Kinematic fitting in $t\bar{t}$ events

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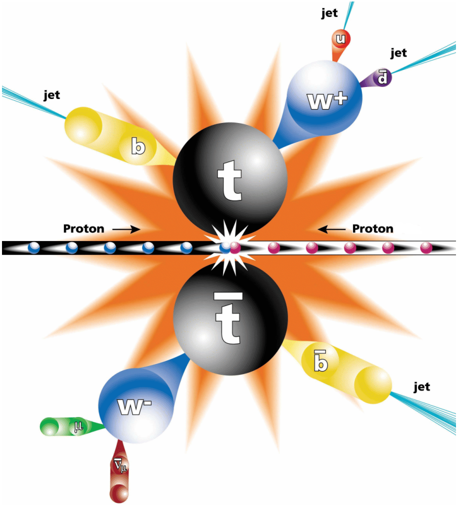
Why a kinematic fit?

- ℓ+jets channel is overconstraint by two (three)
- Measured quantities
  - 4 jets (p_x, p_y, p_z)
  - lepton (p_x, p_y, p_z)
  - x, y component of $\not{E}_T$
- Unmeasured: p_z component of neutrino
- Constraints:
  - W mass $(\times 2)$
  - $m_t = m_{\bar{t}}$ $(= 171.4$ GeV)



⇒ Use these constraints to improve the resolution of the measured quantities and reconstruct the full $t\bar{t}$ system and decay chain

HitFit — kinematic fit from DØ

- Developed by Scott Snyder for his Ph.D. thesis (top quark mass measurement in RunI)
- Converted into c^{++} for RunII of the Tevatron
- Used successfully in many published analyses:
  - Top mass measurement (Template and Ideogram)
  - W helicity in top decays
  - Search for $Z' \to t\bar{t}$
  - ...
- Core Algorithm is experiment independent $\to$ fairly easy to include it into the ATLAS software infrastructure

- $\Rightarrow$ Algorithm has already been extensively tested by DØ
- First working version in the ATLAS framwork exists

How does it work?

Input variables

- Jets: momentum p, azimuthal angle ϕ, pseudorapidity η
- Electrons: p, ϕ, η
- Muons: $1/p, \phi, \eta$
- Missing energy ($\not{E}_T$):
  - Instead of $\not{E}_T$ a different variable is used to minimize the correlation between the variables:

$$\vec{k_T} = \vec{E_T}(\mathrm{lep}) + \vec{\not{E}_T} + \sum_{i \in \mathrm{jets}} \vec{E_T}(\mathrm{jet}_i)$$

  - $\vec{k_T}$ can be interpreted as the transverse momentum of the $t\bar{t}$ system
- Unmeasured: z-component of the neutrino momentum (p_z^ν)

Total: 18 variables + 17 errors on the measured quantities (assumed to be uncorrelated)

Jet permutations

- A fit is performed for each of the 12 possible jet permutations ($4! = 24$ but no need to distinguish the jets from the W)
- In principle it is possible to use more than four jets (ISR/FSR hypothesis) $\rightarrow$ combinatorial problem

  5 Jets $\Rightarrow$ 140 combinations

  6 Jets $\Rightarrow$ 1020 combinations

  ⋮

- This combinatorial explosion is the main reason for restricting the fit to four jets only

Constrained Fit

- Fit χ^2 defined by

$$\chi^2 = (\mathbf{x} - \mathbf{x^m})^T \mathbf{G} (\mathbf{x} - \mathbf{x^m})$$

- x^{m} (x) is the vector of measured (fitted) variables and G is the inverse error matrix
- Minimization is subject to the imposed constraints

  $m_W = 80.4$ GeV and $m_t = m_{\bar{t}}$
- Equation system not linear $\Rightarrow$ iterative procedure is used:
  1. Start with measured quantities
  2. Expand constraints in power series around this point to linearize them
  3. Minimization is solved with these linearized constraints
  4. Result defines starting point for next iteration
  5. Iteration continues until the constraints are satisfied and the χ^2 stops changing

The z component of the neutrino momentum

- To start the procedure the z component of the neutrino must be calculated
- It can be infered from the fact that both top quarks have equal mass

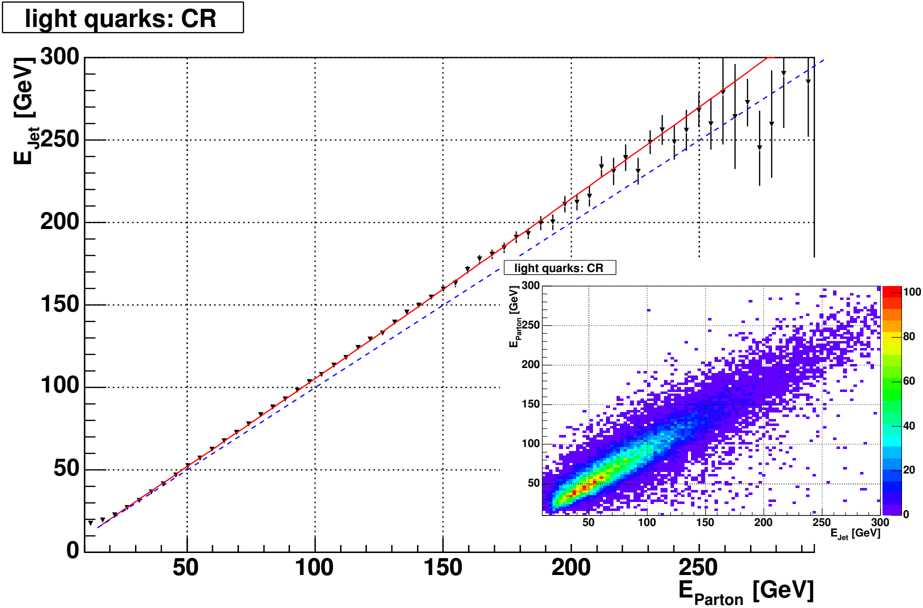
$$
\begin{aligned}
0 &= \left[(p_z^c)^2 - (E^c)^2\right](p_z^\nu)^2 + \alpha p_z^c p_z^\nu - (E^c p_T^\nu)^2 + \frac{\alpha^2}{4} \quad \text{with} \\
\alpha &= m_t^2 - m_c^2 + 2\vec{p_T}^\nu \cdot \vec{p_T}^c
\end{aligned}
$$

- *c* indicates here the sum of the four-vectors of the lepton and the *b*-jet
- $\Rightarrow$ quadratic equation for p_z^ν
- In case of two solutions both are used (i.e. two fits)
- p_z^ν does not enter in the χ^2! The two solutions might only change the local minima that the fit finds!

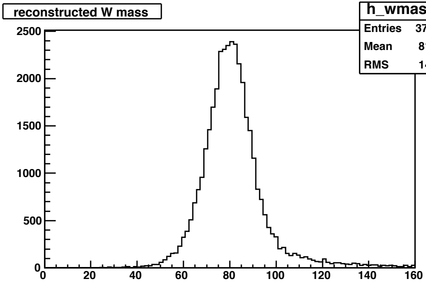
Additional inputs that are needed

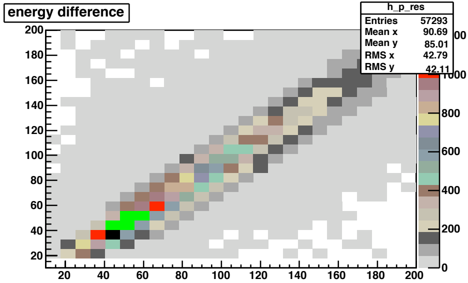
- Resolutions
  - To calculate the inverse error matrix the parameterization for the object resolution needs to be known
  - This is fed into the fit as text file containing the individual terms (constant, sampling, noise)
  - The resolutions can be separated in variable eta bins
- Additional jet corrections
  - The Jet Energy Scale corrections (JES) correct the jets back to particle level (i.e. to what the calorimeter should have seen)
  - The kinematic fit works at the parton level
  - $\Rightarrow$ Need to correct the JES corrected jets back to the parton level (2nd degree polynomial)
  - These corrections (seperately for light/heavy flavour) can be supplied as well via the same textfile as the resolutions

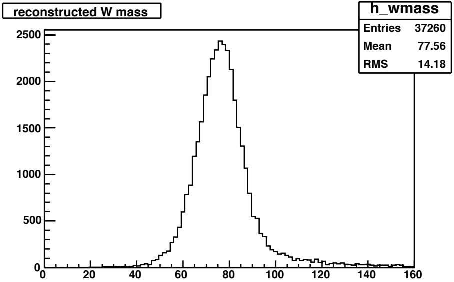
Example of Parton level corrections (DØ)

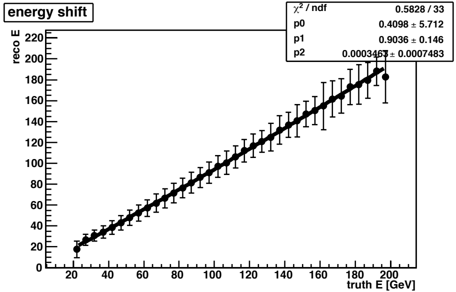


Parton level corrections at ATLAS

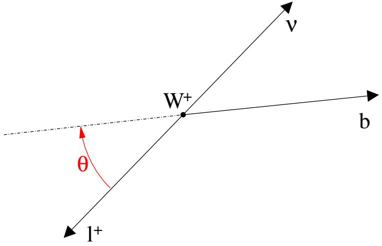




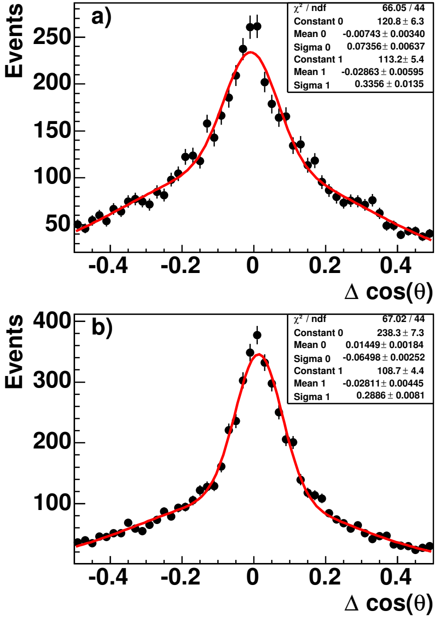




Use of HitFit at DØ: W helicity measurement



- Need four momentum of lepton and $b-$jet from leptonic top decay in the rest frame of the corresponding W boson (jet association as well as resolution)
- Resolutions obtained:
  - a) $\sigma = 0.074$ (m_t not fixed)
  - b) $\sigma = 0.065$ ($m_t = 175$ GeV)



Implementation for ATLAS

- Scott Snyder gave his ok to port HitFit into the ATLAS environment
  - He is also interested to use HitFit in ATLAS
  - Plans to make a public, experiment-independent release of the code
- I've modified the code slightly so that it compiles in the ATLAS framework
  - Code is available in cvs:

    `[atlas] / users / chriss / PhysicsAnalysis / TopPhys / HitFit`
  - Not fully integrated yet: settings via textfile instead of joboptions
  - But it is fully functional
  - No documentation yet
- The default configuration file already contains the resolutions and jet corrections (derived from 11.0.41 ttbar samples)

How to use it

```
// ... instantiate fitter ...
HitFit::Top_Fit_Args args (defs);
HitFit::Top_Fit fitter (args, lepw_mass, hadw_mass, top_mass);

// ... fill event for fitting ...
HitFit::Lepjets_Event ev( m_run, m_evt);

HitFit::Fourvec  lep_fourvec(lep.Px(),lep.Py(),lep.Pz(),lep.E());
ev.add_lep(HitFit::Lepjets_Event_Lep(lep_fourvec,3,io_args.muo_res()));

HitFit::Fourvec met(m_metx, m_mety,missingET,missingET);

// ... add all jets like this giving the jet type (light, b, ISR/FSR) ...
HitFit::Fourvec j_fourvec(jet->Px(), jet->Py(), jet->Pz(), jet->E());
ev.add_jet( HitFit::Lepjets_Event_Jet(j_fourvec,jet_types[idxjet],
                     io_args.jet_res(),has_btag,false)   );

// ... call the fitter ...
HitFit::Fit_Results fitres = fitter.fit(ev);

// ... retrieve the results for the ith permutation ...
chisq=fitres[HitFit::all_list][i].chisq();
const HitFit::Lepjets_Event& ev_fitted = fitres[HitFit::all_list][i].ev();
```

Summary & Conclusions

- Kinematic fit allows the reconstruction of the full $t\bar{t}$ system and decay chain
- Improves the resolutions due to the additional constraints in the lepton+jets channel
- HitFit developed by S.Snyder successfully used by DØ since several years
- Flexible object oriented approach allows to extend this fitter to the full hadronic channel or even other decays in the future
- Ported to ATLAS
  - First version exists and compiles
  - Very rough estimates of resolutions and jet corrections derived
  - In use already by Grant for his studies, will also be used by M. Siebel for his $H \to t\bar{t}$ analysis

Outlook

- Improve the integration into the ATLAS framework
  - Replace textfile parameters by joboptions
  - Make it an AlgTool with a proper interface (other fitters can then use the same interface and the user can use his/her preferred kinematic fit)
- Improve HitFit performance/functionality
  - Extend to all-hadronic channel (basic fitter is very modular, just needs an interface for other decay channels)
  - Derive correct resolution functions/parton corrections for the ATLAS detector
- Increase number of users
  - don't expect bugs in the main implementation (debugged over the last decade!)
  - Get feedback on ATLAS implementation
  - Write documentation/performance note