Challenges for Precision Lattice Flavour Physics

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Towards the Ultimate Precision in Flavour Physics

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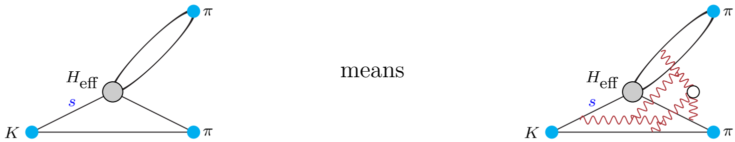
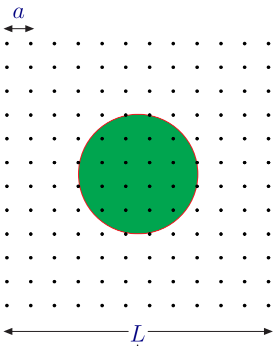
- Precision Flavour Physics is a key approach, complementary to the large E_T searches at the LHC, in exploring the limits of the Standard Model of Particle Physics and in searches for new physics.
  - If the LHC experiments discover new elementary particles BSM, then precision flavour physics will be necessary to understand the underlying framework.
  - The discovery potential of precision flavour physics should also not be underestimated. (In principle, the reach is about two-orders of magnitude deeper than the LHC!)
- Precision flavour physics requires control of hadronic effects for which lattice QCD simulations are essential.
- For example, a schematic cartoon of $K \to \pi\pi$ decays:



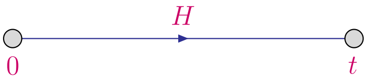


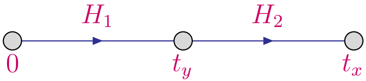
- Lattice phenomenology starts with the evaluation of correlation functions of the form:

$$\langle 0|\, O(x_1,x_2,\cdots,x_n)\,|0\rangle \;=\; \frac{1}{Z}\int [dA_\mu]\,[d\psi]\,[d\bar\psi]\,e^{-S}\,O(x_1,x_2,\cdots,x_n)$$

where $O(x_1,x_2,\cdots,x_n)$ is a multilocal operator composed of quark and gluon fields and Z is the partition function.

- The physics which can be studied depends on the choice of the multilocal operator O.





- The functional integral is performed by discretising Euclidean space-time and using Monte-Carlo Integration.

- In recent years the precision and reliability of lattice calculations has improved remarkably!
- Recent results can be found in the critical compilation “Review of lattice results concerning low energy particle physics”, from the *Flavour Physics Lattice Averaging Group* (FLAG)

  S. Aoki, Y. Aoki, D.Bečirević, C. Bernard, T. Blum, G. Colangelo, M. Della Morte, P. Dimopoulos, S. Dürr, H. Fukaya, M. Golterman, S. Gottlieb, S. Hashimoto, U.M. Heller, R. Horsley, A. Jüttner, T. Kaneko, L. Lellouch, H. Leutwyler, C.-J. Lin, V. Lubicz, E. Lunghi, R. Mawhinney, T. Onogi, C. Pena, C. Sachrajda, S. Sharpe, S. Simula, R. Sommer, A. Vladikas, U. Wenger, H. Wittig. EPJC **77** (2017) 112; arXiv:1607.00299 (383 pages!)

- This third edition is an extension and continuation of the work of the Flavianet Lattice Averaging Group:

  G. Colangelo, S. Dürr, A. Juttner, L. Lellouch, H. Leutwyler, V. Lubicz, S. Necco, C. T. Sachrajda, S. Simula, A. Vladikas, U. Wenger, H. Wittig arXiv:1011.4408

- Motivation - to present to the wider community an *average* of lattice results for important quantities obtained after a critical expert review.
- The closing date for arXiv:1607.00299 was Nov 30th 2015. Fourth edition in preparation.

Quantity	■	$N_f = 2+1+1$	■	$N_f = 2+1$	■	$N_f = 2$
m_s(MeV)	2	$93.9(1.1)$	5	$92.0(2.1)$	2	$101(3)$
m_{ud}(MeV)	1	$3.70(17)$	5	$3.373(80)$	1	$3.6(2)$
m_s/m_{ud}	2	$27.30(34)$	4	$27.43(31)$	1	$27.3(9)$
m_d(MeV)	1	$5.03(26)$	Flag(4)	$4.68(14)(7)$	1	$4.8(23)$
m_u(MeV)	1	$2.36(24)$	Flag(4)	$2.16(9)(7)$	1	$2.40(23)$
m_u/m_d	1	$0.470(56)$	Flag(4)	$0.46(2)(2)$	1	$0.50(4)$
m_c/m_s	3	$11.70(6)$	2	11.82	1	11.74
$f_+^{K\pi}(0)$	1	$0.9704(24)(22)$	2	$0.9667(27)$	1	$0.9560(57)(62)$
f_{K^+}/f_{π^+}	3	$1.193(3)$	4	$1.192(5)$	1	$1.205(6)(17)$
f_K(MeV)	3	$155.6(4)$	3	$155.9(9)$	1	$157.5(2.4)$
f_π(MeV)			3	$130.2(1.4)$		
$\Sigma^{\frac{1}{3}}$(MeV)	1	$280(8)(15)$	4	$274(3)$	4	$266(10)$
F_π/F	1	$1.076(2)(2)$	5	$1.064(7)$	4	$1.073(15)$
$\bar{\ell}_3$	1	$3.70(7)(26)$	5	$2.81(64)$	3	$3.41(82)$
$\bar{\ell}_4$	1	$4.67(3)(10)$	5	$4.10(45)$	2	$4.51(26)$
$\hat{B}_K$	1	$0.717(18)(16)$	4	$0.7625(97)$	1	$0.727(22)(12)$

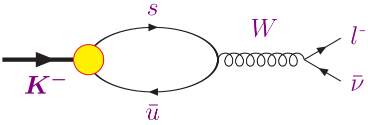
Quantity	■	$N_f=2+1+1$	■	$N_f=2+1$	■	$N_f=2$
f_D(MeV)	2	212.15(1.45)	2	209.2(3.3)	1	208(7)
f_{D_s}(MeV)	2	248.83(1.27)	3	249.8(2.3)	1	250(7)
f_{D_s}/f_D	2	1.716(32)	2	1.187(12)	1	1.20(2)
$f_+^{D\pi}(0)$			1	0.666(29)		
$f_+^{DK}(0)$			1	0.747(19)		
f_B(MeV)	1	186(4)	4	192.0(4.0)	3	188(7)
f_{B_s}(MeV)	1	224(5)	4	228.4(3.7)	3	227(7)
f_{B_s}/f_B	1	1.205(7)	4	1.201(17)	3	1.206(23)
$f_{B_d}\sqrt{\hat{B}_{B_d}}$(MeV)			2	219(14)	1	216(10)
$f_{B_s}\sqrt{\hat{B}_{B_s}}$(MeV)			2	270(16)	1	262(10)
$\hat{B}_{B_d}$			2	1.26(9)	1	1.30(6)
$\hat{B}_{B_s}$			2	1.32(6)	1	1.32(5)
ξ			2	1.239(46)	1	1.225(31)
$\hat{B}_{B_s}/\hat{B}_{B_d}$			2	1.039(63)	1	1.007(21)

$$\alpha^{(5)}_{\overline{\rm MS}}(M_Z) = 0.1182(12) \quad \text{from 5 papers}$$
$$\Lambda^{(5)}_{\overline{\rm MS}} = 211(14)\,\text{MeV} \quad \text{from 5 papers}$$

- In order to improve the precision still further, isospin breaking (IB) effects, including radiative corrections, must be included.

Including IB corrections is the focus of this talk.

Consider $K_{\ell 2}$ decays in pure QCD:



- All QCD effects are contained in a single constant, f_K, the kaon's *(leptonic) decay constant.*

$$\langle 0|\,\bar s\gamma^\mu\gamma^5 u\,|K(p)\rangle = if_K\, p^\mu\ . \qquad (f_\pi \simeq 132\,\mathrm{MeV})$$

- In pure QCD

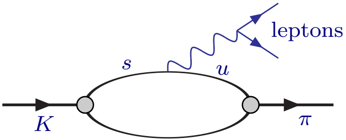
$$\Gamma(K^- \to \ell^-\bar\nu_\ell) = \frac{G_F^2\,|V_{us}|^2 f_K^2}{8\pi}\, m_K\, m_\ell^2 \left(1-\frac{m_\ell^2}{m_K^2}\right)^2 .$$

- From the experimental ratio of the $\pi_{\ell 2}$ and $K_{\ell 2}$ widths we get:

$$\frac{|V_{us}|}{|V_{ud}|}\,\frac{f_K}{f_\pi} = 0.2760(4)\,,$$ M.Moulson, arXiv:1411.5252, J.Rosner, S.Stone & R.Van de Water, arXiv:1509.02220

so that a precise determination of f_K/f_π will yield V_{us}/V_{ud} .

- Every collaboration calculates f_K and f_π (or uses f_π for calibration).



$\Rightarrow \quad V_{us}$

$$\langle\, \pi(p_\pi)\, |\bar{s}\gamma_\mu u\, |\, K(p_K)\,\rangle = f_0(q^2)\,\frac{M_K^2 - M_\pi^2}{q^2}\,q_\mu + f_+(q^2)\,\left[(p_\pi + p_K)_\mu - \frac{M_K^2 - M_\pi^2}{q^2}\,q_\mu\right]$$

where $q \equiv p_K - p_\pi$.

$$\Gamma_{K\to\pi\ell\nu} = C_K^2\,\frac{G_F^2 m_K^5}{192\pi^3}\,I\,S_{\rm EW}[1 + 2\Delta_{\rm SU(2)} + \Delta_{EM}]\,|V_{us}|^2\,|f_+(0)|^2$$

From the experimental measurement of the width we get:

$$|V_{us}|\,f_+(0) = 0.2165(4)\,,$$ M.Moulson, arXiv:1411.5252

so that a precise determination of $f_+(0)$ will yield V_{us}.

Most of the remainder of this talk is based on ongoing work and the following papers:

1. *QED Corrections to Hadronic Processes in Lattice QCD,*
   N.Carrasco, V.Lubicz, G.Martinelli, C.T.Sachrajda, N.Tantalo, C.Tarantino and M.Testa,
   Phys. Rev. D **91** (2015) no.7, 07450 [arXiv:1502.00257 [hep-lat]].
2. *Finite-Volume QED Corrections to Decay Amplitudes in Lattice QCD*,
   V.Lubicz, G.Martinelli, C.T.Sachrajda, F.Sanfilippo, S.Simula and N.Tantalo,
   Phys. Rev. D **95** (2017) no.3, 034504 [arXiv:1611.08497 [hep-lat]].
3. *First Lattice Calculation of the QED Corrections to Leptonic Decay Rates*,
   D.Giusti, V.Lubicz, G.Martinelli, C.T.Sachrajda, F.Sanfilippo, S.Simula, N.Tantalo and C.Tarantino,
   Phys. Rev. Lett. **120** (2018) 072001 [arXiv:1711:06537]

- The precision of “standard" isosymmetric QCD calculations is now such that in order to improve the precision still further isospin breaking (IB) effects (including electromagnetism) need to be included.
- These are
$$O\left(\frac{m_u - m_d}{\Lambda_{\rm QCD}}\right) \quad \text{and} \quad O(\alpha)\,,$$
i.e. $O(1\%)$ or so.
- {The separation of IB corrections into those due to $m_u \neq m_d$ and those due to electromagnetism requires a convention. It is only the sum which is physical.}
- Such calculations for the spectrum have been performed for a few years now, with perhaps the most noteworthy result being BMW Collaboration, arXiv:1406.4088
$$m_n - m_p = 1.51(16)(23)\,\text{MeV}$$
to be compared to the experimental value of 1.2933322(4) MeV.
- I stress that including electromagnetic effects, where the photon is massless of course, required considerable theoretical progress, e.g.
$$\int \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2}\cdots \Rightarrow \frac{1}{L^3T}\sum_k \frac{1}{k^2}\cdots$$
and we have to control the contribution of the zero mode in the sum.

- Calculating electromagnetic corrections to decay amplitudes has an added major complication, not present in computations of the spectrum,

the presence of infrared divergences

- This implies that when studying weak decays, such as e.g. $K^+ \to \ell^+ \nu$ the physical observable must include soft photons in the final state

$$\Gamma(K^+ \to \ell^+ \nu_\ell(\gamma)) = \Gamma(K^+ \to \ell^+ \nu_\ell) + \Gamma(K^+ \to \ell^+ \nu_\ell \gamma)\,.$$

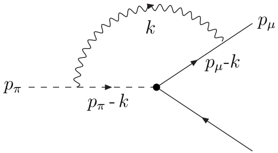
F.Bloch and A.Nordsieck, PR 52 (1937) 54

- The question for the lattice community is how best to combine this understanding with lattice calculations of non-perturbative hadronic effects.
- This is a generic problem if em corrections are to be included in the evaluation of a decay process.
- In 2015 we proposed a method for including electromagnetic corrections in decay amplitudes.

N.Carrasco, V.Lubicz, G.Martinelli, CTS, N.Tantalo, C.Tarantino & M.Testa, arXiv:1502.00257

- I stress (and will explain) that in order to implement this method successfully, it will be be necessary to work with the experimental community to ensure that we are calculating quantities which correspond to the experimental measurements.

Infrared Divergence - Example



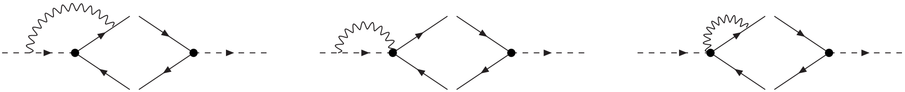
$$
\begin{aligned}
I \quad &\sim \quad \int_{\text{small } k} d^4k \, \frac{1}{(k^2+i\epsilon)((p_\mu-k)^2-m_\mu^2+i\epsilon)((p_\pi-k)^2-m_\pi^2+i\epsilon)} \\
&\sim \quad \int_{\text{small } k} d^4k \, \frac{1}{k^2(-2p_\mu\cdot k)(-2p_\pi\cdot k)} \\
&\sim \quad \int_{\text{small } k} d^4k \, \frac{1}{k^4} \qquad \Rightarrow \text{infrared divergence}\,.
\end{aligned}
$$

- This leads to a contribution to Γ_0 of

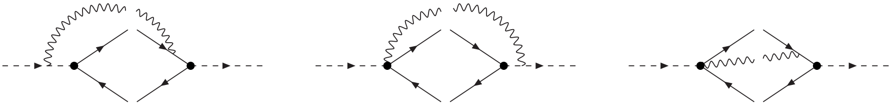
$$
\Gamma_0^{\pi\mu} = \Gamma_0^{\text{tree}} \, \frac{\alpha}{4\pi} \left(\frac{2(1+r_\mu^2)}{1-r_\mu^2} \log r_\mu^2 \log\left(\frac{m_\pi^2}{m_\gamma^2}\right) + \cdots \right) \,,
$$

where the photon mass, m_γ, is introduced to regulate the infrared divergences and $r_\mu = m_\mu/m_\pi$.

Infrared Divergence - Example (Cont)



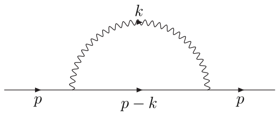
$$\Gamma_0^{\pi\mu} = \Gamma_0^{\rm tree}\,\frac{\alpha}{4\pi}\left(\frac{2(1+r_\mu^2)}{1-r_\mu^2}\log r_\mu^2\,\log\left(\frac{m_\pi^2}{m_\gamma^2}\right)+\cdots\right)\,,$$



(c) (d) (e)

$$\Gamma_1^{\pi\mu} = \Gamma_0^{\rm tree}\,\frac{\alpha}{4\pi}\left(-\frac{2(1+r_\mu^2)}{1-r_\mu^2}\log r_\mu^2\,\log\left(\frac{4\Delta E^2}{m_\gamma^2}\right)+\cdots\right)\,,$$

where $r_\mu = m_\mu/m_\pi$.



- There are no infrared divergence in the em corrections to hadron masses:

$$\begin{aligned} I &\sim \int_{\text{small }k} d^4k\,\frac{1}{(k^2+i\epsilon)((p-k)^2-m^2+i\epsilon)} \\ &\sim \int_{\text{small }k} d^4k\,\frac{1}{k^2(-2p\cdot k)} \\ &\sim \int_{\text{small }k} d^4k\,\frac{1}{k^3} \qquad \Rightarrow \text{no infrared divergence}\,. \end{aligned}$$

- For the spectrum the leading and next-to-leading finite-volume corrections are universal (independent of the hadron H):

$$m_H(L) = m_H\left[1 - Q_H^2\alpha\left(\frac{\kappa}{m_HL}\left(1+\frac{2}{m_HL}\right)\right) + O\left(\frac{1}{(m_HL)^3}\right)\right],$$

where $\kappa = 1.41865$ is a universal constant and the structure dependent terms start at $O(1/L^3)$. S.Borsanyi et al., arXiv:1406.4088

- Calculating electromagnetic corrections to decay amplitudes has an added major complication, not present in computations of the spectrum,

  the presence of infrared divergences

- This implies that when studying weak decays, such as e.g. $K^+ \to \ell^+\nu$ the physical observable must include soft photons in the final state

$$\Gamma(K^+ \to \ell^+\nu_\ell(\gamma)) = \Gamma(K^+ \to \ell^+\nu_\ell) + \Gamma(K^+ \to \ell^+\nu_\ell\gamma)\,.$$

F.Bloch and A.Nordsieck, PR 52 (1937) 54

- The question for the lattice community is how best to combine this understanding with lattice calculations of non-perturbative hadronic effects.
- This is a generic problem if em corrections are to be included in the evaluation of a decay process.
- In 2015 we proposed a method for including electromagnetic corrections in decay amplitudes.

  N.Carrasco, V.Lubicz, G.Martinelli, CTS, N.Tantalo, C.Tarantino & M.Testa, arXiv:1502.00257

- I stress (and will explain) that in order to implement this method successfully, it will be be necessary to work with the experimental community to ensure that we are calculating quantities which correspond to the experimental measurements.

$$\begin{aligned}\Gamma(K^+ \to \ell^+\nu_\ell(\gamma)) &= \Gamma(K^+ \to \ell^+\nu_\ell) + \Gamma(K^+ \to \ell^+\nu_\ell\gamma)\\ &\equiv \Gamma_0 + \Gamma_1\end{aligned}$$

- In principle, it is possible to compute Γ_1 nonperturbatively over a larger range of photon energies.
- At present we do not propose to compute Γ_1 nonperturbatively. Rather we consider only photons which are sufficiently soft for the point-like (pt) approximation to be valid.
  - The calculation of Γ_1 non-perturbatively is however, likely to happen in the medium term.
  - For pions and kaons at least, a cut-off ΔE of $O(10\text{ - }20\,\text{MeV})$ appears to be appropriate both experimentally and theoretically.
    F.Ambrosino et al., KLOE collaboration, hep-ex/0509045. arXiv:0907.3594, NA62
  - Question: What is the best way to translate the photon energy and angular resolutions at LHC, Belle II etc. into the rest frame of the decaying mesons?

- We now write $$\boxed{\Gamma_0 + \Gamma_1(\Delta E) = \lim_{V\to\infty}(\Gamma_0 - \Gamma_0^{\text{pt}}) + \lim_{V\to\infty}(\Gamma_0^{\text{pt}} + \Gamma_1(\Delta E))\,.}$$
  - pt stands for *point-like*.
  - The second term on the rhs in perturbation theory can be calculated in perturbation theory and we have done so. It is infrared convergent, but does contain a term proportional to $\log\Delta E$.
  - The first term is also free of infrared divergences.
  - Γ_0 is calculated non-perturbatively and Γ_0^{pt} in perturbation theory.
- Finite-volume effects take the form:

$$\Gamma_0^{\text{pt}}(L) = C_0(r_\ell) + \tilde{C}_0(r_\ell)\log(m_\pi L) + \frac{C_1(r_\ell)}{m_\pi L} + \dots\,,$$

where $r_\ell = m_\ell/m_\pi$ and m_ℓ is the mass of the final-state charged lepton.

The exhibited L-dependent terms are *universal*, i.e. independent of the structure of the meson!

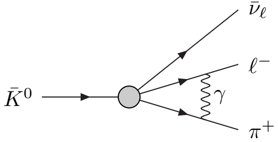
  - We have calculated the coefficients (using the QED_L regulator of the zero mode).
- The leading structure-dependent FV effects in $\Gamma_0 - \Gamma_0^{\text{pt}}$ are of $O(1/L^2)$.

V.Lubicz, G.Martinelli, CTS, F.Sanfilippo, S.Simula, N.Tantalo, arXiv:1611.08497

- Writing
$$\frac{\Gamma(K_{\mu 2})}{\Gamma(\pi_{\mu 2})} = \left|\frac{V_{us}}{V_{ud}}\frac{f_K^{(0)}}{f_\pi^{(0)}}\right|^2 \frac{m_\pi^3}{m_K^3}\left(\frac{m_K^2-m_\mu^2}{m_\pi^2-m_\mu^2}\right)^2 (1+\delta R_{K\pi})$$
where $m_{K,\pi}$ are the physical masses, we find
$$\delta R_{K\pi} = -0.0122(16)\,.$$
D.Giusti et al., arXiv:1711.06537

This first calculation can certainly be improved.
- $f_P^{(0)}$ are the decay constants obtained in iso-symmetric QCD with the renormalized $\overline{\mathrm{MS}}$ masses and coupling equal to those in the full QCD+QED theory extrapolated to infinite volume and to the continuum limit.
- This result can be compared to the PDG value, based on ChPT, is $\delta R_{K\pi} = -0.0112(21)$.
- Our result, together with $V_{ud} = 0.97417(21)$ from super-allowed nuclear β-decays gives $V_{us} = 0.22544(58)$ and
$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.99985(49)\,.$$
- We are now expanding this framework to semileptonic decays, such as $K\to\pi\ell\bar\nu_\ell$, where several new features arise, such as the dependence of the $1/L$ corrections on $df_\pm/dq^2$, which however are physical quantities.

- We are now expanding this framework to semileptonic decays, such as $\bar{K}^0 \to \pi^+ \ell \bar{\nu}_\ell$, where several new features arise. For illustration, one diagram of many is:



- Without QED the amplitude depends on two form factors $f_\pm(q^2)$, where q is the momentum transfer between the $\bar{K}^0$ and the π^+; $q = p_K - p_\pi = p_\ell + p_\nu$.
  - For leptonic decays of a pseudoscalar meson P, without QED the amplitude depends on a single number, the decay constant f_P.
  - With QED corrections to leptonic decays, the universal $1/L$ corrections only depend on f_P, calculated in QCD.
- For semileptonic decays, the $1/L$ corrections depend on $df_\pm/dq^2$ (which however are physical quantities) as well as on the form factors.
  - In both cases these corrections do not depend on the derivative w.r.t. the masses, which are not physical.
- We also need to extend the Lellouch-Lüscher formalism to include QED.

- In this talk I have focussed on one important challenge for the lattice community working *towards the ultimate precision in flavour physics*, i.e. the inclusion of isospin-breaking corrections (including radiative corrections).
  - We need to work with the experimental community to optimise the implementation of these $O(1\%)$ effects to extract precise fundamental information from experimental data.
- For leptonic decays $P \to \ell\bar{\nu}$ we now have a complete framework and the first numerical calculation has shown that it is practicable. $\surd$
  - Technical improvements can still be made.
- For semileptonic decays the same basic framework applies, but there are still technical issues to resolve:
  - The universal $1/L$ corrections need to be evaluated. (We have the expression in terms of integrals and sums which need to be evaluated.)
  - The Lellouch-Lüscher procedure has to be applied for the decays of neutral mesons.

I end by mentioning some novel RBC-UKQCD projects in flavour physics which extend the range of lattice calculations in flavour physics:

- Long-distance contributions to flavour changing processes

$$\iint d^4x\, d^4y\, \langle f\,|\,T[Q_1(x)\,Q_2(y)]\,|\,i\rangle\,.$$

Applications are to

1. the ab initio calculation of Δm_K,
2. long-distance contributions to ϵ_K,
3. rare-kaon decay amplitudes $K\to\pi\ell^+\ell^-$ and
4. the long-distance contributions to $K^+\to\pi^+\nu\bar\nu$ decay amplitudes.

An example of our results, presented at Lattice 2017, is

$$\Delta m_K = (5.5\pm 1.7)\times 10^{-12}\,\mathrm{MeV}\ \text{(preliminary)}$$

(experimental value $\Delta m_K = (3.483\pm 0.006)\times 10^{-12}$ MeV) .

Z.Bai, N.H.Christ and C.T.Sachrajda, EPJ Web Conf. **175** (2018) 13017.

- We continue our study of $K \to \pi\pi$ decays, exploring a number of avenues. An important aim is to improve on our determination of

$$\left.\frac{\epsilon'}{\epsilon}\right|_{\text{RBC-UKQCD}} = (1.38 \pm 5.15 \pm 4.59) \times 10^{-4}$$

to be compared with

$$\left.\frac{\epsilon'}{\epsilon}\right|_{\text{Exp}} = (16.6 \pm 2.3) \times 10^{-4}\,.$$

RBC-UKQCD, arXiv:1505.07863

- This is by far the most complicated project that I have ever been involved with.
- This single result hides much important (and much more precise) information which we have determined along the way.

- I warmly thank my collaborators from Rome and from the RBC-UKQCD collaborations with whom the ideas presented in this talk have been developed.