Model-independent Direct CPV Searches in multi-body decays

Chris Parkes on behalf of LHCb Collaboration

- Techniques
  - On model-independent searches for direct CP violation in multi-body decays, CP, S. Chen, J. Brodzicka, M. Gersabeck, G. Dujany, W. Barter arXiv:1612.04705
  - Calculating p-values and their significances with the Energy Test for large datasets, W. Barter, C. Burr, arXiv:1801.05222, 2018 JINST 13 P04011
- Energy test experimental results from LHCb
  - (LHCb $D^0 \rightarrow \pi^+\pi^-\pi^0$. PLB 740 (2015) 158.)
  - LHCb $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$. PLB 769 (2017) 345

- Multi-body – reduced statistics compared to two-body but

LHCb B➔hhh results excellent example:

Large local CPV
Small(er) global CPV

- Strong phases vary across resonances
- Rich structure of resonances across phase-space
- Local CPV search in phase space of 3-body / 4-body decays

Contents

- Methods used: Energy Test
- Parity-Even & Parity-Odd CP violation Tests
- $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ LHCb analysis results

LHCb >1fb^{-1} model-indep. Multi-body Charm

- Limited Number – "ultimate precision" will take time with current effort
- Current emphasis – charged hadrons, singly Cabibbo suppressed

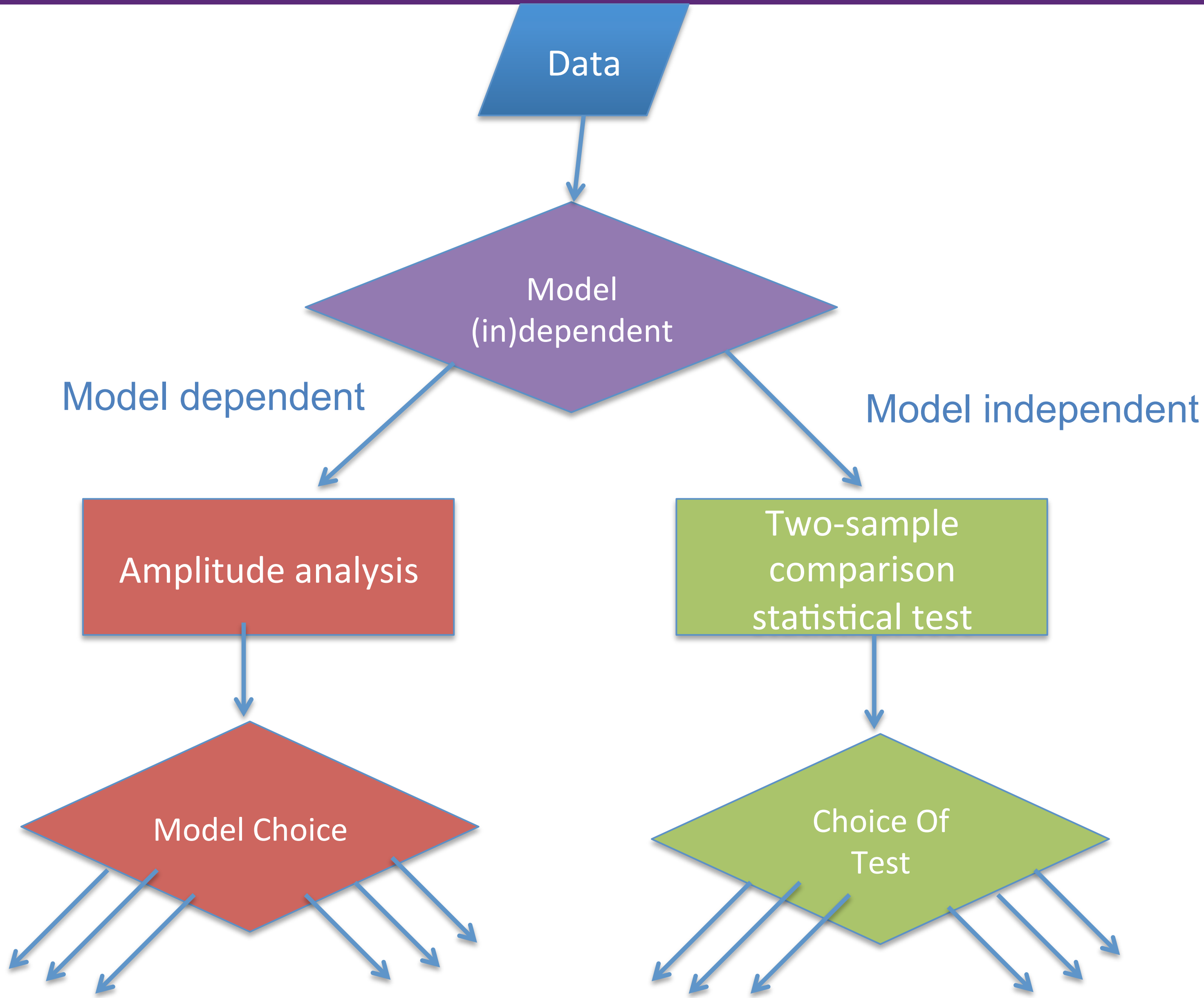
3-body

A measurement of the CP asymmetry difference in $\Lambda_c^+ \to pK^-K^+$ and $p\pi^-\pi^+$ decays	PAPER-2017-044 arXiv:1712.07051 [PDF]	JHEP
Search for CP violation in $D^0 \to \pi^-\pi^+\pi^0$ decays with the energy test	PAPER-2014-054 arXiv:1410.4170 [PDF]	Phys. Lett. B 740 (2015) 158
Search for CP violation in the decay $D^+ \to \pi^-\pi^+\pi^+$	PAPER-2013-057 arXiv:1310.7953 [PDF]	Physics Letters B 728 (2014) 585

4-body

Search for CP violation in the phase space of $D^0 \to \pi^+\pi^-\pi^+\pi^-$ decays	PAPER-2016-044 arXiv:1612.03207 [PDF]	Phys. Lett. B 769 345-356
Search for CP violation using T-odd correlations in $D^0 \to K^+K^-\pi^+\pi^-$ decays	PAPER-2014-046 arXiv:1408.1299 [PDF]	JHEP 10 (2014) 005
Model-independent search for CP violation in $D^0 \to K^-K^+\pi^-\pi^+$ and $D^0 \to \pi^-\pi^+\pi^+\pi^-$ decays	PAPER-2013-041 arXiv:1308.3189 [PDF]	Phys. Lett. B 726 (2013) 623

Flow Chart: How should I look for CPV ?

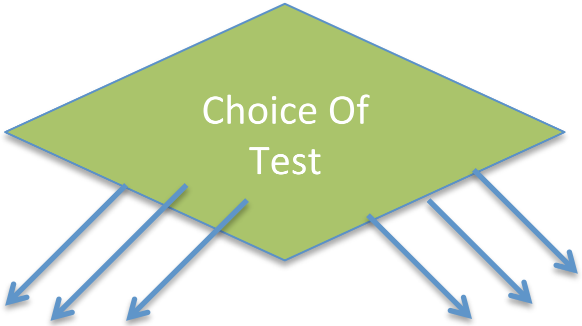


CPV search method

Model
(in)dependent

- If you have not yet observed CPV
  - Model independent
    - Much simpler! Quicker – important for “ultimate”
    - Weak reliance on simulation – important for “ult.”
- If you have observed CPV
  - Model dependent
    - Understand contribution to CPV in more detail
    - BUT is this true for “ultimate” event yields ?

Survey of Model independent tests used



Binned or Unbinned 2-sample tests
Compare CP-related samples in phase space

Techniques used in Exp’tal publications:

- Binned Chisquare
- Nearest Neighbours
- Angular Moments of helicity angles
- T-odd moments - Triple product momenta
- Energy Test

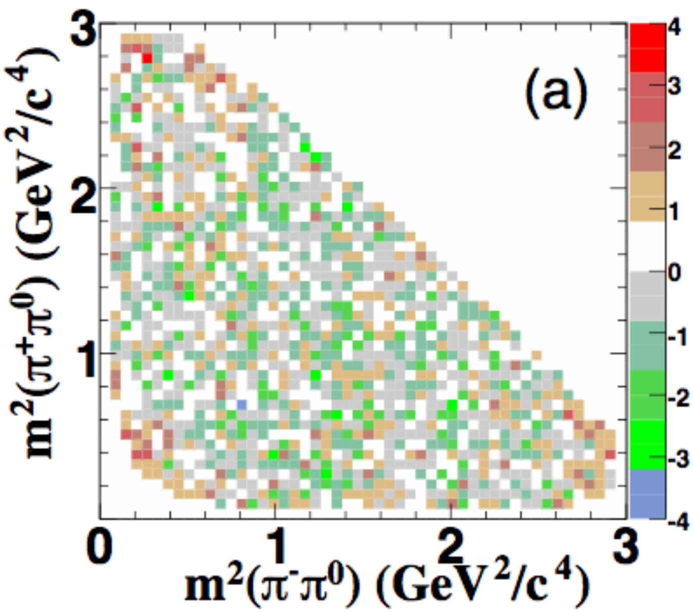
Binned Chi^2 (S_{CP} or Miranda)

Aubert B *et al* (BaBar collaboration) 2008 Search for *CP* violation in neutral D meson cabibbo-suppressed three-body decays *Phys. Rev.* D **78** 051102

$$S_{CP}^i = \frac{N^i(X) - \alpha N^i(\bar{X})}{\sqrt{N^i(X) + \alpha^2 N^i(\bar{X})}}, \qquad \alpha = \frac{N_{\text{tot}}(X)}{N_{\text{tot}}(\bar{X})},$$

$$\boxed{\chi^2 = \sum (S_{CP}^i)^2.}$$

- Compare events in bins
- between D^o /<u>D^o</u>
- α accounts for global asymmetries
  - Production
  - Detection

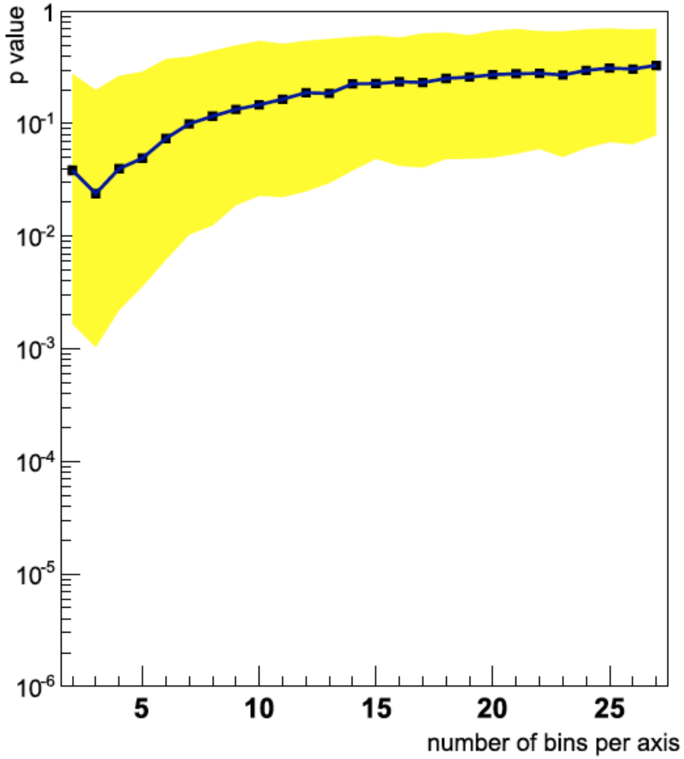


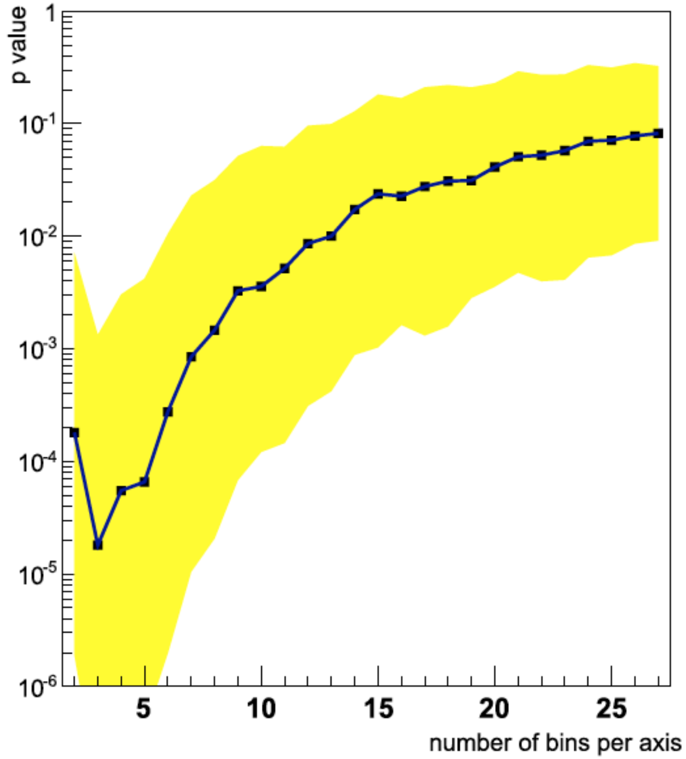
- Can also make asymmetry in bins(equivalent)

Optimizing S_{CP} Test

CP, S. Chen, J. Brodzicka, M. Gersabeck, G. Dujany, W. Barter, arXiv:1612.04705

- Toy amplitude model with CPV added





- Some uses in literature have thousands of bins
  - Each bin adds an extra degree of freedom
- Trade off: bins / local regions
- Recommend: small numbers of bins

Binned / Unbinned

- S_{CP} simple and fast
  - Sensitivity highly dependent on binning / bin boundaries
  - Can optimise: quasi-model dependent. equalising entries…
  - For multi-body n>3, binning should probably be avoided
- Unbinned
  - Many tests available e.g. see

Williams M 2010 How good are your fits? Unbinned multivariate goodness-of-fit tests in high energy physics *J. Instrum.* **5** P09004

Energy Test

On model-independent searches for direct *CP* violation in multi-body decays

J. Phys. G: Nucl. Part. Phys. **44** (2017) 085001 (15pp)

CP et al.

- Statistical Test Proposed in

Aslan B and Zech G 2005 New test for the multivariate two-sample problem based on the concept of minimum energy *J. Stat. Comput. Simul.* **75** 109

Aslan B and Zech G 2005 Statistical energy as a tool for binning-free, multivariate goodness-of-fit tests, two-sample comparison and unfolding *Nucl. Instrum. Methods* A **537** 626

- Suggested for Dalitz plots in

Williams M 2011 Observing *CP* violation in many-body decays *Phys. Rev.* D **84** 054015

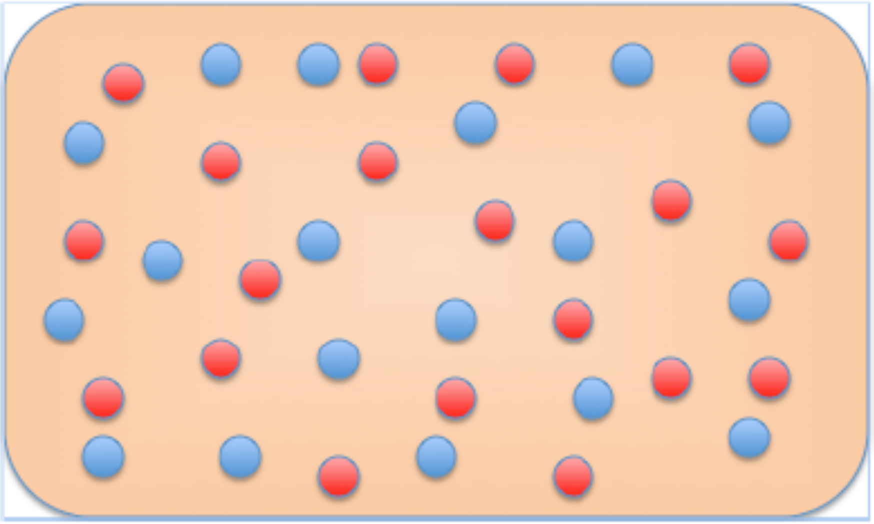
- Measurements performed in

Aaij R *et al* (LHCb collaboration) 2017 Search for *CP* violation in the phase space of $D^0 \to \pi^+\pi^-\pi^+\pi^-$ decays *Phys. Lett.* B **769** 345

Aaij R *et al* (LHCb collaboration) 2015 Search for *CP* violation in $D^0 \to \pi^-\pi^+\pi^0$ decays with the energy test *Phys. Lett.* B **740** 158

Energy Test

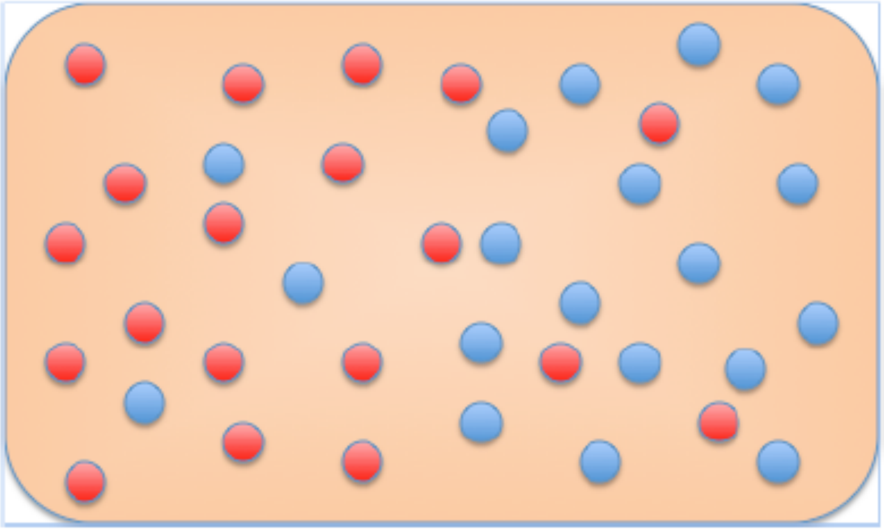
- Compare two distributions statistically
- Idea comes from the calculation of electric potential energy



+q and −q equally distributed,
electric potential energy = 0

Energy Test

- Compare two distributions statistically
- Idea comes from the calculation of electric potential energy



+q and −q equally distributed,
electric potential energy = 0

+q and −q distributions different,
electric potential energy > 0

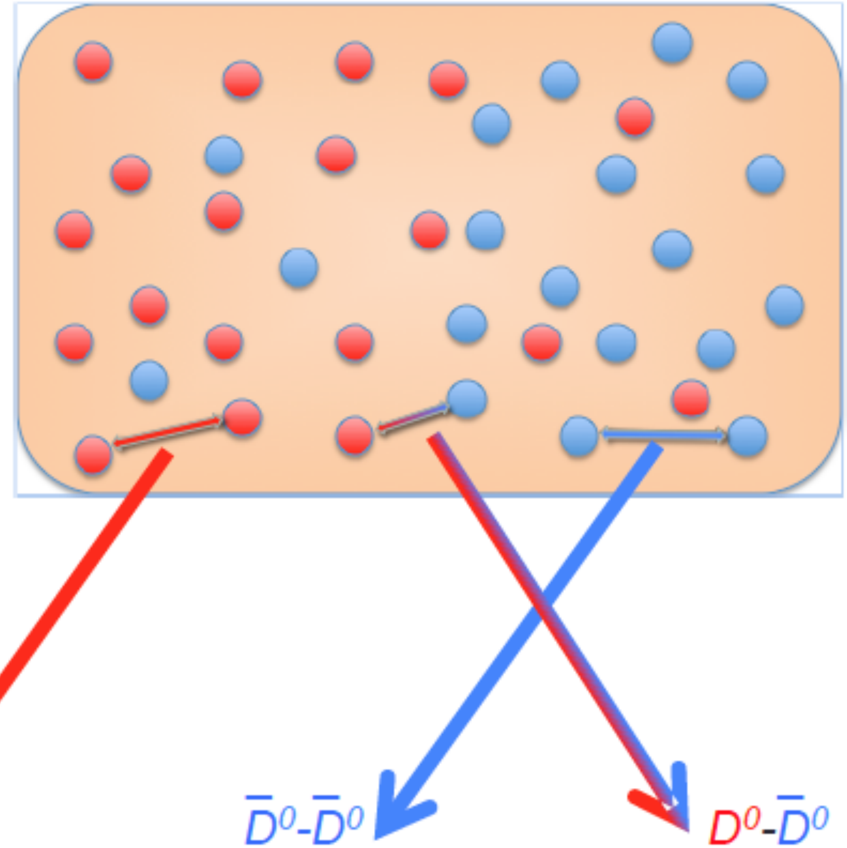
Energy Test

System → phase space
+q /−q → opposite
flavoured decays

$\psi(d_{ij}) = e^{-d_{ij}/2\delta^2}$: interaction potential

$n, \bar{n}$: number of $D^0, \overline{D}^0$ candidates

d_{ij} : distance in phase space

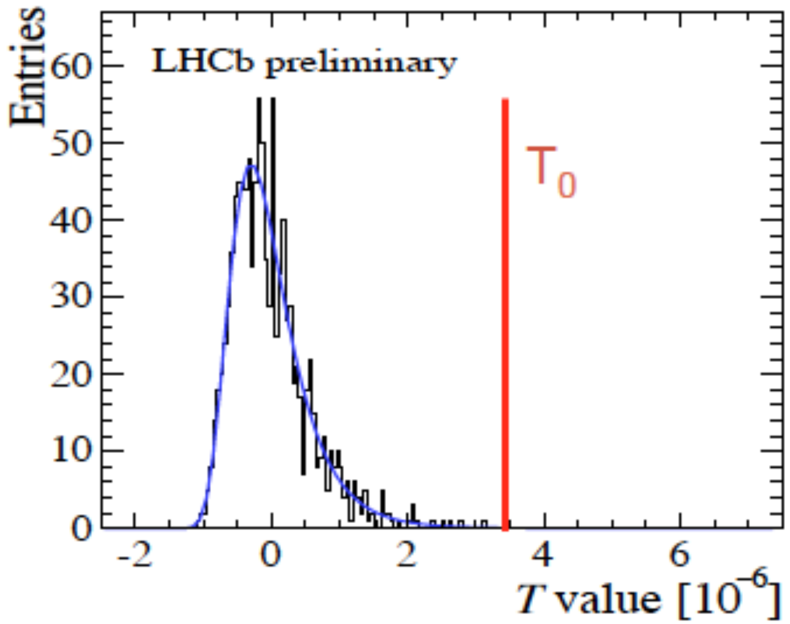


$$\text{Test statistic: } T = \frac{1}{n(n-1)} \sum_{i,j>i}^{n} \psi(d_{ij}) + \frac{1}{\bar{n}(\bar{n}-1)} \sum_{i,j>i}^{\bar{n}} \psi(d_{ij}) - \frac{1}{n\bar{n}} \sum_{i,j}^{n,\bar{n}} \psi(d_{ij})$$

- Each term is an average weighted distance between pairs of events
- The weighting Ψ(d) is a falling function of the distance of events in phase-space

Energy Test : Permutations

- Compare T-value from tested sample (T_0) with T-values from no-*CPV* samples
- No-*CPV* sample from permutation of data: randomly assign flavour tags
- *p*-value: fraction of permutation T-values above T_0



Small *p*-value, evidence of *CPV* !

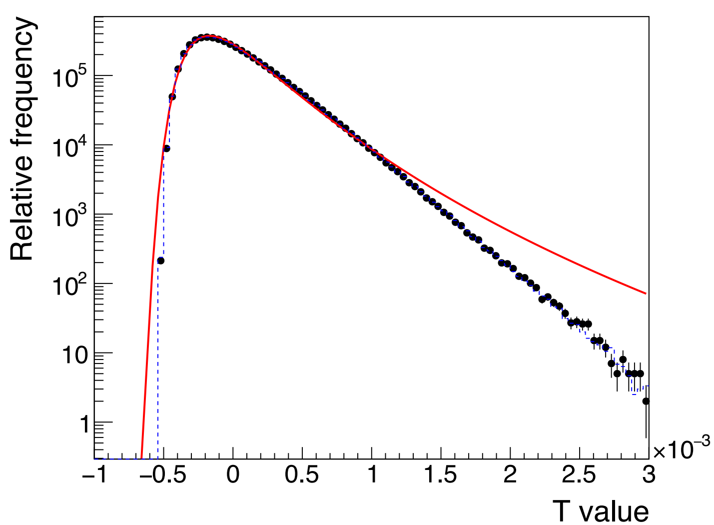
Issue: Calculating T-values can be a significant CPU demand
Solutions: multi-core CPU, GPU optimisation, permutation scaling

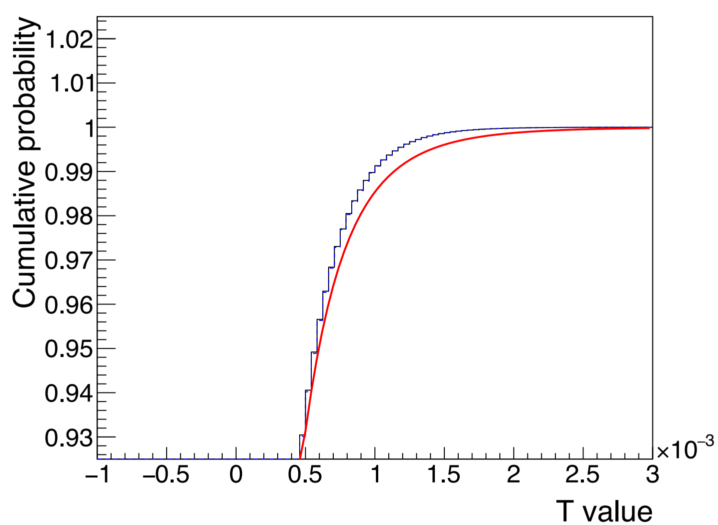
Scaling property of permutations

- Literature suggested Generalised Extreme Value distribution may fit distribution
  - – Not true for tails
- BUT can generate permutation distribution for small number of events and scale to that of large numbers

Black – permutations
Red- GEV function
Blue –scaled permutation distribution

- Helps control CPU/GPU demands for “ultimate” performance

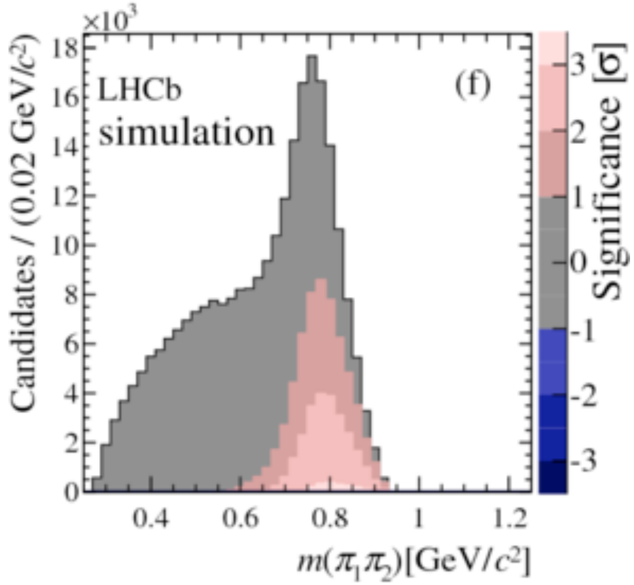


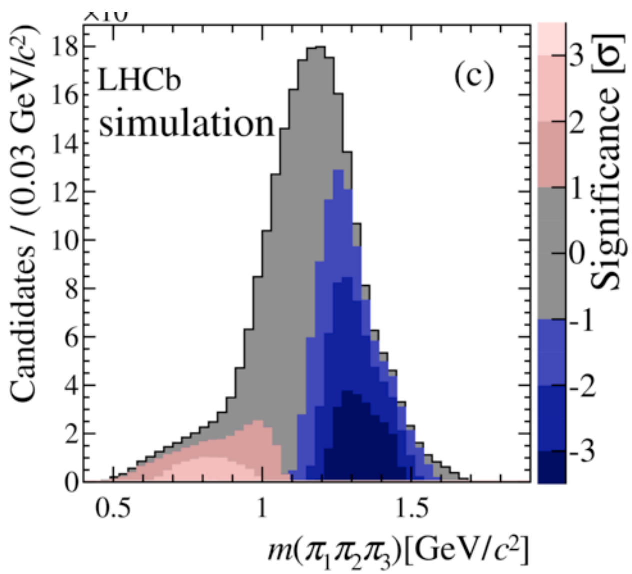


Visualise regions contributing to CPV

- Qualitative not Quantitative

Not a replacement for amplitude analysis





- Plot which events have significant contributions to the test statistic

- Thus localise CPV contributions to resonance regions in phase-space

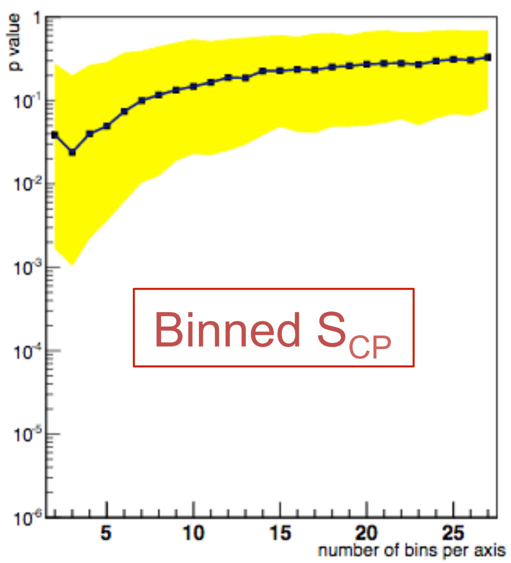
Plots are from LHCb D➔$\pi^+\pi^-\pi^+\pi^-$ analysis

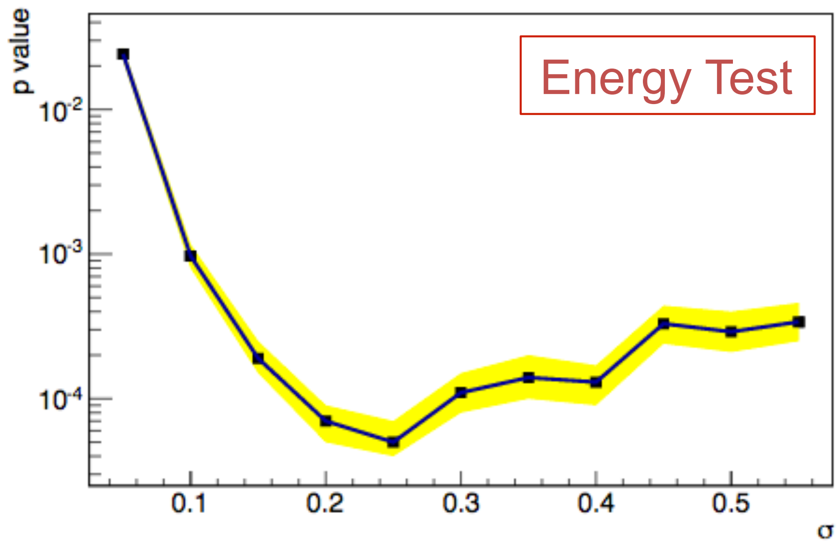
PLB 769 (2017) 345

Comparison of S_{CP}, Energy Test Performance

CP, S. Chen, J. Brodzicka, M. Gersabeck, G. Dujany, W. Barter, arXiv:1612.04705

- Sensitivity of energy test can be much higher than binned approach (not guaranteed)





- Studied here in generic X➔ABC model

with 3% change in amplitude of a main resonance

  – Even bigger differences in 4-body

Systematic Effects for "Ultimate" Precision

- These types of measurements currently check rather than control systematics
- Background Asymmetries
  - Check using events in side-bands in (m, Δm)
  - Can potentially correct for these (see backup)
- Detection Asymmetries
  - Use Check/Control Channel
  - e.g. For $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$, use $D^0 \rightarrow \pi^+\pi^-\pi^+K^-$
  - Correcting pion/kaon differences important
  - Data driven corrections will become more sophisticated, but make analyses more complex

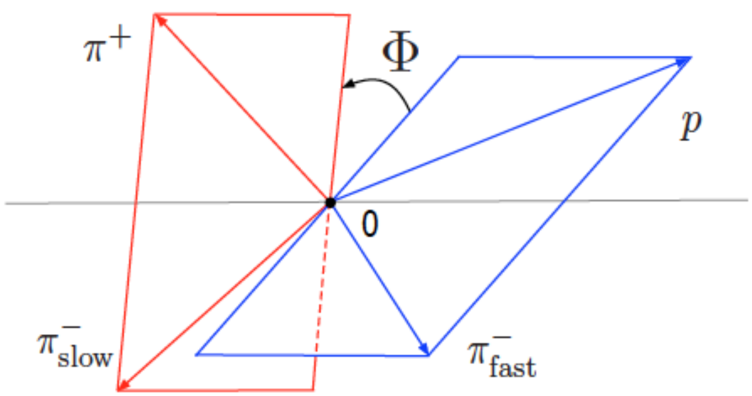
Parity-even, Parity-odd CP Violation

CP, S. Chen, J. Brodzicka, M. Gersabeck, G. Dujany, W. Barter, arXiv:1612.04705

For further discussion of “T-odd” observables see **Maurizio**’s talk

P-even and P-odd CP Violation

- CP Violation can occur in
  - P-parity even or P-parity odd contributions
- **Invariant masses are P-even**
  - **Five masses not fully characterise decay**
- Triple **p** product is a parity sensitive variable

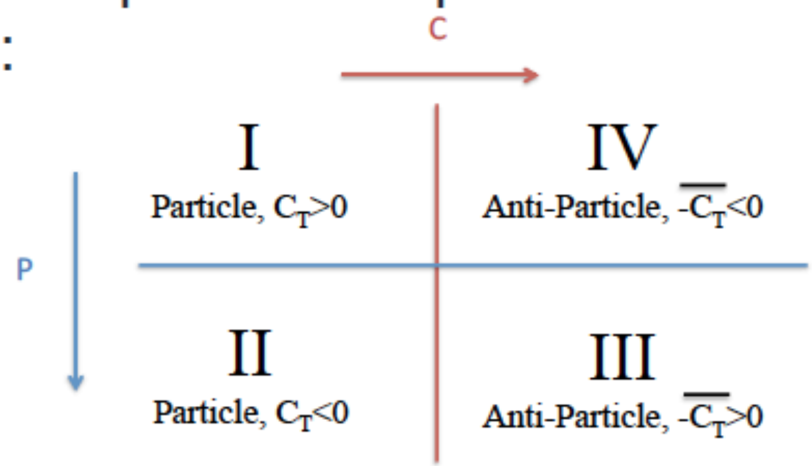


- Can test P-even and P-odd contributions to CP violation separately
  - Thus learn about CPV nature

P-even and P-odd CP Violation

New *P*-odd observables

- The total sample could be divided into four sub-samples according to the particle/antiparticle flavour and the triple product sign:



- Asymmetries may be measured in the C_T regions using the number of events populating the four samples

- P-parity even CPV: D^0 Sample (I+II) $\neq$ $\underline{D}^0$ Sample (III+IV)
- P-parity odd CPV: Sample (I+IV) $\neq$ Sample (II+III)

(Similar comparisons allow tests for Parity violation also)

LHCb $D^0 \to \pi^+\pi^-\pi^+\pi^-$

Search for CP violation in the phase space of $D^0 \to \pi^+\pi^-\pi^+\pi^-$ decays

arXiv:1612.03207v2

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Yield: $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$

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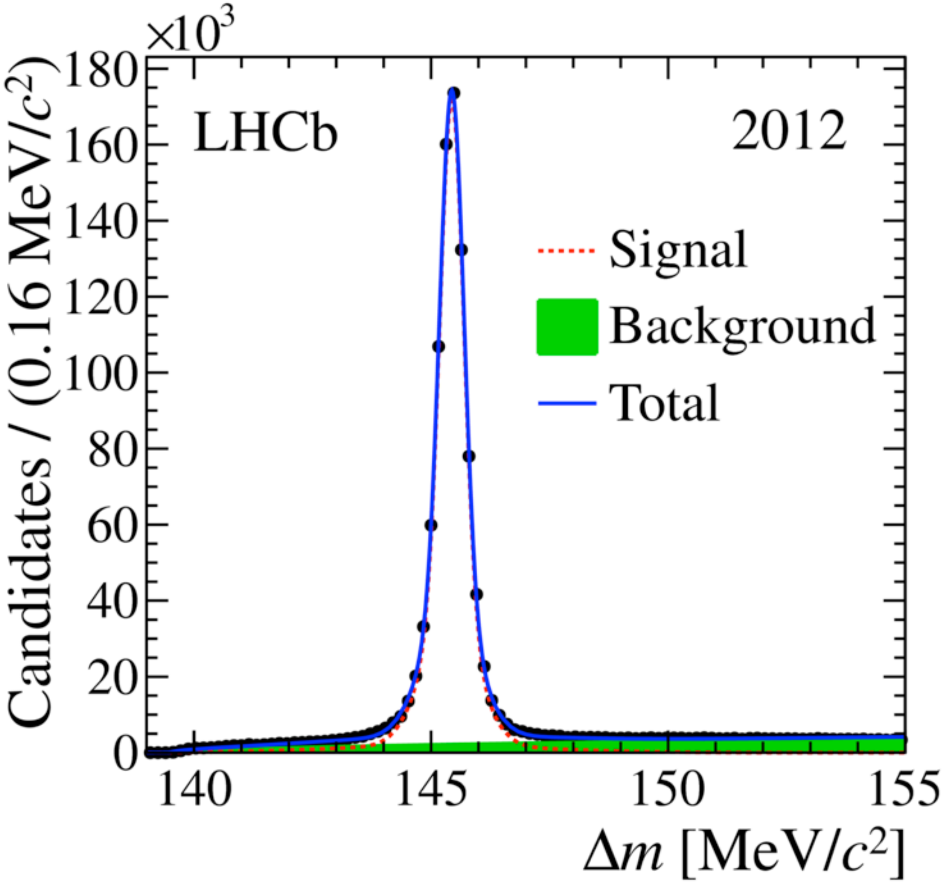

- 1.05M events in Run1 (2011/2012) data sample
- 96% Purity

P-even test: $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$

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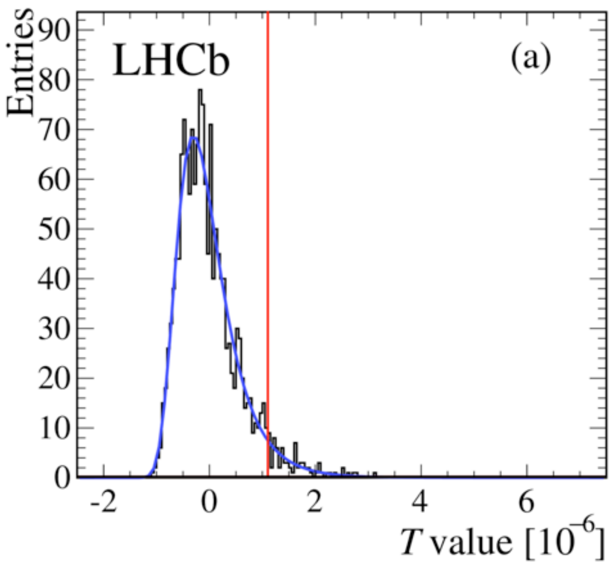

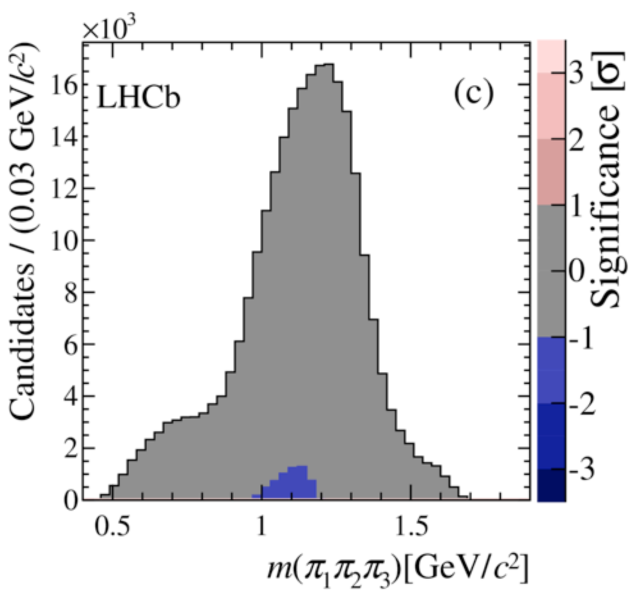

P-value 4.6±0.5%

No evidence for CPV

No strongly contributing regions

P-odd test: $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$

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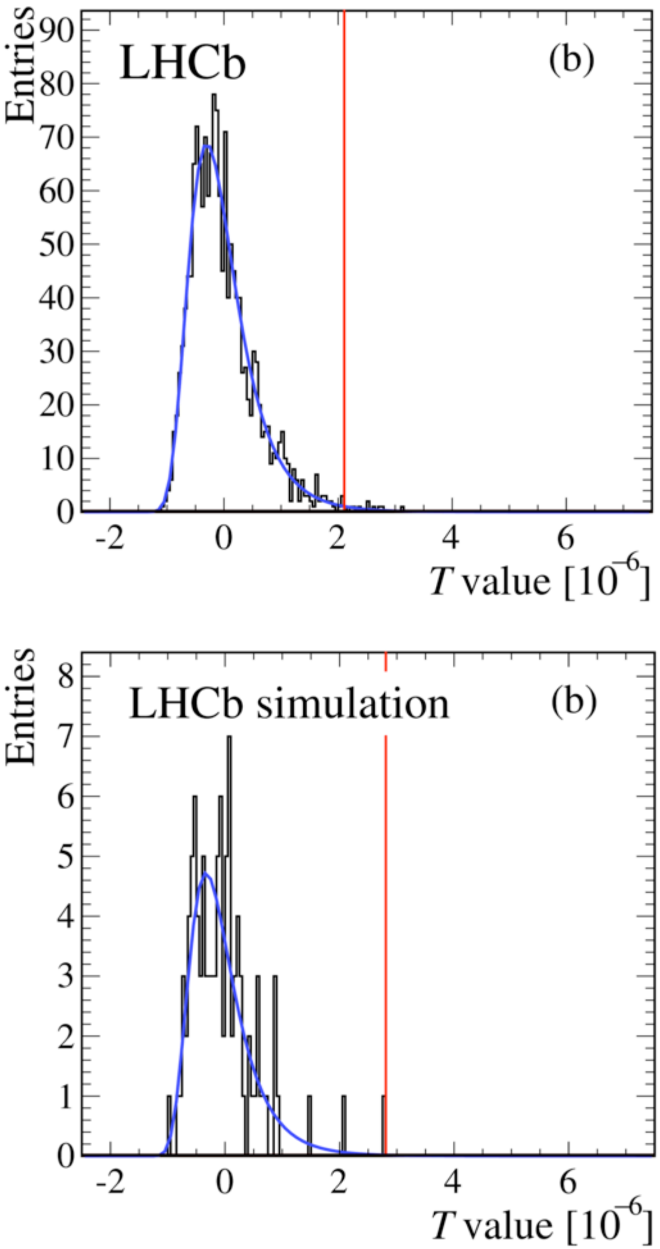

p-value 0.6±0.2%
"marginally consistent with CP symmetry"
2.7σ

Simulation of
P-wave $D^0 \rightarrow \rho^0(770)\rho^0(770)$

3^0 phase shift
between D^0 / $\underline{D^0}$

Simulation based on CLEO-c
Amplitude model

Phase Space Visualisation
$D^0 \rightarrow \pi^+\pi^-\pi^+\pi$ **P-odd** CPV test

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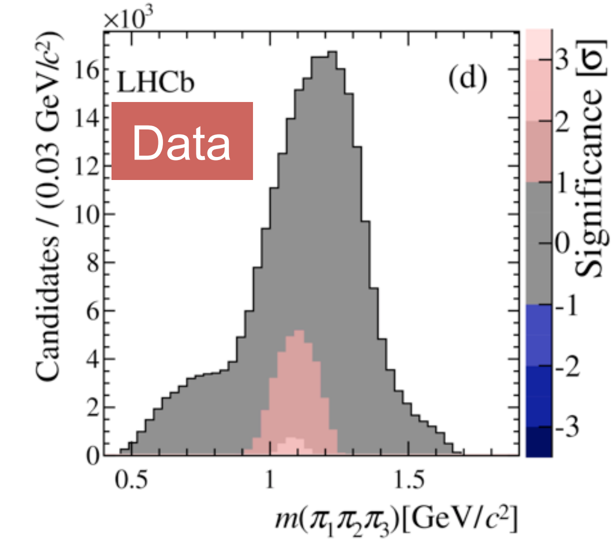

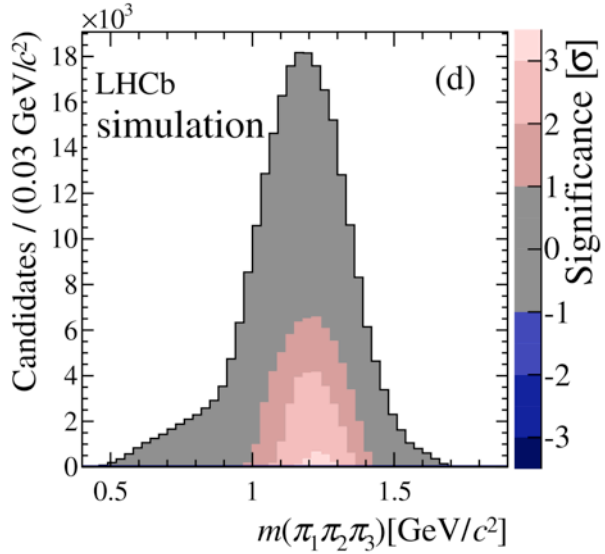

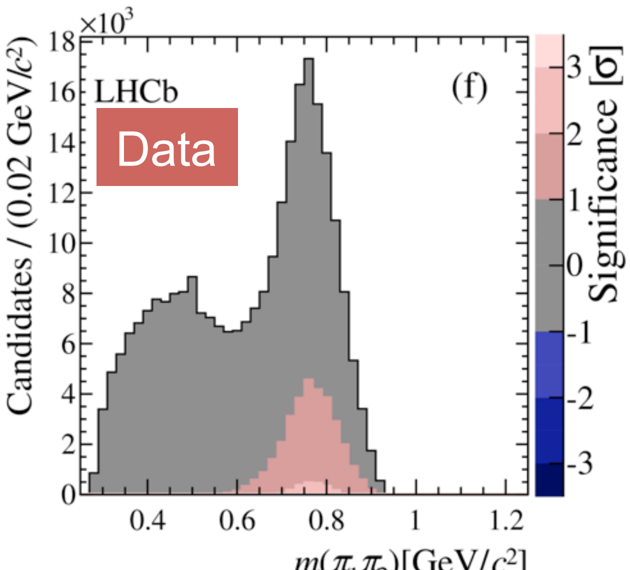

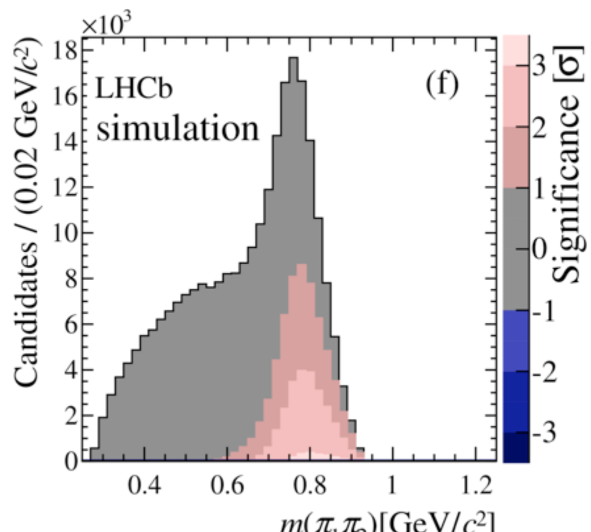

Simulation of P-wave $D^0 \rightarrow \rho^0(770)\rho^0(770)$

3^0 phase shift between D^0 / $\underline{D^0}$

Simulation based on CLEO-c amplitude model

arXiv:1703.08505v1

P. d'Argent[1], N. Skidmore[2], J. Benton[2], J. Dalseno[2], E. Gersabeck[1], S.T. Harnew[2], P. Naik[2], C. Prouve[2], J. Rademacker[2]

Simulation before Data unblinded

CPV search results $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$

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δ [GeV^2/c^4]	p-value P-even Counting	p-value P-even GEV fit	p-value P-odd Counting	p-value P-odd GEV fit
0.3	$(0.88 \pm 0.26)\%$	$(0.78 \pm 0.10)\%$	$(0.24 \pm 0.14)\%$	$(0.28 \pm 0.04)\%$
0.5	$(4.6 \pm 0.5)\%$	$(4.8 \pm 0.3)\%$	$(0.63 \pm 0.20)\%$	$(0.34 \pm 0.05)\%$
0.7	$(16 \pm 2)\%$	$(17 \pm 2)\%$	$(0.83 \pm 0.48)\%$	$(0.52 \pm 0.16)\%$

- Default weighting parameter was optimised **before** unblinding
  - This is the 0.5 GeV^2 counting method in abstract
  - p-value 0.63%
- Alternative weighting values also fixed before unblinding
  - 0.3 GeV^2 result gives
  - p-value < 0.3%, i.e. > 3σ
- Special claims require special evidence….

Summary

- Intriguing LHCb CPV search result in $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$
  PLB 769 (2017) 345
- Multi-body CPV local-asymmetry search methods
  – Unbinned Energy Test is powerful method
- Consider variables to characterise phase-space
- Test for P-even and P-odd CPV contributions across 4-body phase space
- Ever larger event samples will give interesting challenges
  – Trigger/CPU constraints
  – Controlling systematics

Backup

Characterising 4-body Phase-Space

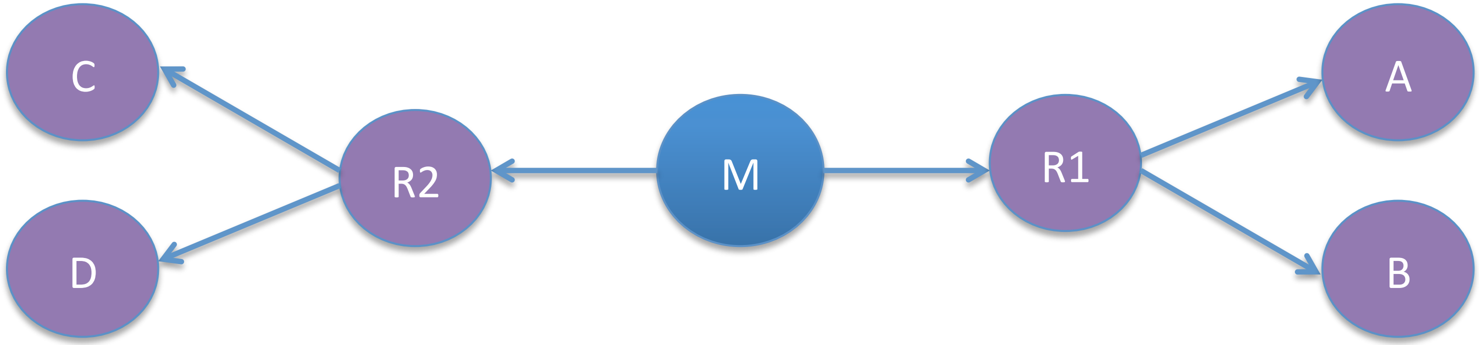
Independent of choice of 2-sample test

Characterising Phase Space

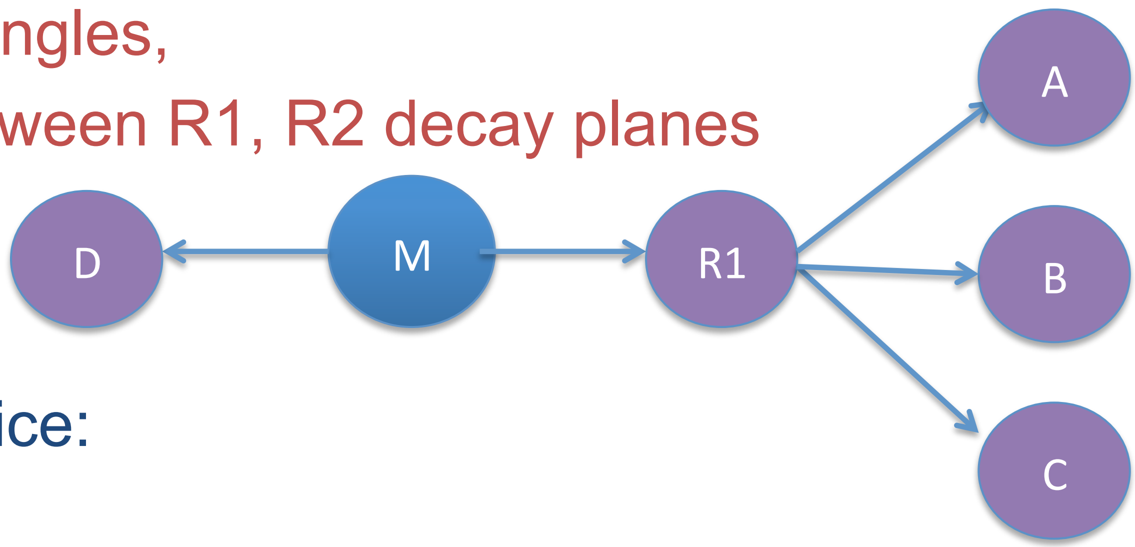
- Pseudo-scalar decays M➔ABC..(n)
  - 4n parameters
  - Impose: masses, E, **p** conservation, rotations
  - Independent Parameters: 3n-7

- Four-body decay requires 5 variables to characterise phase space
  - No preferred choice (unlike 3-body Dalitz plot)
  - Ten mass combinations
  - helicity angles, triple product
  - All contain same information content but will choice will change sensitivity of test

4-body decay topologies: M→ABCD



- A natural choice: transversity basis
  – Invariant mass R1,R2,
  – helicity angles,
  – angle between R1, R2 decay planes



- A natural choice:
  – Mass R1
  – A,B,C, three-body mass combinations…

Characterising Phase Space

- In some decays both topologies contribute
  - $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$
- We choose 5 masses in which primary resonance contributions appear
  - 2-body and 3-body invariant masses
    - 2-body : six $\frac{1}{2}n(n-1)$
    - 3-body: four $\frac{1}{3!}n(n-1)(n-2)$
    - Neglect those that are "unphysical" doubly charge
- Identical final state particles

$\pi_1 \pi_2 \pi_3 \pi_4$

$\pi^+\pi^-$ pair with the largest invariant mass is fixed to be $\pi_3\pi_4$,

Choices

$D^0 \to \pi_1^+\pi_2^-\pi_3^+\pi_4^-$	Two-body masses	Three-body masses
Unphysical	m_{13}, m_{24}	
Physical	m_{12}, m_{14}, m_{23}, m_{34}	m_{123}, m_{124}, m_{134}, m_{234}
Selected	m_{12}, m_{14}, m_{23}	m_{123}, m_{124}
$D^0 \to K_1^+K_2^-\pi_3^+\pi_4^-$	Two-body masses	Three-body masses
Unphysical	m_{13}, m_{24}	
Physical	m_{12}, m_{14}, m_{23}, m_{34}	m_{123}, m_{124}, m_{134}, m_{234}
Selected	m_{12}, m_{14}, m_{23}, m_{34}	m_{134}, (m_{234})

Relies on observing mass distributions or previous amplitude analyses

Brief Model Dependent Remarks

Model Dependent CPV search

Amplitude Analyses of $D^0 \to \pi^+\pi^-\pi^+\pi^-$ and $D^0 \to K^+K^-\pi^+\pi^-$ Decays

arXiv:1703.08505v1

CLEO-c data

P. d'Argent[1], N. Skidmore[2], J. Benton[2], J. Dalseno[2], E. Gersabeck[1], S.T. Harnew[2], P. Naik[2], C. Prouve[2], J. Rademacker[2]

- Good example of state-of-art

- Total amplitude of $D^0 \to h_1\, h_2\, h_3\, h_4$ modelled from:

$$A_{D^0}(\mathrm{x}) = \sum_i a_i\, A_i(\mathrm{x}) . \quad \text{with} \quad a_i = |a_i|\, e^{i\,\phi_i}$$

- Perform fit to total (D^o/$\underline{D^o}$) sample
  - Fix all model parameters

- Fit CPV by introducing D^o/$\underline{D^o}$ difference:

$$a_i \equiv c_i\,(1 + \Delta c_i), \;\; \bar{a}_i \equiv c_i\,(1 - \Delta c_i).$$

Fit for amplitudes and phases

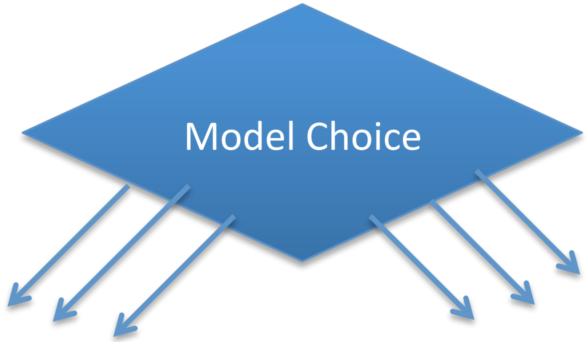
$$\mathcal{A}^i_{CP} \equiv \frac{|a_i|^2 - |\bar{a}_i|^2}{|a_i|^2 + |\bar{a}_i|^2}.$$

Direct CP asymmetry and significance for each component of the $D^0 \to \pi^+\pi^-\pi^+\pi^-$

Decay channel	$\mathcal{A}^i_{CP}$ (%)	Significance (σ)
$D^0 \to \pi^-\, a_1(1260)^+$	$+4.7 \pm 2.6 \pm 4.3 \pm 2.4$	0.9
$D^0 \to \pi^+\, a_1(1260)^-$	$+13.7 \pm 13.8 \pm 9.8 \pm 5.8$	0.8
$D^0 \to \pi^-\, \pi(1300)^+$	$-1.6 \pm 12.9 \pm 5.0 \pm 4.4$	0.1
$D^0 \to \pi^+\, \pi(1300)^-$	$-5.6 \pm 11.9 \pm 25.6 \pm 10.3$	0.2
$D^0 \to \pi^-\, a_1(1640)^+$	$+8.6 \pm 17.8 \pm 16.0 \pm 10.8$	0.3
$D^0 \to \pi^-\, \pi_2(1670)^+$	$+7.3 \pm 15.1 \pm 8.0 \pm 6.6$	0.4
$D^0 \to \sigma\, f_0(1370)$	$-14.6 \pm 16.5 \pm 9.3 \pm 1.3$	0.8
$D^0 \to \sigma\, \rho(770)^0$	$+2.5 \pm 16.8 \pm 13.8 \pm 14.6$	0.1
$D^0 \to \rho(770)^0\, \rho(770)^0$	$-5.6 \pm 5.0 \pm 2.2 \pm 1.9$	1.0
$D^0 \to f_2(1270)\, f_2(1270)$	$-28.3 \pm 12.3 \pm 18.5 \pm 9.7$	1.2

Model Dependent: Issues

$A_{D^0}(\mathbf{x})$



- Isobar, K-matrix…? New approaches ?
- Which resonances to include...?
  - Add a penalty clause to likelihood to favour simplicity

Least Absolute Shrinkage and Selection Operator (LASSO)

$$-2\log\mathcal{L} \to -2\log\mathcal{L} + \lambda\sum_i \sqrt{\int |a_i A_i(\mathbf{x})|^2 \, d\Phi_4},$$

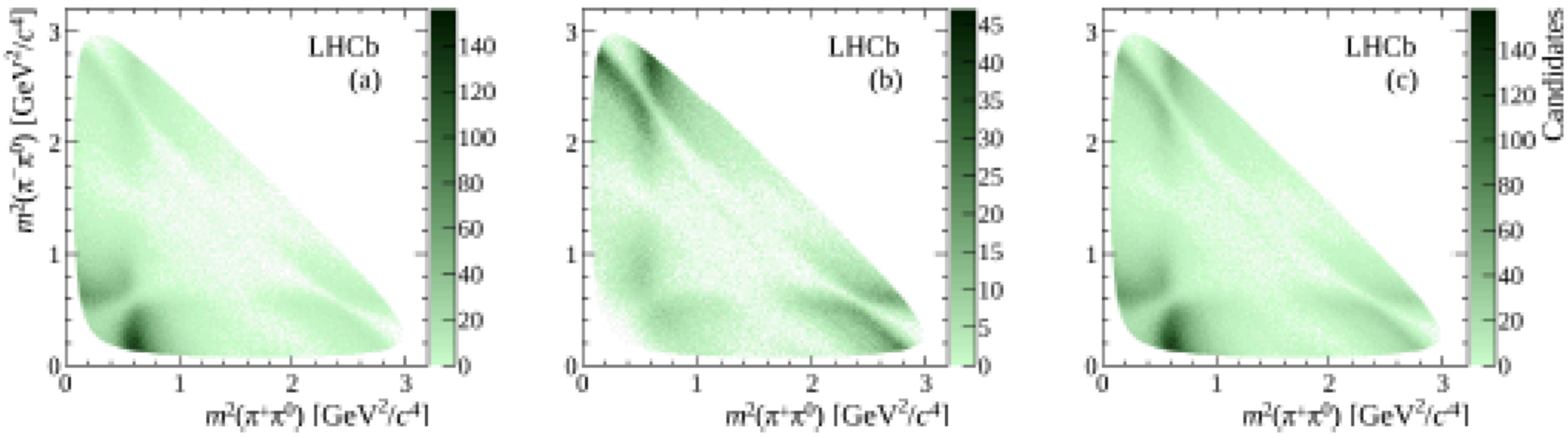
Model selection for amplitude analysis

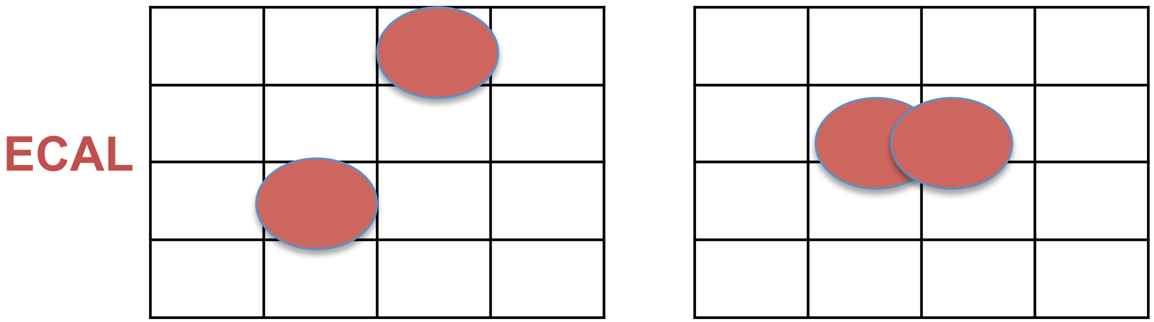
Baptiste Guegan, John Hardin, Justin Stevens and Mike Williams

arXiv:1505.05133v2

  - Cynicism: maybe not quite as clean as this suggests ?
- Modelling the background
- Modelling the efficiency across DP
  - resolution,wrong-tag....

Efficiency can strongly depend on DP position



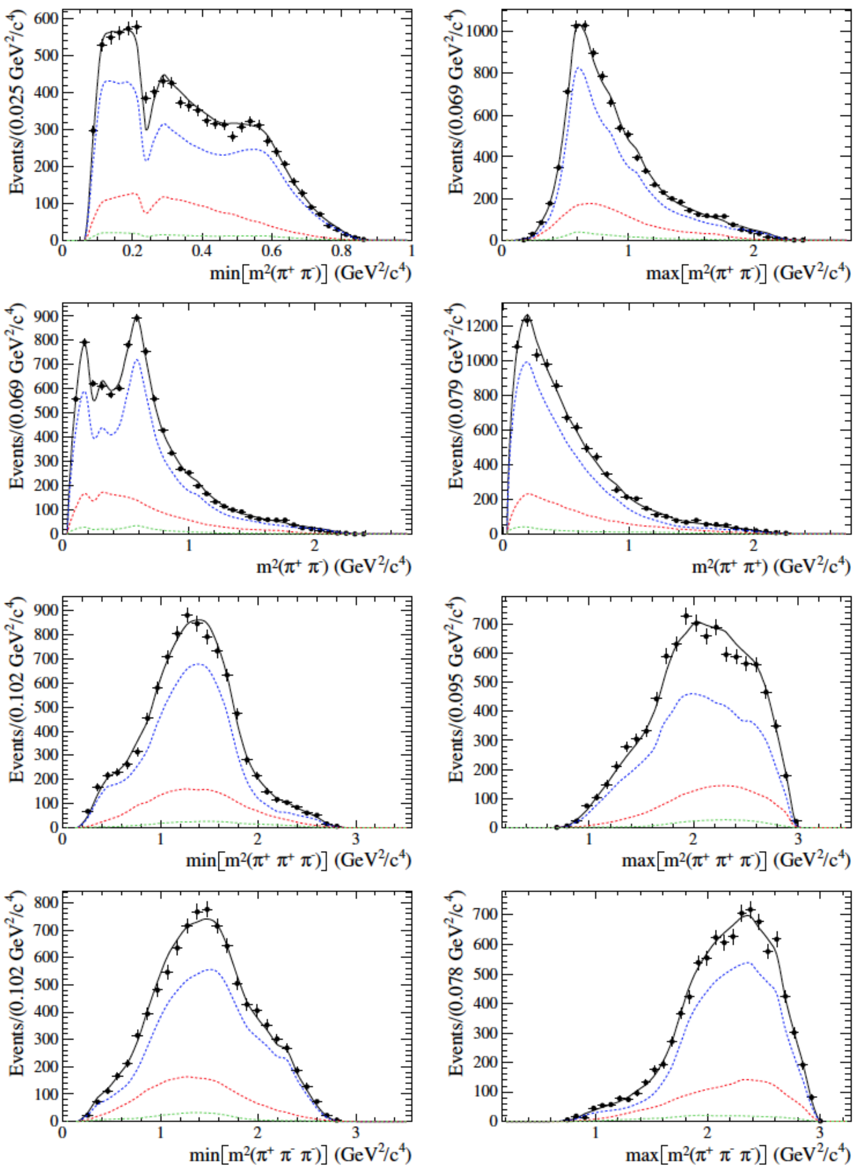


- Combination of samples

- Resolved photons
- Merged photons

Model Dependent: $A_{D^0}(\mathbf{x})$ Model

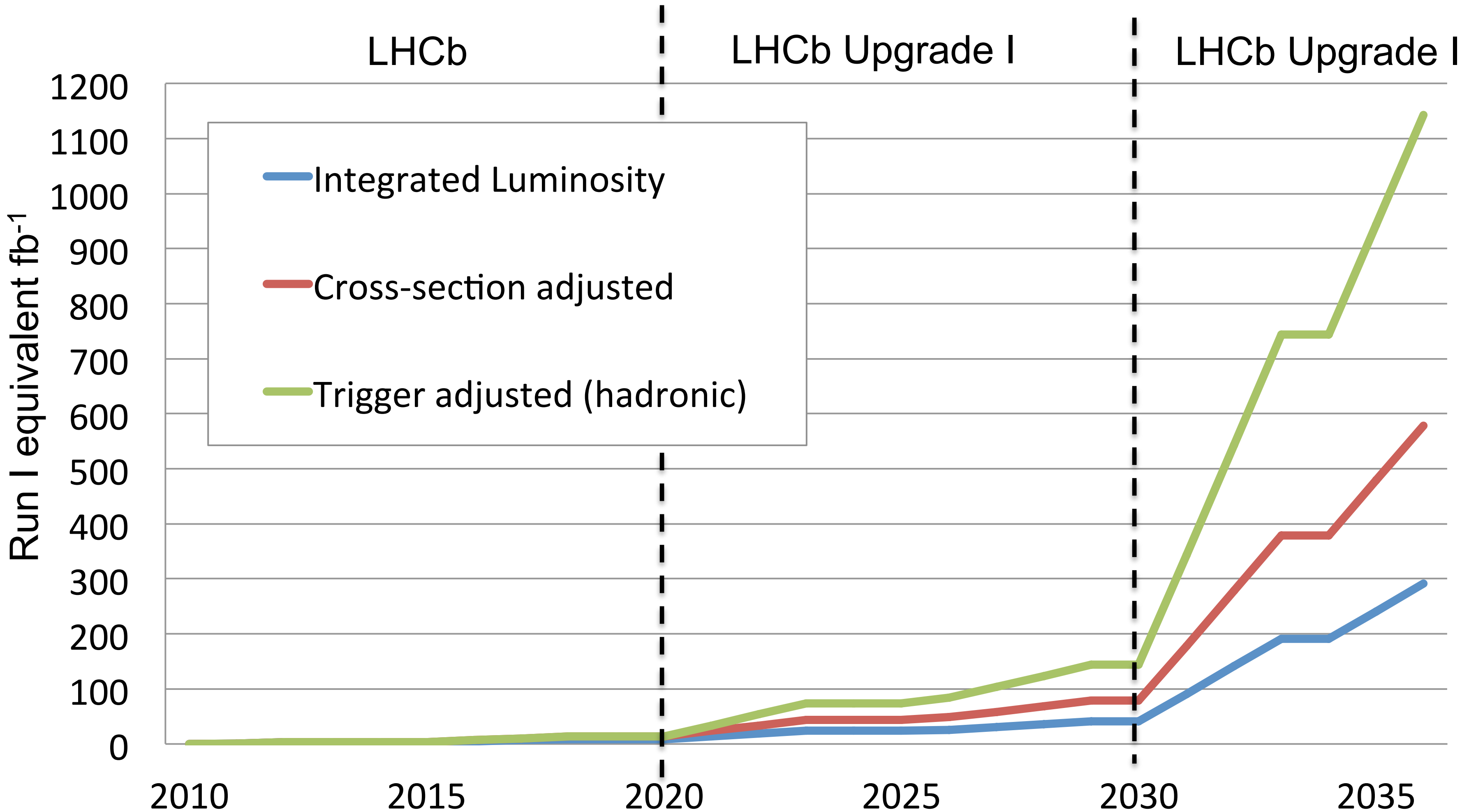
Invariant mass distributions of $D^0 \to \pi^+\pi^-\pi^+\pi^-$



- This is state-of-art
- The pulls aren't shown ☺
- Number of events:
- CLEO-c: $D^0 \to \pi^+\pi^-\pi^+\pi^-$

  $7250 \pm 56\ (\text{stat}) \pm 46\ (\text{syst})$
- LHCb Run 1: $D^0 \to \pi^+\pi^-\pi^+\pi^-$
  - 1.05M (D* tagged)
- LHCb Run 2 (end 2018):
  - ~10M (D* tagged)

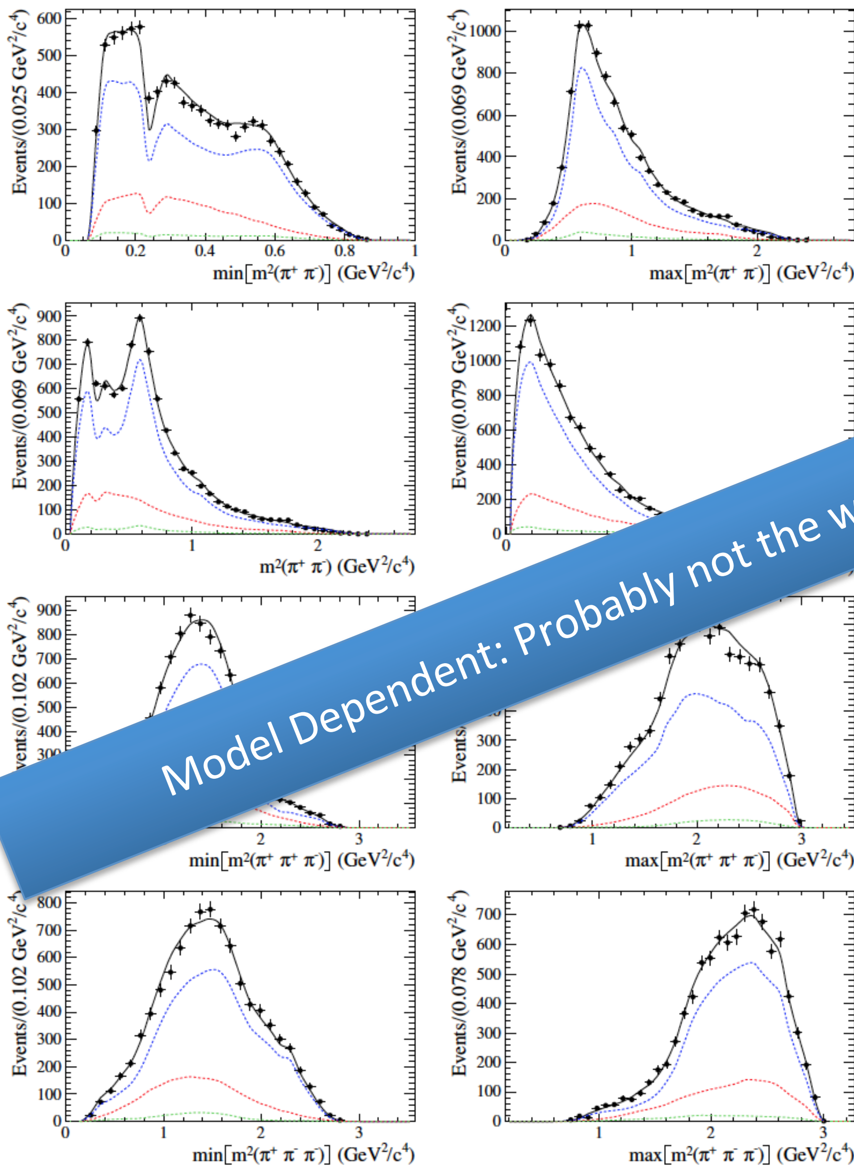
LHCb Statistics- Timeline



- Indicative of potential only
- Assumptions made on relative trigger efficiencies have significant uncertainty

Model Dependent: $A_{D^0}(\mathbf{x})$ Model

Invariant mass distributions of $D^0 \to \pi^+\pi^-\pi^+\pi^-$



- This is state-of-art
- The pulls aren't shown ☺
- Number of events:
- CLEO-c: ... π^-

  $725\ldots$ ...at$) \pm 46$ (syst)
- LHCb Run 1: $D^0 \to \pi^+\pi^-\pi^+\pi^-$
  - 1.05M (D* tagged)
- LHCb Run 2 (end 2018):
  - ~10M (D* tagged)

Model Dependent: Probably not the way to find CPV in charm multi-body decays?

Dealing with Background

Energy Test in the presence of asymmetric background

D^0-D^0 D^0- bckg bckg - bckg

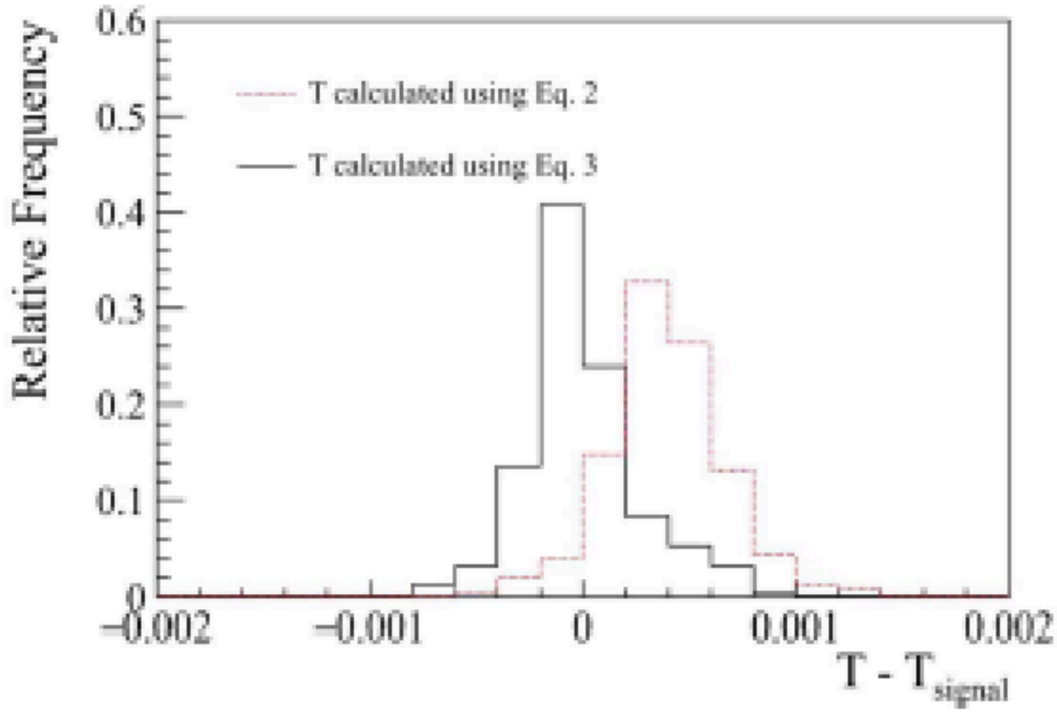
$$T = \frac{1}{2w(w-1)}\left(\sum_i^n\sum_{j\neq i}^n \psi_{ij} - \frac{2b}{b_s}\sum_i^n\sum_j^{b_s}\psi_{ij} + \frac{b(b+1)}{b_s(b_s-1)}\sum_i^{b_s}\sum_{j\neq i}^{b_s}\psi_{ij}\right) \quad \text{“}D^0\text{-}D^0\text{”}$$

$$+ \frac{1}{2\bar{w}(\bar{w}-1)}\left(\sum_i^{\bar{n}}\sum_{j\neq i}^{\bar{n}} \psi_{ij} - \frac{2\bar{b}}{\bar{b}_s}\sum_i^{\bar{n}}\sum_j^{\bar{b}_s}\psi_{ij} + \frac{\bar{b}(\bar{b}+1)}{\bar{b}_s(\bar{b}_s-1)}\sum_i^{\bar{b}_s}\sum_{j\neq i}^{\bar{b}_s}\psi_{ij}\right) \quad \text{“}\underline{D}^0\text{-}\underline{D}^0\text{”}$$

$$- \frac{1}{w\bar{w}}\left(\sum_i^{n}\sum_{j}^{\bar{n}} \psi_{ij} - \frac{\bar{b}}{\bar{b}_s}\sum_i^{n}\sum_j^{\bar{b}_s}\psi_{ij} - \frac{b}{b_s}\sum_i^{b_s}\sum_j^{\bar{n}}\psi_{ij} + \frac{b\bar{b}}{b_s\bar{b}_s}\sum_i^{b_s}\sum_{j}^{\bar{b}_s}\psi_{ij}\right), \quad \text{“}D^0\text{-}\underline{D}^0\text{”}$$

- Add terms for background to each of the three original terms: D^0-D^0, $\underline{D}^0$-$\underline{D}^0$, D^0-$\underline{D}^0$
  - Each term: has signal-signal, signal-bckg, bckg-bckg
- Background distributions from mass side-bands for example

Energy Test in the presence of asymmetric background



- Unbiased distribution after correction in simulation test

Issues

Energy Test – the issues

- (1) Sum runs over all pairs of events
  - As event samples increases can become time-consuming
    - We implemented with GPUs
      - Our implementation available at: https://manet.hepforge.org/
    - and optimised for CPUs
- (2) Distribution of test statistic is not known
  - (though some scaling properties are)
  - Need no-CPV distribution to interpret result
- Sample sizes increasing much faster than computing power

Energy Test: Tuning Free Parameter

Weighted phase-space distance

$$\psi_{ij} \equiv \psi(d_{ij}) = \mathrm{e}^{-d_{ij}^2/2\sigma^2}$$

Falls with distance as searching for local CPV

$$\Delta\vec{x}_{ij} = (m_{12}^{2,j} - m_{12}^{2,i}, m_{23}^{2,j} - m_{23}^{2,i}, m_{13}^{2,j} - m_{13}^{2,i})$$

- Tune weighting factor in phase-space distance
- Relies on amplitude model….

- Results usually not strongly dependent on choice

Systematic Effects

- Background Asymmetries
  - Check using events in side-bands in $(m, \Delta m)$
- Detection Asymmetries
  - Use Control Channel
  - e.g. For $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$, use $D^0 \rightarrow \pi^+\pi^-\pi^+K^-$

Tests

Unbinned Tests

- Nearest Neighbours

  used in Aaij R *et al* (LHCb collaboration) 2014 Search for *CP* violation in the decay $D^+ \to \pi^-\pi^+\pi^+$ *Phys. Lett.* B **728** 585

- Kolmogorov-Smirnov
  – Can be extended to multi-dimensions
  – Popular is astronomy community

- Neither well suited to Dalitz plot CPV searches

- Many other options…here discuss....

P-even/odd Complementary Sensitivity

Simulation D to 4 pi

Asymmetries in $D^0 \to \rho^0\rho^0(P\text{-wave})$	P-even test p-value (fit)	P-odd test p-value (fit)
Δphase 4°, ΔAmplitude 0	$0.30^{+0.03}_{-0.03}$	$1.95^{+0.06}_{-1.95} \times 10^{-4}$
Δphase 0°, ΔAmplitude 4%	$3.02^{+1.2}_{-0.9} \times 10^{-3}$	$0.41^{+0.03}_{-0.03}$

- A number of papers have tested for either:

P-even **or** P-odd contributions to CP Violation

- Should test for BOTH
- In any 4-body decay with potential P-odd contribution