FCC-ee horizontal dynamic aperture with radiation from quadrupoles (FCCee_z_202_nosol_13.seq)

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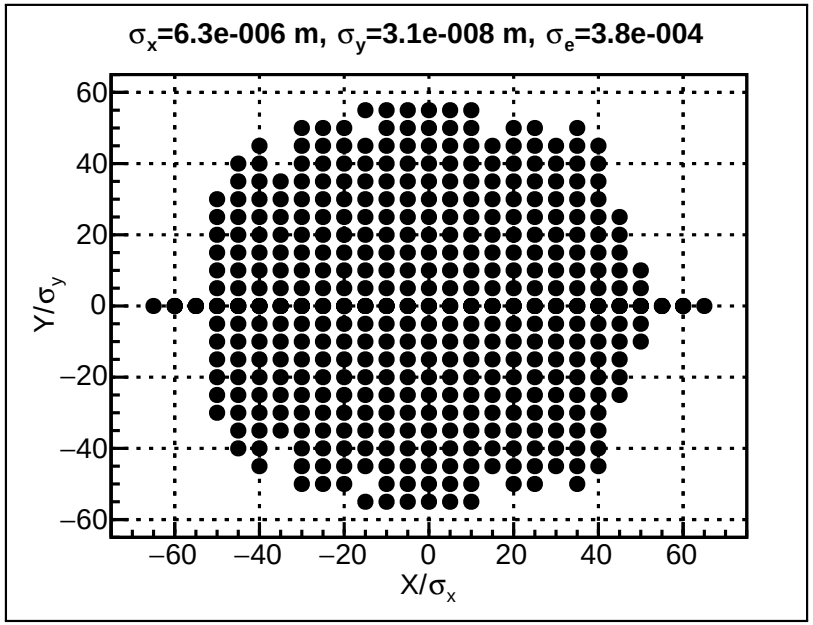
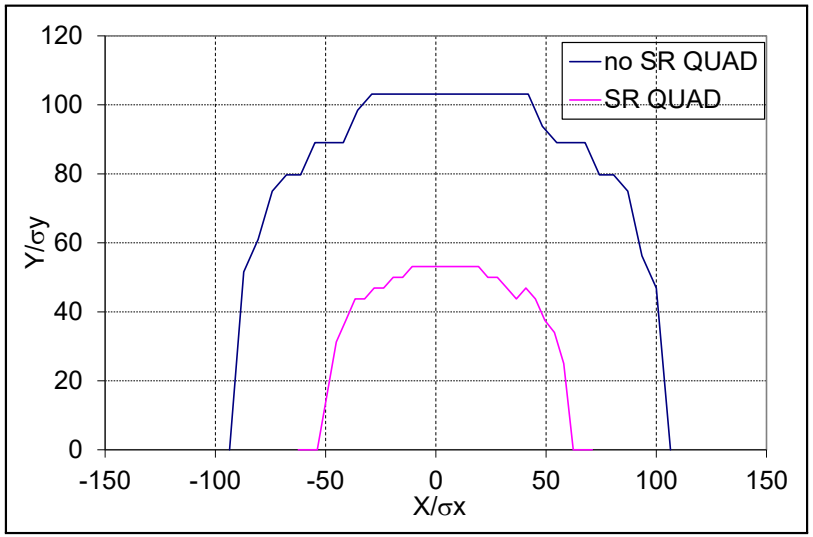
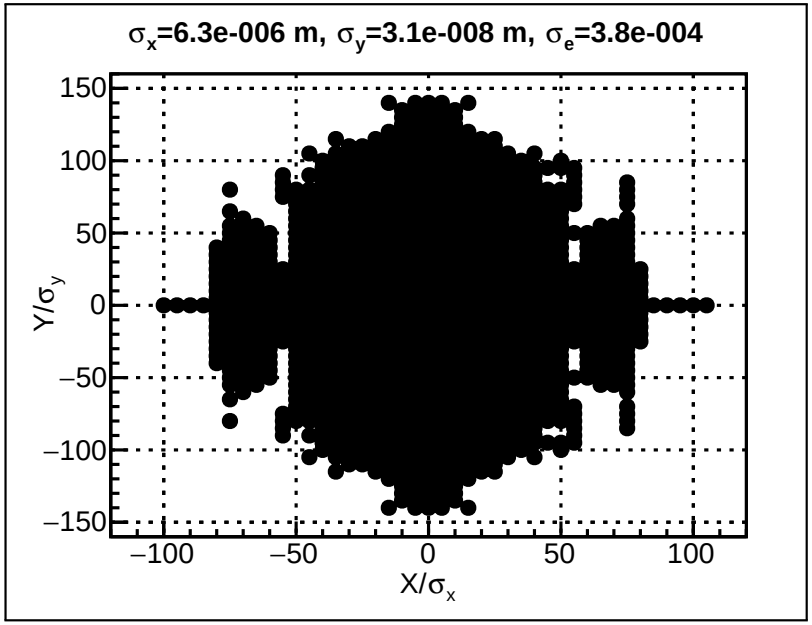
6d (SR from BEND, QUAD) and 6d tracking: *XY*

6d(SR): $R_x = 65\sigma_x$ $R_y = 55\sigma_y$





6d: $R_x = 109\sigma_x$ $R_y = 142\sigma_y$



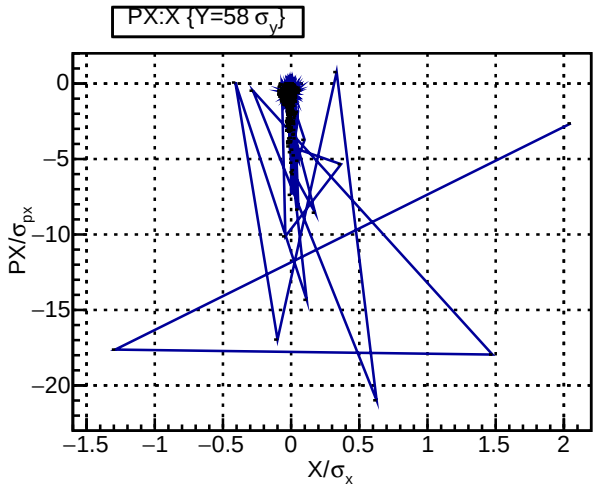
Introduction

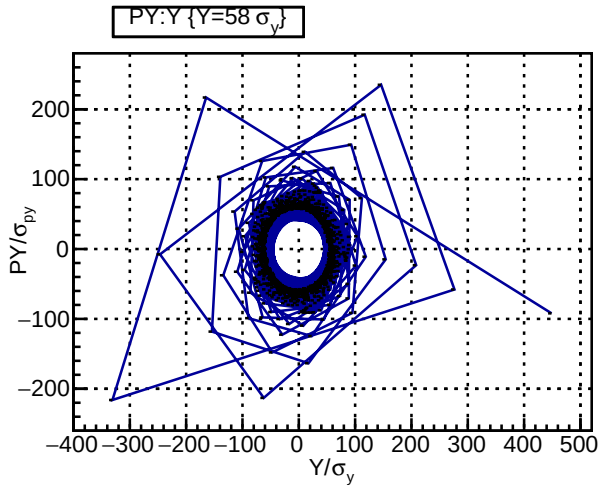
Problem

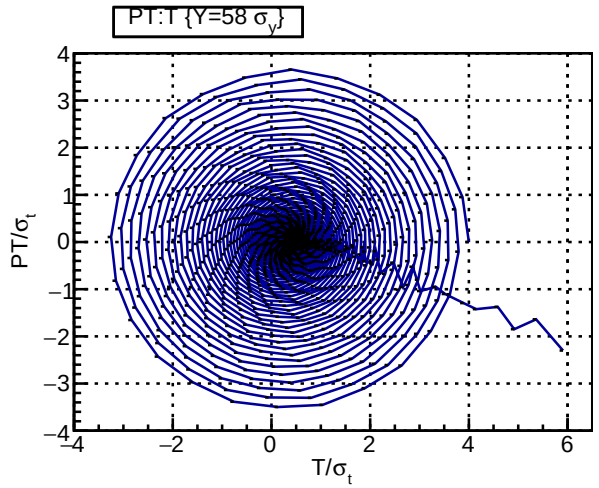
- Why vertical dynamic aperture drops from $R_y = 142\sigma_y$ to $R_y = 55\sigma_y$?
- Why horizontal dynamic aperture drops from $R_x = 109\sigma_x$ to $R_x = 65\sigma_x$?

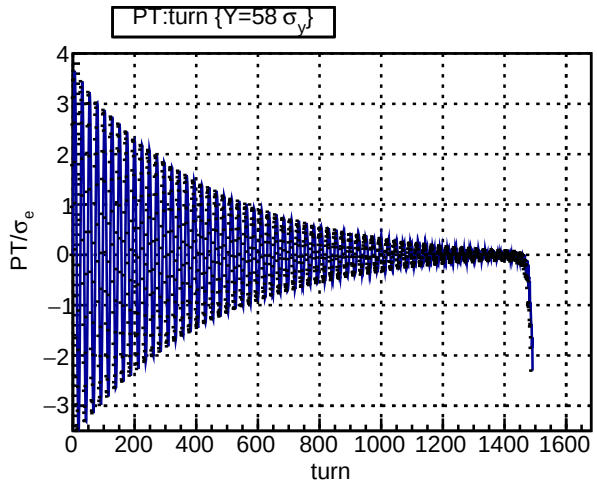
in the FCCee_z_202_nosol_13.seq lattice at 45 GeV

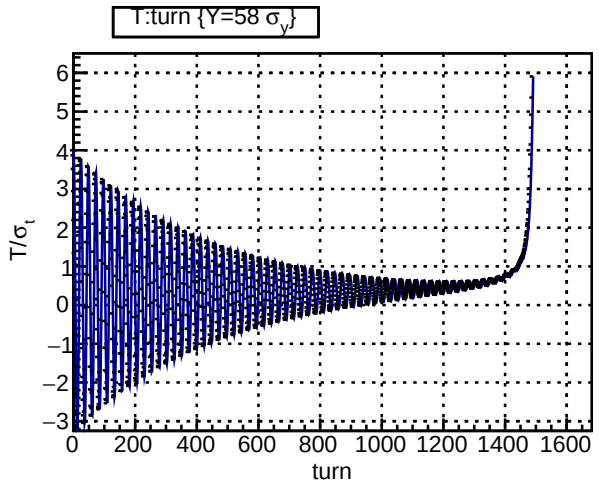
6d (SR from BEND, QUAD; first unstable): $Y_0 = 58\sigma_y$

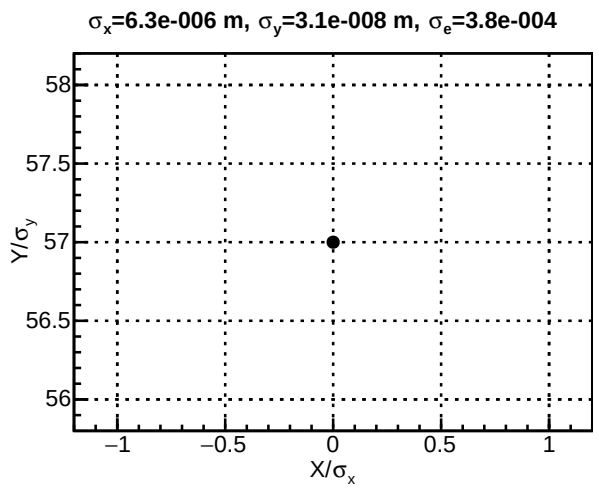




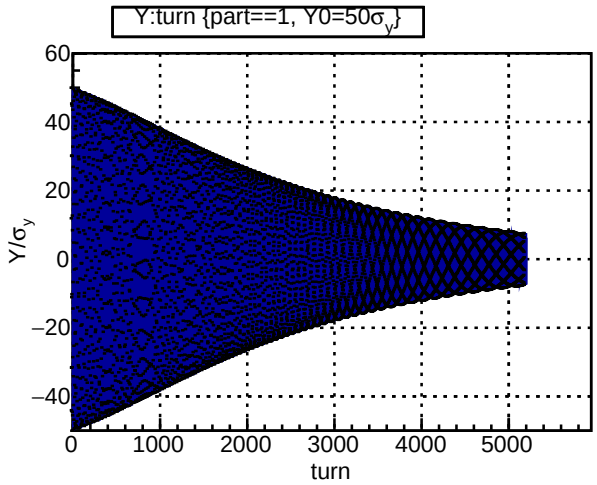


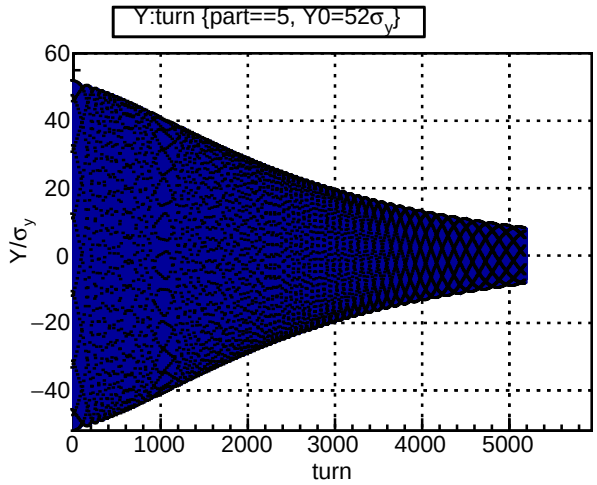


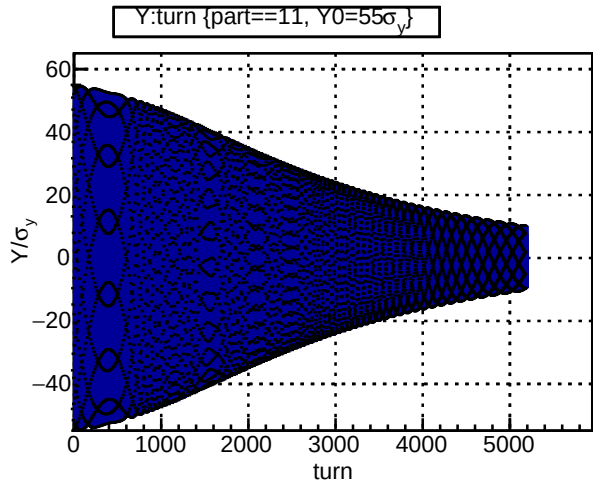


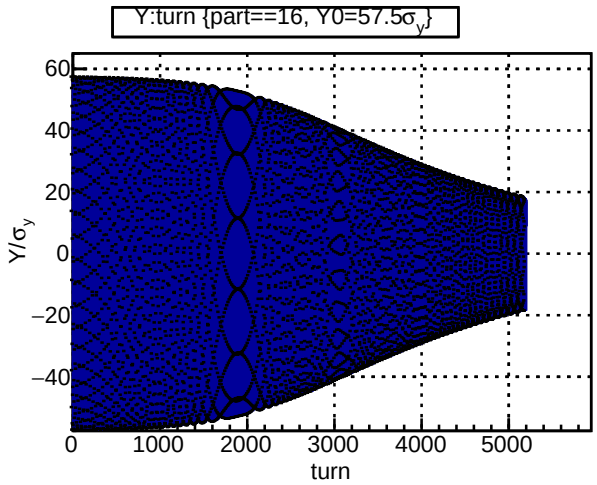


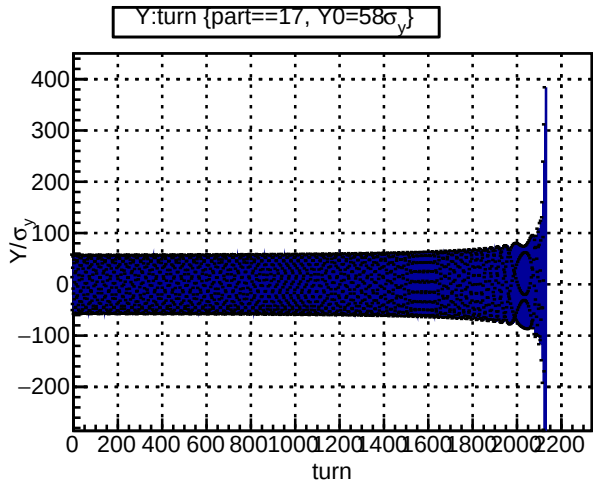
6d (SR from BEND, QUAD) damping

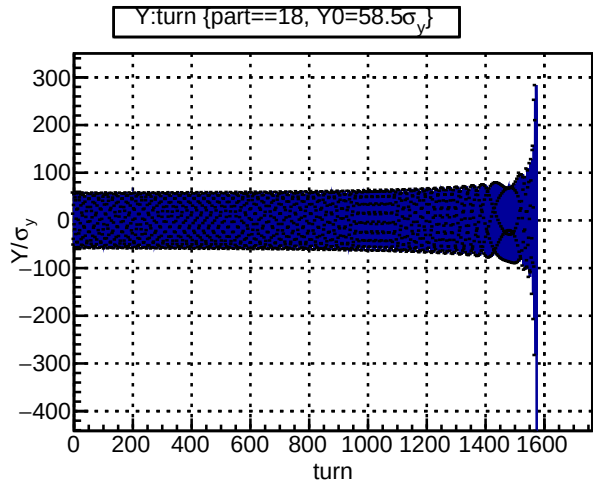








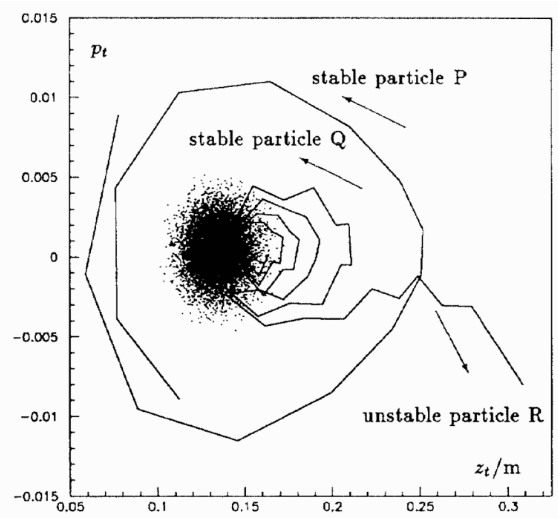




History

References

- John M. Jowett (SLAC), Introductory Statistical Mechanics for Electron Storage Rings, AIP Conf.Proc. 153 (1987) 864-970
- J. Jowett (CERN), Dynamic aperture for LEP: Physics and calculations, Conf.Proc. C9401174 (1994) 47-71, In *Chamonix 1994, LEP performance* 47-71
- F. Barbarin, F. C. Iselin and J. M. Jowett, Particle dynamics in LEP at very high-energy, Conf. Proc. C **940627**, 193 (1994).



Comments

- Some tracking plots are similar.
- There was no mentioning of damping turning into raising.

Parameters (radiation ON, no tappering)

- Energy: $E = 45.6$ Gev.
- Tunes: $\nu_s = 0.0413$, $\nu_y = 0.2217$, $\nu_x = 0.1366$
- Damping times [turns]: $\tau_s = 1300$, $\tau_y = 2600$, $\tau_x = 2600$
- Energy loss: $U_0 = 35.96$ MeV/turn
  - $U_d(B, arc) = 3014 \times 12.4$ keV$= U_0$,
  - $U_q(FF, 50\sigma_y) = 4 \times 0.5$ MeV, $U_q(FF, 50\sigma_x) = 4 \times 3$ MeV,
  - $U_q(QF, 50\sigma_y) = 1470 \times 2.5$ eV, $U_q(QF, 50\sigma_x) = 1470 \times 2.8$ keV,
  - $U_q(QD, 50\sigma_y) = 1468 \times 10$ eV, $U_q(QD, 50\sigma_x) = 1468 \times 1$ keV

Equations of motion: longitudinal

Average, $x_\beta = 0$

$$\sigma' = -\alpha p_t - \frac{J_y \langle\gamma\rangle}{2}$$

$$p_t' = \frac{k_s^2}{\alpha}\sigma - 2\alpha_\sigma p_t - \sum_q \frac{U_q(\sigma_y)}{p_0 c \Pi}\frac{y_q^2}{\sigma_{q,y}^2}$$

$$y_q = \sqrt{2J_y\beta_{q,y}}\cos(\psi_0 + \psi_q + k_y s) = A f_q + A^* f_q^*$$

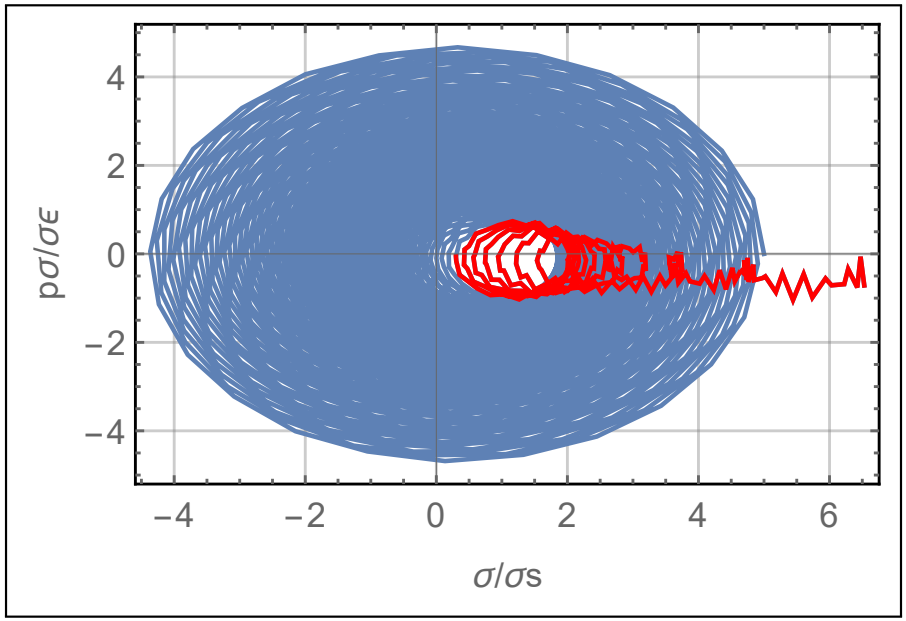
Solution

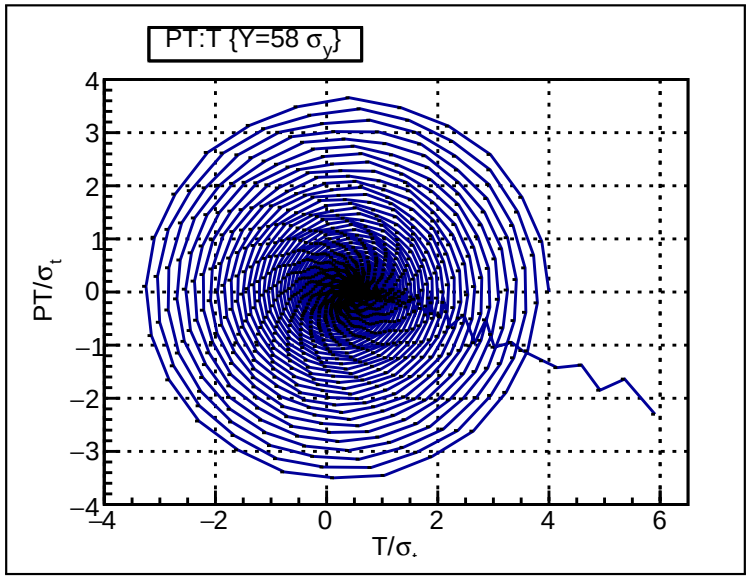
$$p_t = Be^{-\alpha_t s}\cos(k_s s) - \frac{J_y \langle\gamma\rangle}{2\alpha}$$
$$- \sum_q \frac{U_{q,y}(\sigma_y)}{p_0 c}\frac{2k_y}{\Pi(4k_y^2 - k_s^2)}\sin(2\psi_0 + 2\psi_q + 2k_y s)\frac{J_y}{\varepsilon_y}$$

$$k_y = \frac{2\pi\{\nu_y\}}{\Pi}, \quad U_{q,y}(\sigma_y) \text{ is radiation from quadrupole at } 1\sigma_y$$

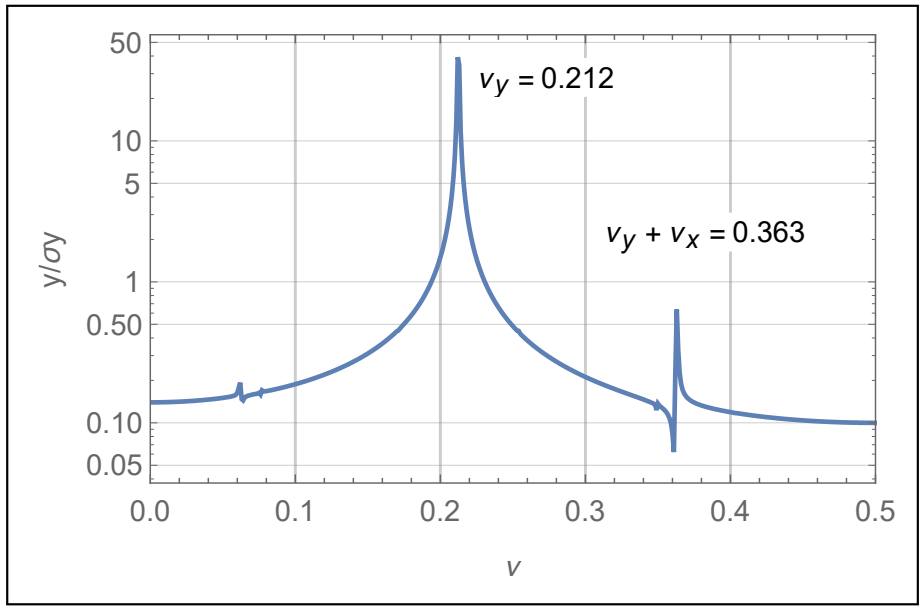
Illustration: $\nu_x = 269.14$, $\nu_y = 267.22$, $\nu_s = 0.0413$

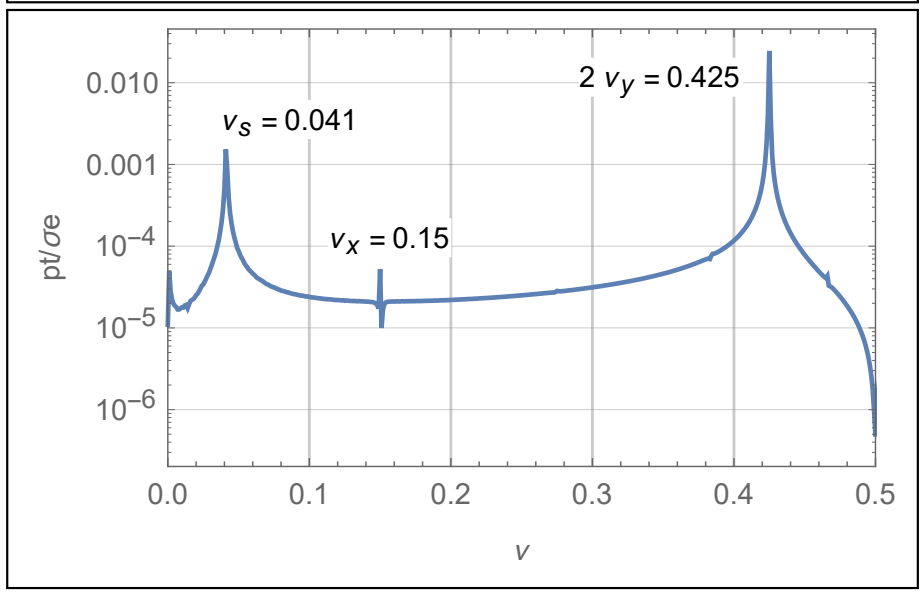
Solution of longitudinal equations





Tracking with $Y_0 = 58\sigma_y$





Vertical dynamic aperture limit

Solving: parameter variation and averaging

$$y(s) = A(s)f(s) + A(s)^* f(s)^*,$$
$$p_y(s) = A(s)f'(s) + A(s)^* f'(s)^*$$

Averaged equation

$$J_y' = -2\alpha_y J_y + \beta J_y^2\,,$$
$$\beta = \frac{k_y}{\varepsilon_y \Pi(4k_y^2 - k_s^2)} \sum_q \frac{U_{q,y}}{p_0 c} \left\langle (K_1 - K_2\eta)\beta_y \cos(2\psi_y - 2\psi_{q,y} - 2k_y s) \right\rangle$$

DA limit

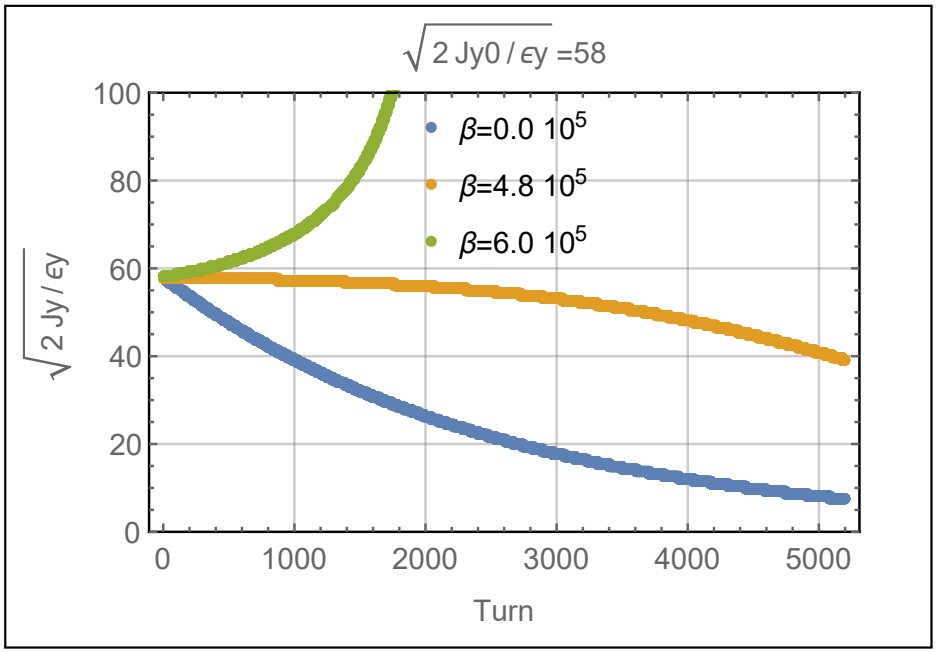
$$J_y' = 0$$
$$\frac{J_y}{\varepsilon_y} = \frac{4k_y^2 - k_s^2}{k_y} \cdot \frac{U_0}{\sum_q U_{q,y} \left\langle (K_1 - K_2\eta)\beta_y \cos(2\psi_y - 2\psi_{q,y} - 2k_y s) \right\rangle}$$

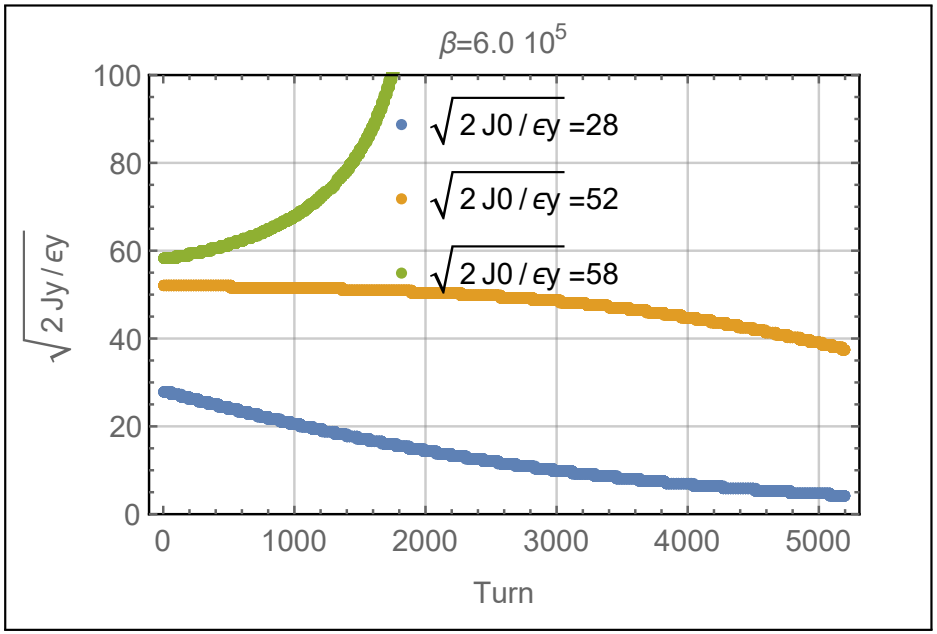
Vertical action

Solution

$$J_y = \frac{J_0 \exp\left(-2\alpha_y s\right)}{1 - \frac{\beta}{2\alpha_y} J_0 \left(1 - \exp\left(-2\alpha_y s\right)\right)}$$

α_y is vertical damping decrement.



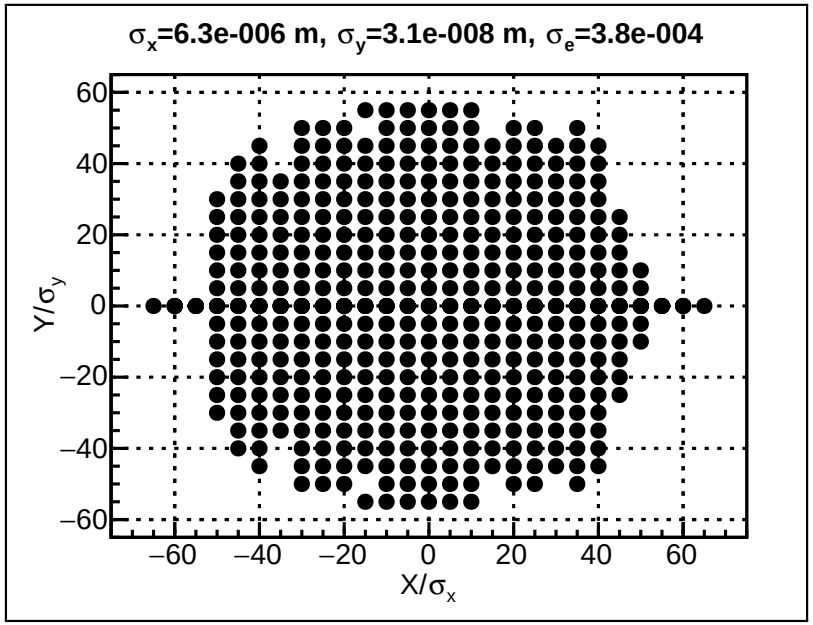


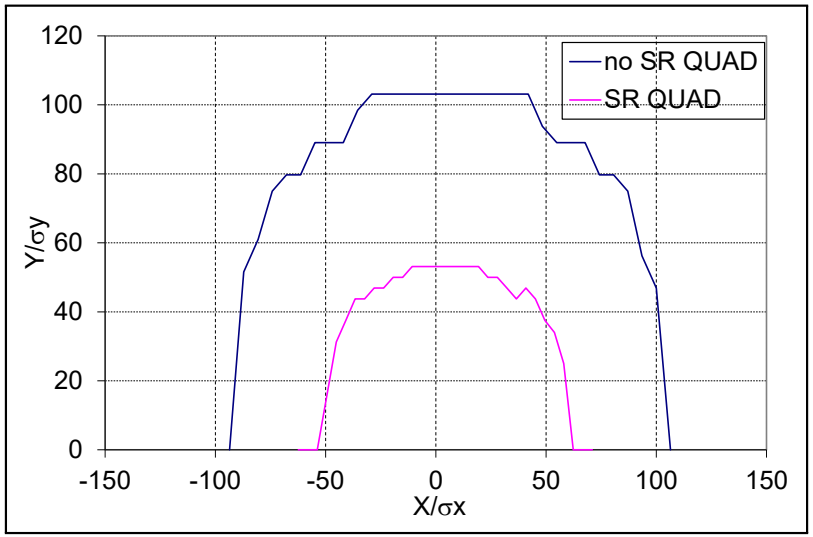
Conclusion for vertical plane at 45.6 GeV

1. Observed and studied a new effect limiting dynamic aperture.
2. Radiation from FF quadrupoles modulates p_t at double betatron frequency.
3. Parametric resonance in vertical motion changes damping. It is observed in tracking and obtained by equations.
4. Estimations with some assumptions predict dynamic aperture limit $J_{y,limit} \approx 30\sigma_y$.
5. Map of radiation from FF quadrupole is similar to quadrupole fringe and kick is larger.
6. $\pi/2$ phase advance between quadrupoles will decrease p_t modulation at double betatron frequency and eliminate parametric resonance.

6d (SR from BEND, QUAD) and 6d tracking: *XY*

6d(SR): $R_x = 65\sigma_x$ $R_y = 55\sigma_y$





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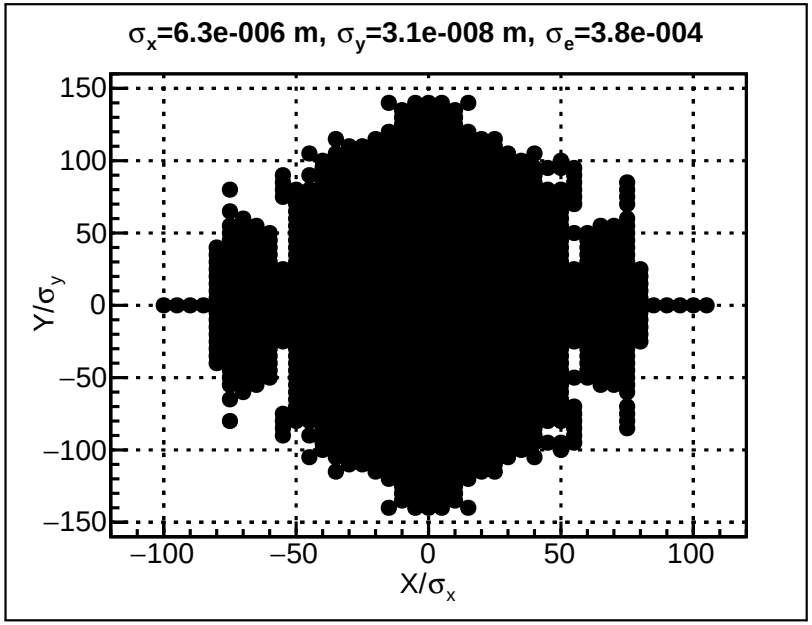

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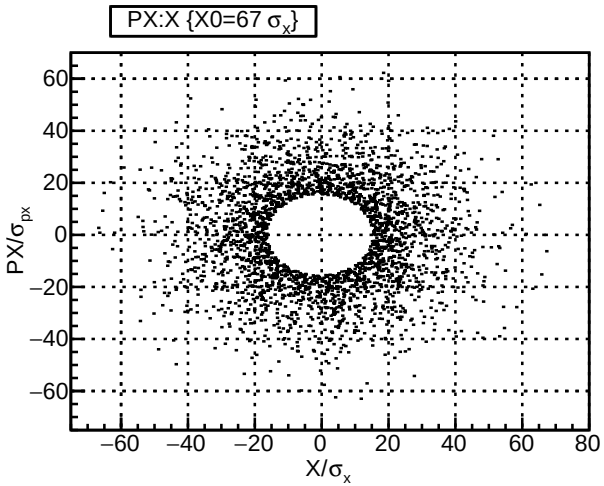
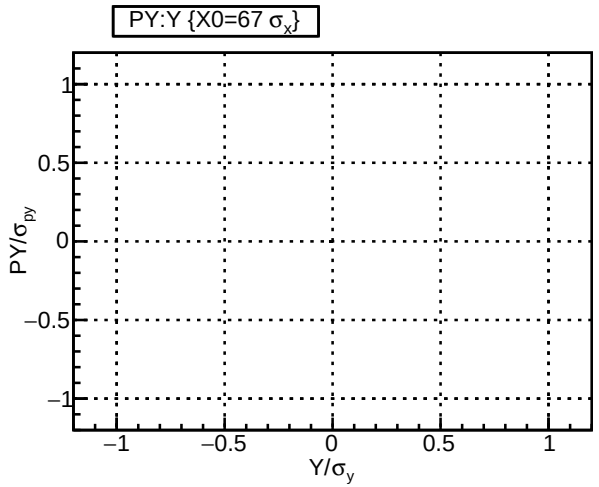
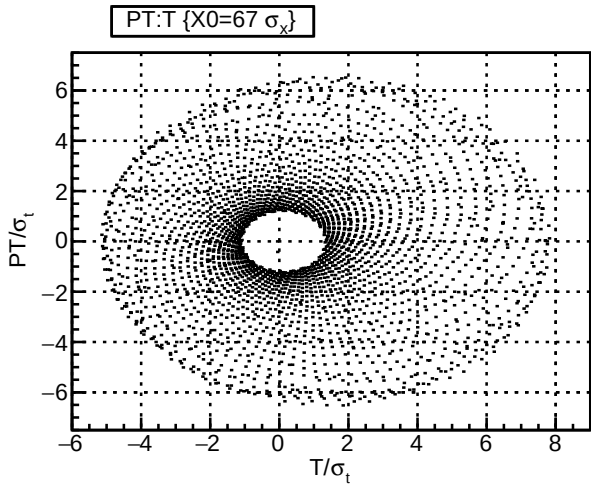
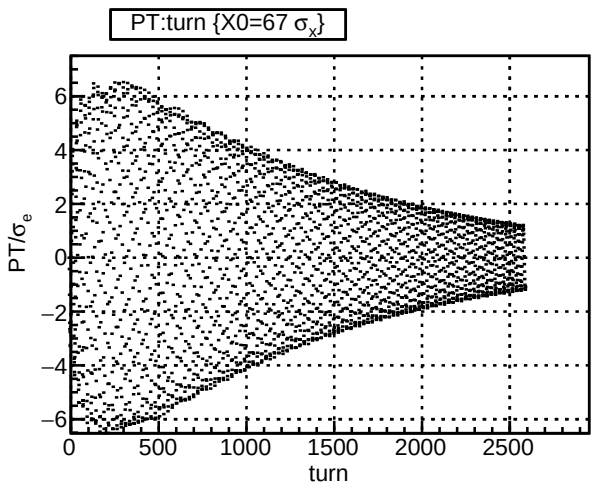
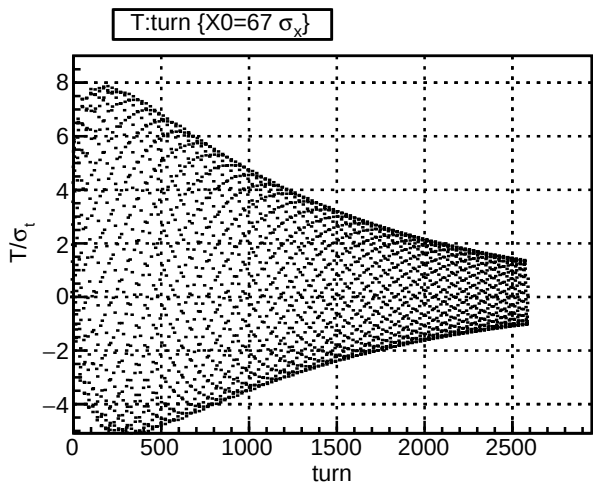
6d (SR from BEND, QUAD; last stable): $X_0 = 67\sigma_x$

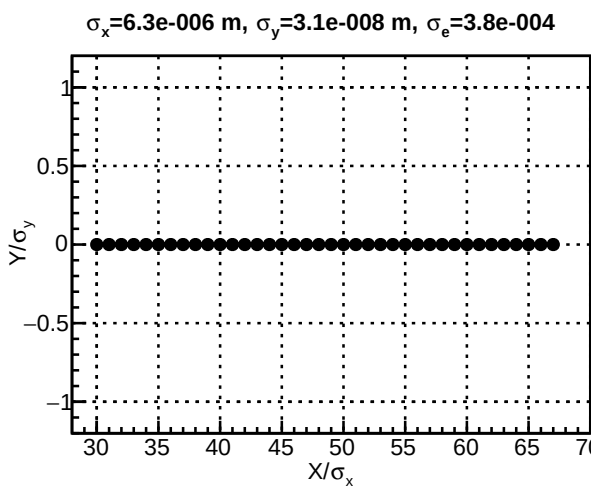




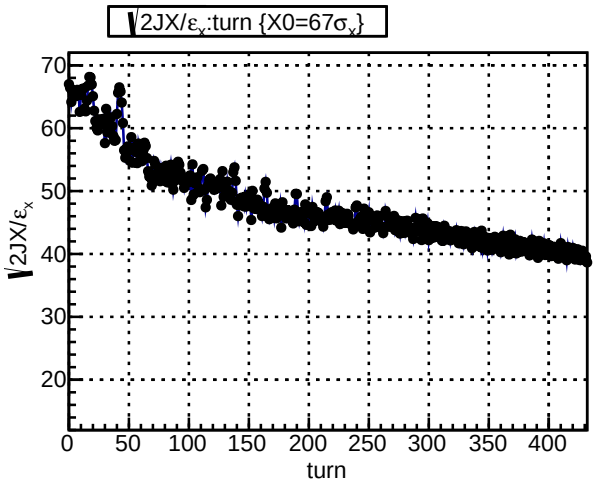


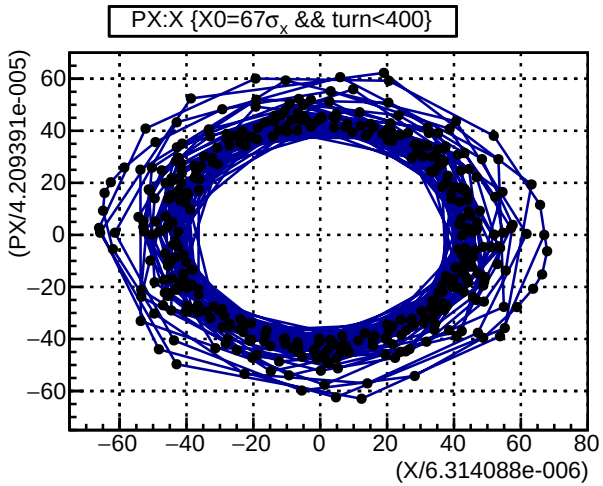


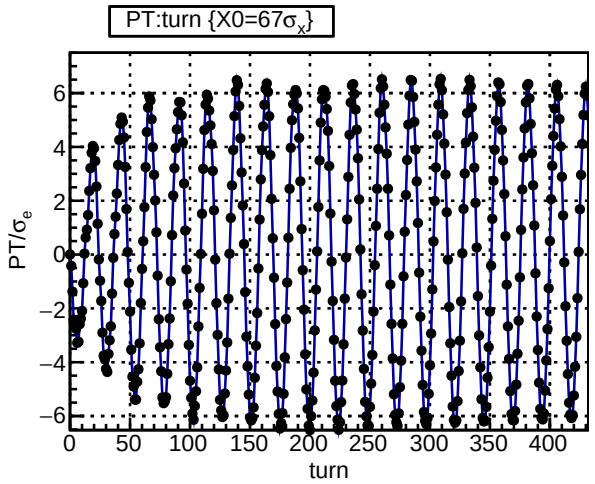


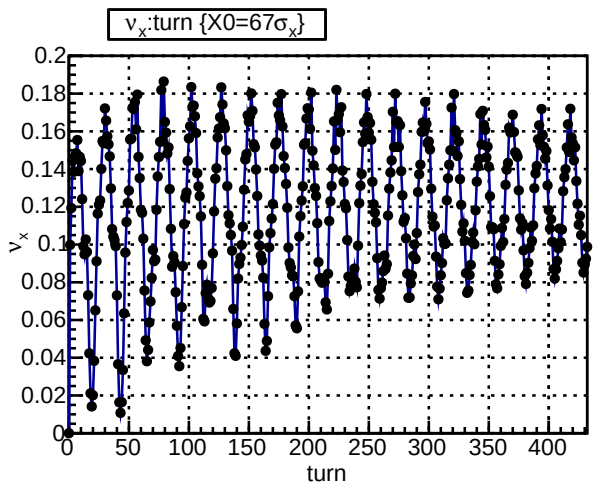


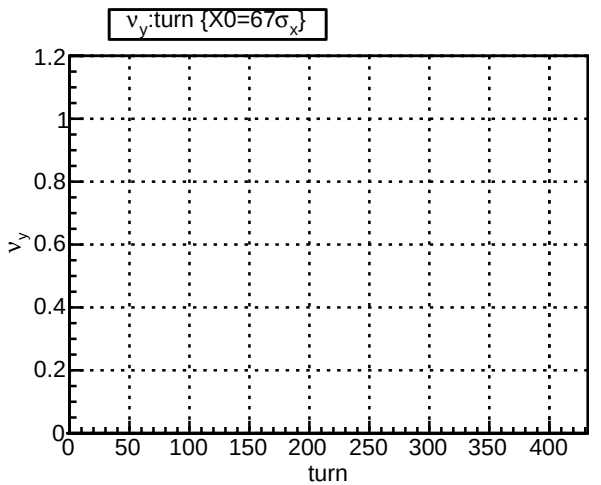
6d (SR from BEND, QUAD; last stable): $X_0 = 67\sigma_x$

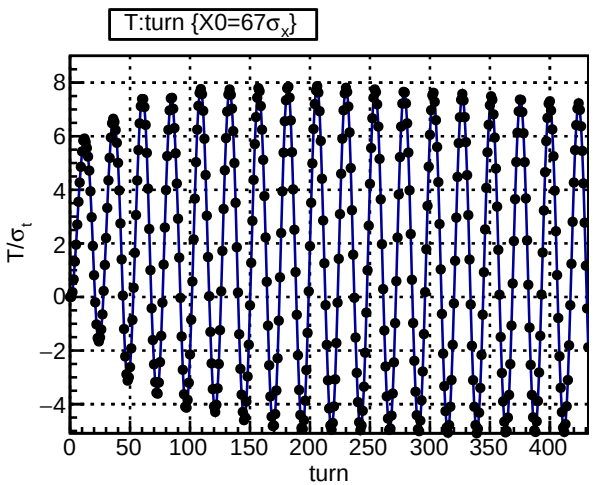












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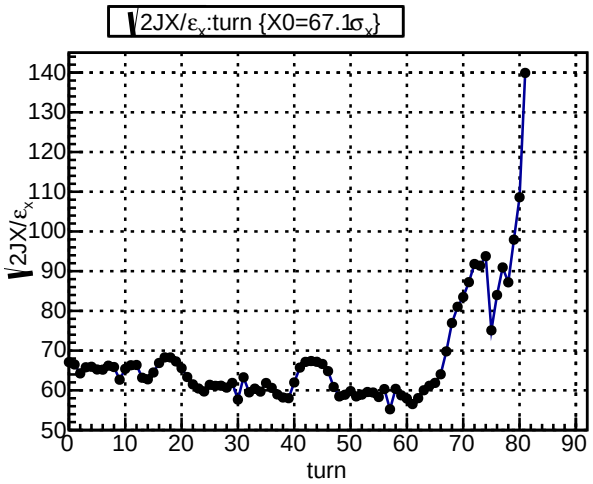
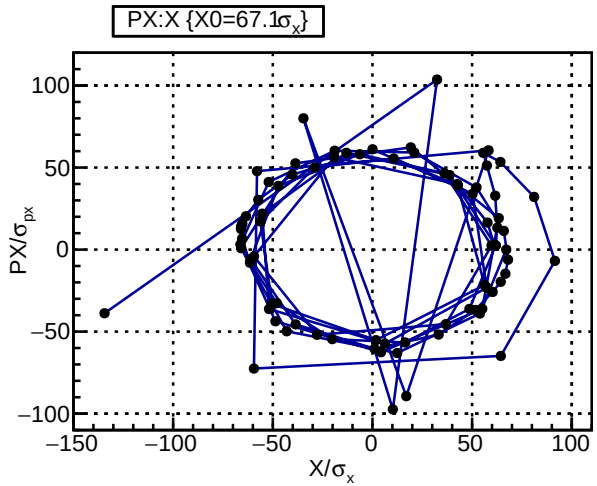
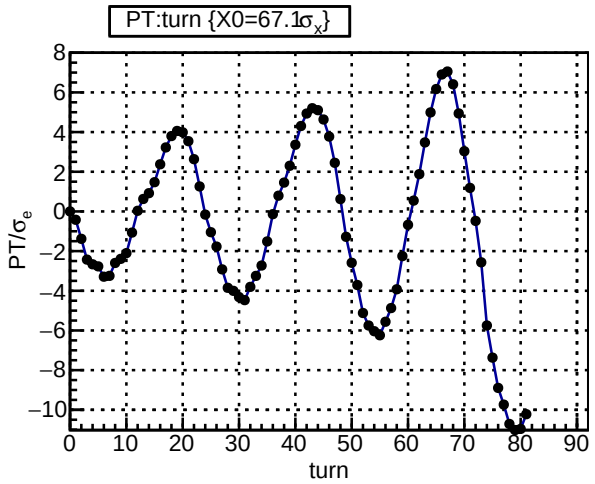
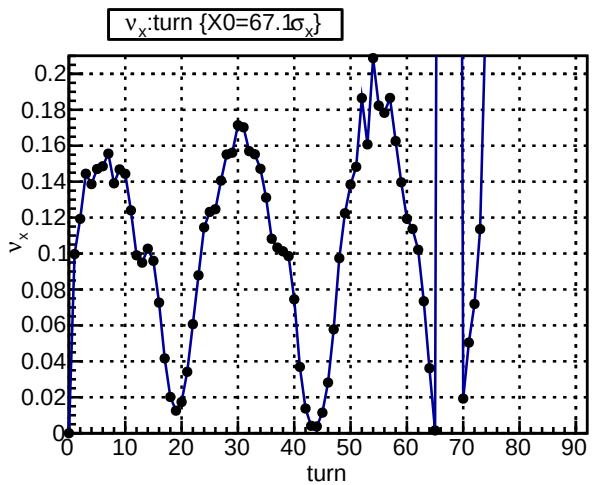
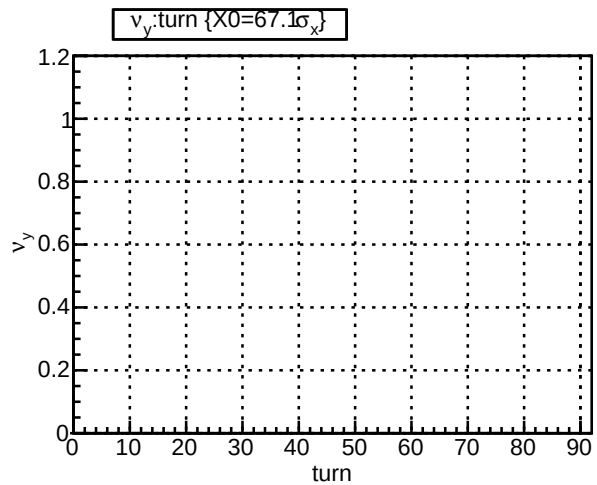
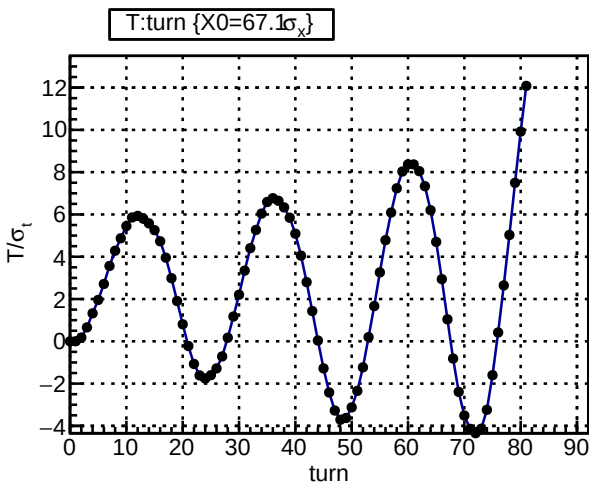












Equations of motion: longitudinal

Exact

$$\begin{cases} \sigma' = -K_0 x - \dfrac{p_x^2}{2}\,, \\ p_t' = \left(-\dfrac{eV_0}{p_0 c}\right) \sin\left[\phi_s + 2\pi\dfrac{\sigma}{\lambda}\right] \delta(s - s_0) - \Gamma K_0^2 \\ \qquad - \Gamma\left(2K_0^2 p_t + 2K_0 K_1 x + K_0^3 x + K_1^2 x^2\right)\,, \qquad \Gamma = \dfrac{C_\gamma}{2\pi}\dfrac{E_0^4}{p_0 c} \end{cases}$$

Average over one turn

$$\begin{cases} \left\langle \sigma' \right\rangle = -\alpha p_t - J_x \dfrac{\langle \gamma_x \rangle}{2} - \left\langle \eta'^2 \right\rangle \dfrac{p_t^2}{2}\,, \\ \left\langle p_t' \right\rangle = \dfrac{k_s^2}{\alpha}\sigma - 2\alpha_\sigma p_t - \Gamma \left\langle K_1^2 \beta_x \right\rangle J_x - \Gamma \left\langle K_1^2 \eta^2 \right\rangle p_t^2\,, \quad k_s = \dfrac{2\pi\nu_s}{\Pi} \end{cases}$$

Synchronous phase

No synchrotron oscillations

Neglecting $\left\langle \eta'^2 \right\rangle \frac{p_t^2}{2}$ and $\Gamma \left\langle K_1^2 \eta^2 \right\rangle p_t^2$

$$\begin{cases} \sigma' = 0 & \Rightarrow \quad p_t = -\dfrac{1}{2\alpha} J_x \left\langle \gamma_x \right\rangle \\ p_t' = 0 & \Rightarrow \quad \sigma = -\dfrac{\alpha_\sigma}{k_s^2} \left\langle \gamma_x \right\rangle J_x + \dfrac{\alpha}{k_s^2} \Gamma \left\langle K_1^2 \beta_x \right\rangle J_x \end{cases}$$

Energy deviation amplitude

$$p_{t,max} = J_x \sqrt{\frac{\left\langle \gamma_x \right\rangle^2}{4\alpha^2} + \left(-\frac{\alpha_\sigma \left\langle \gamma_x \right\rangle}{\alpha k_s} + \frac{\Gamma \left\langle K_1^2 \beta_x \right\rangle}{k_s} \right)^2}$$

$\{X_0, Y_0\}$	$\{1\sigma_x, 0\}$	$\{67\sigma_x, 0\}$	$\{0, 1\sigma_y\}$	$\{0, 58\sigma_y\}$
$p_{t,max}$	$10^{-3}\sigma_\delta$	$4\sigma_\delta$	$10^{-4}\sigma_\delta$	$0.26\sigma_\delta$

Betatron tune modulation

Tune derivatives

$\frac{\partial \nu_x}{\partial J_x}$	-5×10^4
$\frac{\partial^2 \nu_x}{\partial J_x \partial \delta}$	-6.8×10^7

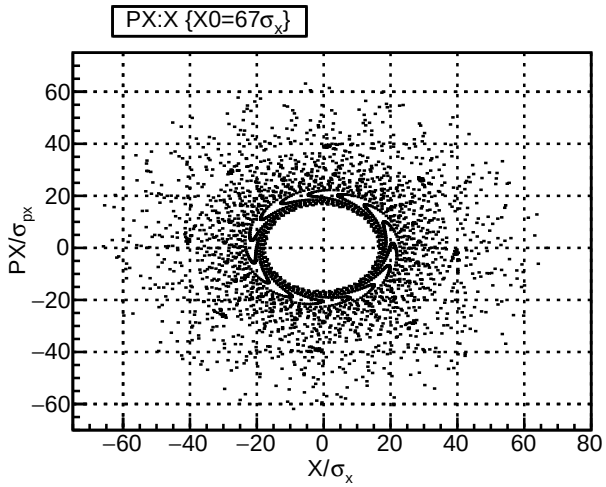
J_x	$67^2 \varepsilon_x / 2$
p_t	$7\sigma_\delta$
$\Delta\nu_x = \frac{\partial \nu_x}{\partial J_x} Jx$	-0.03
$\Delta\nu_x = \frac{\partial^2 \nu_x}{\partial J_x \partial \delta} J_x p_t$	-0.11
ν_x	0.14

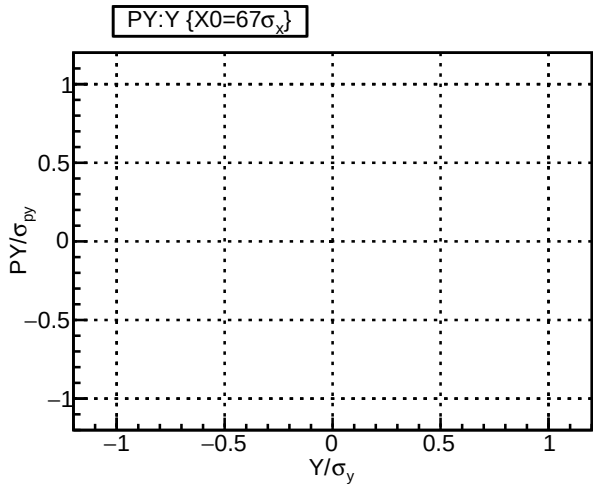
Tune variation from sextupoles

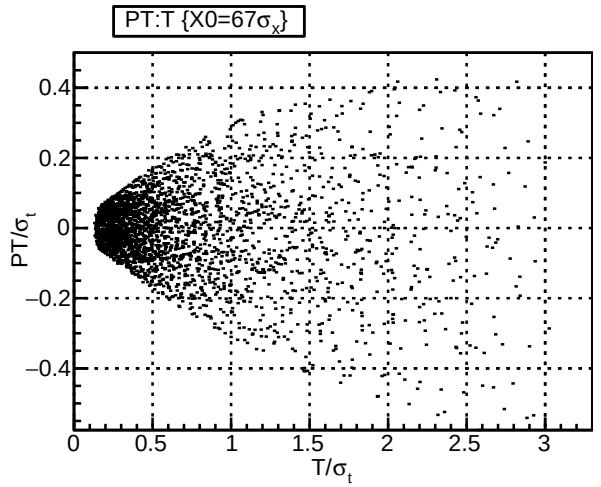
$$\frac{\partial \nu_x}{\partial J_x} = -\frac{1}{64\pi} \oint ds \oint d\tau K2(s) K2(\tau) \beta_x^{\frac{3}{2}}(s) \beta_x^{\frac{3}{2}}(\tau)$$
$$\times \left[\frac{3\cos\left(\pi\nu_x + |\varphi_x(s) - \varphi_x(\tau)|\right)}{\sin(\pi\nu_x)} + \frac{\cos\left(3\pi\nu_x + 3\,|\varphi_x(s) - \varphi_x(\tau)|\right)}{\sin(3\pi\nu_x)} \right]$$

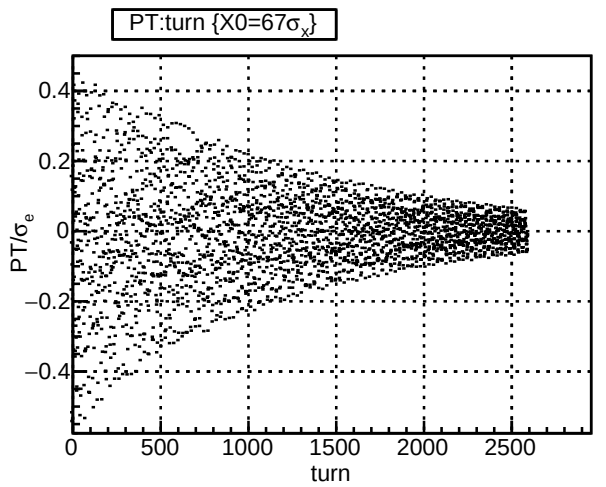
$\frac{\partial^2 \nu_x}{\partial J_x \partial \delta}$ comes from beta chromaticity $\frac{d\beta_x}{d\delta}$

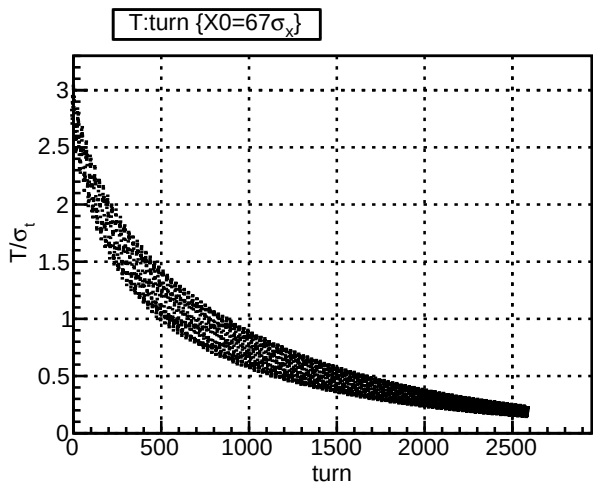
6d (SR from BEND, QUAD; longitudinally adjusted): $X_0 = 67\sigma_x$

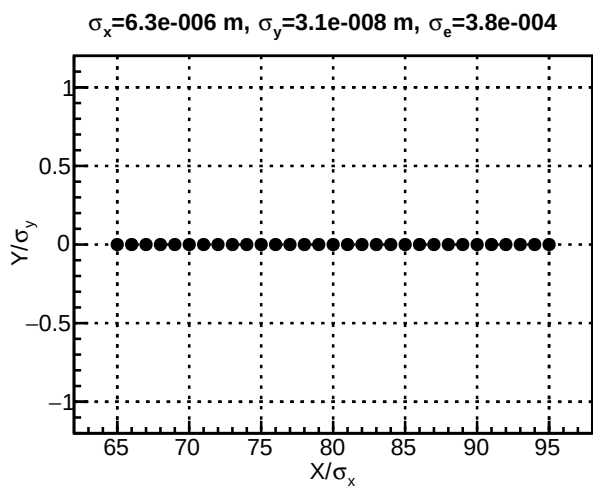




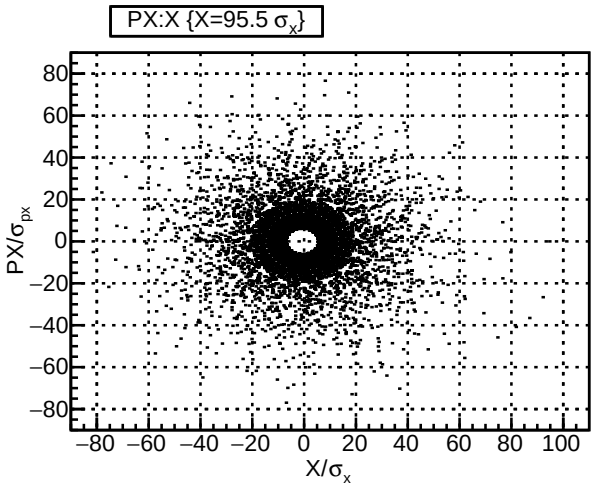


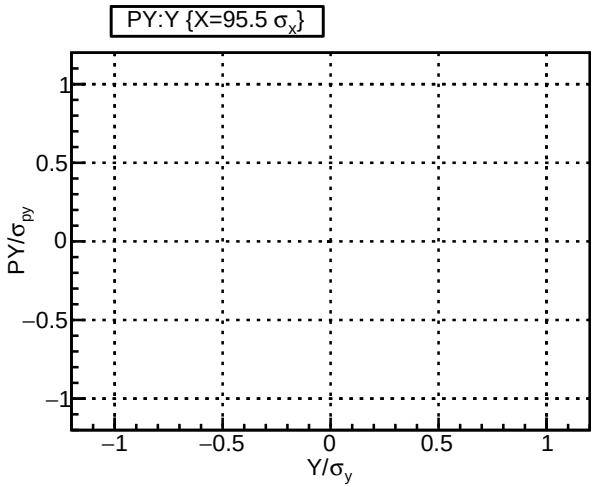


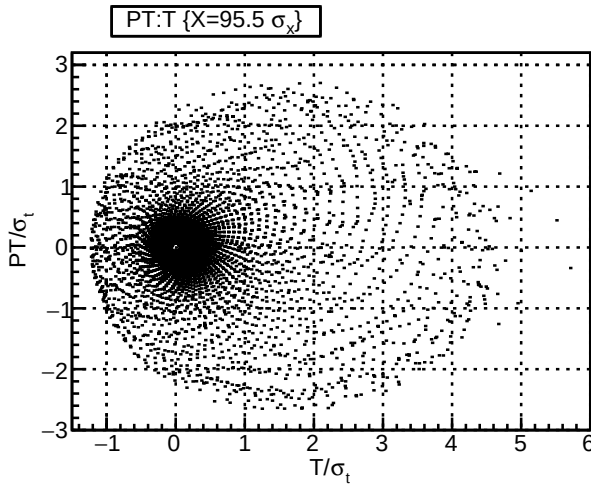


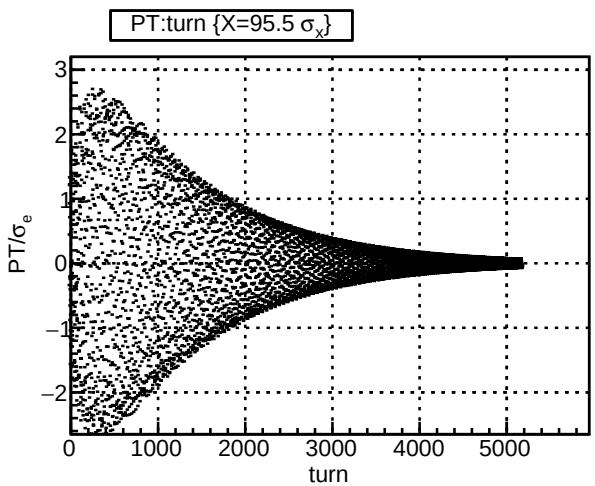


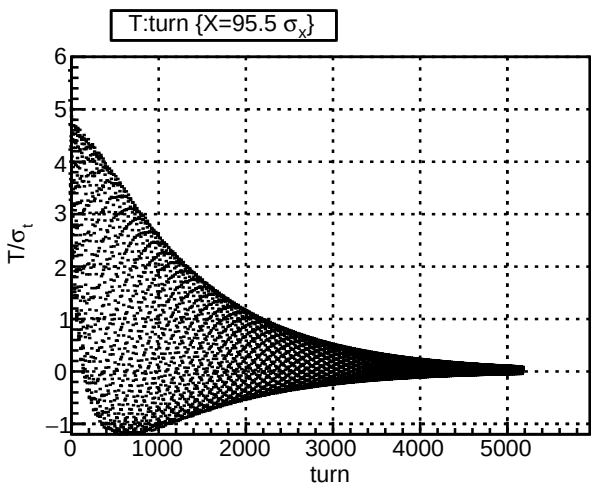
6d (SR from BEND, QUAD; longitudinally adjusted): $X_0 = 95.5\sigma_x$

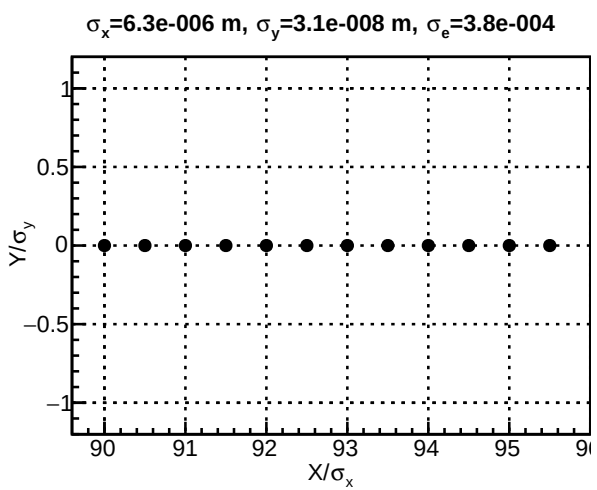






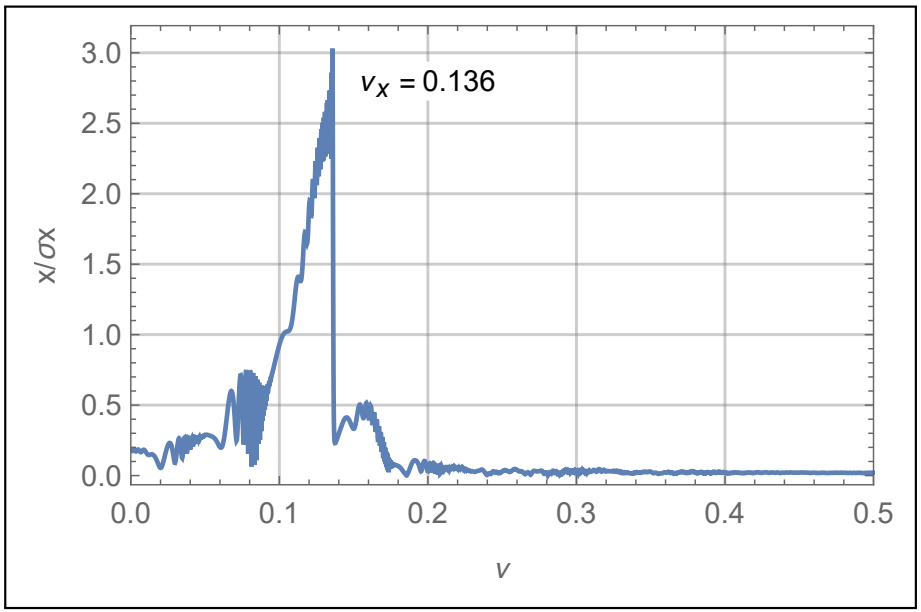


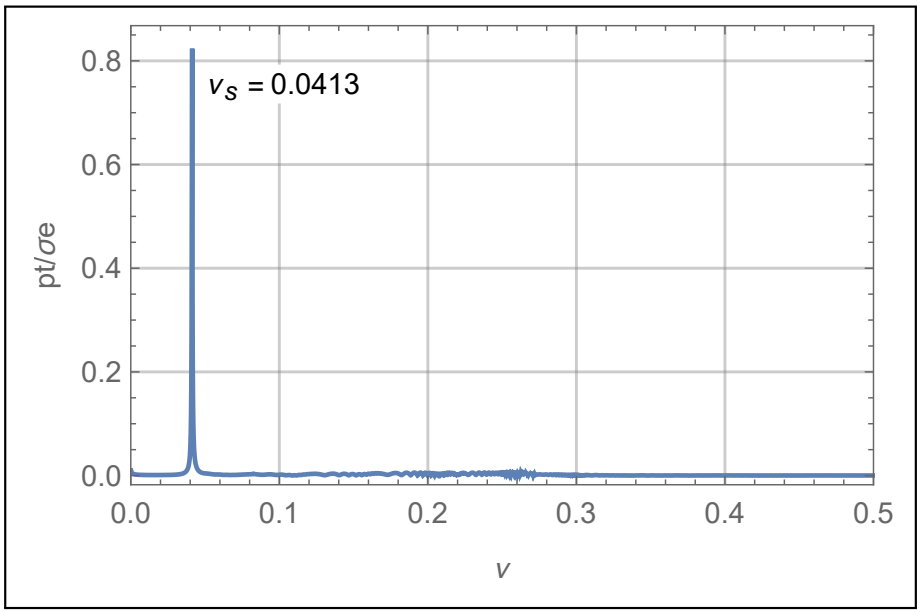




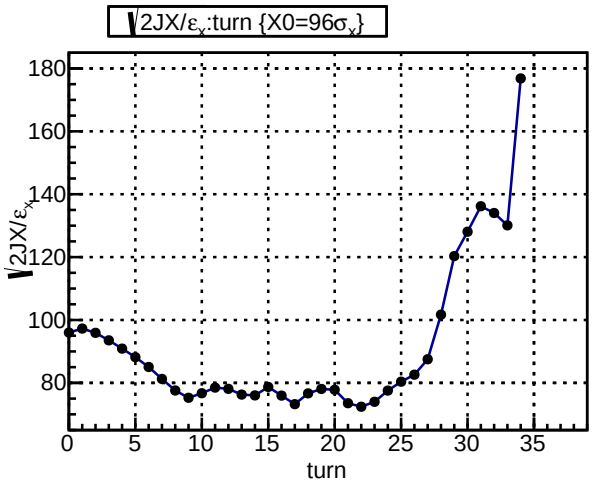
Spectra: $X_0 = 95.5\sigma_x$

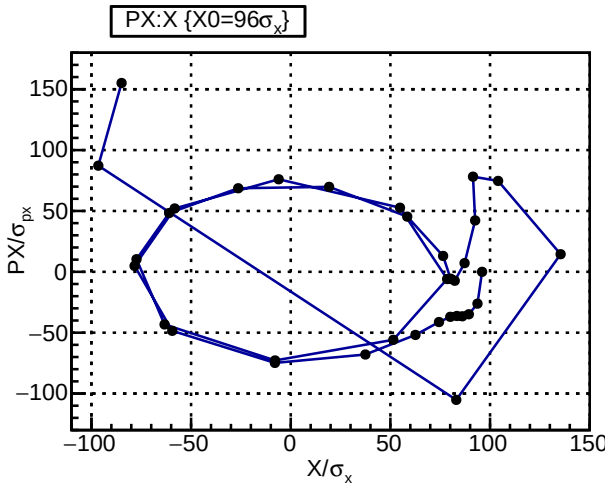
$\nu_x = 269.14$ $\nu_y = 267.22$ $\nu_s = 0.0413$

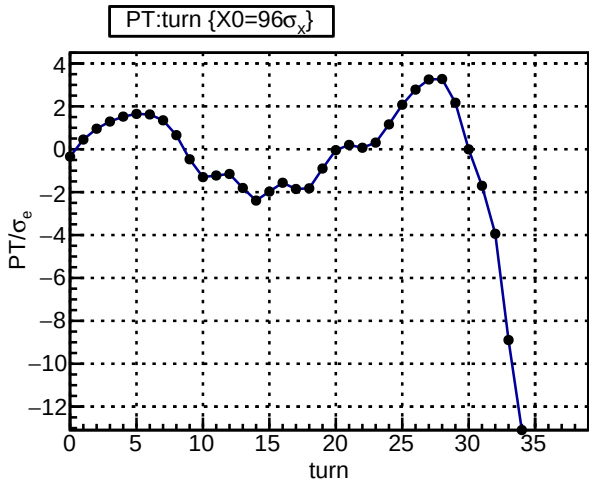


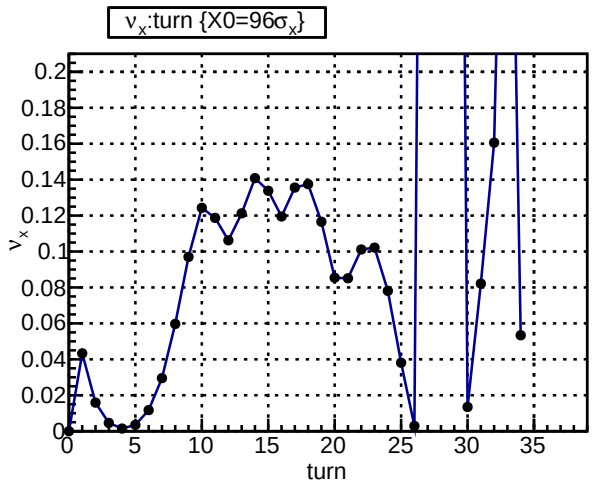


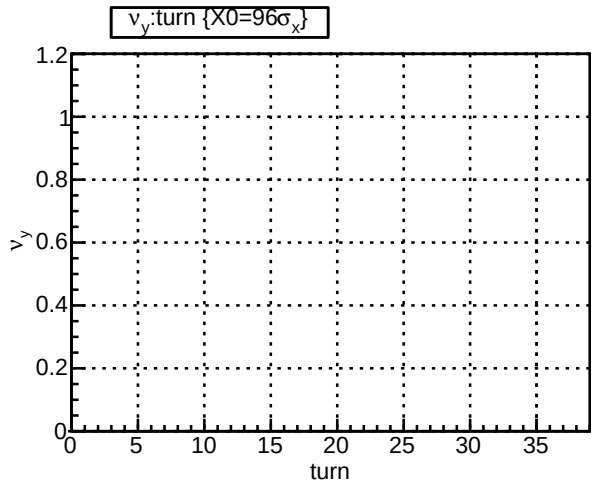
6d (SR from BEND, QUAD; longitudinally adjusted): first unstable, $X_0 = 96\sigma_x$

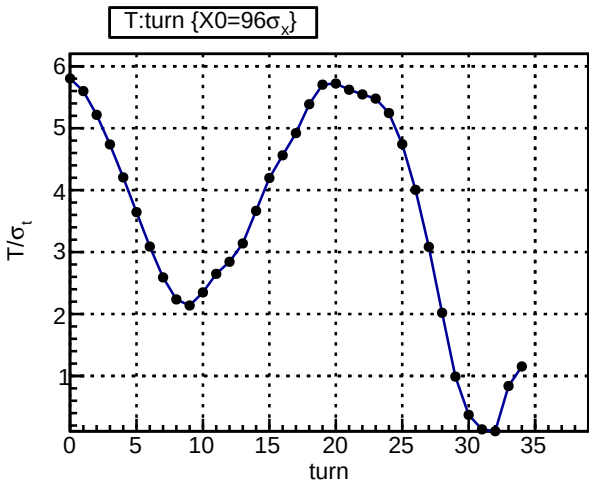




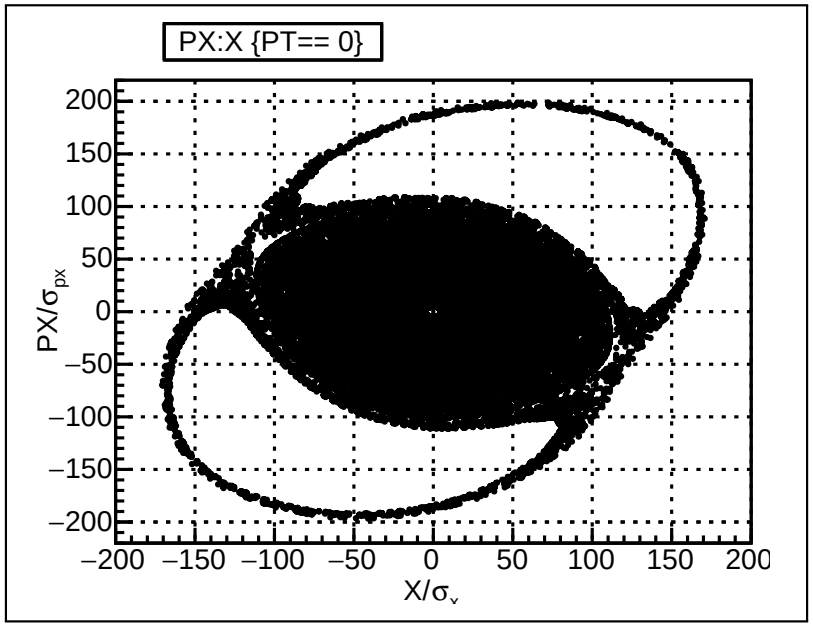


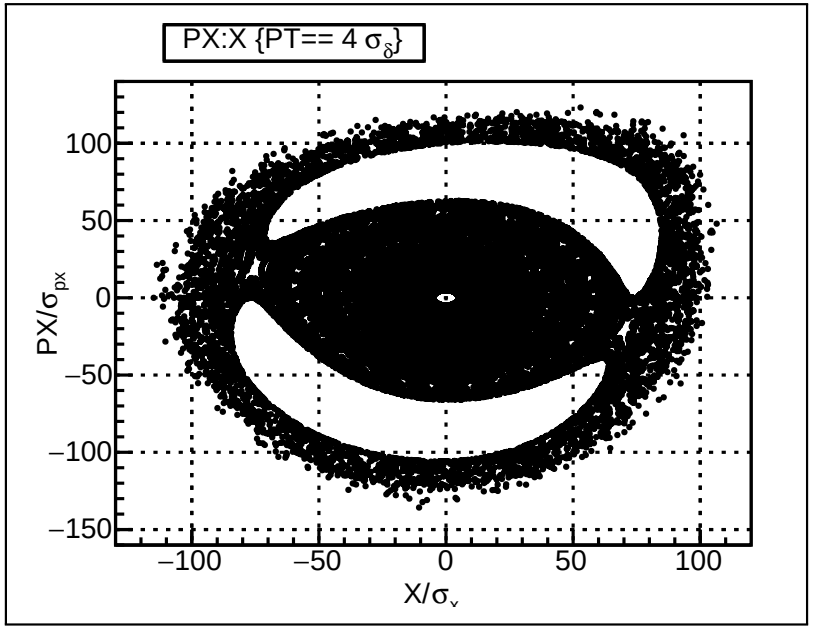


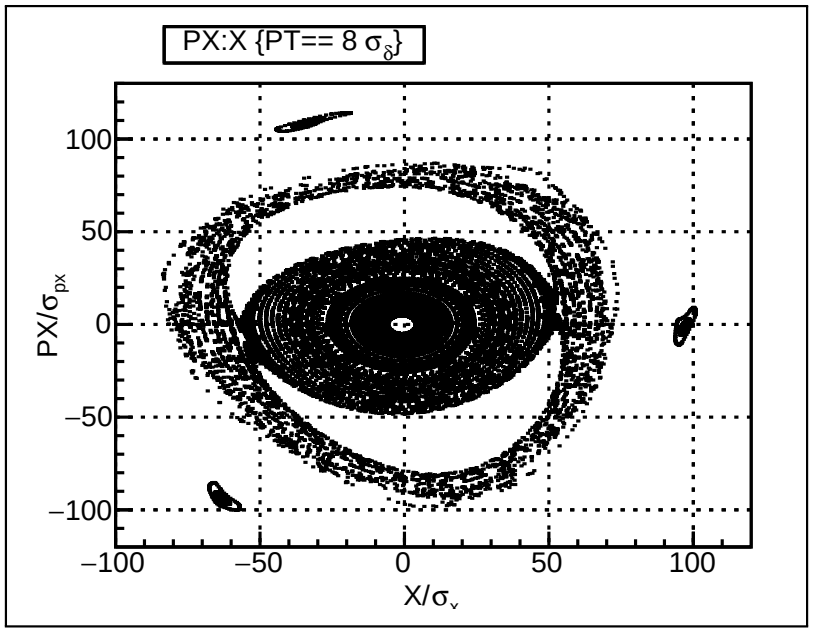


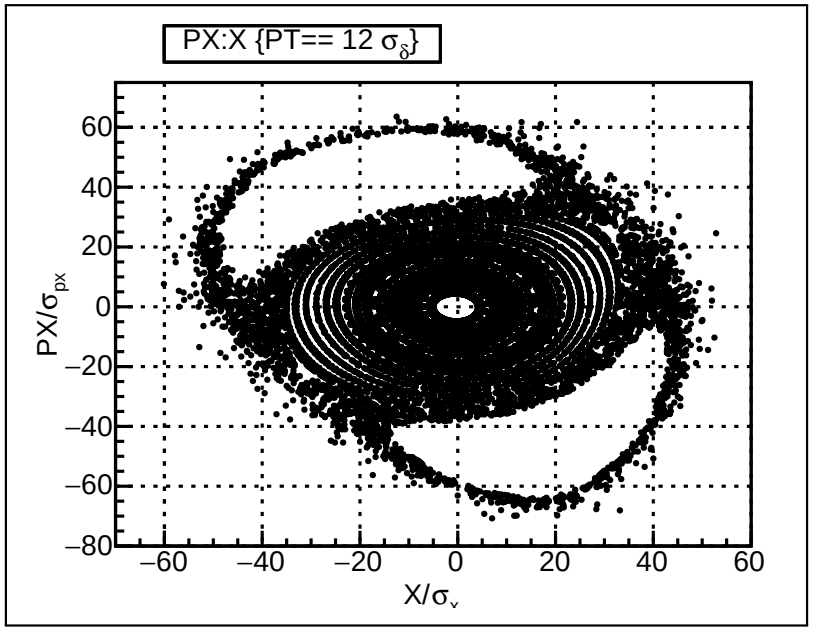


5d tracking: *PX* : *X*









Integer resonance: $PX : X$

$$\begin{pmatrix} \cos(\mu) & \beta \sin(\mu) \\ -\frac{1}{\beta} \sin(\mu) & \cos(\mu) \end{pmatrix} \begin{pmatrix} x \\ p_x \end{pmatrix} + \begin{pmatrix} 0 \\ kx^3 \end{pmatrix} = \begin{pmatrix} x \\ p_x \end{pmatrix}$$

Solution

$$\{x, p_x\} = \{0, 0\}$$

$$\{x, p_x\} = \left\{ \frac{\sqrt{2}}{\sqrt{k\beta}} \left(\tan \frac{\mu}{2} \right)^{\frac{1}{2}}, \frac{\sqrt{2}}{\sqrt{k\beta^3}} \left(\tan \frac{\mu}{2} \right)^{\frac{3}{2}} \right\}$$

$$\{x, p_x\} = \left\{ -\frac{\sqrt{2}}{\sqrt{k\beta}} \left(\tan \frac{\mu}{2} \right)^{\frac{1}{2}}, -\frac{\sqrt{2}}{\sqrt{k\beta^3}} \left(\tan \frac{\mu}{2} \right)^{\frac{3}{2}} \right\}$$

History

J. Jowett (CERN), Dynamic aperture for LEP: Physics and calculations, Conf.Proc. C9401174 (1994) 47-71, In *Chamonix 1994, LEP performance* 47-71

”Here I shall briefly describe a new effect which I propose to call Radiative Beta-Synchrotron Coupling (RBSC). It is a non-resonant effect. A particle with a large betatron amplitude make an extra energy loss by radiating in quadrupoles. ...you can say that its ”effective stable phase angle" will change to reflect the greater energy loss. The particle will tend to oscillate about a displaced fixed point in the synchrotron phase plane. This results in a growth of the oscillation amplitude which may eventually lead the particle outside the stable region in synchrotron phase space.”

Conclusion for horizontal plane at 45.6 GeV

1. Radiation from quadrupoles shifts the synchronous phase and energy proportional to the horizontal action,
2. therefore synchrotron oscillations arise.
3. Horizontal tune dependence on action and energy of the particle shifts the tune to integer resonance.
4. Minimization of beta function chromaticity will reduce $\frac{\partial^2 \nu_x}{\partial J_x \partial \delta}$ and enhance horizontal dynamic aperture.