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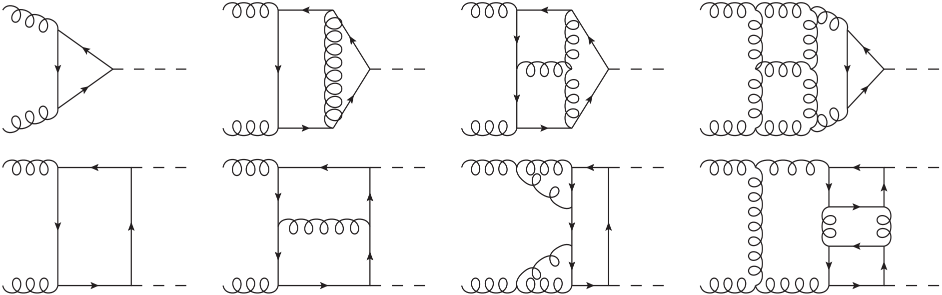
Decoupling in QCD and Higgs effective field theory

Florian Herren | 03.10.2018

in collaboration with M. Gerlach, M. Steinhauser

INSTITUT FÜR THEORETISCHE TEILCHENPHYSIK



Higgs-Boson production in Gluon fusion

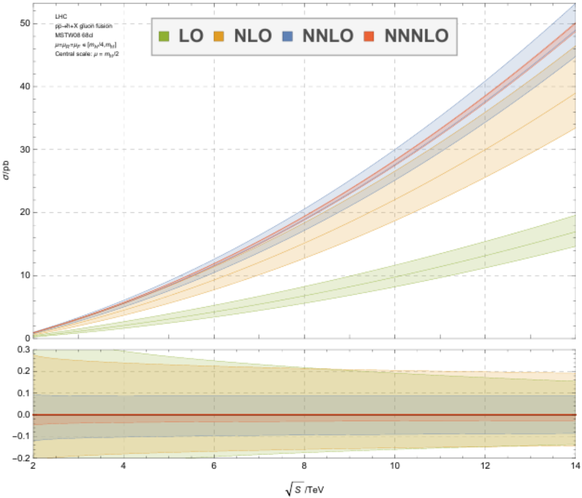
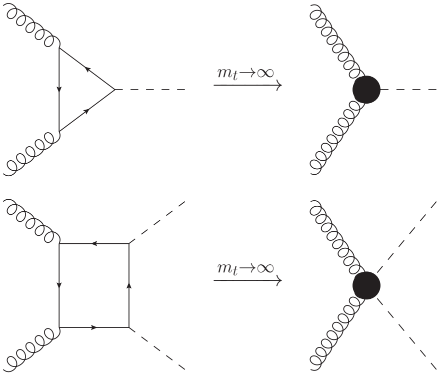


Figure: [Anastasiou et. al 2015]

Wilson coefficients for Higgs-Boson production



Wilson coefficients for Higgs-Boson production

- In the limit $m_t \to \infty$ the coupling of Higgs bosons to gluons is given by

$$\mathcal{L}_{\mathrm{eff}} = -C_H \frac{H}{v} \mathcal{O}_1 + \frac{C_{HH}}{2} \left(\frac{H}{v}\right)^2 \mathcal{O}_1$$

$$\mathcal{O}_1 = G^a_{\mu\nu} G^{a\mu\nu}$$

- Inclusive Higgs-boson production known up to $\mathrm{N^3LO}$
  $\to C_H$ needed at 4 loops
- C_{HH} to 4 loops $\to \mathrm{N^3LO}$ Higgs-pair production

Interlude: Running & Decoupling in QCD

QCD beta function

QCD typically renormalized in $\overline{\mathrm{MS}}$-scheme $\to \alpha_s(\mu)$

$$\beta_{\alpha_s} = \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \frac{\alpha_s}{\pi} \qquad \beta_{\alpha_s} = -\sum_{i \geq 0} \beta_i \left(\frac{\alpha_s}{\pi} \right)^{i+2}$$

$$\beta_0 = \frac{1}{4} \left(11 - \frac{2}{3} n_f \right)$$

[Gross, Wilczek 1973], [Politzer 1973]

QCD beta function

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β_4 (5-loop) recently calculated:
[Baikov, Chetyrkin, Kühn 2016],
[Herzog, Ruijl, Ueda, Vermaseren 2017],
[Luthe, Maier, Marquard, Schroder 2017]

QCD beta function

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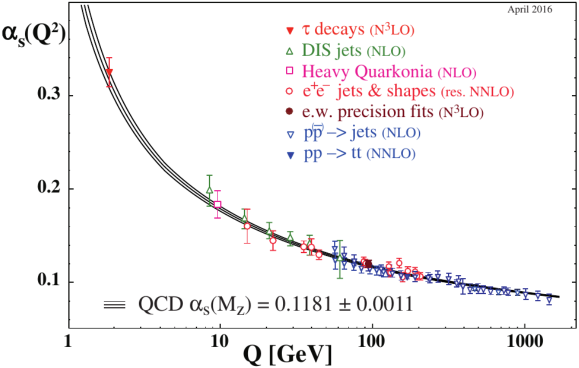


Figure: [PDG 2018]

Decoupling of heavy quarks

- $\sqrt{s} \ll m_t \to$ work in effective 5-flavour theory
  $\to$ decouple top quark from running of α_s
- $\overline{\mathrm{MS}}$ scheme:

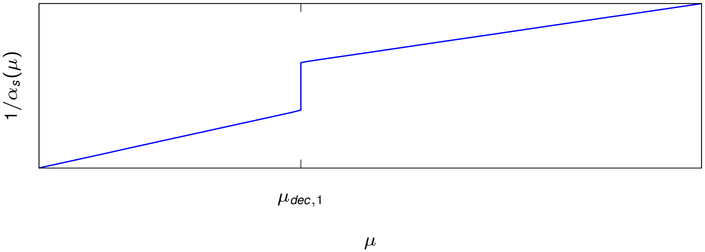
$$\alpha_s^{(5)}(\mu_{\mathrm{dec}}) = \zeta_{\alpha_s}\left(\ln\left(\frac{\mu_{\mathrm{dec}}^2}{m_t^2}\right)\right)\alpha_s^{(6)}(\mu_{\mathrm{dec}})$$

- Matching introduces unphysical decoupling scale μ_{dec}
- ζ_{α_s} known to 4 loops for $N_C = 3$:
  [Chetyrkin, Kühn, Sturm 2006], [Schroder, Steinhauser 2006]

Decoupling of heavy quarks

$$\alpha_s^{(5)}(\mu_{\mathrm{dec}}) = \zeta_{\alpha_s}\left(\ln\left(\frac{\mu_{\mathrm{dec}}^2}{m_t^2}\right)\right)\alpha_s^{(6)}(\mu_{\mathrm{dec}})$$

$\rightarrow \alpha_s(\mu)$ is not continuous:



Decoupling of heavy quarks

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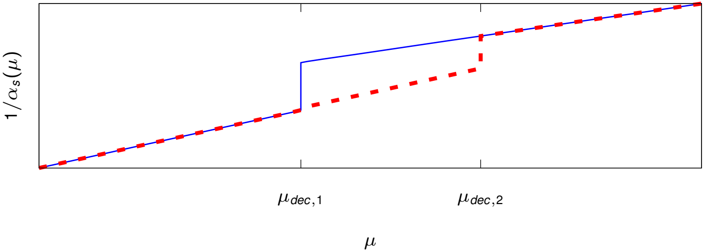


RunDec 3 & CRunDec 3 [FH, Steinhauser, 2017]

- QCD running & decoupling
- Relations between various mass schemes: $\overline{\mathrm{MS}}$, OS, PS, 1S, ...
- highest perturbative accuracy available
- Mathematica and C++

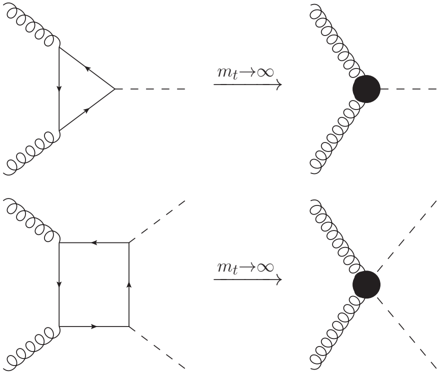
Previous versions:

RunDec: [Chetyrkin, Kühn, Steinhauser, 2000]

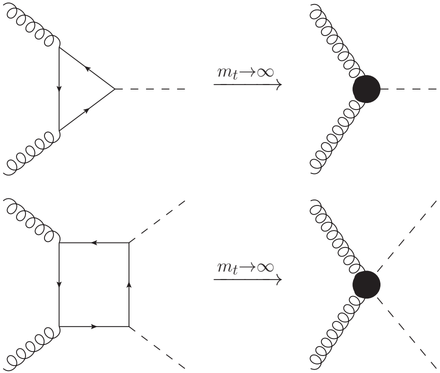
CRunDec: [Schmidt, Steinhauser, 2012]

Decoupling & Higgs physics

$\mathcal{L}_{\text{eff}}$



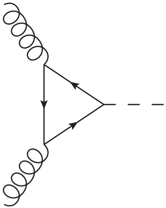
$\mathcal{L}_{\text{eff}}$



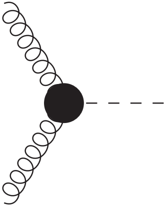
$$\mathcal{L}_{\text{eff}} = -C_H \frac{H}{v} \mathcal{O}_1 + \frac{C_{HH}}{2} \left(\frac{H}{v} \right)^2 \mathcal{O}_1$$

$$\mathcal{O}_1 = G^a_{\mu\nu} G^{a\mu\nu}$$

C_H



$$= \epsilon_1^\mu \epsilon_2^\mu \left(g_{\mu\nu}(q_1.q_2) - q_{1\,\nu} q_{2\,\mu} \right) F(m_t, m_H)$$

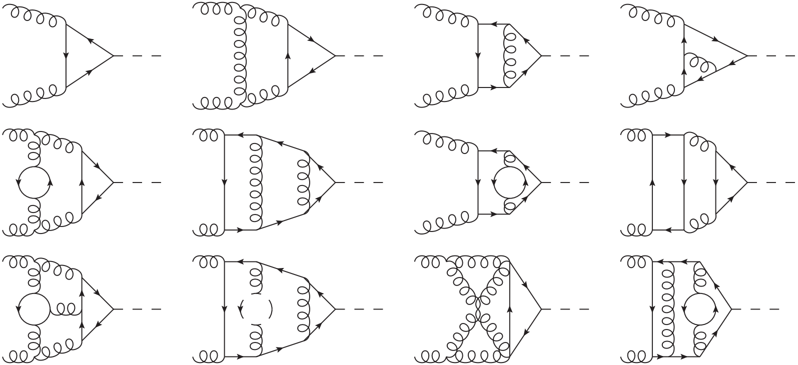


$$= \epsilon_1^\mu \epsilon_2^\mu \left(g_{\mu\nu}(q_1.q_2) - q_{1\,\nu} q_{2\,\mu} \right) \frac{C_H}{v}$$

$\rightarrow$ At leading order: $C_H = vF|_{m_t \rightarrow \infty}$

C_H

23251 diagrams:



Technicalities

- Diagrams generated with `graf` [Nogueira 1993]
- Amplitudes generated and mapped onto topologies with `q2e` and `exp` [Harlander, Seidensticker, Steinhauser 1998]
- Diagrams with same colour factors and topologies combined into superdiagrams
- Expansion in $1/m_t$ and subsequent tensor reduction in `FORM` [Ruijl, Ueda, Vermaseren 2017]

$\rightarrow$ 4-loop scalar Tadpole integrals

Technicalities

- Reduction to master integrals with `LiteRed` [Lee 2012] and `FIRE5` [Smirnov 2015]
- Master integrals reduced to minimal set using `FindRules` of `FIRE5`
- Master integrals known [Lee, Terekhov 2011], [Schroder, Vuorinen 2005], [Chetyrkin, Faisst, Sturm, Tentyukov 2006]

$$
\begin{aligned}
C_H^{(4)} = {} & C_A^3\left(\frac{110041}{41472} - \frac{1577}{3072}\zeta(3)\right) + C_A^2 C_F\left(-\frac{99715}{6912} + \frac{5105}{512}\zeta(3)\right) \\
& + C_A^2 T_F\left(-\frac{1081}{3456} + \frac{1}{384}\zeta(3)\right) + C_A C_F^2\left(\frac{2963}{384} - \frac{407}{128}\zeta(3)\right) \\
& + C_A C_F T_F\left(\frac{4537}{1728} - \frac{115}{64}\zeta(3)\right) + C_A T_F^2\left(\frac{2}{27} - \frac{7}{64}\zeta(3)\right) - \frac{471}{128}C_F^3 \\
& + C_F^2 T_F\left(-\frac{5}{12} + \frac{13}{32}\zeta(3)\right) + C_F T_F^2\left(\frac{113}{432} - \frac{7}{32}\zeta(3)\right) \\
& + \frac{d_R^{abcd} d_A^{abcd}}{N_A T_F}\left(-\frac{2}{3} + \frac{13}{2}\zeta(3)\right) + \frac{d_R^{abcd} d_R^{abcd}}{N_A T_F}\left(\frac{11}{12} - 2\zeta(3)\right) \\
& + \left[\frac{1993}{1152}C_A^3 - \frac{275}{72}C_A^2 C_F - \frac{55}{576}C_A^2 T_F + \frac{99}{64}C_A C_F^2 - \frac{11}{72}C_A C_F T_F\right]\ln\left(\frac{\mu^2}{M_t^2}\right) \\
& + \left[\frac{77}{192}C_A^3 - \frac{121}{192}C_A^2 C_F\right]\ln^2\left(\frac{\mu^2}{M_t^2}\right) + n_l\frac{d_R^{abcd} d_R^{abcd}}{N_A T_F}\left(\frac{11}{6} - 4\zeta(3)\right) \\
& + n_l T_F\left[C_A^2\left(-\frac{12421}{10368} - \frac{151}{256}\zeta(3)\right) + C_A C_F\left(\frac{9605}{2592} - \frac{1145}{384}\zeta(3)\right)\right. \\
& \left. + C_A T_F\left(\frac{7}{216} - \frac{7}{64}\zeta(3)\right) + C_F^2\left(\frac{215}{288} + \frac{127}{96}\zeta(3)\right) + C_F T_F\left(-\frac{29}{144} - \frac{7}{32}\zeta(3)\right)\right] \\
& + n_l^2 T_F^2\left[-\frac{161}{2592}C_A - \frac{677}{1296}C_F\right] + n_l T_F\left[-\frac{55}{288}C_A^2 + \frac{55}{36}C_A C_F + \frac{5}{144}C_A T_F - \frac{5}{8}C_F^2 + \frac{1}{18}C_F T_F\right]\ln\left(\frac{\mu^2}{M_t^2}\right) \\
& + n_l^2 T_F^2\left[\frac{5}{144}C_A + \frac{1}{18}C_F\right]\ln\left(\frac{\mu^2}{M_t^2}\right) + n_l T_F\left[-\frac{7}{48}C_A^2 + \frac{11}{16}C_A C_F\right]\ln^2\left(\frac{\mu^2}{M_t^2}\right) - \frac{1}{6}n_l^2 C_F T_F^2 \ln^2\left(\frac{\mu^2}{M_t^2}\right)
\end{aligned}
$$

C_H

- In total 19 colour factors
- Fully analytic result
- One transcendental constant: $\zeta(3)$

$$C_H = -\frac{\alpha_s^{(5)}}{3\pi}\left\{1 + ... + \left(\frac{\alpha_s^{(5)}}{\pi}\right)^3 \left[{C_A}^3\left(-\frac{110041}{124416} + \frac{1577}{9216}\zeta(3) - \frac{1993}{3456}\ln\frac{\mu^2}{m_t^2} - \frac{77}{576}\ln^2\frac{\mu^2}{m_t^2}\right) + ...\right]\right\}$$

Low-energy theorem

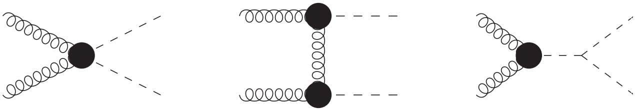
- Another way to compute C_H [Chetyrkin, Kniehl, Steinhauser 1997]:
- Low-energy Theorem (LET):

$$C_H = -\frac{m_t}{\zeta_{\alpha_s}} \frac{\partial}{\partial m_t} \zeta_{\alpha_s}$$

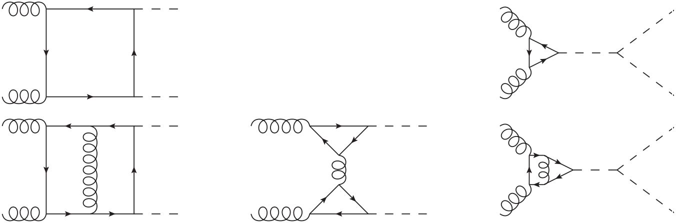
- Explicit calculation of C_H agrees with LET
- Good cross-check of setup

C_{HH}

$$C_{HH} Z_{\mathcal{O}_1} \mathcal{A}^{\text{eff}}_{1PI} + C_H^2 Z^2_{\mathcal{O}_1} \mathcal{A}^{\text{eff}}_{1PR,\lambda=0} + C_H Z_{\mathcal{O}_1} \mathcal{A}^{\text{eff}}_{1PR,\lambda\neq 0}$$



$$= \frac{1}{\zeta_3^0} \left(\mathcal{A}^{\text{full}}_{1PI} + \mathcal{A}^{\text{full}}_{1PR,\lambda=0} + \mathcal{A}^{\text{full}}_{1PR,\lambda\neq 0} \right) + \mathcal{O}\left(1/m_t\right)$$



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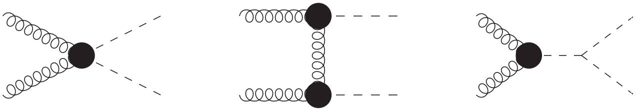

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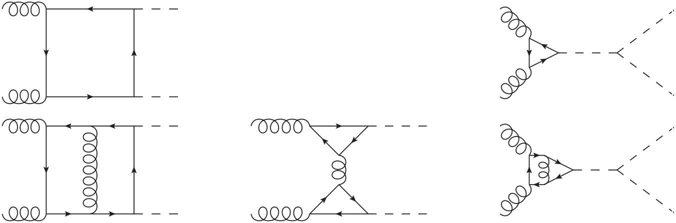

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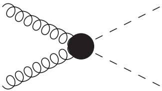

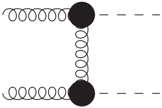

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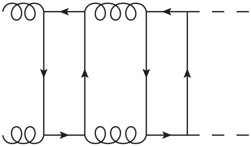

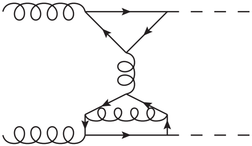

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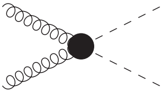

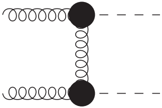

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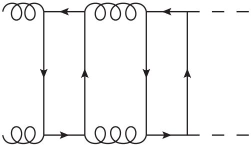
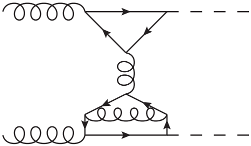
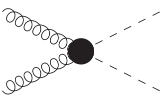
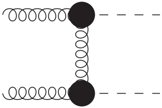




C_{HH}

$$\left(C_{HH} Z_{\mathcal{O}_1} + C_H^2 Z_{11}^L\right) \mathcal{A}^{\text{eff}}_{1PI} + C_H^2 Z^2_{\mathcal{O}_1} \mathcal{A}^{\text{eff}}_{1PR,\lambda=0}$$





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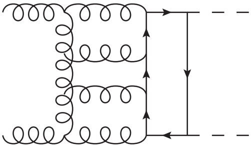
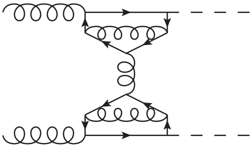
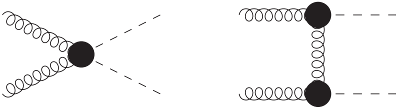




C_{HH}

$$\left(C_{HH} Z_{\mathcal{O}_1} + C_H^2 Z_{11}^L\right) \mathcal{A}_{1PI}^{\text{eff}} + C_H^2 Z_{\mathcal{O}_1}^2 \mathcal{A}_{1PR,\lambda=0}^{\text{eff}}$$



- Where does this new term come from?

$$\mathcal{A}_{(\mathcal{O}_1)^2}^{\text{eff}} = Z_{\mathcal{O}_1}^2 \mathcal{A}_{(\mathcal{O}_1^0)^2}^{\text{eff}} + Z_{11}^L \mathcal{A}_{\mathcal{O}_1^0}^{\text{eff}}$$

$$\frac{Z_{11}^L}{Z_{\mathcal{O}_1}} = -\frac{\beta_1}{\epsilon} \frac{\alpha_s^2}{(4\pi)^2} + \mathcal{O}\left(\alpha_s^3\right)$$

- Z_{11}^L found by [Zoller 2016] only contibutes starting from $\mathcal{O}\left(\alpha_s^4\right)$

C_{HH}

- 145942 diagrams
- 1 and 2 loop $C_{HH} = C_H$
- 3 loops:

$$C_{HH} - C_H = \frac{7}{8} C_A^2 - \frac{5}{6} C_A T_F - \frac{11}{8} C_A C_F + \frac{1}{2} C_F T_F + C_F T_F n_l$$

[Grigo, Melnikov, Steinhauser 2014]

- Again, 19 colour factors in total

$$
\begin{aligned}
C_{HH}^{(4)} - C_H^{(4)} = &\frac{1993}{576} C_A^3 - \frac{1289}{144} C_A^2 C_F - \frac{3191}{864} C_A^2 T_F + \frac{165}{32} C_A C_F^2 \\
&+ \frac{67}{18} C_A C_F T_F + \frac{5}{72} C_A T_F^2 - \frac{3}{2} C_F^2 T_F + \frac{1}{9} C_F T_F^2 \\
&+ \left[\frac{77}{48} C_A^3 - \frac{121}{48} C_A^2 C_F - \frac{7}{12} C_A^2 T_F + \frac{11}{12} C_A C_F T_F \right] \ln\left(\frac{\mu^2}{M_t^2}\right) \\
&+ n_l T_F \left[-\frac{55}{144} C_A^2 + \frac{55}{18} C_A C_F + \frac{109}{216} C_A T_F - \frac{11}{4} C_F^2 + \frac{19}{36} C_F T_F \right] \\
&+ n_l^2 T_F^2 \left[\frac{5}{72} C_A + \frac{1}{9} C_F \right] \\
&+ n_l T_F \left[-\frac{7}{12} C_A^2 + \frac{11}{4} C_A C_F - \frac{2}{3} C_F T_F \right] \ln\left(\frac{\mu^2}{M_t^2}\right) \\
&- \frac{2}{3} n_l^2 C_F T_F^2 \ln\left(\frac{\mu^2}{M_t^2}\right)
\end{aligned}
$$

Low-energy theorem

- [Spira 2016]:

$$C_{HH} = \frac{m_t^2}{\zeta_{\alpha_s}} \frac{\partial^2}{\partial m_t^2} \zeta_{\alpha_s} - 2 \left(\frac{m_t}{\zeta_{\alpha_s}} \frac{\partial}{\partial m_t} \zeta_{\alpha_s} \right)^2$$

- We find complete agreement at four loops

Summary

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$$\mathcal{L}_{\text{eff}} = -C_H \frac{H}{v}\mathcal{O}_1 + \frac{C_{HH}}{2}\left(\frac{H}{v}\right)^2 \mathcal{O}_1$$

- Explicit calculation of C_H and C_{HH} at 4 loops
  $\rightarrow$ ingredient for $\mathrm{N}^3\mathrm{LO}$ production cross-sections
- Agreement with LET
- Issues with renormalization of operator products understood
  $\rightarrow$ [Banerjee, Borowka, Dhani, Gehrmann, Ravindran 2018]
- Byproduct: all decoupling constants in QCD at 4 loops
  $\rightarrow$ [Gerlach, Herren, Steinhauser 2018]