Production of Higgs bosons with large transverse momentum

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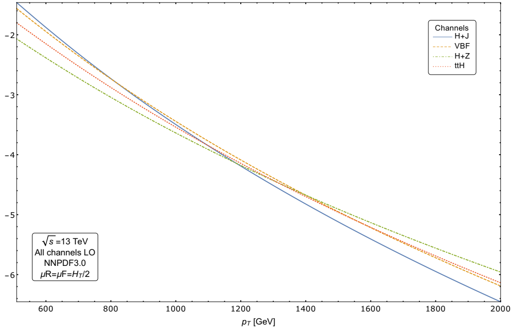
High Precision for Hard Processes 2018

Higgs bosons with large transverse momentum

- Higgs transverse momentum distribution is used to constrain Higgs couplings
- Few channels for the top-Yukawa coupling
- Boosted Higgs $H \to b\bar{b}$ channel is competitive (at least complementary) to the $t\bar{t}H$ *C.Grojean,et al. 2013*
- Boosted Higgs provides an alternative approach to study the top Yukawa
- CMS probed already the high - $p_T > 450$ GeV region



LO rates for H + j (2j)

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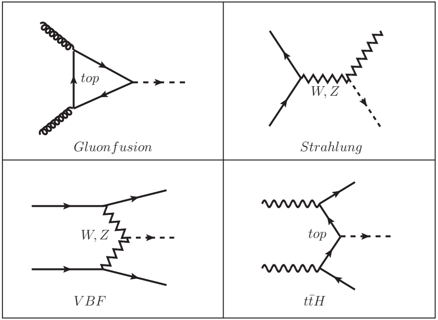

Main channels

Higgs bosons with large transverse momentum

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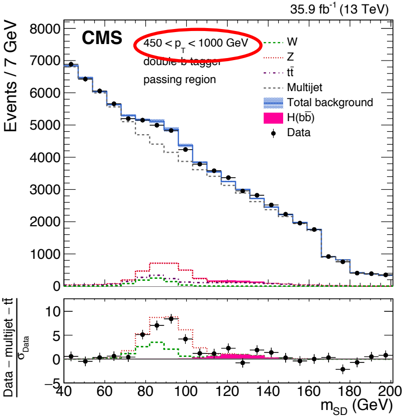


CMS Physics Analysis Summary; ArXiv: 1709.05543

$$\sigma^{CMS}_{incl}(H \to b\bar{b}) = 74^{+51}_{-49}\text{fb}$$

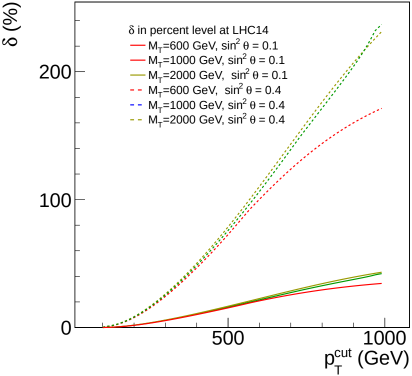
Since, only the LO cross-section was available (*R.K. Ellis,et al. ,1988; U. Baur, et al., 1990*) it led to huge theoretical uncertainties.

Table 1: Summary of the systematic uncertainties affecting the signal, W and Z+ jets processes. Instances where the uncertainty does not apply are indicated by "—".

Systematic source	W/Z	H
Integrated luminosity	2.5%	2.5%
Trigger efficiency	4%	4%
Pileup	<1%	<1%
$N_2^{1,\mathrm{DDT}}$ selection efficiency	4.3%	4.3%
Double-b tag	4% (Z)	4%
Jet energy scale / resolution	10/15%	10/15%
Jet mass scale (p_T)	0.4%/100 GeV (p_T)	0.4%/100 GeV (p_T)
Simulation sample size	2–25%	4–20% (ggF)
H p_T correction	—	30% (ggF)
NLO QCD corrections	10%	—
NLO EW corrections	15–35%	—
NLO EW W/Z decorrelation	5–15%	—

Anomalous couplings

- BSM physics $\rightarrow$ deforming the top-Yukawa sector (since, the top-Yukawa coupling is known to about 50 %)
- $$\frac{m_t}{v}\bar{t}tH \rightarrow - \kappa_g \frac{\alpha_s}{12\pi v} G^a_{\mu\nu} G^{\mu\nu,a} H + \kappa_t \frac{m_t}{v}\bar{t}tH$$
- **Problem**: $\sigma_{gg\rightarrow H} \sim \alpha_s^2/v^2(\kappa_g + \kappa_t)^2$
- **Solution**: to go beyond inclusive cross-section $\sigma_{gg\rightarrow H+g} \sim \left(\kappa_g + \kappa_t \frac{4m_t^2}{p_\perp^2}\right)^2$

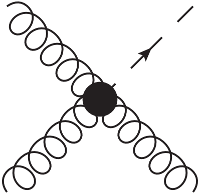


Anrea Banfi, et al., 2013; Additional heavy fermion will increase the number of events relative to the SM

Low pt vs High pt

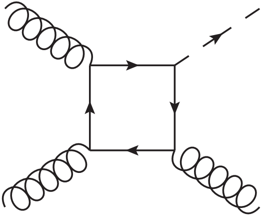
Why only LO results?

$p_{T,H}^2 \ll 4m_t^2$



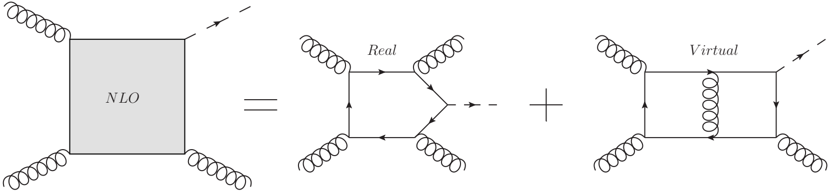
- $\{m_h, s, t, u\} \ll m_t$
- well established algorithmic approach to calculations (Large mass expansion)
- Higgs Effective Field Theory(HEFT) $m_t \to \infty$

$p_{T,H}^2 \gg 4m_t^2$



- $m_h < m_t \ll \{s, t, u\}$ hierarchy
- it is not clear what the degrees of freedom are here
- no robust algorithm *until recently*: *K. Melnikov, L. Tancredi, C. Wever, 2016*

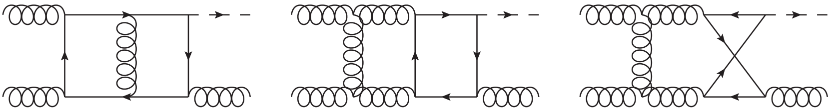
NLO corrections to Higgs + Jet



NLO corrections consist of two parts

- Real corrections are known analytically (*V. Del Duca, et al.,2001*)
- We used *OpenLoops* implementation of real corrections (*F. Cascioli, et al.,2012*)
- It is well understood how to combine these two pieces together *(S. Frixione, Z. Kunstz, A. Signer, 1995; S. Catani, M.H. Seymour, 1996)*
- Missing analytical results for the virtual amplitude

Virtual corrections



top topologies for H + Jet

- A four-scale problem: three external (s, p_T, m_h) and one internal (m_t)
- 264 Feynman integrals
- Complicated reduction
- No complete analytic result with the full top mass dependence (*R. Bonciani, et al., 2016)*
- Only numerical results with the full top mass available (*S. P. Jones, et al., 2018*)
- Different approach instead of exact results

Virtual corrections

Hierarchy $m_h < m_t \ll \{s, t, u\}$ suggests $\rightarrow$ Expansion in small parameters $(-\frac{m_h^2}{4m_t^2}, -\frac{m_t^2}{s})$ using Differential Equation approach (DEQ). It allows to calculate the virtual amplitude for H + jet production.

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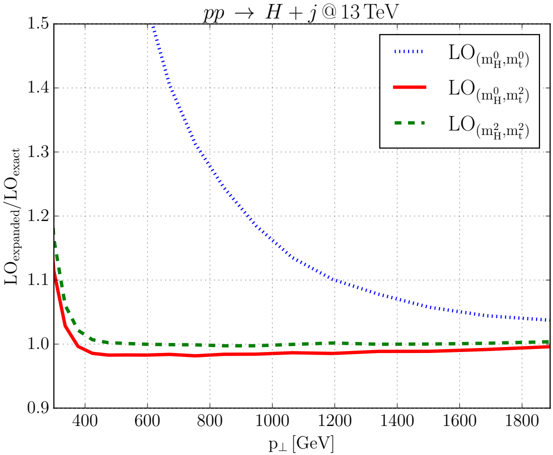


Amplitudes

Amplitudes for $H \to ggg$ & $H \to q\bar{q}g$

$$\mathcal{A}_{H\to ggg}\,(p_1^{a_1}, p_2^{a_2}, p_3^{a_3}) = f^{a_1 a_2 a_3}\; \epsilon_1^{\mu}\, \epsilon_2^{\nu}\, \epsilon_3^{\rho}\, \mathcal{A}^{g}_{\mu\nu\rho}(s,t,u,m_t)\,,$$

$$\mathcal{A}_{H\to q\bar{q}g}(p_1^{j}, p_2^{k}, p_3^{a}) = i\; T^{a}_{jk}\; \epsilon_3^{\mu}(p_3)\, \bar{u}(p_1)\, \mathcal{A}^{q}_{\mu}(s,t,u,m_t)\, v(p_2)\,.$$

Tensor decomposition(*T. Gehrmann, et al., 2011*)

$$\mathcal{A}^{g}_{\mu\nu\rho}(s,t,u,m_t) = F_1^{g}\, g_{\mu\nu}\, p_{2\rho} + F_2^{g}\, g_{\mu\rho}\, p_{1\nu} + F_3^{g}\, g_{\nu\rho}\, p_{3\mu} + F_4^{g}\, p_{3\mu} p_{1\nu} p_{2\rho}\,,$$

$$\mathcal{A}^{q}_{\mu} = F_1^{q}\, (\not{p}_3\, p_{2\mu} - p_2\cdot p_3\, \gamma^{\mu}) + F_2^{q}\, (\not{p}_3\, p_{1\mu} - p_1\cdot p_3\, \gamma_{\mu})\,.$$

where $F_j^i = \sum_k R^i_{jk}(s,t,u,m_t;\epsilon)\mathcal{I}_k$ are form factors; a linear combination of rational arguments and scalar integrals

Reduction of scalar integrals

Scalar integrals

$$\mathcal{I}_{\rm top}(a_1, a_2, ..., a_8, a_9) = \int \frac{\mathfrak{D}^d k \mathfrak{D}^d l}{[1]^{a_1}[2]^{a_2}[3]^{a_3}[4]^{a_4}[5]^{a_5}[6]^{a_6}[7]^{a_7}[8]^{a_8}[9]^{a_9}}$$

Integration-by-parts (IBP; *K. G. Chetyrkin and F. V. Tkachov, 1981*)

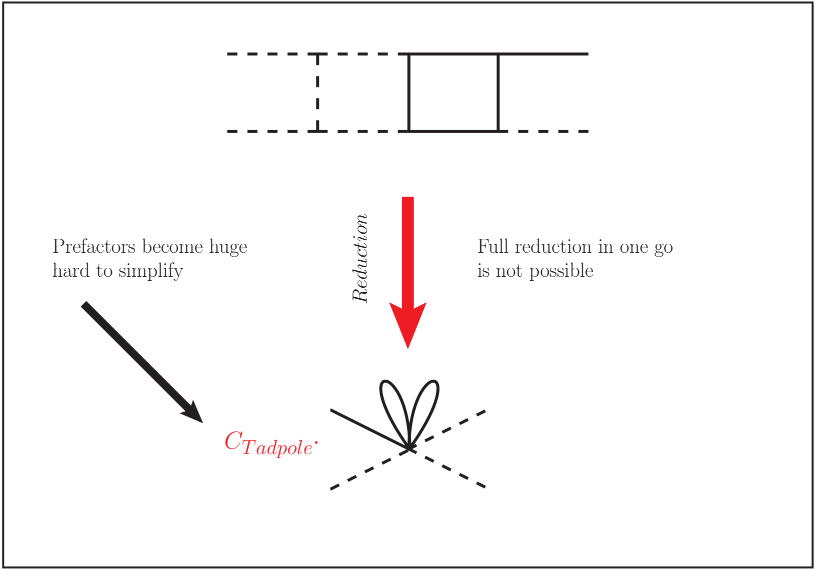
$$\int \frac{\partial}{\partial k^\mu}(q^\mu \prod_{j=1}^{J} \frac{1}{[N]^{a_l}})\mathfrak{D} k \mathfrak{D} l = 0 \qquad q^\mu = \{k^\mu, p^\mu\}$$

via IBP the following mapping is done

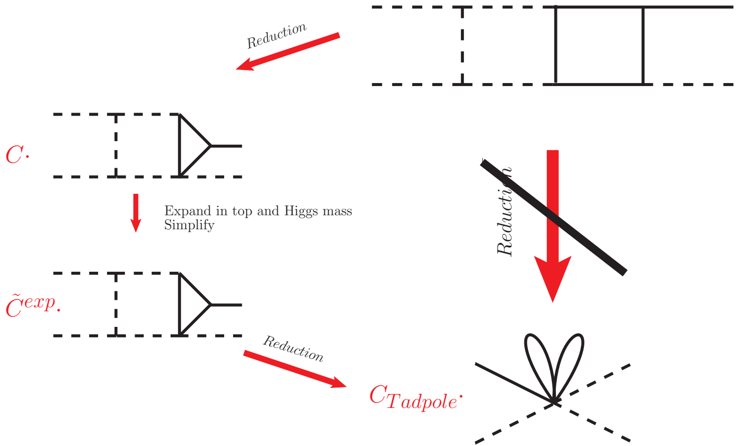
$$\{\mathcal{I}_1, \mathcal{I}_2, ..., \mathcal{I}_N\} \to \{\mathfrak{I}_1, \mathfrak{I}_2, ..., \mathfrak{I}_{\tilde{N}}\}$$

$\mathfrak{I}_i$ are Master Integrals(MI); 458 Master integrals to compute (crossings included)

Reduction of the scalar integrals



Reduction of the scalar integrals



Many thanks to Lorenzo Tancredi

Differential equations

DEQs

Taking derivatives w.r.t m_t, s, t, u and applying IBPs

$$\partial_k \mathfrak{I}_i(\kappa, \eta, z, \epsilon) = \sum_j A^k_{ij}(\kappa, \eta, z, \epsilon)\, \mathfrak{I}_j(\kappa, \eta, z, \epsilon), \quad k \in \{\kappa,\ \eta,\ z\}.$$

"Normalized" variables

$$\kappa = -\frac{m_h}{4m_t}, \eta = -\frac{m_t}{s}, z = \frac{u}{s}, \qquad 0 < \kappa, \eta \ll 1, z > 0, s < 0$$

Note that both κ, η are numerically small!

Ansatz

Once constructed, DEQs are analyzed. DEQs admit the following solutions

$$\mathfrak{I}_i(\kappa, \eta, z, \epsilon) = \sum_{j,k,l,m \in \mathbb{Z}, n \in \mathbb{N}} c_{i,j,k,l,m,n}(z, \epsilon)\, \eta^{j-k\epsilon} \kappa^{l/2-m\epsilon} \log^n(\kappa).$$

Solving DEQs

Inserting Ansatz

By inserting the ansatz into DEQs we simplify the problem significantly. Namely, we mapped DEQs onto algebraic equations

$$\mathfrak{I} \to c_{i,j,k,l,m,n}(z,\epsilon)$$

Combining this with $\kappa, \eta \ll 1$ we get a finite system of linear equations after truncating linear series; in practice it is a very sparse system. This takes care of κ, η DEQs .

z - integration

Ansatz helps, but we still need to integrate z-DEQs

$$\frac{\partial}{\partial z} c(z,\epsilon) = \epsilon M(z) c(z,\epsilon)$$

Integrated in terms of Goncharov's polylogarithms

$$\mathrm{G}(\underbrace{l_1, \cdots, l_n}_{\text{weight n}}; z) := \int_0^z dz' \frac{\mathrm{G}(l_2, \cdots, l_n; z')}{z' - l_1},$$

$$\mathrm{G}(;z) = 1, \qquad \mathrm{G}(\underbrace{0, \cdots, 0}_{\text{n times}}; z) = \frac{1}{n!} \log^n(z).$$

Comparison with full result

Simplifications

- 49 letters $\to$ $\{-1, 0, 1\}$ letters (correspond to HPLs)
- Elliptic sectors "dissapear"

Complications

- it is a different system $\to$
- not all mathematical limits are accessible
- not in a canonical form

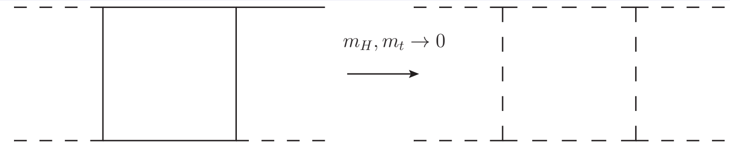
Determining boundary conditions is difficult.

Last step is always the hardest one

Massless two-loop master diagrams

$$\mathfrak{I} = M0 + M1 \cdot \eta + ...$$
$$\frac{\partial \mathfrak{I}}{\partial \eta} = (A + B \cdot \eta + ...)\mathfrak{I} + \text{subtopologies}$$

Convenient to solve for M0, since they are known (T. Gehrmann, E. Remiddi, 2000 and 2001). In other words, master integrals for Higgs + Jet are master integrals in massless limit (*not always*)

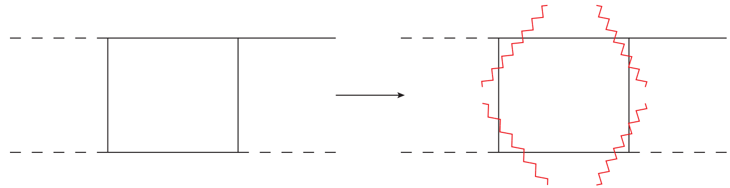


Massless limit

Last step is always the hardest one

Singularities

Master-integrals admits certain singularities



cut for a triangle integral

At equation level, we can check that is satisfied

$$\frac{\partial \mathfrak{I}}{\partial \eta} = \dots + \frac{\sum b_j M_j}{p_1 \cdot p_4} + \dots$$

Hence, we should $\sum b_j M_j \to 0$

Last step is always the hardest one

Mellin-Barns transformation (M. Czakon and A. Smirnov; V. Smirnov,1999;J.B Tausk, 1999)

- we can express a sum as a contour integral (*MBtools*)

$$(A_1 + A_2)^{-\nu} = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} d\sigma A_1^{\sigma} A_2^{-\nu-\sigma} \frac{\Gamma(-\sigma)\Gamma(\nu+\sigma)}{\Gamma(\nu)}$$

- we want a particular branch $((m_t^2)^{-\epsilon}, (m_t^2)^{-2\epsilon}, \dots)$
- The main difficulty $\rightarrow$ Mellin-Barns representation
- Advantage $\rightarrow$ many constants could be found simultaneously by pinching

$$\int \frac{\mathfrak{D}^d k \mathfrak{D}^d l}{[1]^{\#+a_1}[2]^{\#+a_2}[3]^{\#+a_3}\dots}$$

that is putting $a_i \rightarrow 0$ we can find boundaries for subtopologies.

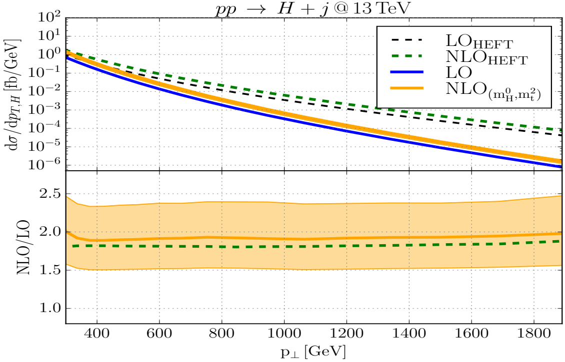
Last step is always the hardest one

Computing Feynman integral in a kinematic point

It is last resort method. Just few suggestions

- We have to extract a particular branch $((m_t^2)^{-\epsilon}, (m_t^2)^{-2\epsilon}, \dots)$
- Change of variables
- Bring integrand to a such a form where one can integrate it to hypergeometric functions
- Hypergeometric functions are well understood.
- Look for asymptotics of hypergeometric functions to extract the branch you need

Higgs + Jet at NLO



$p_{H,T}$ distribution above the top mass threshold

PDF:NNPDF3.0

$m_t = 173.2\text{GeV}$

$m_h = 125\text{GeV}$

$\mu = \{1/2, 2\}\mu_0$

the top mass effect

$4 - 6\%$

	LO_{HEFT} [fb]	NLO_{HEFT} [fb]	K	LO [fb]	NLO [fb]	K
$p_\perp > 400$ GeV	$33.8^{+44\%}_{-29\%}$	$61.4^{+20\%}_{-19\%}$	1.82	$12.4^{+44\%}_{-29\%}$	$23.6^{+24\%}_{-21\%}$	1.90
$p_\perp > 450$ GeV	$22.0^{+45\%}_{-29\%}$	$39.9^{+20\%}_{-19\%}$	1.81	$6.75^{+45\%}_{-29\%}$	$12.9^{+24\%}_{-21\%}$	1.91
$p_\perp > 500$ GeV	$14.7^{+44\%}_{-28\%}$	$26.7^{+20\%}_{-19\%}$	1.81	$3.80^{+45\%}_{-29\%}$	$7.28^{+24\%}_{-21\%}$	1.91
$p_\perp > 1000$ GeV	$0.628^{+46\%}_{-30\%}$	$1.14^{+21\%}_{-19\%}$	1.81	$0.0417^{+47\%}_{-30\%}$	$0.0797^{+24\%}_{-21\%}$	1.91

Summary

- The two loop amplitude has been computed up to the subleading order in the top mass squared
- We combined existing real amplitude with our result, to produce the higgs pT distribution
- We found that the NLO QCD corrections increase the LO prediction by 100 % and the ration of NLO over LO is of O(2) and stable at large values of the transverse momentum
- Scale uncertainty $\pm 20\%$

Outlook

- This method can be used in different calculations related to (HL)-LHC physics
- Since the exact solution is not available, one can combine results from two regions to have a decent differential distribution for any value of pT
- Constraining anomalous couplings $\mathfrak{L} \sim c_t t\bar{t}h + c_g G^a_{\mu\nu} G^{\mu\nu}_a$

Thank you!

Backup slides

Real corrections with open loops

- Partonic channels contributing to the H + j @NLO

$$q\bar{q} \to Hgg, gg \to Hgg, qg \to Hqg, \ldots$$

- Receives contributions from kinematical regions where one parton become soft or collinear to another parton
- This requires a delicate approach of these regions in phase space integral
- *Openloops* algorithm is publicly available program which is capable of dealing with these singular regions in a numerically stable way
- Crucial ingredient is tensor integral reduction performed via expansions in small Gram determinants (*Cascioli et al., 2012* and *Denner et al., 2003 - 2017*)
- Exact top mass dependence kept throughout for one-loop computations

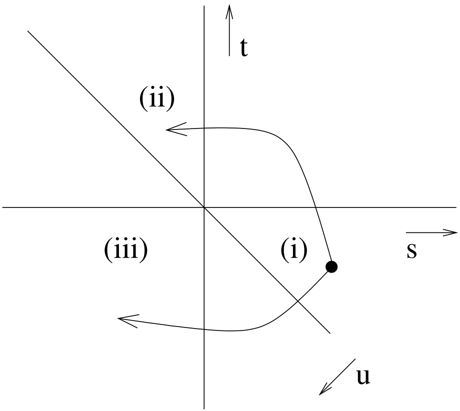
Pole substructure

S. Catani, M.H. Seymour, 1996

$$(F^i)_j^{(1),\mathrm{UV}} = (F^i)_j^{(1),\mathrm{fin}}, \quad (F^i)_j^{(2),\mathrm{UV}} = I_1^i(\epsilon)(F^i)_j^{(1),\mathrm{UV}} + (F^i)_j^{(2),\mathrm{fin}}$$

Keep in mind that $(F^i)_j^{(1),\mathrm{UV}}$ kept exact through out the calculations. In the end the whole expression is expanded in the Higgs and top mass.

Analytic continuation



C. Anastasiou et al., 2016

$$(i) \; : \; s \to s + i\,0\,,$$
$$(ii) \; : \; t \to t + i\,0\,,$$
$$(iii) \; : \; u \to u + i\,0\,.$$

The analytic continuation is done in two steps;
Consider analytic continuation from (i) $\to$(ii). First we cross $t = 0$, which would require to do a transformation of the kind: $x \to 1/x$. $t \to t + i0$. Next, we cross $s = 0$, hence a value of $s \to s + i0$. Apply complex analisys, we can get the following formulas for transformation:
$log(-t/s) \to log(-t/s) - 2i\pi$ and
$log(-u/s) \to log(u/s) - i\pi$