Soft and Coulomb effects in top-quark pair production beyond NNLO

Christian Schwinn

— RWTH Aachen —

03.10.2018

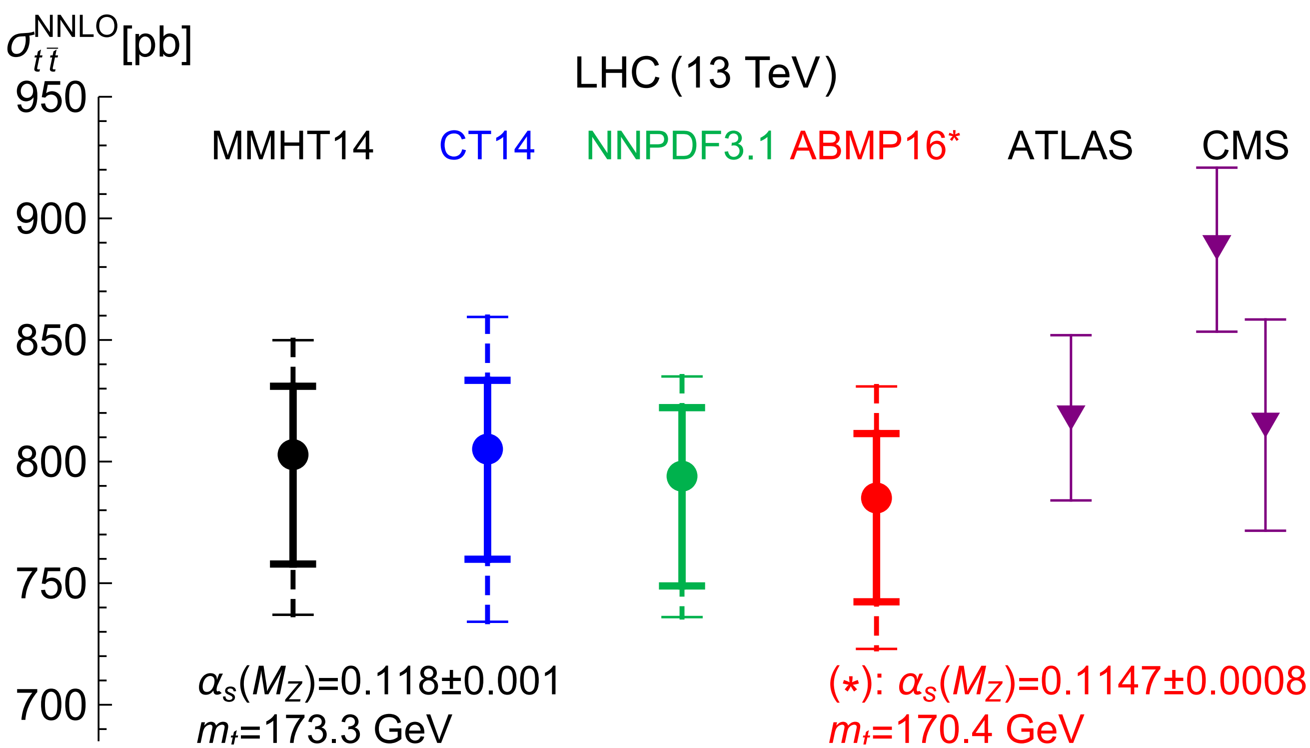
based on

J.Piclum, CS, JHEP 1803 (2018) 164, arXiv:1801.05788 [hep-ph]

Heisenberg-Programm
DFG Deutsche Forschungsgemeinschaft

Total $t\bar{t}$ cross section test of QCD and nature of top-quark:

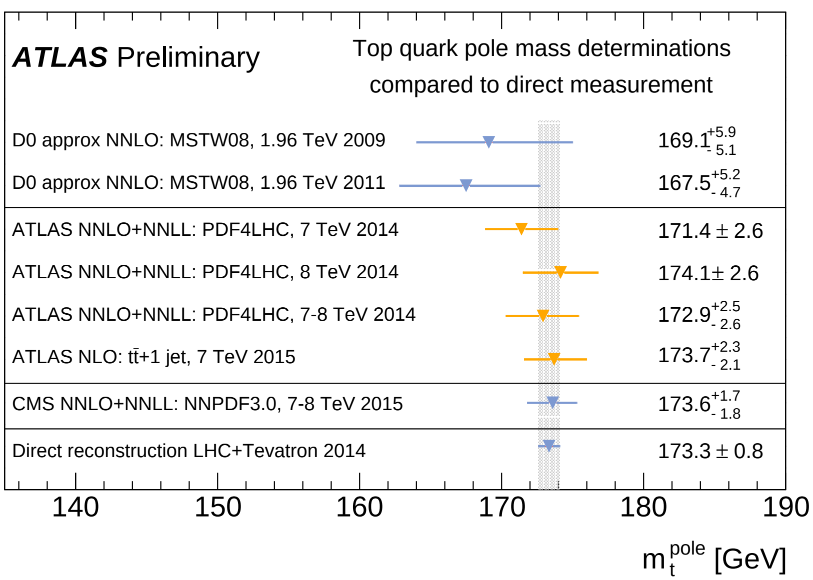
- Experimental precision $3-4\%$ smaller than uncertainty of NNLO QCD prediction (Bärnreuther/Czakon/Fiedler/Mitov 12–13)

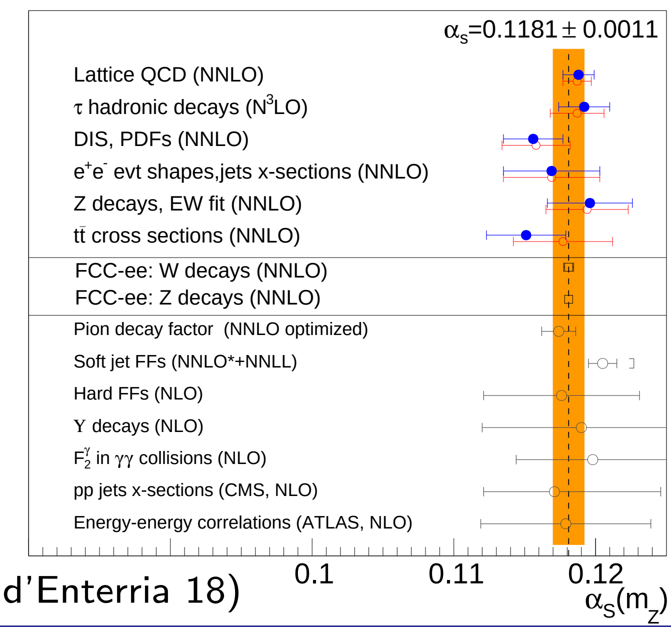


(dashed: 68% confidence-level PDF+α_s uncertainty)

Total $t\bar{t}$ cross section test of QCD and nature of top-quark:

- Experimental precision $3-4\%$ smaller than uncertainty of NNLO QCD (Bärnreuther/Czakon/Fiedler/Mitov 12–13)
- Sensitive to m_t, α_s, PDFs
  - included in MMHT14, NNPDF3.1, ABMP16
  - pole mass $m_t = 173.8^{+1.7}_{-1.8}\mathrm{GeV}$ from σ_{tt} measurement (CMS 16)
  - determination of $\alpha_s(M_Z) = 0.1177^{+0.0034}_{-0.0036}$ (Klijnsma et al. 17)





(d'Enterria 18)

Introduction

Resummation of threshold-enhanced corrections, $\beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}} \to 0$

$$\hat{\sigma}_{pp'} \propto \sigma^{(0)} \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \underbrace{\alpha_s^2 g_3(\alpha_s \ln\beta)}_{\text{(N}^3\text{LL)}} + \ldots\Big]$$

$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \times \Big\{ \underbrace{1}_{\text{(LL,NLL)}} \;;\; \underbrace{\alpha_s, \beta}_{\text{(NNLL)}} \;;\; \underbrace{\alpha_s^2, \alpha_s\beta, \beta^2}_{\text{(NNLL', N}^3\text{LL)}} \;; \ldots \Big\} :$$

Resummation of threshold-enhanced corrections, $\beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}} \to 0$

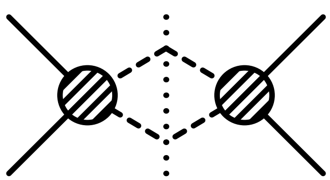
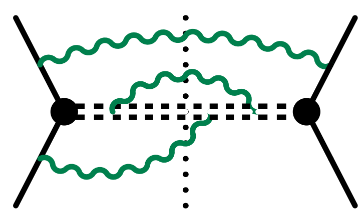
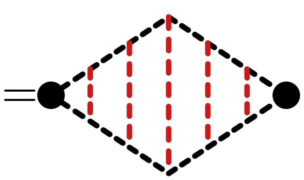
$$\hat{\sigma}_{pp'} \propto \sigma^{(0)} \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \underbrace{\alpha_s^2 g_3(\alpha_s \ln\beta)}_{\text{(N}^3\text{LL)}} + \dots\Big]$$

$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \times \Big\{ \underbrace{1}_{\text{(LL,NLL)}} \;;\; \underbrace{\alpha_s, \beta}_{\text{(NNLL)}} \;;\; \underbrace{\alpha_s^2, \alpha_s\beta, \beta^2}_{\text{(NNLL', N}^3\text{LL)}} \;; \dots \Big\} :$$

Factorization of cross section for $\beta \to 0$ (Beneke, Falgari, CS 09/10)

$$\Rightarrow \hat{\sigma}_{pp' \to t\bar{t}}|_{\hat{s} \to 4m_t^2} = \sum_{R=1,8} H_R(m_t, \mu) \int d\omega\, J_R(\sqrt{\hat{s}} - 2m_t - \frac{\omega}{2}) W^R(\omega, \mu)$$

Hard, soft and Coulomb functions:

$H_R =$  , $W^R =$  , $J^R =$ 

Soft-gluon resummation from evolution equations for H, W;
Coulomb resummation using non.-rel. Schrödinger equation.

NNLL corrections: (13TeV, MMHT2014)

reduced scale uncertainty, estimate of resum. uncertainty?

$$\sigma_{t\bar{t}}^{\rm NNLO} = 802.8^{+28.1(3.5\%)}_{-44.9(5.6\%)}\text{pb} \Rightarrow \begin{cases} \text{NNLL(top++)}: & 821.4^{+20.3(2.5\%)}_{-29.6(3.6\%)}\text{pb} \\ \text{NNLL(topixs)}: & 807.1\underbrace{{}^{+15.6(1.9\%)}_{-36.8(4.6\%)}}_{\text{scale}}\underbrace{{}^{+19.2(2.5\%)}_{-12.9(1.8\%)}}_{\text{resum}}\text{pb} \end{cases}$$

`top++`: Mellin-space resummation of threshold logarithms

(Czakon/Mitov/Sterman 09/Cacciari et al. 11)

`topixs:` momentum-space resummation of threshold logs combined with Coulomb corrections α_s/β (Beneke/Falgari/(Klein)/CS 09/11)

Main numerical differences:

- α_s^2 hard coefficient in `top++`: (NNLL'): $\Delta\sigma \approx 9\text{pb}$
- bound-state effects in `topixs`: $\Delta\sigma_{\rm BS} \approx 3\text{pb}$

NNLL corrections: (13TeV, MMHT2014)

reduced scale uncertainty, estimate of resum. uncertainty?

$$\sigma_{t\bar{t}}^{\text{NNLO}} = 802.8^{+28.1(3.5\%)}_{-44.9(5.6\%)}\text{pb} \Rightarrow \begin{cases} \text{NNLL(top++)}: & 821.4^{+20.3(2.5\%)}_{-29.6(3.6\%)}\text{pb} \\ \text{NNLL(topixs)}: & 807.1 \underbrace{{}^{+15.6(1.9\%)}_{-36.8(4.6\%)}}_{\text{scale}} \underbrace{{}^{+19.2(2.5\%)}_{-12.9(1.8\%)}}_{\text{resum}} \text{pb} \end{cases}$$

`top++`: Mellin-space resummation of threshold logarithms

(Czakon/Mitov/Sterman 09/Cacciari et al. 11)

`topixs`: momentum-space resummation of threshold logs combined with Coulomb corrections α_s/β (Beneke/Falgari/(Klein)/CS 09/11)

$\Rightarrow$ **Upgrade** `topixs` to partial N^{3}LL

- First step: expansion to N^{3}LO
- estimate of unknown N^{3}LL ingredients

$\Rightarrow$ complementary estimate of higher-order corrections

Joint soft/Coulomb resummation for $\alpha_s \log\beta \sim 1$, $\frac{\alpha_s}{\beta} \sim 1$

- interplay of Coulomb $(\alpha_s/\beta)^n$ and power corrections $\sim \beta^l$
- logarithmic NNLO contributions from $\alpha_s\beta$ potentials
- no $\alpha_s/\beta \times \alpha_s \log^{2,1}\beta \times \beta$ corrections
  to soft NNLL resummation for σ_{tot}, $d\sigma/dM_{t\bar{t}'}$ (Beneke/Falgari/CS 10)
- Known corrections relevant for N³LO threshold expansion

$$\frac{\alpha_s^2}{\beta^2} \times \alpha_s\beta^2 \log\beta \sim \alpha_s^3 \log\beta$$

  – "next-to-eikonal" effects in DY/Higgs

  (Krämer/Laenen/Spira 98; Laenen et al. 10)

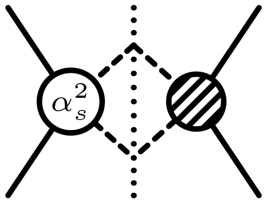
  – (ultra)-soft corrections as in $e^-e^+ \to t\bar{t}$ (Beneke/Kiyo 08)

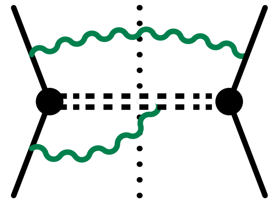
- systematic treatment: extended factorization

$$\sigma = \sum_{ijklm} B_1^{(i)} B_2^{(j)} H^{(k)} \otimes W^{(l)} J^{(m)}$$

(LL resummation for DY: Beneke et al. 18)

Input to resummation formula at N³LL

$H_i^{(2)} =$ 

, $W_i^{(2)R} =$ 
, $J^R_{\rm NNLO} =$ 

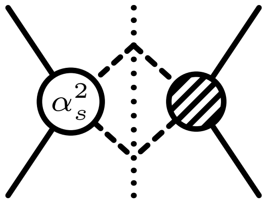
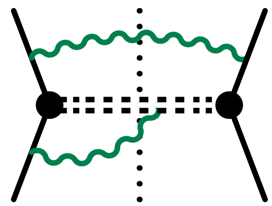
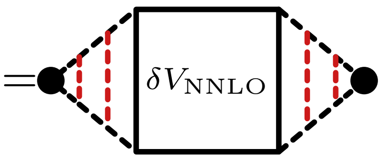
Hard function

- NNLL: one-loop H_i (Czakon/Mitov 08; also Hagiwara et.al. 08)
- NNLL'/N³LL: two-loop H_i
  from constant in NNLO threshold expansion
  (Bärnreuther/Czakon/Fiedler 13)

Soft function

- NNLL: 1-loop soft function for arbitrary R (Beneke/Falgari/CS 09)
- NNLL'/N³LL: 2-loop soft function for singlet/octet
  (Belitzky 98;Becher/Neubert/Xu 07; Czakon/Fiedler 13)

Input to resummation formula at N³LL

$H_i^{(2)} =$  , $W_i^{(2)R} =$  , $J^R_{\rm NNLO} =$ 

RGEs

- known for N³LL:
  - 4-loop $\gamma_{\rm cusp}$ (Moch et al. 17/18; not needed for $\rm N^3LO_{app}$)
  - anomalous dimensions extracted from 3-loop splitting functions and quark and gluon form factors (Moch/Vermaseren/Vogt 04/05)
- missing:
  - 3-loop massive soft anomalous dimension $\gamma_{H,s}^{(2)}$ (Massless result: Almelid/Duhr/Gardi 15)
  - NNLL resummation in (p)NRQCD for colour octet

Potential function

PNRQCD colour-and spin projected potential includes "non-Coulomb" potentials suppressed by $\alpha_s \frac{|\boldsymbol{q}|}{M}$, $\frac{\boldsymbol{q}^2}{M^2}$

$$V^{R,S}(\mathbf{p},\mathbf{p}') = \frac{4\pi\alpha_s D_R}{\mathbf{q}^2}\left[\mathcal{V}_C^R - \mathcal{V}_{1/m}^R \frac{\pi^2\,|\mathbf{q}|}{m_t} + \mathcal{V}_{1/m^2}^{R,S}\frac{\mathbf{q}^2}{m_t^2} + \mathcal{V}_p^R \frac{\mathbf{p}^2+\mathbf{p}'^2}{2m_t^2}\right] + \frac{\pi\alpha_s}{m_t^2}\nu_{\rm ann}^{R,S},$$

($R = 1,8,\ S = 1,3,$ Coulomb coefficients $D_1 = -C_F;\ D_8 = \frac{1}{2}(C_A - 2C_F) = \frac{1}{2N_C}$)

Known for colour singlet and octet:

- Two-loop Coulomb $\mathcal{V}_C^R$ (singlet: Schröder 98; octet: Kniehl et al. 04)
- One-loop spin-dependent $\mathcal{V}_{1/m^2}^{R,S}$
  (Wüster 03; colour-singlet: Beneke/Kiyo/Schuller 13, colour octet: Piclum/CS 18)
- One-loop annihilation contributions $\nu_{\rm ann}^{R,S}$ (Pineda/Soto 98)

Unknown for octet

- Two-loop $\mathcal{V}_{1/m}^R$ (singlet: Kniehl et al. 01)
  estimate by naive replacement $C_F \to (C_F - C_A/2)$

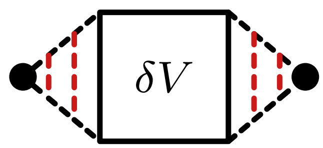
Resummation of $\frac{\alpha_s}{\beta}$ corrections: (Fadin, Khoze 87; Peskin, Strassler 90)

solve NR-Schrödinger equation for Green's function

$$-\left(\frac{\vec{\partial}_r^{\,2}}{2m_r}+E\right)G_R^{(0)}(E,\vec{r},\vec{r}\,')-\frac{\alpha_s D_R}{r}G_R^{(0)}(E,\vec{r},\vec{r}\,')=(2\pi)^3\delta^3(\vec{r}-\vec{r}\,')$$

NLO potential function from perturbation theory

$\delta G_R^{(1)}(0,0,E) =$



$$= \int d^3z\, G_R^{(0)}(0,\vec{z},E)\,(i\delta V^R(\vec{z}))\, iG_R^{(0)}(\vec{z},0,E)$$

- all terms $\alpha_s(\alpha_s/\beta)^n$

NNLO Green function: (using implementation of Beneke/Kiyo/Maier/Piclum 16)

- double/single insertions of NLO/NNLO potentials
- expansion to $\mathcal{O}(\alpha_s^3)$: (for colour-singlet agreement with Kiyo et al. 09)

$$\Delta J_R^{S(3)} \sim \alpha_s^3\left\{\frac{1}{\beta^2}\ln\left(\frac{\beta m_t}{\mu}\right),\frac{1}{\beta^2},\frac{1}{\beta}\ln^2\left(\frac{\beta m_t}{\mu}\right),\frac{1}{\beta}\ln\left(\frac{\beta m_t}{\mu}\right),\frac{1}{\beta}\right\}$$

Potential function

N³LO Green function: (using implementation of Beneke/Kiyo/Maier/Piclum 16)

contributions relevant at N³LL:

$$\Delta J^{S(3)}_{R,\mathrm{N^3LO}} \sim \alpha_s^3 \left\{ \ln^2\left(\frac{\beta m_t}{\mu}\right), \ln\left(\frac{\beta m_t}{\mu}\right)\ln\left(\frac{m_t}{\mu}\right), \ln\left(\frac{\beta m_t}{\mu}\right) \right\}$$

- colour singlet completely known;
  scale-dependence reproduces known results (Kniehl et al. 02)
- colour octet: not completely known
  - two-loop $1/m$-potential
  - (ultra)-soft corrections (singlet: Beneke/Kiyo 08)
    with chromoelectric vertex $\psi^\dagger \vec{x} \cdot \vec{E}_{us} \psi'^\dagger$

⇒ unknown contributions to cross section

$$\alpha_s^3(\delta c^{(2,0)}_{J,3} \ln^2 \beta + \delta c^{(1,0)}_{J,3} \ln \beta + \dots)$$

- Estimate $\delta c^{(i,0)}_{J,3}$ for octet by naive replacement $C_F \to (C_F - C_A/2)$

- No 3-loop Coulomb correction $\sim \alpha_s^3/\beta^3$ for $\Gamma_t \to 0$

  Careful treatment in distributional sense: (Beneke/Ruiz-Femenia 16)

$$\Delta J_{R,\mathrm{LO}}^{S(3)}(E) = -\alpha_s^3 D_R^3 \frac{m_t^3}{8} \zeta_3\, \delta(E)$$

  Small correction to cross section: $\Delta\sigma = 0.6\,\mathrm{pb}$ at 13 TeV.

- P-wave contributions $\sigma_{gg}^{(0)}((t\bar{t})^P) \sim \beta^3$

  Coulomb corrections different from S-wave (Bigi/Fadin/Khoze 92)

  $\Rightarrow$ contributions $\sim \alpha_s^2 \times$ const., $\sim \frac{\alpha_s^3}{\beta}$ relative to leading S-wave

- Sub-leading soft corrections to DY/Higgs production:

  (Krämer/Laenen/Spira 96; Laenen et al. 10)

$$\left[\frac{\ln(1-x)}{1-x}\right]_+ \to \left[\frac{\ln(1-x)}{1-x}\right]_+ - \ln(1-x)$$

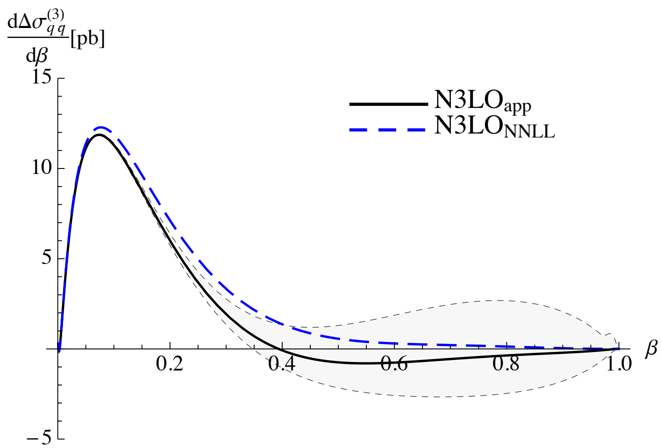
  enhancement by second Coulomb correction $\Rightarrow \sim \alpha_s^3 \ln\beta$ effect

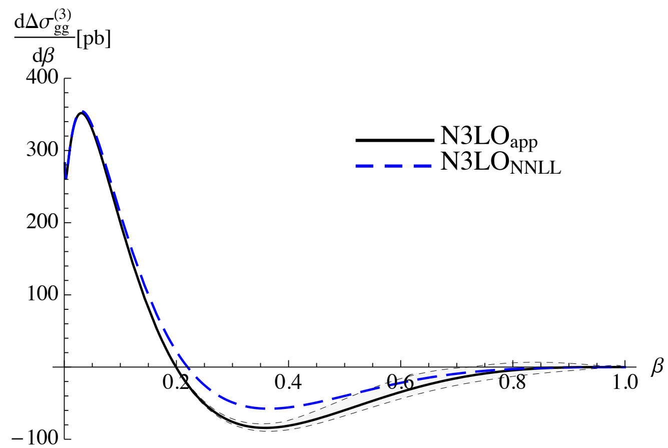
  Numerical effect $0.9\,\mathrm{pb}$ at 13 TeV

Partonic cross sections

Terms predicted by expansion to N³LL

$$\Delta\sigma^{(3)}_{gg_8,\mathrm{N^3LL}} = \Delta\sigma^{(3)}_{gg_8,\mathrm{NNLL}} + \sigma^{(0)}_{gg_8}\frac{\alpha_s^3}{(4\pi)^3}\Big\{(-298530 + 157.914\,\delta c^{(2,0)}_{J,3})\ln^2\beta$$
$$+ (48175.5 + 12\gamma^{(2)}_{H,s} + 157.914\,\delta c^{(1,0)}_{J,3})\ln\beta - \frac{2775.05}{\beta} + C^{(3)}_{gg(8)}\Big\}$$



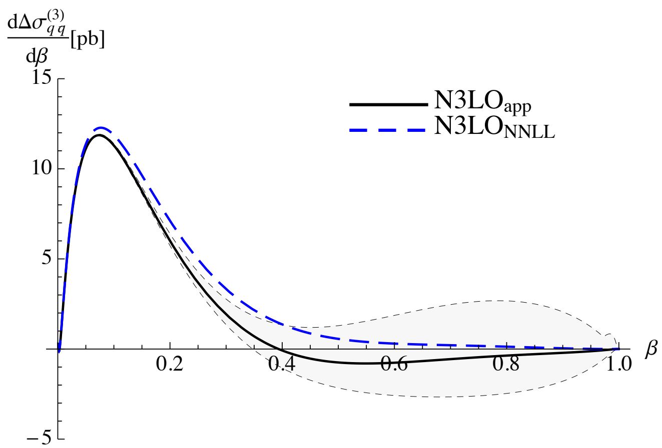


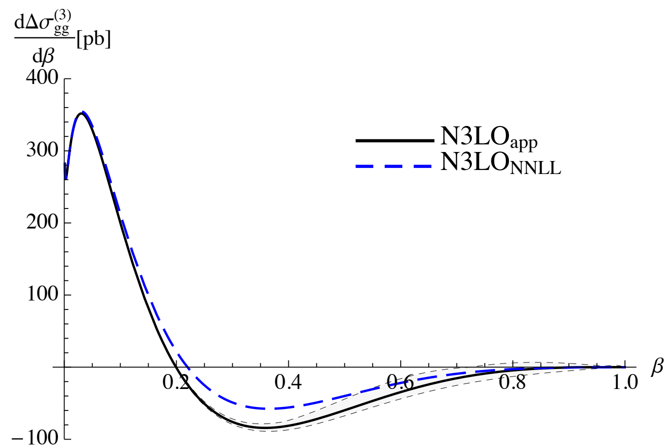
Estimate of uncertainty:

- Variation of $\delta c^{(i,0)}_{J,3}$ by ± 2; estimate $\gamma^{(2)}_{H,s} = \pm(\gamma^{(1)}_{H,s})^2/\gamma^{(0)}_{H,s}$
- Estimate of constants $C^{(3)}(\mu_h, \mu_s)$ by scale variation
- Expansion in $v = \sqrt{\sqrt{\hat{s}}/m_t - 2} = \beta\left(1 + \frac{3}{8}\beta^2 + \ldots\right)$ instead of β

Terms predicted by expansion to N³LL

$$\Delta\sigma^{(3)}_{gg_8,\mathrm{N^3LL}} = \Delta\sigma^{(3)}_{gg_8,\mathrm{NNLL}} + \sigma^{(0)}_{gg_8}\frac{\alpha_s^3}{(4\pi)^3}\Big\{(-298530 + 157.914\,\delta c^{(2,0)}_{J,3})\ln^2\beta + (48175.5 + 12\gamma^{(2)}_{H,s} + 157.914\,\delta c^{(1,0)}_{J,3})\ln\beta - \frac{2775.05}{\beta} + C^{(3)}_{gg(8)}\Big\}$$





Corrections to cross-section $+1.6\%$ relative to NNLO (MMHT14 PDFs)

$$\Delta\sigma^{\mathrm{N^3LO_{app}}}_{t\bar{t}}(13\mathrm{TeV}) = 12.25\underbrace{{}^{+7.87}_{-6.24}}_{C^{(3)}_{pp'}}\underbrace{{}^{+5.3}_{-0.0}}_{\mathrm{kin.}}\pm\underbrace{0.11}_{\gamma^{(2)}_{H,s}}\pm\underbrace{0.60}_{\delta c^{(i,0)}_{J,3}}\,\mathrm{pb},$$

Scale-dependence of approximate N³LO cross section

$$\hat{\sigma}^{(3),\text{app}}_{pp',R}(\beta,\mu_f) = \hat{\sigma}^{(0)}_{pp',R}\left(\frac{\alpha_s(\mu_f)}{4\pi}\right)^3 \sum_{m=0}^{3} f^{(3,m)}_{pp'(R)} \ln^m\left(\frac{\mu_f}{m_t}\right)$$

obtained in two ways:

- Expansion of resummation formula
- Direct computation using Altarelli-Parisi equations

Convolution of scaling functions $g^{(n,m)}_{pp'}(\rho) = \beta f^{(n,m)}_{pp'}(\rho)$
with $x \to 1$ limit of splitting functions
$P_{p/\tilde{p}}(x) \approx \left(2\Gamma^r_{\text{cusp}}(\alpha_s)\frac{1}{[1-x]_+} + 2\gamma^{\phi,r}(\alpha_s)\delta(1-x)\right)\delta_{p\tilde{p}}$

$$\begin{aligned}
g^{(3,3)}_{pp} &= \frac{1}{3}\left[8\beta_0\, g^{(2,2)}_{pp} - 2g^{(2,2)}_{pp} \otimes P^{(0)}_{p/p}\right], \\
g^{(3,2)}_{pp} &= 4\beta_0\, g^{(2,1)}_{pp} + 3\beta_1\, g^{(1,1)}_{pp} - g^{(2,1)}_{pp} \otimes P^{(0)}_{p/p} - g^{(1,1)}_{pp} \otimes P^{(1)}_{p/p}, \\
g^{(3,1)}_{pp} &= 8\beta_0\, g^{(2,0)}_{pp} + 6\beta_1\, g^{(1,0)}_{pp} + 4\beta_2\, g^{(0,0)}_{pp} \\
&\quad - g^{(2,0)}_{pp} \otimes P^{(0)}_{p/p} - g^{(1,0)}_{pp} \otimes P^{(1)}_{p/p} - g^{(0,0)}_{pp} \otimes P^{(2)}_{p/p},
\end{aligned}$$

Factorization scale dependence

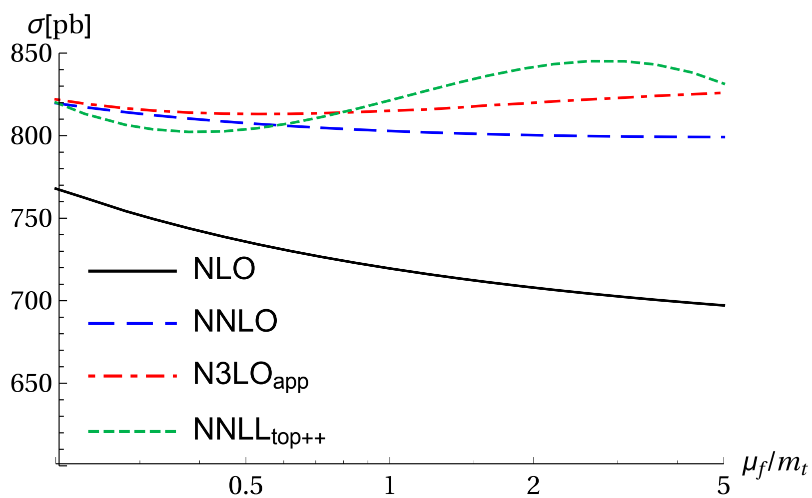
Scale-dependence of approximate N³LO cross section

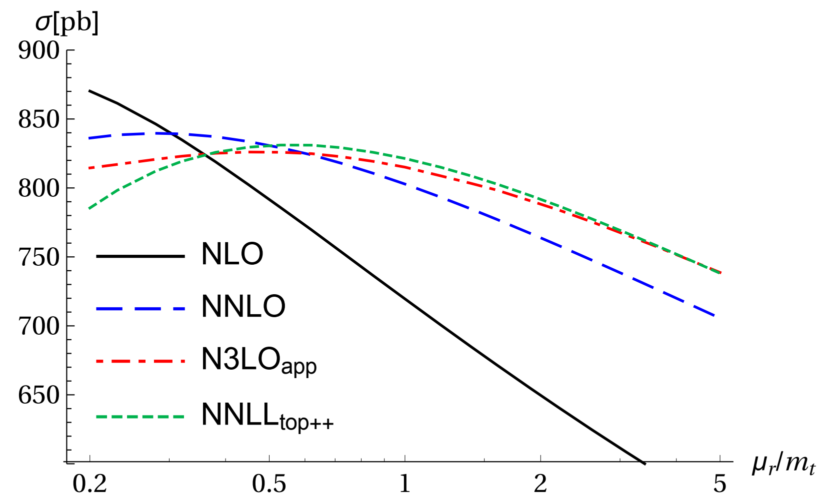
$$\hat{\sigma}^{(3),\text{app}}_{pp',R}(\beta,\mu_f) = \hat{\sigma}^{(0)}_{pp',R}\left(\frac{\alpha_s(\mu_f)}{4\pi}\right)^3 \sum_{m=0}^{3} f^{(3,m)}_{pp'(R)} \ln^m\left(\frac{\mu_f}{m_t}\right)$$

obtained in two ways:

- Expansion of resummation formula
- Direct computation using Altarelli-Parisi equations

Scale dependence similar to NNLL from `top++`



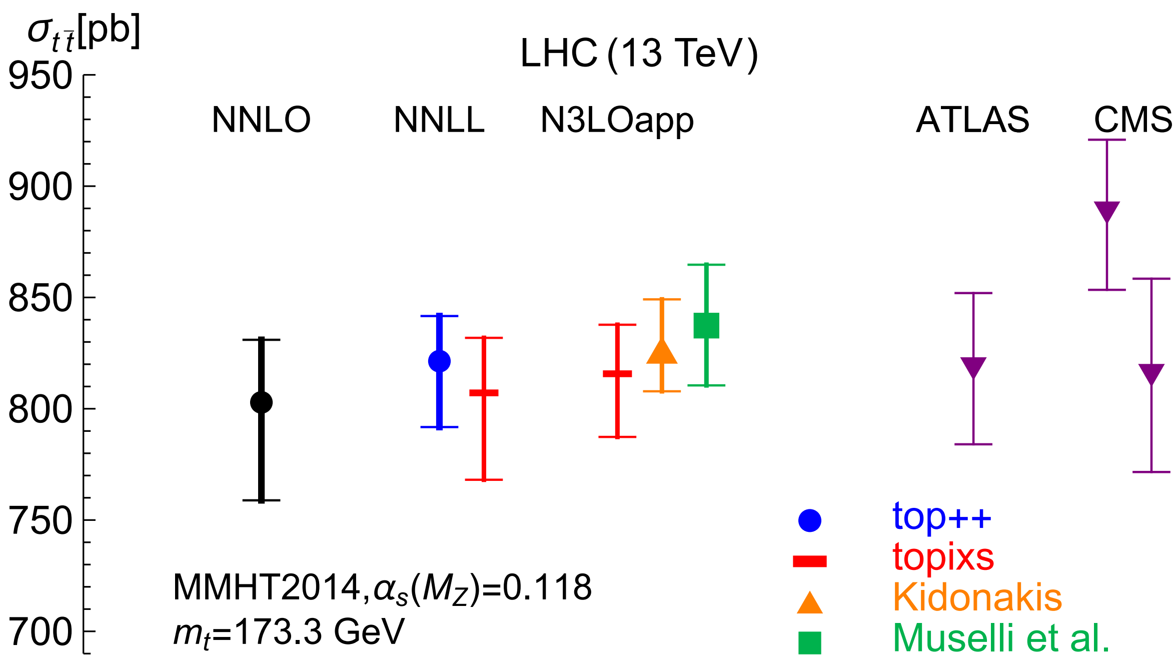


Final prediction ("approx" uncertainties added in quadrature, $\Delta\alpha_s = 0.002$)

$$\sigma_{t\bar{t}}^{\mathrm{N^3LO_{app}}}(13\mathrm{TeV}) = 815.70^{+19.88(2.4\%)}_{-27.69(3.4\%)}(\mathrm{scale})^{+9.49(1.2\%)}_{-6.27(0.8\%)}(\mathrm{approx})^{+42.67(5.2\%)}_{-30.37(3.7\%)}(\mathrm{PDF}+\alpha_s)\mathrm{pb}$$

Other approximate N^{3}LO predictions:

- NNLL in one-particle inclusive kinematics (Kidonakis 14)
- Including subleading collinear; $\beta \to 1$ terms, (Muselli et al. 15) includes $\pm 1.9\%$ approx. uncertainty



- **partial N^{3}LL** soft/Coulomb resummation
  - unknown: 3-loop massive soft anomalous dimension, logarithmic terms in N^{3}LO Coulomb Green function
  - kinematically suppressed contributions enter $\alpha_s^3 \ln^{2,1}\beta$ terms

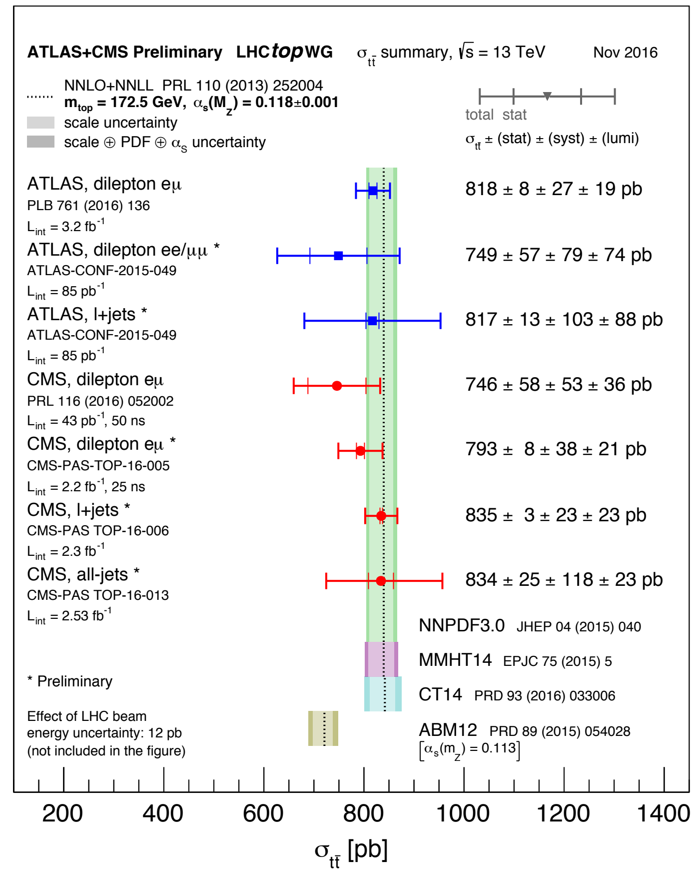
    (P-wave contributions, next-to-eikonal corrections, ultrasoft potential corrections)
- **N^{3}LO$_{\rm app}$ results**
  - complementary to NNLL resummation

    (includes input beyond NNLL; estimate uncertainty due to unknown input)
  - moderate correction $\sim 1.6\%$ compared to NNLO
  - smaller than other N^{3}LO$_{\rm app}$ predictions but consistent within $1-2\%$ systematic uncertainties of approximations
  - available at `http://users.ph.tum.de/t31software/topixs/`
- **Outlook**
  - implement resummed N^{3}LL$_{\rm part}$ prediction.

Fixed-order prediction in QCD (Bärnreuther/Czakon/Fiedler/Mitov 12–13)

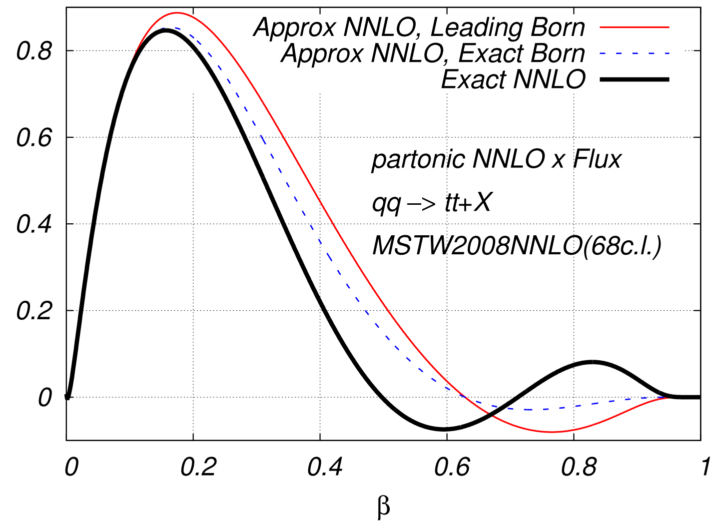
$$\sigma_{t\bar{t}}^{\rm NNLO}(13{\rm TeV}) = \begin{cases} 802.85^{+28.12\,+42.03}_{-44.97\,-29.15}\,{\rm pb} & \text{MMHT2014} \\ 805.14^{+28.28\,+46.01}_{-45.29\,-45.35}\,{\rm pb} & \text{CT14} \\ 794.00^{+28.18\,+17.13}_{-45.13\,-17.35}\,{\rm pb} & \text{NNPDF3.1} \\ 785.02\underbrace{{}^{+26.50}_{-42.68}}_{\rm scale}\underbrace{{}^{+19.37}_{-19.37}}_{{\rm PDF}+\alpha_s}{\rm pb} & \text{ABMP16} \end{cases}$$

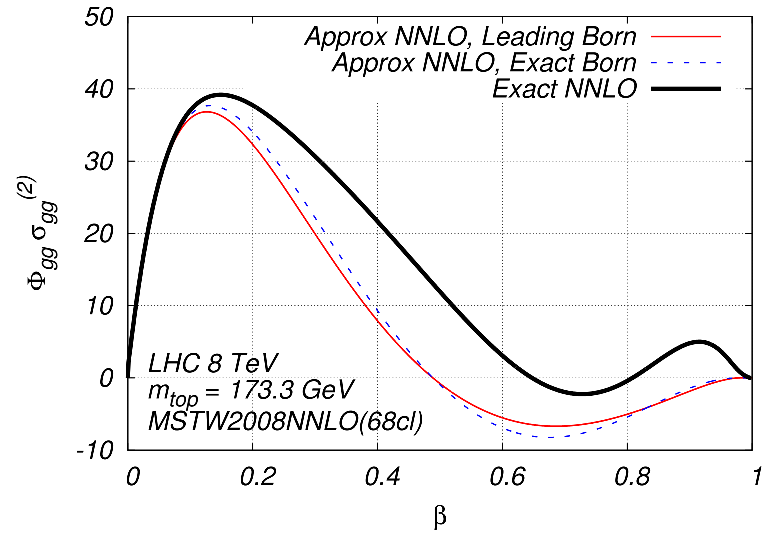
$m_t = 173.3\,{\rm GeV},\ \alpha_s(M_Z) = 0.118 \pm 0.002;$
ABPM16: $m_t = 170.4\,{\rm GeV},\ \alpha_s(M_Z) = 0.1147 \pm 0.0008$



- $\sigma_{t\bar{t}}$ included in PDF fits
- Scale uncertainty $\sim 5\% \gtrsim$ PDF+α_s uncertainty
- Experimental uncertainty reaches $\sim 3-4\%$

- Top-pair production dominated by $\beta \sim 0.6$
  $\Rightarrow$ justification of threshold approximation?





$\frac{d\sigma}{d\beta} = \frac{8\beta m_t^2}{s(1-\beta^2)^2} L(\beta,\mu_f)\hat{\sigma}$, (Bärnreuther/Czakon/Mitov 12; Czakon/Fiedler/Mitov 13)

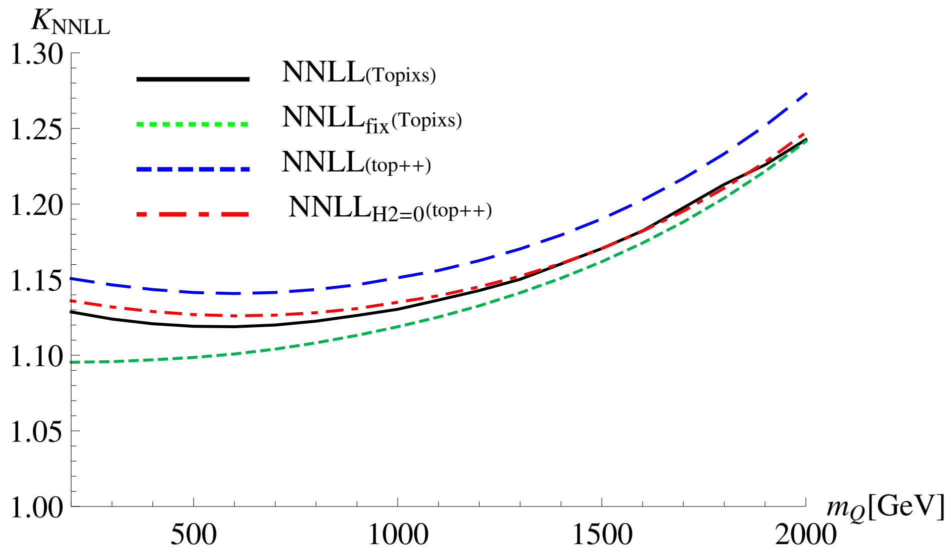
$\Rightarrow$ threshold corrections give estimate of higher-order corrections

$\Rightarrow$ careful estimate of uncertainties necessary

- resummation not mandatory for $t\bar{t}$ production at LHC

$\Rightarrow$ compare resummed results to fixed-order expansions

Heavy Quarks as test case for resummation methods



($K_{\text{NNLL}} = \sigma^{\text{NNLL}}/\sigma^{\text{NLO}}$, LHC $\sqrt{s} = 8$ TeV)

NNLL: momentum-space, running $\mu_s = 2m_Q\beta^2$ (Topixs default)

NNLL$_{\text{fix}}$: momentum-space, fixed μ_s (Topixs)

NNLL (top++): Mellin-space (Cacciari et al. 11; Czakon/Mitov 11-13)

NNLL$_{H_2=0}$ (top++): Mellin-space, two-loop constant term set to zero

$\Rightarrow$ resummation methods agree well for larger masses

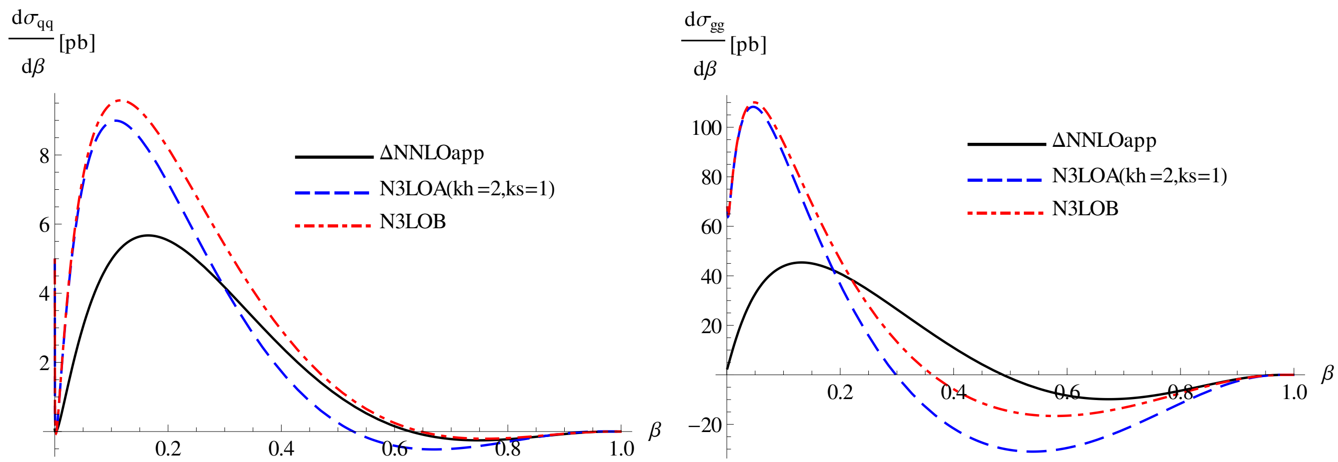
- differences at m_t: estimate of resummation ambiguities
- main difference: treatment of $H_2 \Rightarrow \alpha_s^3 \log \beta^2$ terms (NNLL')

Expand NNLL to $\mathcal{O}(\alpha_s^3)$, e.g. (Beneke/Falgari/Klein/CS 11)

$$\Delta\sigma^{(3)}_{gg_8,\text{NNLL}} = \sigma^{(0)}_{gg_8}\frac{\alpha_s^3}{(4\pi)^3}\Big\{147456.\ln^6\beta - 169658.\ln^5\beta - 140834.\ln^4\beta + 524210.\ln^3\beta + \frac{1}{\beta}\left[-15159.7\ln^4\beta - 5364.82\ln^3\beta + 19598.9\ln^2\beta - 17054.7\ln\beta\right] + \frac{1}{\beta^2}\left[346.343\ln^2\beta + 522.978\ln\beta - 71.7884\right]\Big\} + \underbrace{\left\{\log\beta^{1,2}, 1/\beta, C^{(3)}\right\}}_{\text{not known exactly}} + \text{scale dep.}$$

N³LO$_A$: keep all terms, including μ_s, μ_h-dependence and constants

N³LO$_B$: only keep terms known exactly



P-wave processes

P-wave contributions to $gg \to t\bar{t}$ in $R = 1, 8_s$ colour representations:

$$\sigma^{R(0)}(gg \to (t\bar{t})^{^3P_0}) = \sigma^{R(0)}(gg \to (t\bar{t})^{^1S_0})\ \beta^2,$$

$$\sigma^{R(0)}(gg \to (t\bar{t})^{^3P_2}) = \sigma^{R(0)}(gg \to (t\bar{t})^{^1S_0})\ \frac{4}{3}\beta^2.$$

LO-Coulomb Green function for P-waves: (Bigi/Fadin/Khoze 92)

$$\begin{aligned} J_R^P(E) &= m_t E\left(1 + \frac{(\alpha_s D_R)^2 m_t}{4E}\right) J_R(E) \\ &= m_t^4\left(\frac{E}{m_t}\right)^{3/2}\left[1 + \frac{\alpha_s(-D_R)}{2}\sqrt{\frac{m_t}{E}} + \frac{\alpha_s^2 D_R^2 (3+\pi^2)}{12}\frac{m_t}{E} \right. \\ &\qquad \left. + \frac{\alpha_s^3 \pi (-D_R)^3}{8}\left(\frac{m_t}{E}\right)^{-3/2} \ldots\right] \end{aligned}$$

$\Rightarrow$ contributions $\sim \alpha_s^2 \times$ const., $\sim \frac{\alpha_s^3}{\beta}$ relative to leading S-wave

- NLL resummation sufficient for N^3LO_{app}
- no formal proof for NNLL resummation (see Falgari/CS/Wever 12)

NNLO potential function explicitly scale-dependent:

$$\frac{d}{d\ln\mu} J_R^S(E) = -\gamma_J^{R,S} J_R^S(E)$$

$$\gamma_J^{R,S(1)} = -(4\pi)^2 D_R \left(2D_R\left(\nu_{\rm spin}^S + \frac{5}{4}\right) + \frac{\nu_{\rm ann}^{R,S}}{2} + b_1^R\right)$$

$\mathcal{O}(\alpha_s^2)$ limit of NLL anomalous dimension in pNRQCD (Pineda 01)

Expansion of NNLO potential function to α_s^3

$$\Delta J_{R,NNLO}^{S(3)}(E) = J^{(0}(E)\frac{\alpha_s^3(\mu)}{4\pi}\left\{\frac{m_t}{E}\frac{D_R^2}{6}\left[\pi^2\left(2\beta_0 L_E + a_1\right) - 12\beta_0\zeta_3\right]\right.$$
$$\left. + \sqrt{\frac{m_t}{E}} D_R\left[-\frac{1}{2}\beta_0^2 L_E^2 + \frac{1}{8}\left(\gamma_J^{R,S(1)} - 2\beta_1 - 4a_1\beta_0\right) L_E + \text{const.}\right]\right\}$$

with $L_E = -\frac{1}{2}\ln\left(\frac{4Em_t}{\mu^2}\right)$