Probing CP in multi-body and baryon decays

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(DESY)

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with Yuval Grossman, spin-0 multibody decays

JHEP 10 (2016) 005, [1608.03288]
spin-1/2 multibody decays

16 April 2018, Warwick
Towards the ultimate precision in flavour physics

Multibody decays

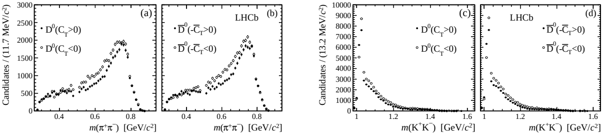
- Large statistics

$B_s^0 \to \phi\phi \to K^+K^-K^+K^-$	3950 ± 67 candidates	[1407.2222]
$B_s^0 \to K^+\pi^-K^-\pi^+$	6080 ± 83	[1712.08683]
$D^0 \to K^+K^-\pi^+\pi^-$	$171\,300 \pm 600$	[1408.1299]
$D^0 \to K^-\pi^+\pi^+\pi^-$	$890\,701 \pm 927$	[1712.08609]
$D^0 \to \pi^+\pi^-\pi^+\pi^-$	$1\,040\,000$	[1612.03207]
$\Lambda_b^0 \to p\pi^-\pi^+\pi^-$	$6\,646 \pm 105$	[1609.05216]
$\Lambda_b \to pK^-J/\psi \to pK^-\mu^+\mu^-$	$28\,834 \pm 204$	[1603.06961]

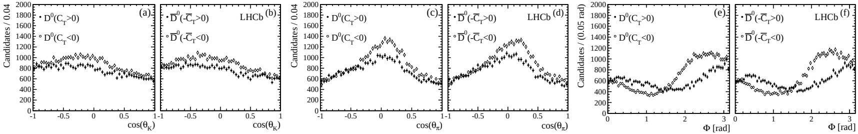
...

- Multidimensional phase space

e.g. 5d for a four-body meson decay like $D^0 \to K^+K^-\pi^+\pi^-$:

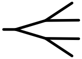
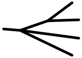
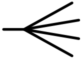


[1408.1299]



The paradox of richness and complexity

- Rich variety of interfering contributions

	Intermediate states in $D^0 \to K^+K^-\pi^+\pi^-$	Br $/10^{-4}$
	$(\phi\rho^0)_S, \quad \phi \to K^+K^-, \quad \rho^0 \to \pi^+\pi^-$	9.3 ± 1.2
	D	0.83 ± 0.23
	$(K^{*0}\overline{K}^{*0})_S, \quad K^{*0} \to K^\pm\pi^\mp$	1.48 ± 0.30
	$\phi(\pi^+\pi^-)_S, \quad \phi \to K^+K^-$	2.50 ± 0.33
	$(K^-\pi^+)_P(K^+\pi^-)_S$	2.6 ± 0.5
	$K_1^+K^-, \quad K_1^+ \to K^{*0}\pi^+$	1.8 ± 0.5
	$K_1^-K^+, \quad K_1^- \to \overline{K}^{*0}\pi^-$	0.22 ± 0.12
	$K_1^+K^-, \quad K_1^+ \to \rho^0K^+$	1.14 ± 0.26
	$K_1^-K^+, \quad K_1^- \to \rho^0K^-$	1.46 ± 0.25
	$K^*(1410)^+K^-, \; K^*(1410)^+ \to K^{*0}\pi^+$	1.02 ± 0.26
	$K^*(1410)^-K^+, \; K^*(1410)^- \to \overline{K}^{*0}\pi^-$	1.14 ± 0.25

[CLEO '12]

$\Longrightarrow$ Opportunities for CP violation searches
but also modelling challenges!

CP violation in differential distributions

Motion reversal $\hat{\text{T}}$ (often called *naive time reversal*)

$\hat{\text{T}}$ flips $\vec{p}$ and $\vec{s}$.

$\hat{\text{T}}$-oddity arises from $\epsilon_{\mu\nu\rho\sigma}\ p^\mu q^\nu r^\rho s^\sigma$ contractions ...

... of four independent momenta or spin vectors.

$\rightarrow$ minimal multiplicity

e.g. spinless four-body decays

In the p restframe,

$$\epsilon_{\mu\nu\rho\sigma}\ \ p^\mu q^\nu r^\rho s^\sigma \quad \propto \quad \vec{q}\cdot(\vec{r}\times\vec{s})$$

is a scalar *triple product*.

Differential CP violation

Compare the CP-conjugate amplitudes (squared)

$$\mathcal{M}(\{\vec{p}_i, \sigma_i\}) \quad \text{and} \quad \bar{\mathcal{M}}(\{-\vec{p}_{\bar{\imath}}, -\sigma_{\bar{\imath}}\})\Big|_{\vec{p}_{\bar{\imath}}=\vec{p}_i, \sigma_{\bar{\imath}}=\sigma_i}$$

phase-space point by phase-space point.

Contributions of definite
- *strong* δ and *weak* φ phases
- $\hat{\mathrm{T}}$ transformation properties

$$\mathcal{M}(\{\vec{p}_i, \sigma_i\}) = \\ +a(\{\vec{p}_i, \sigma_i\})\, e^{i(\delta_a+\varphi_a)} \\ +b(\{\vec{p}_i, \sigma_i\})\, e^{i(\delta_b+\varphi_b)} \\ +c(\{\vec{p}_i, \sigma_i\})\, e^{i(\delta_c+[\varphi_c+\pi/2])} \\ +\cdots$$

$$\bar{\mathcal{M}}(\{-\vec{p}_i, -\sigma_i\}) = \\ +a(\{-\vec{p}_i, -\sigma_i\})\, e^{i(\delta_a-\varphi_a)} \\ +b(\{-\vec{p}_i, -\sigma_i\})\, e^{i(\delta_b-\varphi_b)} \\ +c(\{-\vec{p}_i, -\sigma_i\})\, e^{i(\delta_c-[\varphi_c+\pi/2])} \\ +\cdots$$

with
$$a(\{-\vec{p}_i, -\sigma_i\}) = +a(\{\vec{p}_i, \sigma_i\}) \quad \hat{\mathrm{T}}\text{-even}$$
$$b(\{-\vec{p}_i, -\sigma_i\}) = +b(\{\vec{p}_i, \sigma_i\}) \quad \hat{\mathrm{T}}\text{-even}$$
$$c(\{-\vec{p}_i, -\sigma_i\}) = -c(\{\vec{p}_i, \sigma_i\}) \quad \hat{\mathrm{T}}\text{-odd}$$
$$\cdots$$

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with
$a(\{-\vec{p}_i, -\sigma_i\}) = +a(\{\vec{p}_i, \sigma_i\})$ $\hat{\mathrm{T}}$-even
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$c(\{-\vec{p}_i, -\sigma_i\}) = -c(\{\vec{p}_i, \sigma_i\})$ $\hat{\mathrm{T}}$-odd
$\cdots$

$\Longrightarrow$ The φ phases are defined to contain all ‘CP-oddity’.

CP violation and strong phases

Distributions of definite CP and $\hat{T}$ transformation properties

$$\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{T}\text{-}{}^{\text{even}}_{\text{odd}}}_{CP\text{-}{}^{\text{even}}_{\text{odd}}} \equiv \frac{\mathbb{I} \pm \hat{T}}{2}\,\frac{\mathbb{I} \pm CP}{2}\,\frac{d\Gamma}{d\Phi}$$

- $\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{T}\text{-even}}_{CP\text{-even}} \propto \;\; a\,a + b\,b + c\,c$
  $\quad +2\,a\,b\,\cos(\delta_a - \delta_b)\cos(\varphi_a - \varphi_b)$
- $\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{T}\text{-odd}}_{CP\text{-even}} \propto \;\; 2\,a\,c\,\sin(\delta_a - \delta_c)\,\cos(\varphi_a - \varphi_c)$
  $\quad +2\,b\,c\,\sin(\delta_b - \delta_c)\,\cos(\varphi_b - \varphi_c)$
- $\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{T}\text{-even}}_{CP\text{-odd}} \propto -2\,a\,b\,\sin(\delta_a - \delta_b)\,\sin(\varphi_a - \varphi_b)$
- $\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{T}\text{-odd}}_{CP\text{-odd}} \propto \;\; 2\,a\,c\,\cos(\delta_a - \delta_c)\sin(\varphi_a - \varphi_c)$
  $\quad +2\,b\,c\,\cos(\delta_b - \delta_c)\sin(\varphi_b - \varphi_c)$

$\Longrightarrow$ Four different sensitivities to strong and weak phases.

CP violation and strong phases

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$\left.\frac{d\Gamma}{d\Phi}\right\|^{\hat{\mathrm{T}}\text{-even}}_{\mathrm{CP}\text{-odd}} \propto$ '$\sin\delta \sin\varphi$'	Sensitivity to small differences of CP-odd phases between decay amplitudes of different CP-even phases.

$\left.\frac{d\Gamma}{d\Phi}\right\|^{\hat{\mathrm{T}}\text{-odd}}_{\mathrm{CP}\text{-odd}} \propto$ '$\cos\delta \sin\varphi$'	Sensitivity to small differences of CP-odd phases between decay amplitudes of identical —or vanishing— CP-even phases.

$\Longrightarrow$ Four different sensitivities to strong and weak phases.

Untagged samples / self-conjugate states

- Tagging CP-conjugate processes may cost efficiency, especially with a self-conjugate final state.
- Defining E^*, the permutation of CP-conjugate particles, an untagged sample is: [as in 1503.05362]

e.g. $$\begin{cases} B_s^0 \to & K^+(+\vec{p}_1) & \pi^-(+\vec{p}_2) & K^-(+\vec{p}_3) & \pi^+(+\vec{p}_4) \\ \bar{B}_s^0 \to & K^-(-\vec{p}_1) & \pi^+(-\vec{p}_2) & K^+(-\vec{p}_3) & \pi^-(-\vec{p}_4) \end{cases}$$

$$\frac{\mathbb{I} + \mathrm{CP}}{2} \, \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi}$$

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$$\frac{\mathbb{I} + \mathrm{CP\hat{T}E}^*}{2} \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi}$$

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$$\frac{\mathbb{I} + \mathrm{CP\hat{T}E^*}}{2} \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi}$$

It has two CP-odd distributions, $\hat{\mathrm{T}}$-odd or E^*-odd:

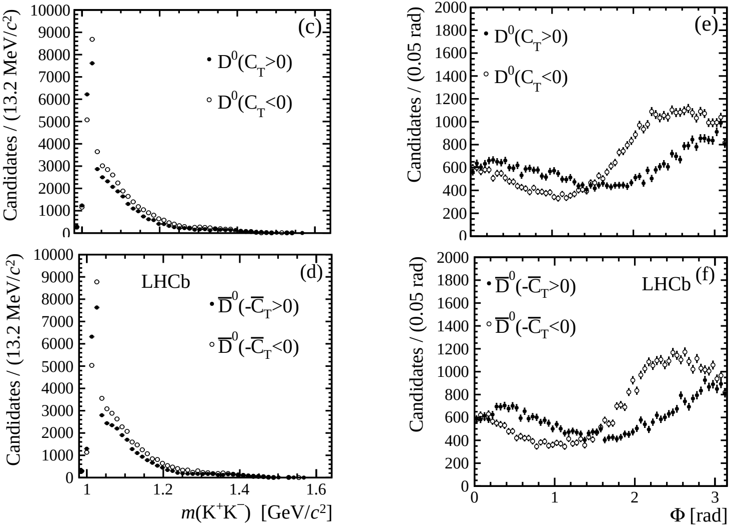
$$\frac{\mathbb{I} \pm \hat{\mathrm{T}}}{2} \; \frac{\mathbb{I} \mp \mathrm{E}^*}{2} \; \left(\frac{\mathbb{I} + \mathrm{CP\hat{T}E^*}}{2} \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi} \right) = \frac{\mathbb{I} \pm \hat{\mathrm{T}}}{2} \; \frac{\mathbb{I} \mp \mathrm{E}^*}{2} \; \frac{\mathbb{I} - \mathrm{CP}}{2} \; \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi}$$

Multibody meson decays

$\hat{T}$-folding of the phase space

$$\frac{d\Gamma}{d\Phi}(D^0 \to K^+K^-\pi^+\pi^-)$$

[1408.1299]



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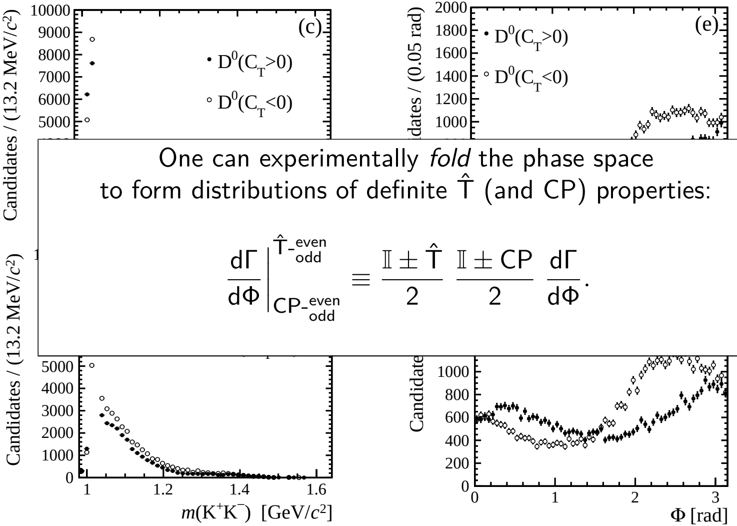
[1408.1299]



One can experimentally *fold* the phase space
to form distributions of definite $\hat{\mathrm{T}}$ (and CP) properties:

$$\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{\mathrm{T}}\text{-}^{\text{even}}_{\text{odd}}}_{\mathrm{CP}\text{-}^{\text{even}}_{\text{odd}}} \equiv \frac{\mathbb{I} \pm \hat{\mathrm{T}}}{2}\,\frac{\mathbb{I} \pm \mathrm{CP}}{2}\,\frac{d\Gamma}{d\Phi}.$$

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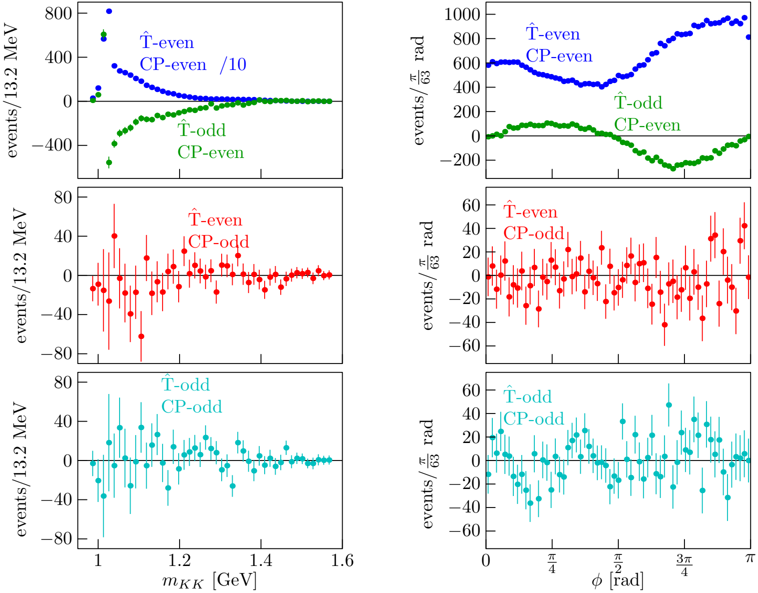

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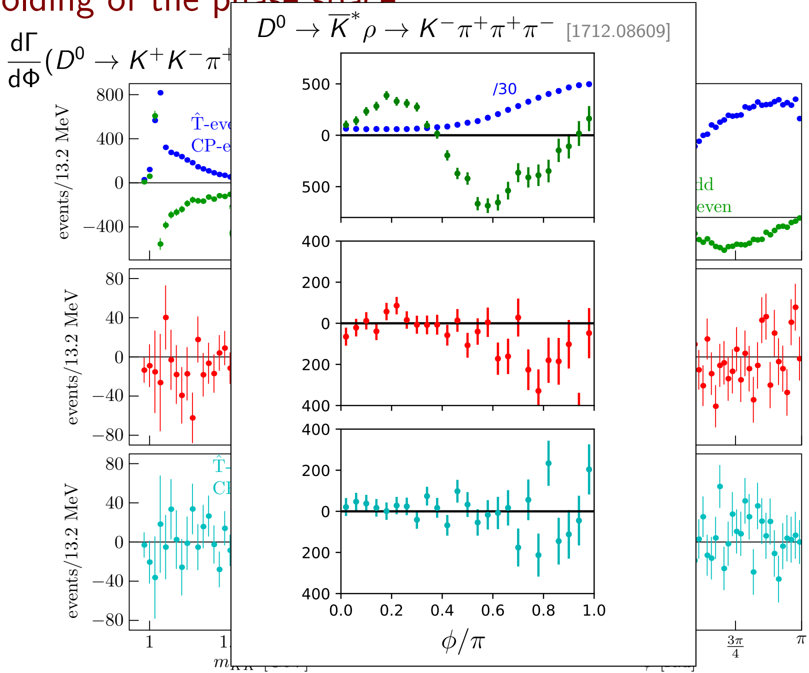


Exploring the phase space

Relying on phenomenological parametrisations

- full unbinned likelihood fits
  - − limited by our modelling abilities
  - + optimal for *expected* CPV
  - − may miss *unexpected* CPV

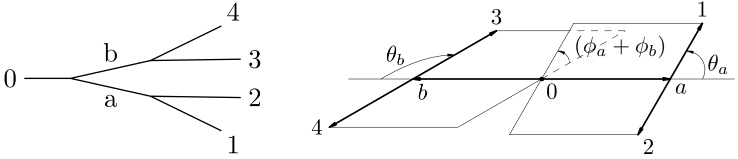
More systematically and using less dynamical information

- phase-space binnings
  - + simple
  - − granularity requires statistics
  - − arbitrariness in bin boundaries
- series of asymmetries or moments
  - ± exploits information about resonance structure
- so-called *energy test* [1105.5338, 1612.04705]
  - − still arbitrariness in phase-space parametrization

Series of asymmetries

For each resonance structure,

and the corresponding phase-space parametrization



define CP-odd asymmetries —or moments— systematically

$$\mathcal{A}_{no}^{kl} \equiv \int d\Omega \left(\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega} - \frac{1}{\bar{\Gamma}}\frac{d\bar{\Gamma}}{d\Omega} \right) \text{sign} \left\{ f_k(\cos\theta_a)\, f_l(\cos\theta_b)\, \sin\left(n\phi_a + n\phi_b + o\frac{\pi}{2} \right) \right\}$$

with $o = 0$: $\hat{\text{T}}$-odd
$o = 1$: $\hat{\text{T}}$-even

- each sensitive to different interferences
- sometimes statistically correlated
- suboptimal for interferences between different resonance structures
- add sign flips at known resonances in invariant mass distributions

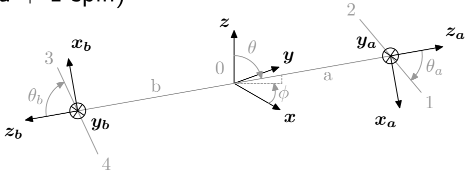
Multibody baryon decays

Polarization: richness and ambiguities

+ $\hat{\mathrm{T}}$-oddity with lower multiplicity

  in the three-body decay of a polarized particle
  ($\epsilon_{\mu\nu\rho\sigma} p^\mu q^\nu r^\rho s^\sigma$ with 3 momenta + 1 spin)

  e.g. $\Lambda_b \to \Lambda^* \gamma \to pK\gamma$
  $\Lambda_b \to N^* K \to p\pi K$



+ new angular distributions

  broken rotation symmetries

+ new $\hat{\mathrm{T}}$-odd variables
  - angular variables: $\cos\theta$, $\cos\phi_a$, $\cos\phi_b$
  - polarization component $\perp$ to the production plane

− coupled production and decay

  Asymmetries in decay angles probe CPV in both.

− polarization not directly measurable like a kinematic variable

  Decay kinematics is insufficient to define distributions of definite $\hat{\mathrm{T}}$.

Resolving ambiguities

e.g. for $\frac{1}{2} \to \frac{1}{2}\,1 \to (\frac{1}{2}\,0)\,(0\,0)$ like $\Lambda_b \to N^*\rho \to p\pi\pi\pi$, $\Lambda_b \to \Lambda\,\phi \to p\pi KK$

$\frac{d\Gamma}{d\Phi} \propto$

$+3$	$\lvert A_+\rvert^2 + \lvert A_-\rvert^2$					$\cos^2\theta_b$	
$+3/2$	$\lvert B_+\rvert^2 + \lvert B_-\rvert^2$						
$+3$	$\lvert A_+\rvert^2 - \lvert A_-\rvert^2$	α_a					
$+3/2$	$\lvert B_+\rvert^2 - \lvert B_-\rvert^2$	α_a			$\cos\theta_a$	$\sin^2\theta_b$	
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_-\} - \mathrm{Re}\{A_-^* B_+\}$	α_a			$\sin\theta_a$	$\sin 2\theta_b$	$\cos(\phi_a+\phi_b)$
$+3$			P_z	$\cos\theta$		$\cos^2\theta_b$	
$-3/2$			P_z	$\cos\theta$		$\sin^2\theta_b$	
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_+\} - \mathrm{Re}\{A_-^* B_-\}$		P_z	$\sin\theta$		$\sin 2\theta_b$	$\cos\phi_b$
$+3$	$\lvert A_+\rvert^2 + \lvert A_-\rvert^2$	α_a	P_z				
$-3/2$	$\lvert B_+\rvert^2 + \lvert B_-\rvert^2$	α_a	P_z				
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_-\} + \mathrm{Re}\{A_-^* B_+\}$	α_a	P_z	$\cos\theta$	$\sin\theta_a$	$\sin 2\theta_b$	$\cos(\phi_a+\phi_b)$
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_+\} + \mathrm{Re}\{A_-^* B_-\}$	α_a	P_z	$\sin\theta$	$\cos\theta_a$	$\sin 2\theta_b$	$\cos\phi_b$
-6	$\mathrm{Re}\{A_+^* A_-\}$	α_a	P_z	$\sin\theta$	$\sin\theta_a$	$\cos^2\theta_b$	$\cos\phi_a$
$+3$	$\mathrm{Re}\{B_+^* B_-\}$	α_a	P_z	$\sin\theta$	$\sin\theta_a$	$\sin^2\theta_b$	$\cos(\phi_a+2\phi_b)$
$-3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_+\} + \mathrm{Im}\{A_-^* B_-\}$		P_z	$\sin\theta$		$\sin 2\theta_b$	$\sin\phi_b$
$+3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_-\} - \mathrm{Im}\{A_-^* B_+\}$	α_a	P_z	$\cos\theta$	$\sin\theta_a$	$\sin 2\theta_b$	$\sin(\phi_a+\phi_b)$
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$+3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_-\} + \mathrm{Im}\{A_-^* B_+\}$	α_a			$\sin\theta_a$	$\sin 2\theta_b$	$\sin(\phi_a+\phi_b)$

$\hat{\mathrm{T}}$-even angular distrib.

$\to$ '$\sin\delta\sin\varphi$'

$\hat{\mathrm{T}}$-odd angular distrib.

$\to$ '$\cos\delta\sin\varphi$'

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e.g. for $\frac{1}{2} \to \frac{1}{2}\,1 \to (\frac{1}{2}\,0)\,(0\,0)$ like $\Lambda_b \to N^*\rho \to p\pi\pi\pi$, $\Lambda_b \to \Lambda\,\phi \to p\pi KK$

$\frac{d\Gamma}{d\Phi} \propto$

$+3$	$\lvert A_+\rvert^2 + \lvert A_-\rvert^2$					$\cos^2\theta_b$	
$+3/2$	$\lvert B_+\rvert^2 + \lvert B_-\rvert^2$					$\sin^2\theta_b$	
$+3$	$\lvert A_+\rvert^2 - \lvert A_-\rvert^2$	α_a			$\cos\theta_a$	$\cos^2\theta_b$	
$+3/2$	$\lvert B_+\rvert^2 - \lvert B_-\rvert^2$	α_a			$\cos\theta_a$	$\sin^2\theta_b$	
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_-\} - \mathrm{Re}\{A_-^* B_+\}$	α_a			$\sin\theta_a$	$\sin 2\theta_b$	$\cos(\phi_a + \phi_b)$
$+3$	$\lvert A_+\rvert^2 - \lvert A_-\rvert^2$		P_z	$\cos\theta$		$\cos^2\theta_b$	
$-3/2$	$\lvert B_+\rvert^2 - \lvert B_-\rvert^2$		P_z	$\cos\theta$		$\sin^2\theta_b$	
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_+\} - \mathrm{Re}\{A_-^* B_-\}$		P_z	$\sin\theta$		$\sin 2\theta_b$	$\cos\phi_b$
$+3$	$\lvert A_+\rvert^2 + \lvert A_-\rvert^2$	α_a	P_z	$\cos\theta$	$\cos\theta_a$	$\cos^2\theta_b$	
$-3/2$	$\lvert B_+\rvert^2 + \lvert B_-\rvert^2$	α_a	P_z	$\cos\theta$	$\cos\theta_a$	$\sin^2\theta_b$	
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_-\} + \mathrm{Re}\{A_-^* B_+\}$	α_a	P_z	$\cos\theta$	$\sin\theta_a$	$\sin 2\theta_b$	$\cos(\phi_a + \phi_b)$
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_+\} + \mathrm{Re}\{A_-^* B_-\}$	α_a	P_z	$\sin\theta$	$\cos\theta_a$	$\sin 2\theta_b$	$\cos\phi_b$
-6	$\mathrm{Re}\{A_+^* A_-\}$						
$+3$	$\mathrm{Re}\{B_+^* B_-\}$						
$-3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_+\} + \mathrm{Im}\{A_-^* B_-\}$						
$+3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_-\} - \mathrm{Im}\{A_-^* B_+\}$						
$-3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_+\} - \mathrm{Im}\{A_-^* B_-\}$						
-6	$\mathrm{Im}\{A_+^* A_-\}$						
$+3$	$\mathrm{Im}\{B_+^* B_-\}$						
$+3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_-\} + \mathrm{Im}\{A_-^* B_+\}$						

$$P_{x,y} = 0$$

$$A_\pm \equiv \mathcal{M}_{\frac{1}{2}\to\frac{1}{2}1}(\pm\tfrac{1}{2}, 0)$$
$$B_\pm \equiv \mathcal{M}_{\frac{1}{2}\to\frac{1}{2}1}(\pm\tfrac{1}{2}, \pm 1)$$

$$1 \equiv \mathcal{M}_{\frac{1}{2}\to\frac{1}{2}0}(+\tfrac{1}{2}, 0) + \mathcal{M}_{\frac{1}{2}\to\frac{1}{2}0}(-\tfrac{1}{2}, 0)$$
$$\alpha_a \equiv \mathcal{M}_{\frac{1}{2}\to\frac{1}{2}0}(+\tfrac{1}{2}, 0) - \mathcal{M}_{\frac{1}{2}\to\frac{1}{2}0}(-\tfrac{1}{2}, 0)$$

$$1 \equiv \mathcal{M}_{1\to 00}(0, 0)$$

Resolving ambiguities

by isolating a resonance structure
and examining the helicity amplitude decomposition

$$\mathcal{A}^{jkl}_{mno} \equiv \int d\Omega \left(\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega} - \frac{1}{\bar{\Gamma}}\frac{d\bar{\Gamma}}{d\Omega} \right) \text{sign}\left\{ f_j(\cos\theta)\, f_k(\cos\theta_a)\, f_l(\cos\theta_b)\, \sin\left(m\phi_a + n\phi_b + o\frac{\pi}{2} \right) \right\}$$

with $o \in \{0,1\}$, $j+m+n+o \in 2\mathbb{Z} \to \hat{\mathrm{T}}$-odd
$j+m+n+o \in (2\mathbb{Z}+1) \to \hat{\mathrm{T}}$-even

+ further understanding gained:

e.g., in strongly-produced $\frac{1}{2} \to \frac{1}{2}\; 1 \to (\frac{1}{2}\; 0)\; (0\; 0)$,

- vanishing 'classic' $\sin(\phi_a + \phi_b)$ asymmetry (integrated $a_{CP}^{\hat{T}\text{-odd}}$)
- vanishing asymmetries based on 'special' angles [hep-ph/0602043]

$$\cos\Phi_a = \frac{\cos\theta\cos\phi\sin\phi_a + \sin\phi\cos\phi_a}{\sqrt{1-\sin^2\phi_a\sin^2\theta}}, \quad \sin\Phi_a = \frac{\cos\theta\sin\phi\sin\phi_a - \cos\phi\cos\phi_a}{\sqrt{1-\sin^2\phi_a\sin^2\theta}},$$
$$\cos\Phi_b = \frac{\cos\theta\cos\phi\sin\phi_b - \sin\phi\cos\phi_b}{\sqrt{1-\sin^2\phi_b\sin^2\theta}}, \quad \sin\Phi_b = \frac{\cos\theta\sin\phi\sin\phi_a + \cos\phi\cos\phi_b}{\sqrt{1-\sin^2\phi_b\sin^2\theta}}.$$

measured in $\Lambda_b \to \Lambda\phi \to p\pi KK$ [1603.02870]

Summary

Probing CP in multi-body and baryon decays

The differential distributions of multibody decays are rich of opportunities in CPV searches.

Trade-offs between sensitivity and modelling-independence are unavoidable.

The systematic construction of asymmetries provides an interesting compromise in meson decays.

Helicity amplitude decompositions help resolving the ambiguities generated by polarization and identify useful asymmetries in baryon decays.

Can one obtain (B)SM predictions for CPV in some of those channels?