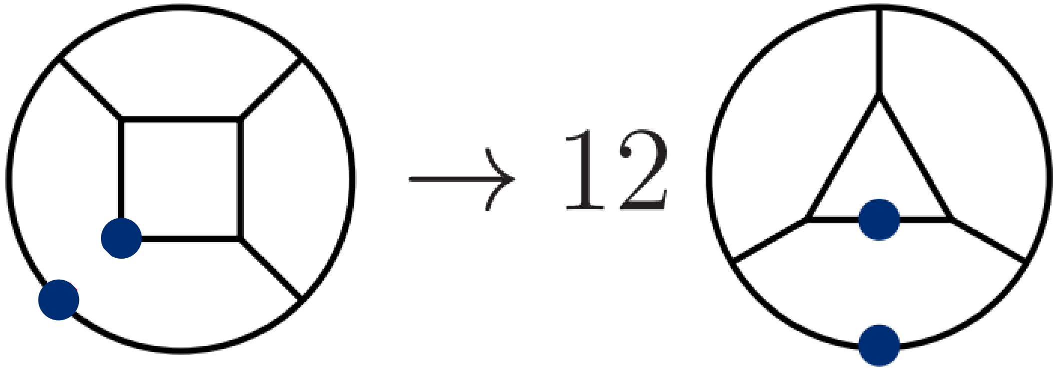
$$\mathcal{M}_4^{(4)}\Big|_{\text{leading}} = -\frac{23}{2}\,\mathcal{K}_{\text{G}}\left(\frac{\kappa}{2}\right)^{10} (s^2+t^2+u^2)^2\left(\frac{1}{4}\right.$$  $$+\frac{1}{2}$$  $$+\frac{1}{4}$$  $\Big)$,

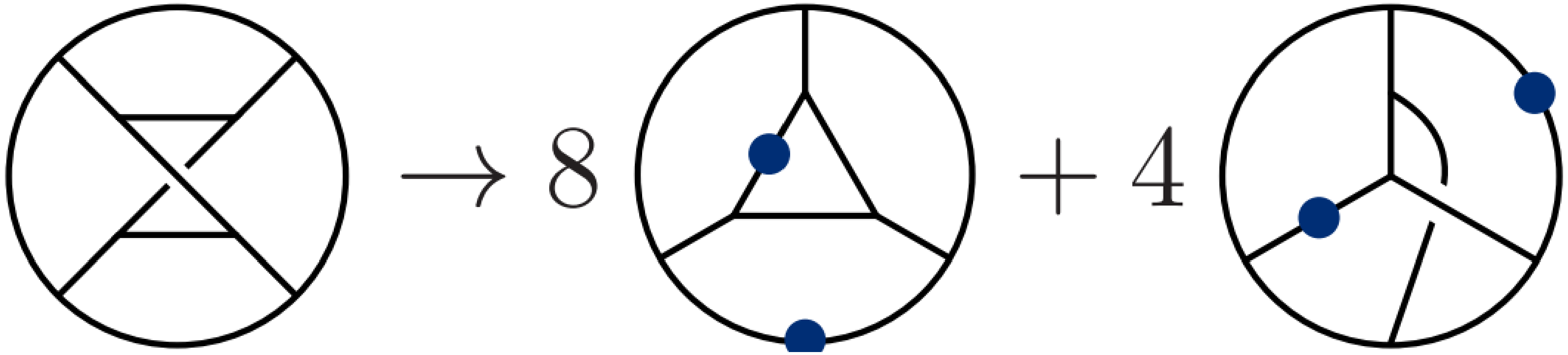
$$\mathcal{M}_4^{(5)}\Big|_{\text{leading}} = -\frac{16\times 629}{25}\,\mathcal{K}_{\text{G}}\left(\frac{\kappa}{2}\right)^{12} (s^2+t^2+u^2)^2\left(\frac{1}{48}\right.$$  $$+\frac{1}{16}$$  $\Big)$,

No triangles + BCJ-like symmetry factors

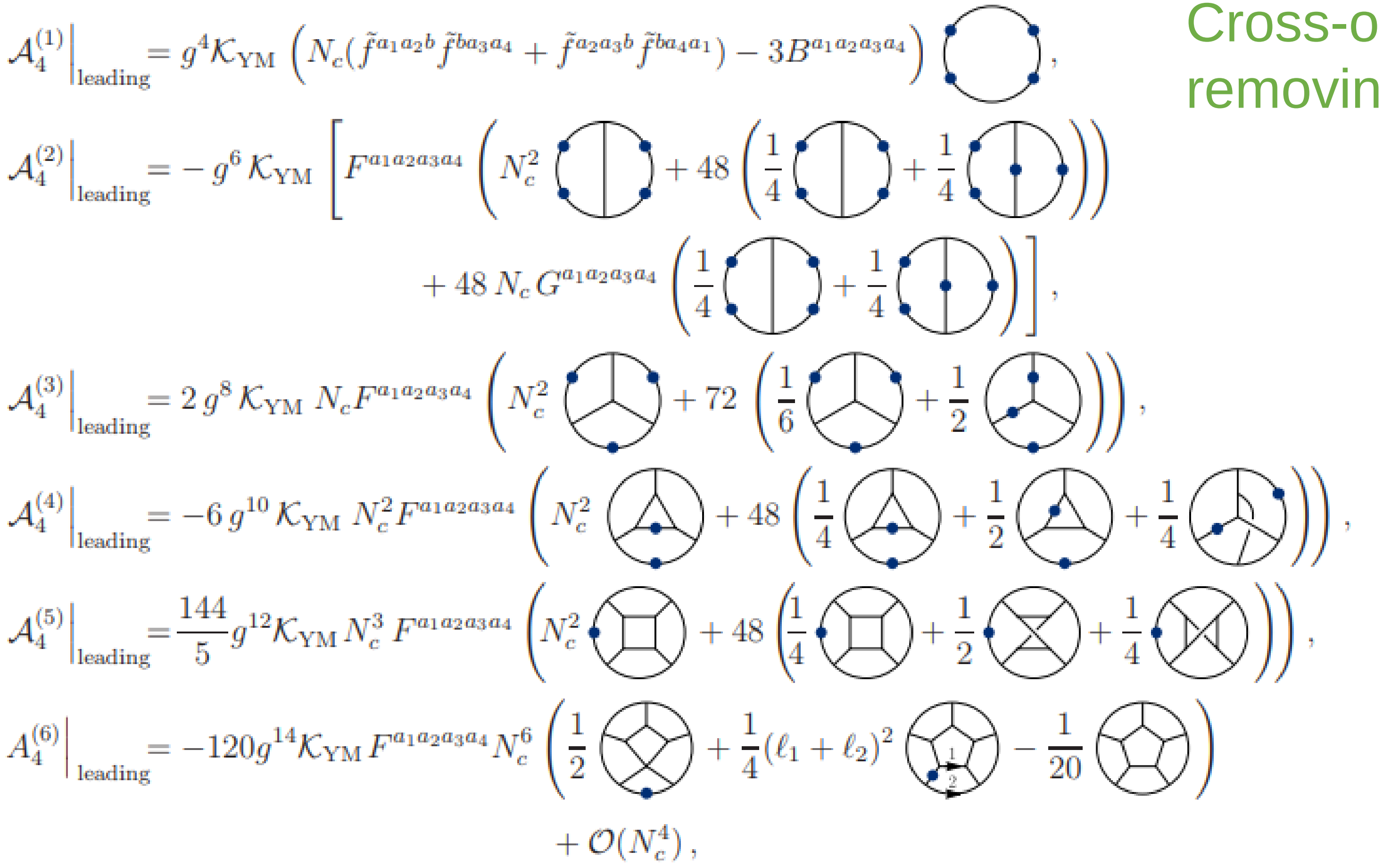
All-loop patterns: $\mathcal{N} = 8$ SUGRA

Cross-order relations from removing propagators





All-loop patterns: $\mathcal{N}=4$ SYM



Cross-order relations from removing propagators

No triangles + BCJ-like symmetry factors

Conclusions

- UV behavior of $\mathcal{N} = 8$ SUGRA worse than $\mathcal{N} = 4$ SYM at 5 loops

- Implication for 4D unclear, but further progress within reach.

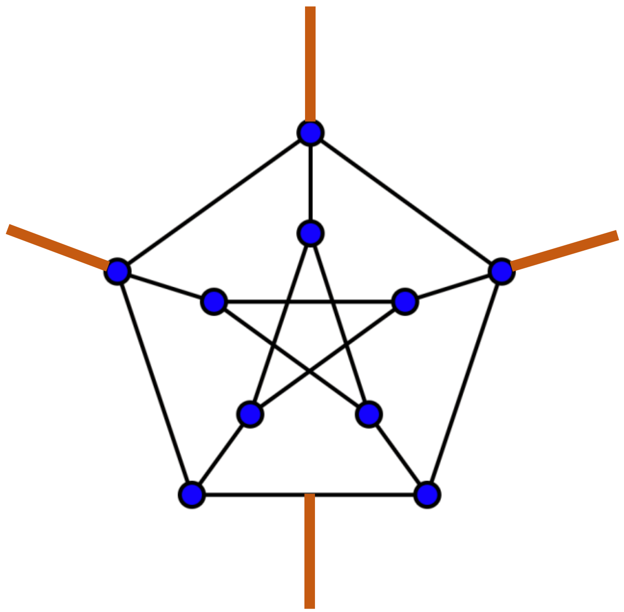
- Simplifications from vacuum cuts suggest paths to higher loops.

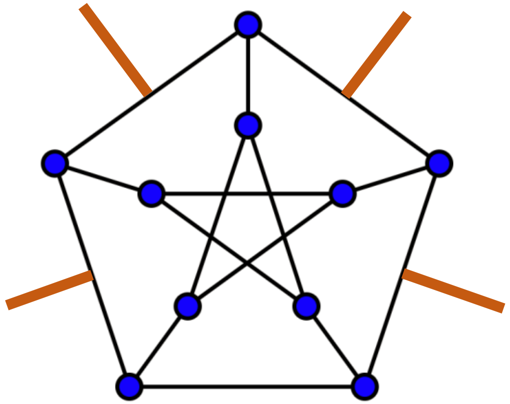
- All-loop patterns: symmetry coefficients & cross-order relations

Thank you!

Outlook – higher loops

- Gauge invariant sub-component of UV divergence (max. cut vacuum coefficients) may be extracted from a subset of 4-point diagrams.





[Center: Peterson graph, Wikipedia]

- Conjectural all-loop patterns constrain results – explore other theories too.