

Heavy Ions Interactions

FLUKA Beginner's Course

Overview

Introduction

The physics models DPMJET

RQMD BME

Input options Beam definition

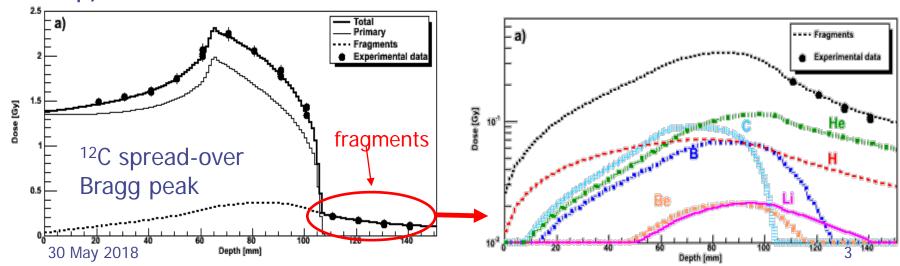
Transport thresholds

Energy ranges in which the models

are valid (optional)

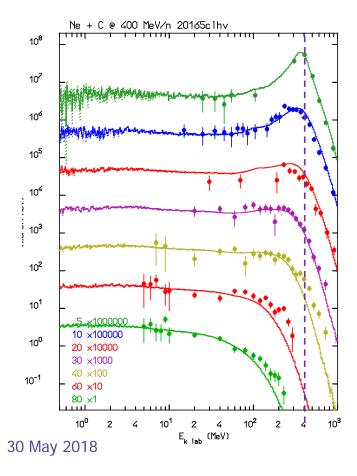
Introduction - 1

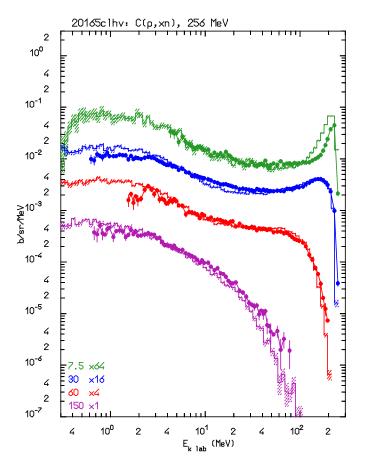
- In hadron nucleus interactions, reaction products and residuals come mostly from the TARGET nucleus
- In nucleus-nucleus interactions, reaction products and residuals come from both TARGET and PROJECTILE nuclei.
- Indeed, except for complete fusion, one often refers to "projectile-like" and "target-like" fragments
- projectile-like fragments travel with approximately the same projectile speed, thus they can be energetic, and travel longer or shorter than the average projectile range (range ≈÷ A/Z² at given β)



Introduction - 2

Neutron energy spectra at different angles for Ne+C at 400 MeV/n (left), and p+C at 256 MeV (right). Note the high energy (>E/A) tails, and the different shapes. Also, different "effects" of reaction stages: in A-A, evaporation products can be fast (from proj like)!





Heavy ion interaction models in FLUKA

E > 5 GeV/n

Dual Parton Model (DPM)
DPMJET-III (by R.Engel, A.Fedynitch, J.Ranft and S.Roesler,
FLUKA-implementation by T.Empl et al.)

~0.1 GeV/n < E < 5 GeV/n

Relativistic Quantum Molecular Dynamics Model (RQMD) RQMD-2.4 (original code by H.Sorge et al., FLUKA-implementation by A.Ferrari et al.)

E < ~0.1 GeV/n

Boltzmann Master Equation (BME) theory BME (original code by E.Gadioli et al., FLUKA-implementation by F.Cerutti et al.)

At ALL Energies:

DPMJET

E > 5 GeV/n

Dual Parton Model (DPM)

DPMJET-III (original code by R.Engel, J.Ranft and S.Roesler, present version by A.Fedynitch

FLUKA-implementation by T.Empl et al.)

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DPMJET – Overview

DPMJET = Dual Parton Model and JETs

Monte Carlo **event-generator** for the simulation of high-energy hadronic interactions

DPMJET - Version III.1

 hadron-nucleus collisions nucleus-nucleus collisions photon-nucleus collisions off nuclei

It uses PHOJET for

 hadron-hadron collisions photon-hadron collisions

energy range: 5 GeV/nucleon – 10¹¹ GeV/nucleon

programming language: Fortran 77 size of the code: about 180000 lines

30 May 2018 authors: R.Engel, A.Fedynitch, J.Ranft, S.Roesler

DPMJET - Basic physics (review)

DPMJET: (as well as the FLUKA high energy h—A generator) is based on the Dual Parton Model in connection with the Gribov-Glauber formalism.

Parton model: to analyze high-energy hadron collisions. Hadrons are considered made of "partons".

Glauber formalism: elastic, quasi-elastic and absorption hadronnucleus (h-A) cross sections are derived from the hadron-nucleon (h-N) cross sections.

Inelastic interactions are equivalent to multiple interactions of the projectile with the target nucleons.

Gribov theory: the elastic hadron-nucleus (h-A) amplitude is obtained by the Glauber model (multiple elastic rescatterings) plus all possible diffractive excitations of the initial hadron.

DPMJET - The Gribov-Glauber formalism

Fundamental idea: nucleus-nucleus collision expressed in terms of individual nucleon-nucleon interactions

• nucleus-nucleus

- total cross section
- elastic cross section

- scattering amplitude

$$\sigma_{AB}^{tot}(s) = 4 \int d^2 \vec{B} \Im [A_{AB}(s, \vec{B})]$$

$$\sigma_{AB}^{el}(s) = 4 \int d^2 \vec{B} \left| A_{AB}(s, \vec{B}) \right|^2$$

$$A_{AB} = \frac{i}{2} \left[1 - \exp(X_{AB}) \right]$$

eikonal function

$$\chi_{AB} = \sum_{k,l} \chi_{N_{k}N_{l}}$$

- nucleon-nucleon
 - scattering amplitude

$$a_{N_k N_l} = \frac{i}{2} \left[1 - \exp(\chi_{N_k N_l}) \right]$$

DPMJET – Main steps (1)

1. Interaction of high energy nuclei:

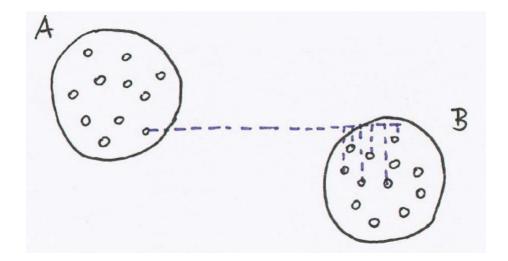
Individual nucleon-nucleon scatterings

Dual Parton Model as a two components model using:

Gribov's reggeon field theory for soft and perturbative interactions

QCD improved parton model for hard interactions

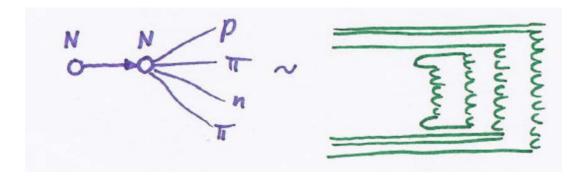
Formation of strings between valence and sea partons (quarks, gluons)



DPMJET - Main steps (2)

2. Hadronization process

Creation of hadrons / resonances from string fragmentation

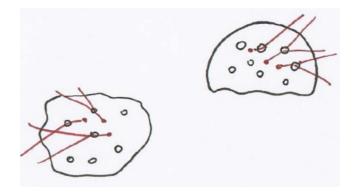


Results are **hadrons** and **spectators**, where the latter ones are nucleons from the projectile and target nuclei that did not take part in the interactions

DPMJET - Main steps (3)

3. Intranuclear Cascade

Secondary low-energy interactions of hadrons with spectator nucleons



Hadrons are followed in space and time as straight trajectories Hadrons may re-interact after certain *formation time*

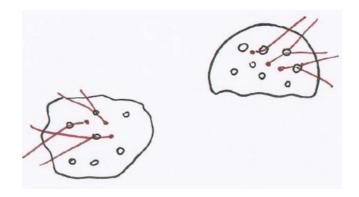
→ Emission of nucleons

Spread of excitation energy

DPMJET - Main steps (4)

3. Intranuclear Cascade

Secondary low-energy interactions of hadrons with spectator nucleons



Note: DPMJET has its own implementation of intranuclear cascade. It is similar in its fundamental ideas to the one of FLUKA but it is much more simplified.

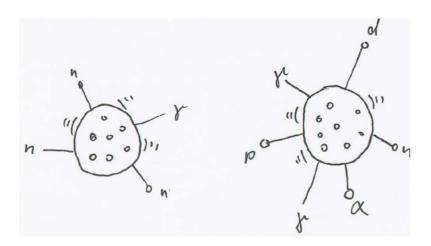
PEANUT is activated at all energies < 20 TeV for hN and hA interactions, above DPMJET-III is used

DPMJET - Main steps (5)

4. Break-up of excited spectator nuclei

Excited fragments are treated by **PEANUT** in FLUKA by

- Fermi break-up for light residual nuclei
- high-energy fission
- γ -deexcitation \longrightarrow production of final residual nuclei



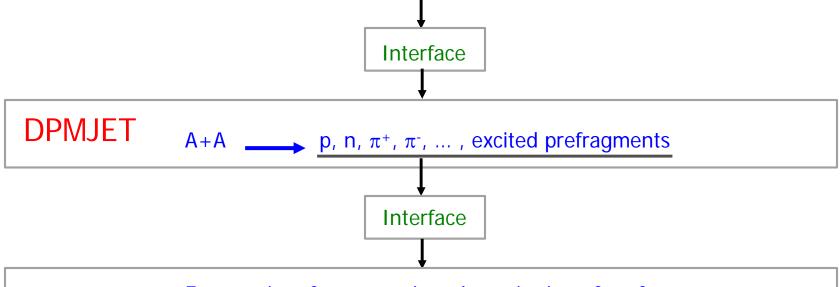
DPMJET - Interface to FLUKA

Initialization of DPMJET

FLUKA

pre-computed cross sections and impact parameter distributions read in from data files (complete matrix of projectile-target combinations up to A=246 and entire energy range)

Call for: Single nucleus-nucleus interaction (E>5 GeV/n), hadron-hadron and hadron-nucleus interactions (E>20 TeV)



FLUKA

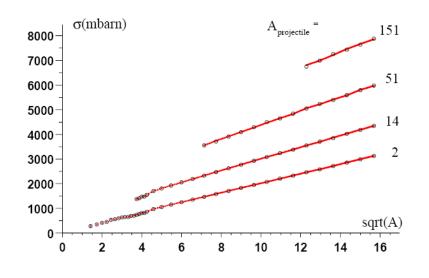
Evaporation, fragmentation, de-excitation of prefragments Transport of produced hadrons

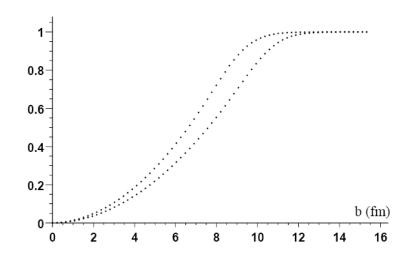
DPMJET – Pre-computed parameters

Examples for pre-initialised data:

Inelastic cross sections

Impact parameter distribution

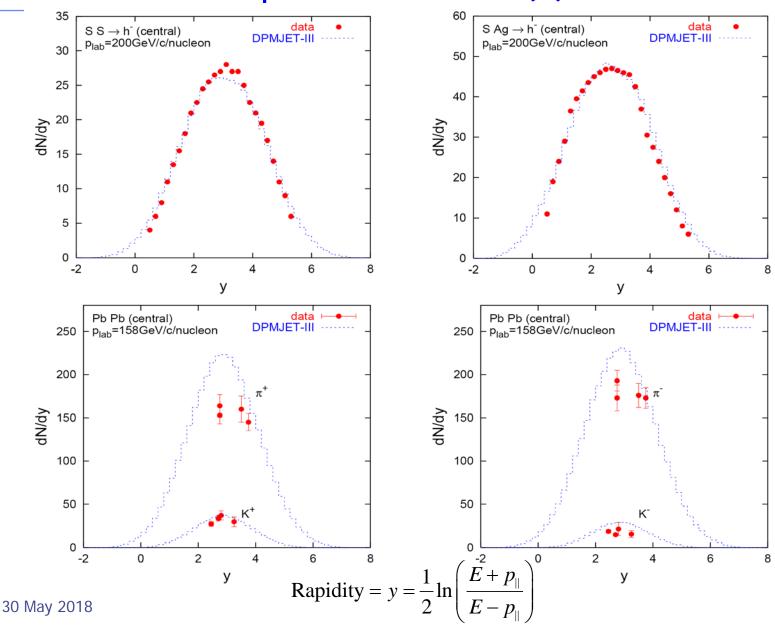




 $E_{Lab}^{=}$ 6.3×10^{9} GeV/nucleon

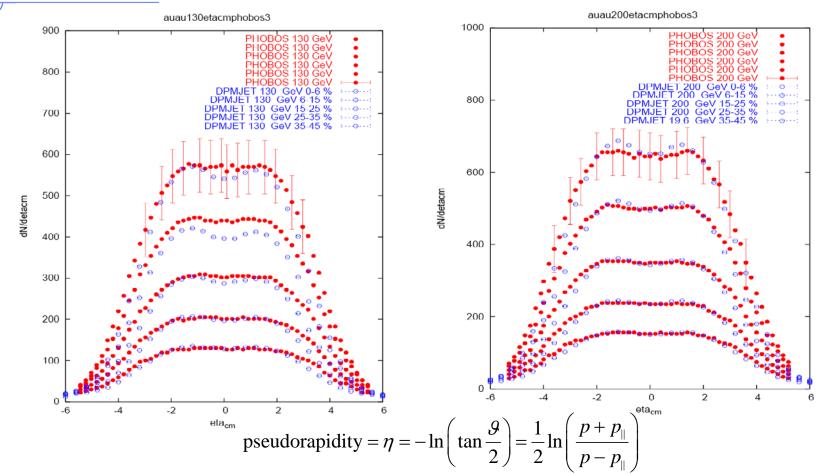
e.g., for highest and lowest energy at one fixed projectile-target configuration

DPMJET – Comparison to data (1)



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DPMJET – Comparison to data (2)

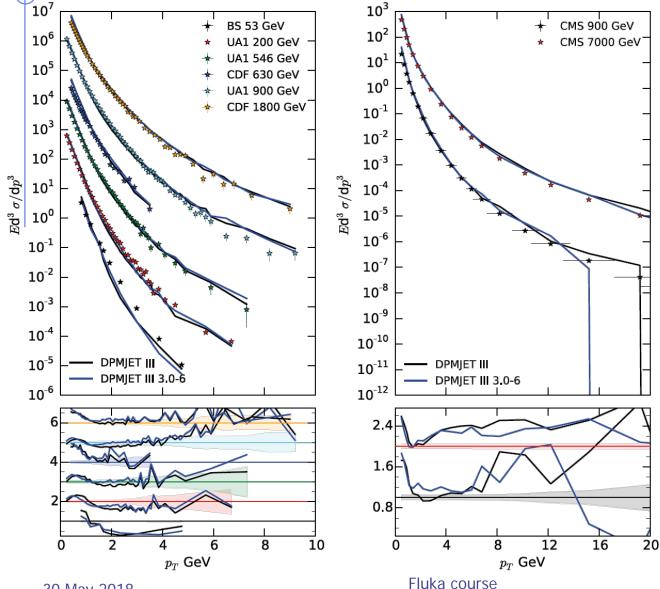


Pseudorapidity distribution of charged hadrons produced in Au-Au collisions at a c.m. energy of 130GeV/A (left) and 200GeV/A (right) for different ranges of centralities.

Exp. data: PHOBOS-Collaboration

J.Ranft, in Proceedings of the Hadronic Shower Simulation Workshop, CP896, Batavia, Illinois (USA), 6-8 September 2006

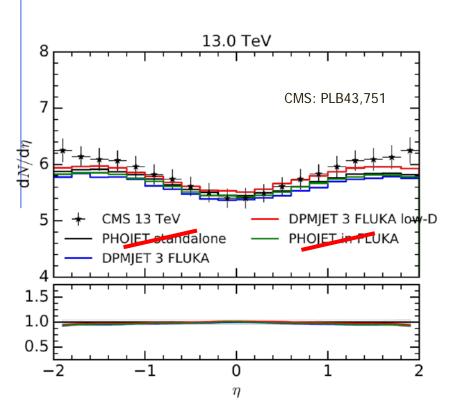
... Phojet/Dpmjet vs LHC results



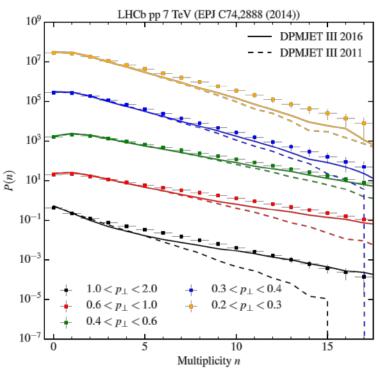
Invariant cross
section for
charged particles
as a function of
transverse
momentum for
pp collisions at
various centreof-mass energies

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... Phojet/Dpmjet vs LHC results

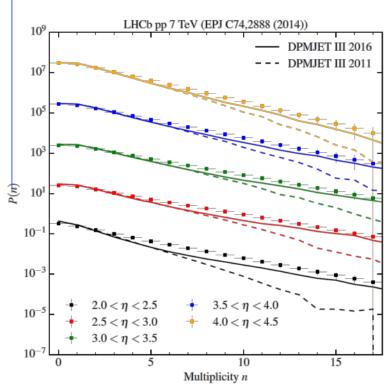


Average charged particle multiplicity as a function of pseudo rapidity η in the central region as measured by CMS @ \sqrt{s} =13 TeV

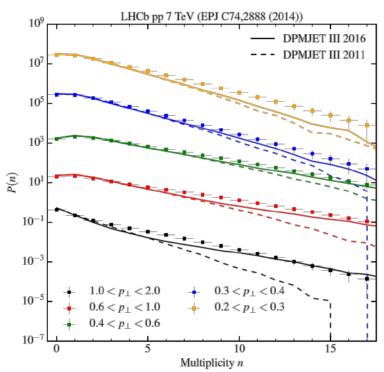


Charged particle multiplicity distribution for different p_T ranges in the forward region (2< η <4.5) as measured by LHCb @ \sqrt{s} =7 TeV

... Phojet/Dpmjet vs LHC results

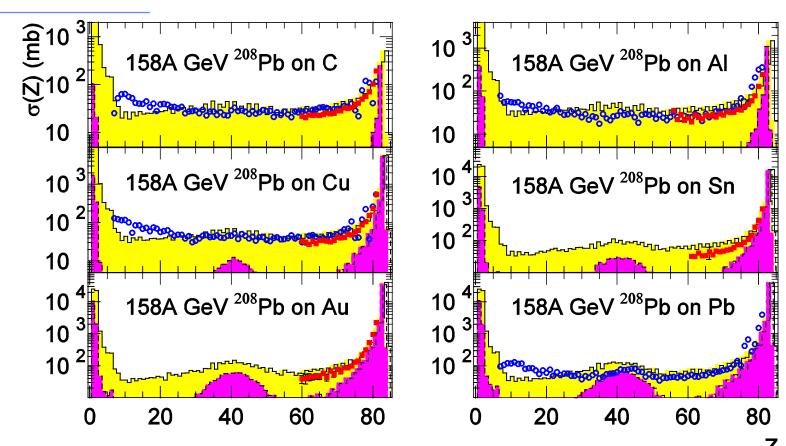


Charged particle multiplicity distribution for different η ranges in the forward region (0<p_T<2) as measured by LHCb @ \sqrt{s} =7 TeV



Charged particle multiplicity distribution for different p_T ranges in the forward region (2< η <4.5) as measured by LHCb @ \sqrt{s} =7 TeV

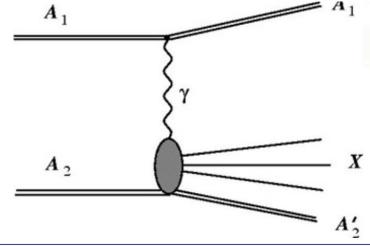
DPMJET – FLUKA benchmarks



Fragment charge cross sections for 158GeV/n Pb ions on various targets. FLUKA: solid histogram (line total, purple EMD, yellow DPMJET-III) Exp. data: NPA662, 207 (2000), NPA707, 513 (2002) (blue circles), C.Scheidenberger et al. PRC70, 014902 (2004), (red squares)

Electromagnetic dissociation (review)

- Very peripheral collisions
- Break-up of one of the colliding nuclei in the electromagnetic field of the other nucleus



```
        PHYSICS
        2.0
        0.0
        0.0
        0.0
        0.0
        0.0 EM-DISSO

        PHYSICS
        Type: EM-DISSO ▼
        EM Disso: Proj&Target EM-Disso ▼
```

WHAT(1): flag for activating ion electromagnetic-dissociation

```
=< -1.0 : resets to default (no em-dissociation)
```

= 0.0 : ignored

= 1.0 : (default) no em-dissociation

= 2.0 : projectile and target em-dissociation activated

= 3.0 : projectile only em-dissociation activated

= 4.0 : target only em-dissociation activated

WHAT(2)-WHAT(6): not used

ROMD

E > 5 GeV/n

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E < ~0.1 GeV/n

Boltzmann Master Equation (BME) theory

BME (original code by E.Gadioli et al.,

FLUKA-implementation by F.Cerutti et al.)

RQMD - FLUKA implementation (1)

RQMD, a relativistic QMD model, adapted to FLUKA: RQMD-2.4

```
H. Sorge, Phys. Rev. C 52, 3291 (1995);
```

H. Sorge, H. Stöcker, and W. Greiner, Ann. Phys. 192, 266 (1989), Nucl. Phys. A 498, 567c (1989)

 QMD: Follows the Time evolution of the combined A+A system performing n-n interactions

mean field effects short range interactions

Re-calculation of the nuclear potentials from sum of two-body fields fields due to the nucleons of the same nuclei fields due to the nucleons of the other particle



In FLUKA used in its faster cascade-like version

RQMD - FLUKA implementation (2)

 A-posteriori identification of residual fragments and their excitation was not provided by the original RQMD: added in the FLUKA implementation.
 Fragment de-excitation (evaporation, fission, Fermi break-up) is performed in PEANUT



Statistical approach

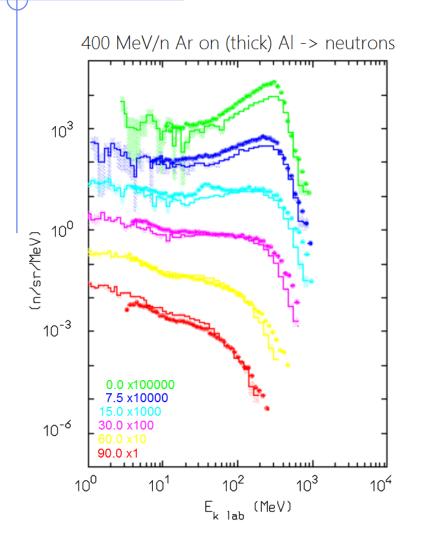
Correct energy/momentum conservation:

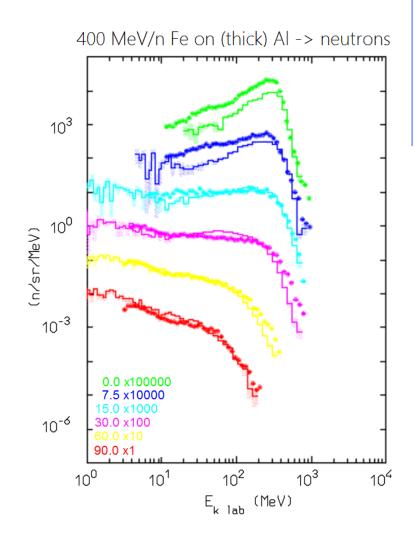
Nuclear final state reworked out of the information on spectators

Excitation energy deduced from the holes left

Use of experimental binding energies

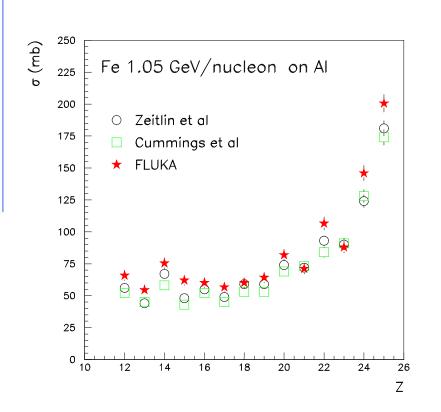
RQMD - FLUKA benchmarks (1)

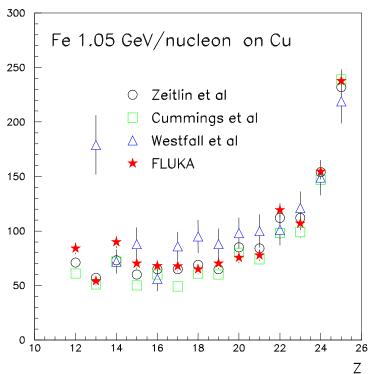




Exp. Data from T. Kurosawa *et al*, Phys. Rev. **C 62**, 044615 (2000)

RQMD - FLUKA benchmarks (2)





Fragment charge cross section for 1.05 GeV/n Fe ions on AI (left) and Cu (right).

Exp. data from PRC56, 338 (1996), PRC42,5208(1990) and PRC19, 1309 (1979)

BME

E > 5 GeV/n

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~0.1 GeV/n < E < 5 GeV/n

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E < ~0.1 GeV/n

Boltzmann Master Equation (BME) theory BME (original code by E.Gadioli et al., FLUKA-implementation by F.Cerutti et al.)

BME - References

interface to a Monte Carlo code founded on the BME theory (E. Gadioli et al.)

[M. Cavinato et al., Nucl. Phys. A 679, 753 (2001),

M. Cavinato et al., Phys. Lett. **B 382**, 1 (1996)]

BME – The nuclear interaction processes

- Complete Fusion: projectile and target nuclei interact and merge in a composite nucleus (P+T → C)
- Transfer: pickup reaction where the smaller nucleus is fully overlapped by the density distribution of the bigger one and collects some of the partner nucleons
- 3body: projectile and target nuclei interact with partial overlap of the density distributions, a hot region is produced (middle source X) and 3 outgoing fragments result (P+T → B+Y+X, with B and Y proj- and target-like)
- Incomplete FUSion: as 3 body, with the middle source absorbed by one nucleus $(P+T \rightarrow B+W \text{ or } P+T \rightarrow Z+Y)$
- "Inelastic" collisions: either the projectile or the target loses a single nucleon, possibly absorbed by the partner nucleus

BME – The implemented code

pre-equilibrium de-excitation of the produced fragment(s)
according to the BME theory (where available)
or the PEANUT exciton model

NB interface to PEANUT pre-eq
not yet distributed!

In order to get the multiplicities of the pre-equilibrium particles and their double differential spectra, the BME theory is applied to several representative systems at different bombarding energies and the results are parameterized.

```
12C + 8Li, 8B, <sup>12</sup>C, <sup>27</sup>AI, <sup>40</sup>Ca

16O + 6Li, 8Li, 8B, <sup>10</sup>B, <sup>12</sup>C, <sup>14</sup>N, <sup>16</sup>O, <sup>19</sup>F, <sup>20</sup>Ne, <sup>24</sup>Mg, <sup>27</sup>AI, <sup>56</sup>Fe, <sup>197</sup>Au at 12, 30, 50, 70, 100 MeV/n
```

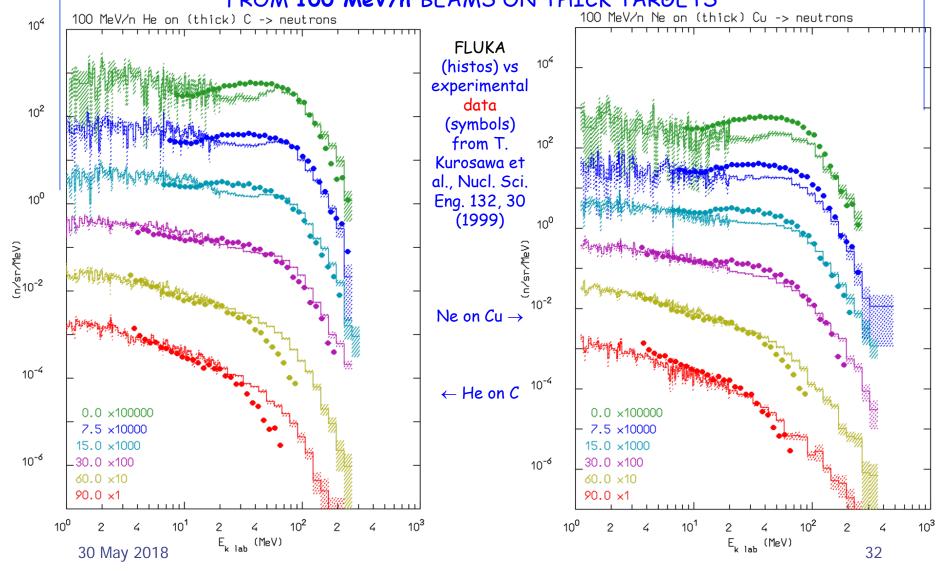
Work is ongoing to extend it to more massive systems, e.g.

and consequently review the fitting functions and the extrapolation recipes over a significantly larger mass range

FLUKA evaporation/fission/fragmentation/gamma de-excitation

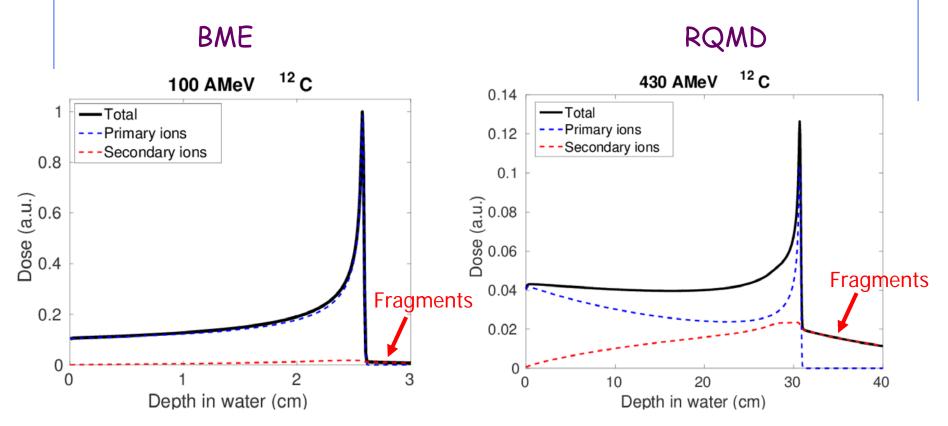
BME – Benchmarking (1)





BME – Medical applications

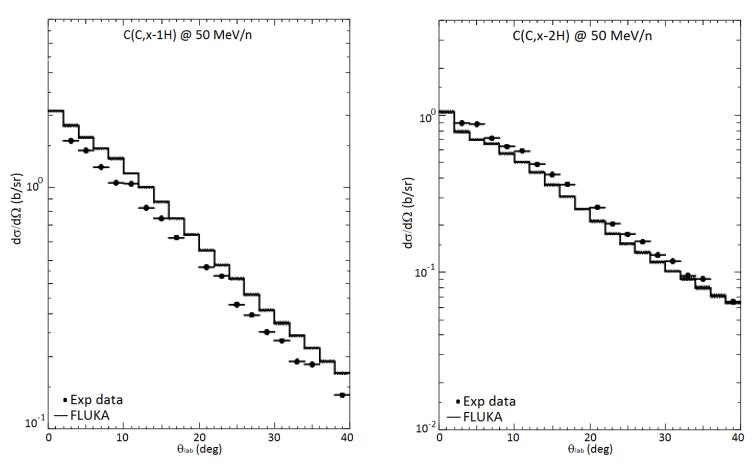
CARBON ION BRAGG PEAKS IN WATER



Primary ion fragmentations affect the dose delivered in the patient. Fragments has to be considered in the simulations for accurate dose calculations.

BME – Benchmarking (2)

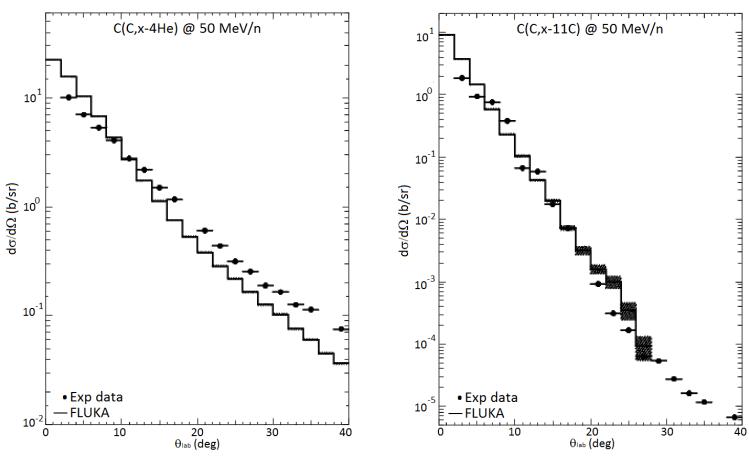
SINGLE DIFFERENTIAL FRAGMENT SPECTRA FROM C+C at 50 MeV/n



Experimental data from Divay et al (2017) Phys Rev C 95 044602

BME – Benchmarking (3)

SINGLE DIFFERENTIAL FRAGMENT SPECTRA FROM C+C at 50 MeV/n



Experimental data from Divay et al (2017) Phys Rev C 95 044602

a) define momentum / energy

```
-10.0
                                    0.0
                                                  0.0
                                                              0.0
                                                                           0.0
                                                                                       0.0HEAVYION
BEAM
                              Beam:Energy ▼
                                                            E: 10.0
                                                                                      Part: HEAVYION ▼
₩BEAM
        Δp:Flat ▼
                                Δp: 0.0
                                                           Δφ:Flat ▼
                                                                                      Δφ: 0.0
   Shape (X): Rectangular ▼
                                                       Shape (Y): Rectangular ▼
                                ∆x: 0.0
                                                                                       ∆y: 0.0
```

```
Note: for SDUM = HEAVYION units <u>per nucleon</u> (in fact per nmu) for SDUM = 4-HELIUM, etc. per nucleus
```

```
WHAT(2) beam momentum spread (GeV/c)
WHAT(3)-WHAT(6) (as for any other particle)
SDUM = HEAVYION
also    4-HELIUM    alpha
    3-HELIUM    3-helium
```

TRITON tritium

DEUTERON deuterium 30 May 2018

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b) define charge and mass (required for BEAM/SDUM=HEAVYION)

If:

```
BEAM -10.0 0.0 0.0 0.0 0.0 0.0HEAVYION
```

by default: 12C

```
WHAT(1) = Atomic number Z of the heavy ion, Default: 6.0
```

WHAT(2) = Mass number A of the heavy ion, Default: 12.0

WHAT(3) = if < 0 isomeric state of the heavy ion</pre>

Otherwise, to define another heavy ion (e.g. ⁷⁹Au)

HI-PROPE	79.0	197.0	0.0	0.0	0.0	0.0	
**************************************		Z: 7 9.0		^{A:} 197.0		Isom: 0.0	

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c) switch on heavy ion transport and interactions

```
IONTRANS -2.0

IONTRANS Transport: Full transport ▼
```

(pleonastic in case of ion beams)

```
WHAT(1) >= 1 : no transport, full or approximate, of any light/heavy ion
= 0 : ignored
= -1 : approximate transport (without interactions) of all light and heavy ions
= -2 : full transport of all light and heavy ions
>= -6 and <= -3 : full transport of light ions with FLUKA id >= WHAT(1)

(-3=d,-4=t,-5=3-He,-6=4-He), and approximate transport of all other ions
```

Default: 0.0 (no ion transport, unless a ion beam is requested by the BEAM card

WHAT(2-6) and SDUM not used.

c) switch on heavy ion transport and interactions



When requested, interactions at energies larger than 100MeV/n are performed *provided that the external event generators DPMJET and RQMD are linked* (through the script \$FLUPRO/flutil/ldpmqmd).

In the presence of a heavy ion beam, full transport of all ions is enabled by default

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IMPORTANT:

 the DPMJET/RQMD event generators are EXTERNAL, they are distributed with FLUKA but not included in the main library neither in the standard executable

 Don't forget to link the DPMJET/RQMD event generators for enabling ion-ion interactions above 125MeV/n either using FLAIR or the script \$FLUPRO/flutil/ldpmqmd

 The BME event generator, covering the low energy range up to 150MeV/n does not need to be linked since it's already embedded in the main \$FLUPRO/libflukahp.a library and linked in the standard \$FLUPRO/flukahp executable

Card: PHYSICS

Please activate the following two cards if residuals are of interest:

switch to activate evaporation of heavy fragments (up to A=24, CPU expensive)

PHYSICS3.0EVAPORATPHYSICS1.0COALESCE

→ special options for coalescence treatment

Card: PHYSICS

Please activate the following two cards if residuals are of interest:

→ switch to activate evaporation of heavy fragments (up to A=24, CPU expensive)

PHYSICS 3.0 EVAPORAT PHYSICS 1.0 COALESCE

For SDUM = EVAPORATion:

WHAT(1): flag for FLUKA evaporation model

=< -1.0 : resets to default (new model, no heavy fragment evaporation)

= 0.0: ignored

= 1.0 : old evaporation model (OBSOLETE: kept for developers)

= 2.0 : new evaporation model, no heavy fragment evaporation

= 3.0 : new evaporation model, with heavy fragment evaporation (CPU expensive)

Default = 2.0 (new evaporation model, no heavy fragment evaporation)

WHAT(2)-WHAT(6): not used

Card: PHYSICS

Please activate the following two cards if residuals are of interest:

PHYSICS 3.0 EVAPORAT
PHYSICS 1.0 COALESCE

special options for coalescence treatment

For SDUM = COALESCEnce:

WHAT(1): coalescence flag

< 0.0 : false (no coalescence)

= 0.0: ignored

> 0.0 : true, coalescence is activated

Default = no coalescence

WHAT(2)-WHAT(3): reserved to developers' use

Warning: deuterons

- Deuteron interactions are NOT modelled in BME, therefore
 ** NO DEUTERON interactions are available in FLUKA below a few hundreds MeV ***
- RQMD performs the interaction, however reliability is not ensured due to the "special" nature of deuteron interactions

Current Francesc Salvat-Pujol's work:

- Reaction cross sections for d-A collisions
- Elastic break-up
- Stripping to bound state
- Stripping into continuum (→ PEANUT)

Transport thresholds

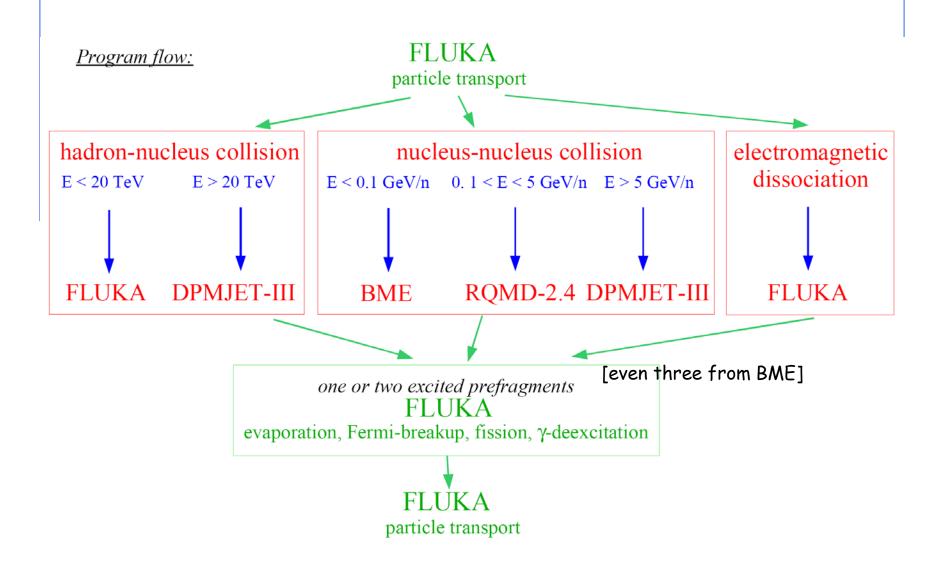
• The transport momentum threshold for ions $(p_{th,HI})$ is linked to that of alphas $(p_{th,\alpha})$

$$p_{th,HI} = p_{th,\alpha} \times m_{HI}/m_{\alpha}$$
 (GeV/c)

- The transport threshold for light ions (alpha, He-3, t, d) is set equal to total kinetic energy = 10 MeV (100 keV) if DEFAULTS=NEW-DEFA (PRECISIO).
- To change the transport threshold use the PART-THR card (requiring GeV and not GeV per nucleon)
- When the energy of an ion becomes lower than the transport threshold, and if such threshold is lower than 100 MeV/n, the ion is not stopped, but it is ranged out to rest

Additional information

Heavy ion interaction models in FLUKA - 2



DPMJET - Main steps of a high energy interaction

1. Interaction of high-energy nuclei

- individual nucleon-nucleon scatterings
- formation of »strings« between valence and sea partons (quarks, gluons)

2. Hadronization process

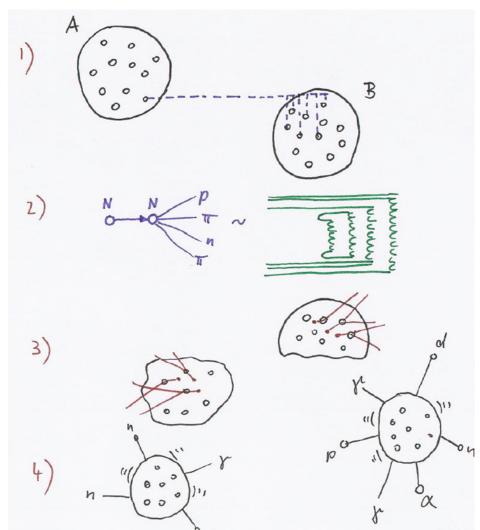
→ creation of hadrons / resonances

3. Intranuclear cascade

→ low-energy interactions of hadrons in spectator nuclei

4. Fragmentation of excited spectator nuclei

- → evaporation of light fragments (e.g., p, n, d, ³H, ³He, ⁴He,..),
- → fragmentation, fission
- → production of residual nuclei



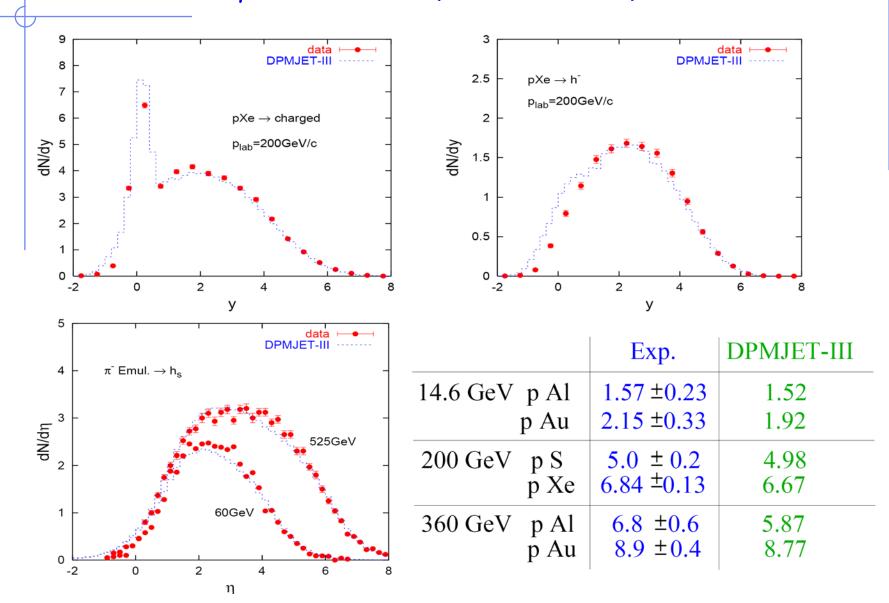
DPMJET - Intranuclear cascade and fragmentation

- nuclear model: Fermi-gas of nucleons in potential well
 nuclear densities: shell model (A ≤18)
 Wood-Saxon (A>18)
- hadrons are followed in space and time on straight trajectories
- hadrons may re-interact after certain formation-time (assume local nuclear density and vacuum cross sections)
- calculation of excitation energies (in AA interactions for both spectators)

<u>Note</u>: DPMJET has its own implementation of intranuclear cascades. It is similar in its fundamental ideas to that one of FLUKA but is much more simplified.

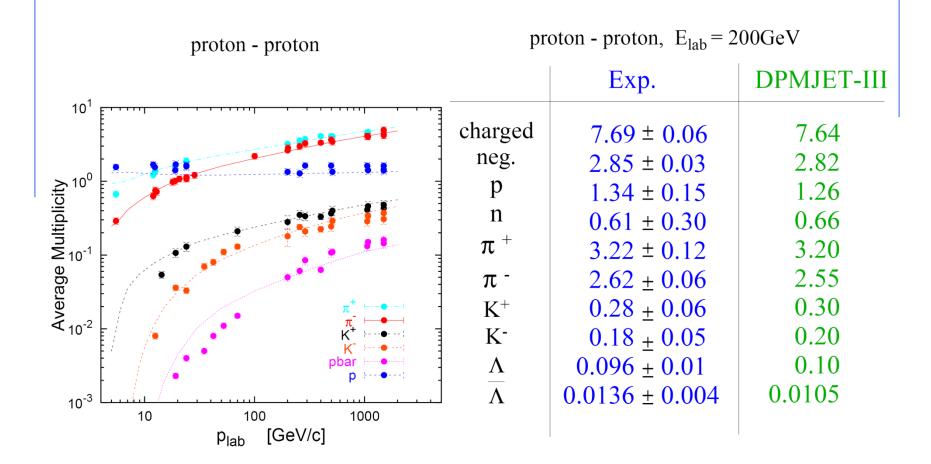
fragmentation by - nuclear evaporation
 Fermi-breakup
 high-energy fission
 γ-deexcitation

DPMJET - Comparison to data (hadron-nucleus)



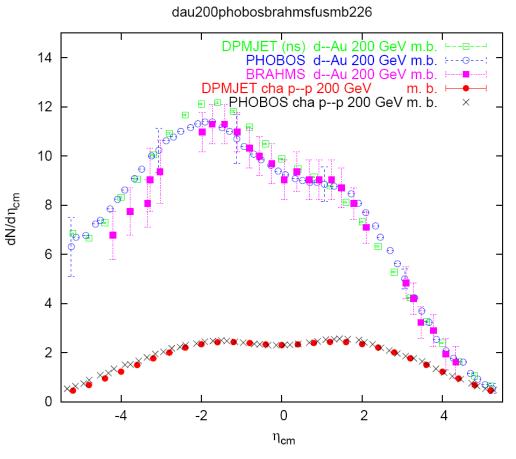
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DPMJET - Comparison to data (hadron-hadron)



DPMJET - Comparison to data (nucleus-nucleus)

Pseudorapidity distribution of charged hadrons produced in minimum bias d-Au and p-p collisions at a c.m. energy of 200GeV/A.



Exp. data: BRAHMS- and PHOBOS-Collaborations J.Ranft, in Proceedings of the Hadronic Shower Simulation Workshop, CP896, Batavia, Illinois (USA), 6-8 September 2006

RQMD - The original code

The RQMD-2.4 code

INITIAL CONDITION two Fermi gases (projectile and target)

Fermi momentum
$$p_{F0} = \hbar \left(3\pi^2 \frac{A}{2V}\right)^{1/3}$$
 $V = (4/3) \pi \left(r_0 A^{1/3}\right)^3$ $r_0 = 1.12 \, \text{fm} \Rightarrow \rho = 0.17 \, \frac{\text{nucl.}}{\text{fm}^3}$

nucleon momentum
$$p=p_{F0}\left(\frac{\rho\left(r\right)}{\rho_{0}}\right)^{\frac{1}{3}}\epsilon^{1/3} \qquad \epsilon\in\left[0,1\right] \text{ random}$$

$$\phi=2\pi\epsilon \qquad \qquad \cos\theta=1-2\epsilon$$

$$p_x = p \sin \theta \cos \phi$$
 $-(\sum p_x)/A$
 $p_y = p \sin \theta \sin \phi$ $-(\sum p_y)/A$ so $\sum p_x = \sum p_y = \sum p_z = 0$
 $p_z = p \cos \theta$ $-(\sum p_z)/A$

FINAL STATE

- (p^0, p_x, p_y, p_z) for nucleons (and produced particles) in the LAB frame
- the spectators are marked
- no residue and fragment identification
- energy non-conservation issues, particularly when run in full QMD mode

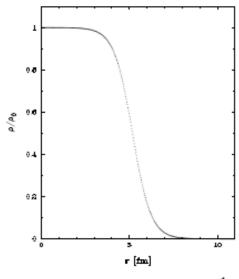
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RQMD - The interfaced code

Implemented developments

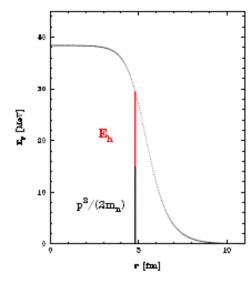
construct the projectile- and target-like nuclei by gathering spectator nucleons,

assuming
$$E_{PL}^{\star} = \sum_{pa.\ P} E_h$$
 (TL)



$$\rho(r) \propto \left(1 + \exp\left(\frac{r - R}{a}\right)\right)^{-1}$$

$$R = 1.19 A^{1/3} - 1.61 A^{-1/3} fm \qquad a = 0.52 fm$$



$$E_h = \frac{1}{2m_n} \left\{ \left[p_{F0} \left(\rho (r) / \rho_0 \right)^{1/3} \right]^2 - \rho^2 \right\}$$

$$r, p (t = 0)$$

- fix the remaining energy-momentum conservation issues taking into account experimental binding energies
- use the FLUKA evaporation/fission/fragmentation module

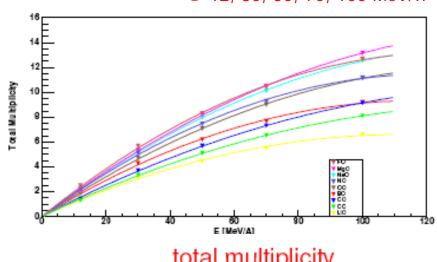
BME - The database for the pre-equilibrium emissions

In order to get the multiplicities of the pre-equilibrium particles and their double differential spectra, the BME theory is applied to several representative systems at different bombarding energies and the results are parameterized.

16O + ⁶Li, ⁸Li, ⁸B, ¹⁰B, ¹²C, ¹⁴N, ¹⁶O, ¹⁹F, ²⁰Ne, ²⁴Mg, ²⁷Al, ⁵⁶Fe, ¹⁹⁷Au

¹²C + ⁸Li, ⁸B, ¹²C, ²⁷Al, ⁴⁰Ca

@ 12, 30, 50, 70, 100 MeV/n



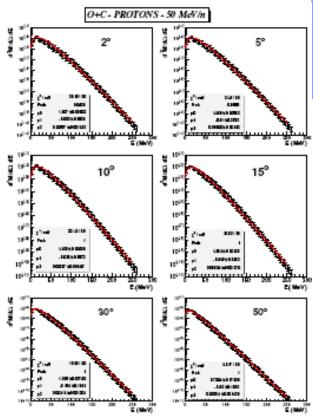
total multiplicity

$$M = P_1 E_{nucl} - P_2 E_{nucl}^2$$

Work is ongoing to extend it to more massive systems, i.e.

40
Ca + 120 Sn

and consequently review the fitting functions and the extrapolation recipes over a significantly larger mass range 30 May 2018



energy spectra

$$d^{2}M/(dEd\Omega) = E^{P_{0}(\theta)} \exp(-P_{1}(\theta) - P_{2}(\theta)E)$$

BME - Peripheral collisions

We integrate the nuclear densities of the projectile and the target over their overlapping region, as a function of the impact parameter, and obtain a preferentially excited "middle source" and two fragments (projectile- and target-like). The kinematics is suggested by break-up studies.

i. selection of the *impact parameter* <u>b</u>

ii. kinematics determination

 θ_{PL} , θ_{TL} chosen according to $[d\sigma/d\Omega]_{cm}$ ~ exp(-k θ_{cm})

 θ_{MS} momentum conservation

 p_{PL} , p_{TL} chosen according to a given energy loss distribution

p_{MS} momentum conservation

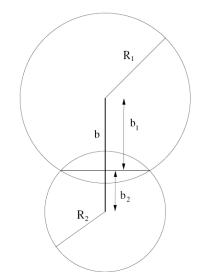
 ϕ_{Pl} free

PL

MS

TL

 ϕ_{TL} , ϕ_{MS} same reaction plane



iii. excitation energy sharing

$$E_{MS}^{\star} = (A_{MS}/A_{tot})E_{tot}^{\star} \sum_{n=0}^{k} (1 - A_{MS}/A_{tot})^{n}$$

$$m{E}_{\scriptscriptstyle PL}^{\star} = f(m{A}_{\scriptscriptstyle PL},m{A}_{\scriptscriptstyle TL}) \left(m{E}_{tot}^{\star} - m{E}_{\scriptscriptstyle MS}^{\star}
ight)$$

$$m{\mathcal{E}}_{\scriptscriptstyle TL}^{\star} = \left(m{\mathcal{E}}_{\scriptscriptstyle tot}^{\star} - m{\mathcal{E}}_{\scriptscriptstyle MS}^{\star} - m{\mathcal{E}}_{\scriptscriptstyle PL}^{\star}
ight)$$

forced on the experimental values in the discrete level region

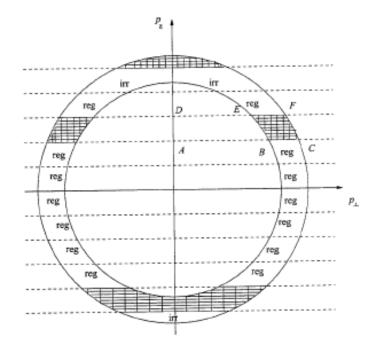
BME - Theoretical framework

Calculation of preequilibrium for the composite nucleus

proton and neutron momentum spaces divided into bins

$$\left\{ (p_X, p_Y, p_Z) : p_Z \in [p_{Zi}, p_{Zi} + \Delta p_Z), \ \varepsilon = (2m)^{-1} \left(p_X^2 + p_Y^2 + p_Z^2 \right) \in [\varepsilon_i, \varepsilon_i + \Delta \varepsilon) \right\}$$
(Z is the beam direction)

of volume $2\pi m \Delta \varepsilon \Delta p_Z$



BME - Theoretical framework

The BME system

$$N_i = n_i g_i$$
number of states in bin i
occupation probability

$$\frac{d(n_{i}^{\pi}g_{i}^{\pi})}{dt} = \sum_{jlm} \left[\omega_{lm\to ij}^{\pi\pi} g_{l}^{\pi} n_{l}^{\pi} g_{m}^{\pi} n_{m}^{\pi} (1 - n_{i}^{\pi}) (1 - n_{j}^{\pi}) \right] \\
- \omega_{ij\to lm}^{\pi\pi} g_{i}^{\pi} n_{i}^{\pi} g_{j}^{\pi} n_{j}^{\pi} (1 - n_{l}^{\pi}) (1 - n_{m}^{\pi}) \right] \\
+ \sum_{jlm} \left[\omega_{lm\to ij}^{\pi\nu} g_{l}^{\pi} n_{l}^{\pi} g_{m}^{\nu} n_{m}^{\nu} (1 - n_{i}^{\pi}) (1 - n_{j}^{\nu}) \right] \\
- \omega_{ij\to lm}^{\pi\nu} g_{i}^{\pi} n_{i}^{\pi} g_{j}^{\nu} n_{j}^{\nu} (1 - n_{l}^{\pi}) (1 - n_{m}^{\nu}) \right] \\
- n_{i}^{\pi} g_{i}^{\pi} \omega_{i\to i'}^{\pi} g_{i'}^{\pi} \delta(\varepsilon_{i}^{\pi} - \varepsilon_{i'}^{\pi} - \varepsilon_{F}^{\pi} - B^{\pi}) - \frac{dD_{i}^{\pi}}{dt}$$

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BME - Theoretical framework

Multiplicity spectra

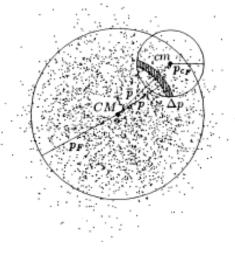
of emitted nucleons

$$\frac{d^2 M(\varepsilon',\theta)}{d\varepsilon' d\Omega} = \frac{1}{2\pi \sin \theta} \int_0^{t_{eq}} n(\varepsilon,\theta,t) \frac{\sigma_{inv} V}{V} \rho(\varepsilon',\theta) dt$$

$$\frac{d^{2}M_{c}\left(E_{c}^{\prime},\theta_{c}\right)}{dE_{c}^{\prime}d\Omega}=\frac{R_{c}}{2\pi\sin\theta}\int_{0}^{t_{eq}}N_{c}\left(E_{c},\theta_{c},t\right)\frac{\sigma_{inv,c}\,v_{c}}{V}\rho_{c}\left(E_{c}^{\prime},\theta_{c}\right)\,dt$$

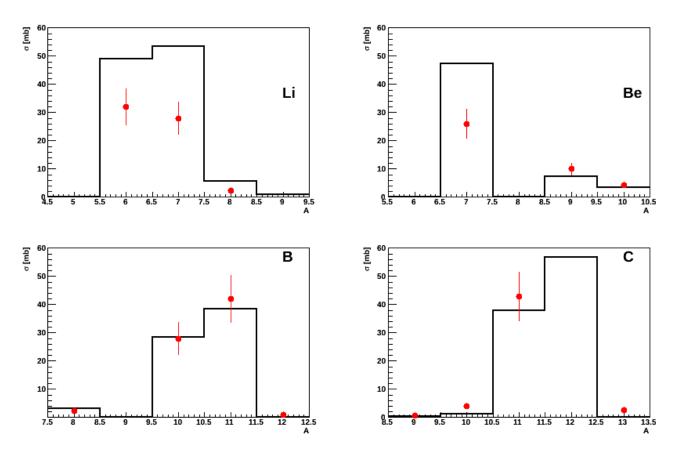
$$N_c(E_c, \theta_c, t) = \prod_i (n_i^{\pi}(\varepsilon, \theta, t))^{P_i(E_c, \theta_c)Z_c} \cdot \prod_i (n_i^{\nu}(\varepsilon, \theta, t))^{P_i(E_c, \theta_c)N_c}$$

joint probability



BME – Benchmarking (2)

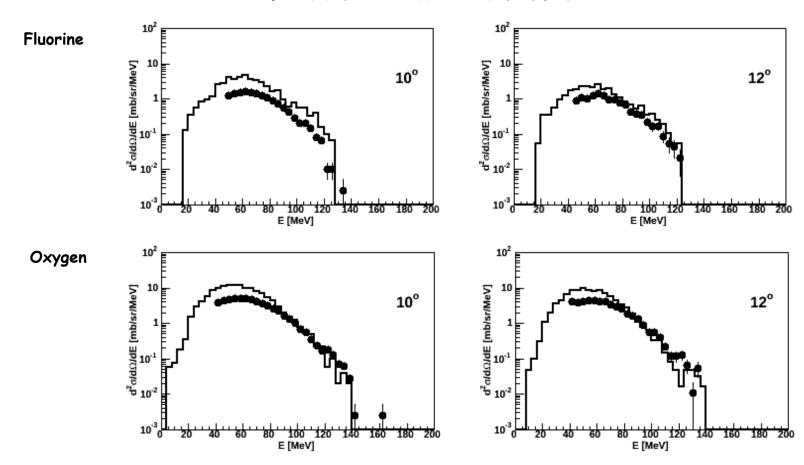
ISOTOPE YIELDS FROM C+C at 86 MeV/n



experimental data from H. Ryde, Physica Scripta T5, 114 (1983)

BME – Benchmarking (3)

DOUBLE DIFFERENTIAL FRAGMENT SPECTRA FROM C+C at 13 MeV/n



experimental data by courtesy of S. Fortsch et al., iThemba Labs, South Africa