# Factorization of double Drell–Yan.

Riccardo Nagar

Deutsches Elektronen-Synchrotron (DESY)

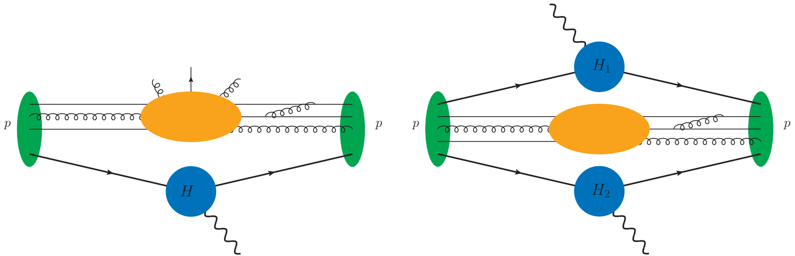
Workshop on Resummation, Evolution, Factorization 2018
19 November 2018

work in collaboration with Markus Diehl (DESY)

[DESY 18-204]

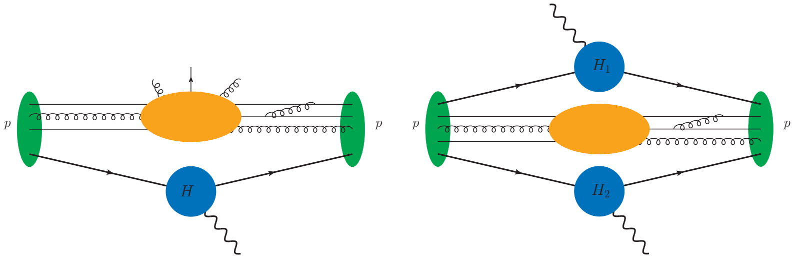
HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

# Multi-parton scattering




- hadron-hadron collision with more than a single hard partonic interaction
- partonic interactions can be
  - soft → underlying event
  - hard → multiple hard scattering

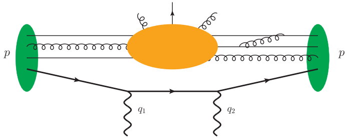
# Multi-parton scattering




- hadron-hadron collision with more than a single hard partonic interaction
- partonic interactions can be
  - soft → underlying event
  - hard → multiple hard scattering
- factorized formula for double-parton scattering:

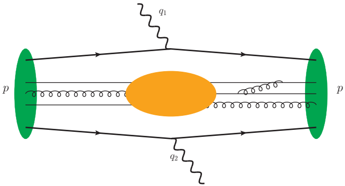
$$d\sigma = \Big( \underbrace{d\sigma_1 \times d\sigma_2}_{\text{hard partonic cross-sections}} \Big) \otimes \Big( \underbrace{F(h_1) \times F(h_2)}_{\text{double-parton distributions}} \Big) + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$

# Double Drell–Yan




Double Drell–Yan in the SPS case.

$$|\mathbf{q}_1 + \mathbf{q}_2| \ll Q$$




Double Drell–Yan in the DPS case.

$$|\mathbf{q}_1| \ll Q \text{ and } |\mathbf{q}_2| \ll Q$$

## Comparing SPS and DPS

- integrated XS: $\frac{\sigma_{\text{DPS}}}{\sigma_{\text{SPS}}} \sim \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right) \implies$ phase-space suppressed
- differential XS: $\frac{\mathrm{d}^2\sigma_{\text{SPS}}}{\mathrm{d}^2\boldsymbol{q}_1\mathrm{d}^2\boldsymbol{q}_2} \sim \frac{\mathrm{d}^2\sigma_{\text{DPS}}}{\mathrm{d}^2\boldsymbol{q}_1\mathrm{d}^2\boldsymbol{q}_2} \implies$ same power counting

  [Diehl et al. '11]
- other enhancements:
  - coupling suppression in SPS ($W^+W^+$ production)
  - small $x$

# Factorization proof

Factorization theorem for single Drell–Yan proven. [Collins, Soper, Sterman '84, '85], [Bodwin '85]

With two partons, more complex color structure [Manohar, Waalewijn '12] and need to drop some approximations [Diehl et al. '12, '15]

## Factorization proof

1. identify leading regions & introduce appropriate approximations
2. establish a subtraction mechanism [Collins '11]
3. treat Glauber gluons [Diehl et al. '15]
4. use Ward identities to obtain Wilson lines
5. factorization of the soft gluons [Diehl, RN]
6. obtain operator definitions of soft & collinear factors

## Scaling of momenta

$\ell = (\ell^+, \ell^-, |\boldsymbol{\ell}|)$

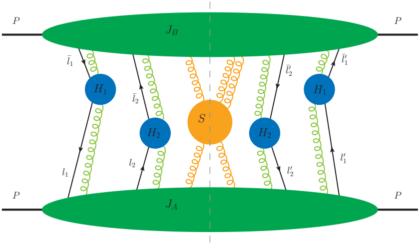
- hard: $(Q, Q, Q)$
- right-collinear: $(Q, \Lambda^2/Q, \Lambda)$
- left-collinear: $(\Lambda^2/Q, Q, \Lambda)$
- good soft: $(\Lambda, \Lambda, \Lambda)$ or $(\Lambda^2/Q, \Lambda^2/Q, \Lambda^2/Q)$
- bad soft (Glauber region): $(\Lambda, \Lambda^2/Q, \Lambda)$ or $(\Lambda^2/Q, \Lambda, \Lambda)$

# Factorization proof: leading diagrams

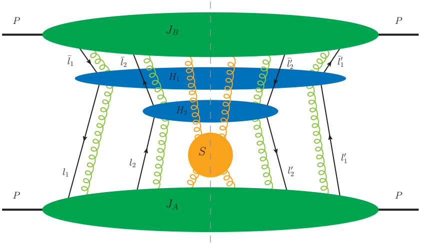
The identification of the leading diagrams depend on the observable.

- TMD factorization: for transverse-momentum dependent cross-section
- collinear factorization: for transverse-momentum integrated cross-section

**TMD factorization**




**collinear factorization**

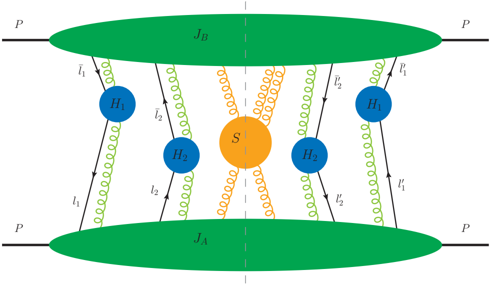



Here we will focus on TMD factorization.

# Soft & collinear approximations

The TMD cross section is given by

$$d\sigma \simeq \frac{1}{C}\frac{1}{4P\cdot\bar{P}}\left[\frac{d^4 q_1}{(2\pi)^4}\frac{d^4 q_2}{(2\pi)^4}\right]\int[\text{all soft momenta } \ell_i \text{ and } \bar{\ell}_i]$$
$$\times\int[\text{all collinear momenta } k_j \text{ and } \bar{k}_j]\int[\text{all collinear momenta } l_m, l'_m, \bar{l}_m \text{ and } \bar{l}'_m]$$
$$\times\left[\prod_{m=1}^{N} H_m(q_m; l_m, \bar{l}_m, l'_m, \bar{l}'_m; k_j, \bar{k}_j)\right] S_{\ldots\alpha_i\ldots\beta_{\bar{i}}\ldots}(\ell_i, \bar{\ell}_i)$$
$$\times J_A^{\alpha_1\ldots\alpha_n}(\ell_i; l_1, \ldots, l_N, l'_1, \ldots, l'_N; k_j)$$
$$\times J_B^{\beta_1\ldots\beta_{\bar{n}}}(\bar{\ell}_i; \bar{l}_1, \ldots, \bar{l}_N, \bar{l}'_1, \ldots, \bar{l}'_N; \bar{k}_j)$$



# Soft & collinear approximations

The TMD cross section is given by

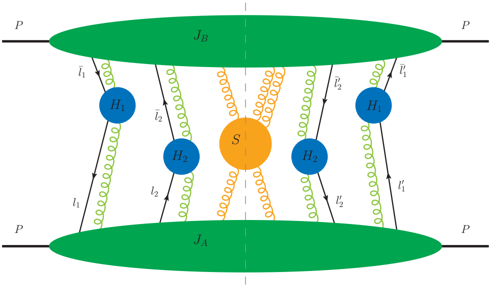
$$\mathrm{d}\sigma \simeq \frac{1}{C}\frac{1}{4P\cdot\bar{P}}\left[\frac{\mathrm{d}^4 q_1}{(2\pi)^4}\frac{\mathrm{d}^4 q_2}{(2\pi)^4}\right]\int[\text{all soft momenta } \ell_i \text{ and } \bar{\ell}_i]$$

$$\times \int[\text{all collinear momenta } k_j \text{ and } \bar{k}_j]\int[\text{all collinear momenta } l_m, l'_m, \bar{l}_m \text{ and } \bar{l}'_m]$$

$$\times \left[\prod_{m=1}^{N} H_m(q_m; l_m, \bar{l}_m, l'_m, \bar{l}'_m; k_j, \bar{k}_j)\right] \langle 0|\ldots A_{\alpha_i}(\ell_i)\ldots A_{\beta_i}(\bar{\ell}_i)\ldots|0\rangle$$

$$\times \langle P|\, q'_N \ldots q'_1 \ldots A^{\alpha_i}(\ell_i)\ldots A(k_j)\ldots q_1\ldots q_N\,|P\rangle$$

$$\times \langle \bar{P}|\, \bar{q}'_N \ldots \bar{q}'_1 \ldots A^{\beta_i}(\bar{\ell}_i)\ldots A(\bar{k}_j)\ldots \bar{q}_1\ldots \bar{q}_N\,|\bar{P}\rangle$$



# Soft & collinear approximations

The TMD cross section is given by

$$\mathrm{d}\sigma \simeq \frac{1}{C}\frac{1}{4P\cdot\bar{P}}\left[\frac{\mathrm{d}^4 q_1}{(2\pi)^4}\frac{\mathrm{d}^4 q_2}{(2\pi)^4}\right]\int[\text{all soft momenta } \ell_i \text{ and } \bar{\ell}_i]$$
$$\times\int[\text{all collinear momenta } k_j \text{ and } \bar{k}_j]\int[\text{all collinear momenta } l_m, l'_m, \bar{l}_m \text{ and } \bar{l}'_m]$$
$$\times\left[\prod_{m=1}^{N} H_m(q_m; l_m, \bar{l}_m, l'_m, \bar{l}'_m; k_j, \bar{k}_j)\right] S_{\dots\alpha_i\dots\beta_{\bar{i}}\dots}(\ell_i, \bar{\ell}_i)$$
$$\times J_A^{\alpha_1\dots\alpha_n}(\ell_i; l_1,\dots,l_N, l'_1,\dots,l'_N; k_j)$$
$$\times J_B^{\beta_1\dots\beta_{\bar{n}}}(\bar{\ell}_i; \bar{l}_1,\dots,\bar{l}_N, \bar{l}'_1,\dots,\bar{l}'_N; \bar{k}_j)$$

**Kinematic approximations**

- collinear-to-$J_A$ gluons in $H$: $\ell_c = (\ell^+, 0, \mathbf{0})$
- soft gluons in $J_A$: $\ell_s = (0, \ell^-, \boldsymbol{\ell})$
  NB: we do NOT neglect the transverse momenta!

**Grammer–Yennie approximation** [Grammer, Yennie '73]

Examples:

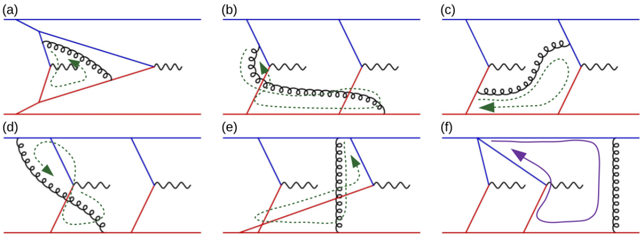
$$J_{A\mu}(\ell)H^\mu(\ell_c) \simeq J_{A\mu}(\ell)\frac{v_A^\mu \ell_c^\nu}{\ell\cdot v_A + i\epsilon}H_\nu(\ell_c)$$

$$S_\mu(\ell)J_A^\mu(\ell_s) \simeq S_\mu(\ell)\frac{v_R^\mu \ell_s^\nu}{\ell\cdot v_R + i\epsilon}J_{A\nu}(\ell_c)$$

where $v_i = (v_i^+, v_i^-, 0, 0)$, $v_A^- \gg v_A^+$ and $v_R^+ \gg v_R^-$.

# Glauber gluons cancellation

- The Grammer–Yennie approximation is not valid in correspondence of the Glauber region, hence we must find a way to avoid this region.
- Glauber cancellation does not depend on the colour structure, but only on the kinematics and topology.
- After checking all possible cases, only non-trivial case connects spectators before final state cut (see figure).
- Cancellation analogous to the one in single Drell–Yan, i.e. due to unitarity (sum over final states at the cut).

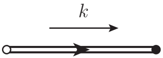



from [Diehl, Gaunt, Ostermeier, Plößl, Schäfer, in Proc. MPI@LHC 2015]

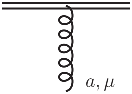
# Eikonal and Wilson lines

- Factorization usually results in the appearance of Wilson lines.
- Eikonal propagators and vertices appear thanks to the Grammer–Yennie factors
- Eikonal lines are the Fourier transform of the Wilson line operator expanded to a given perturbation order in $g$.

**Eikonal Feynman rules:**




$$= \frac{-i}{v \cdot k + i\epsilon}$$




$$= -igv^{\mu}$$

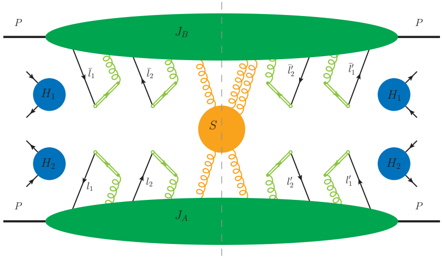
**Past-pointing Wilson lines**

- SU(3) fundamental: $W_{ij}(\xi, v) = \mathcal{P} \exp\left[ig \int_{-\infty}^{0} \mathrm{d}s \, v \cdot A^a(\xi + sv) \, t^a_{ij}\right]$
- SU(3) adjoint: $W_{ab}(\xi, v) = \mathcal{P} \exp\left[-g \int_{-\infty}^{0} \mathrm{d}s \, v \cdot A^c(\xi + sv) \, f^c_{ab}\right]$

# Factorization of collinear gluons

## Substitutions in the collinear amplitudes

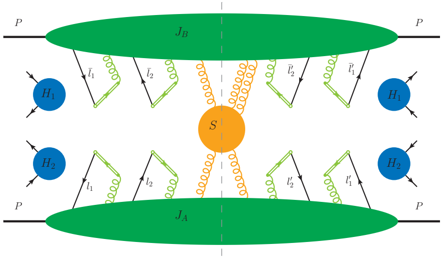
- $q_i(x) \to W_{ij}(x, v)q_j(x)$
- $\bar{q}_i(x) \to \bar{q}_j(x)W^\dagger_{ji}(x, v)$
- $G^a_{\mu\nu}(x) \to W^{ab}(x, v)G^b_{\mu\nu}(x)$



# Factorization of collinear gluons

**Substitutions in the collinear amplitudes**

- $q_i(x) \to W_{ij}(x,v) q_j(x)$
- $\bar{q}_i(x) \to \bar{q}_j(x) W^{\dagger}_{ji}(x,v)$
- $G^a_{\mu\nu}(x) \to W^{ab}(x,v) G^b_{\mu\nu}(x)$




After collinear gluons are factorized:

$$d\sigma \simeq \frac{1}{C}\frac{1}{4P\cdot\bar{P}}\left[\frac{d^4 q_1}{(2\pi)^4}\frac{d^4 q_2}{(2\pi)^4}\right]\int[\text{all soft momenta } \ell_i \text{ and } \bar{\ell}_i]$$
$$\times \int[\text{all hard momenta } l_m, l'_m, \bar{l}_m \text{ and } \bar{l}'_m]$$
$$\times H_1(q_1; l_1, \bar{l}_1, l'_1, \bar{l}'_1)\, H_2(q_2; l_2, \bar{l}_2, l'_2, \bar{l}'_2)\, S_{\dots\alpha_i\dots\beta_{\bar{i}}\dots}(\ell_i, \bar{\ell}_i)$$
$$\times J_A^{\alpha_1\cdots\alpha_n}(\ell_i; l_1, l_2, l'_1, l'_2)\, J_B^{\beta_1\cdots\beta_{\bar{n}}}(\bar{\ell}_j; \bar{l}_1, \bar{l}_2, \bar{l}'_1, \bar{l}'_2)$$

# Ward identities

Given an amplitude with $n$ external gauge bosons, a Ward identity can be proven also for non-Abelian theories

$$\langle h| \ldots A_{\mu_1}(p_1) \ldots A_{\mu_n}(p_n) \ldots |h\rangle \; p_1^{\mu_1} \ldots p_n^{\mu_n} = 0$$

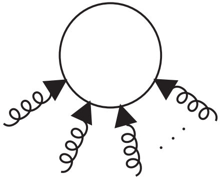
with all momenta contracted, order-by-order but *not* graph-by-graph.
It can also be proven including only certain (internal and external) momentum regions. ['t Hooft 1971, 't Hooft, Veltman 1972]

**Grammer–Yennie factors**

Define

$$Y^{\alpha\beta}(\ell) = \frac{v^\alpha \ell^\beta}{v \cdot \ell + i\epsilon}$$

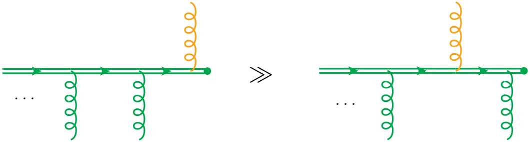
$$Y^{\alpha_1\beta_1}(p_1) \ldots Y^{\alpha_k\beta_k}(p_k) \langle \ldots A_{\beta_1}(p_1) \ldots A_{\beta_n}(p_k) \ldots \rangle =$$



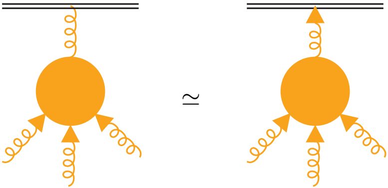
$= 0$

## Simplifications and power-suppressed graphs

- coupling to the middle of a collinear Wilson line

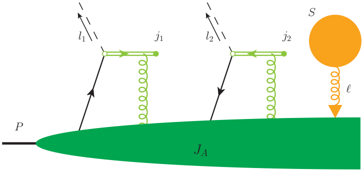


- fixed-order in $\alpha_s$ $\implies$ tree-level $m \to 1$ gluon diagrams only
- moving a Grammer–Yennie approximation through a gluon subgraph



# Ward identities: one gluon attachment

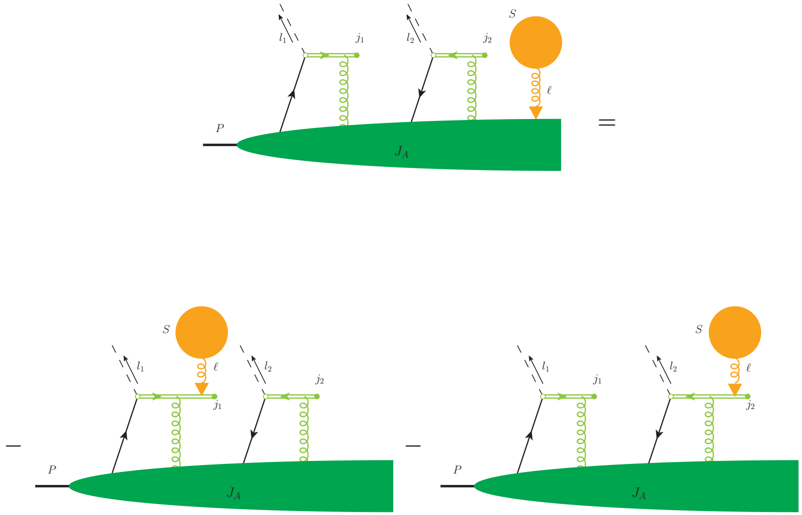
The case of single-gluon attachment is the base case for the recursion.




$=$

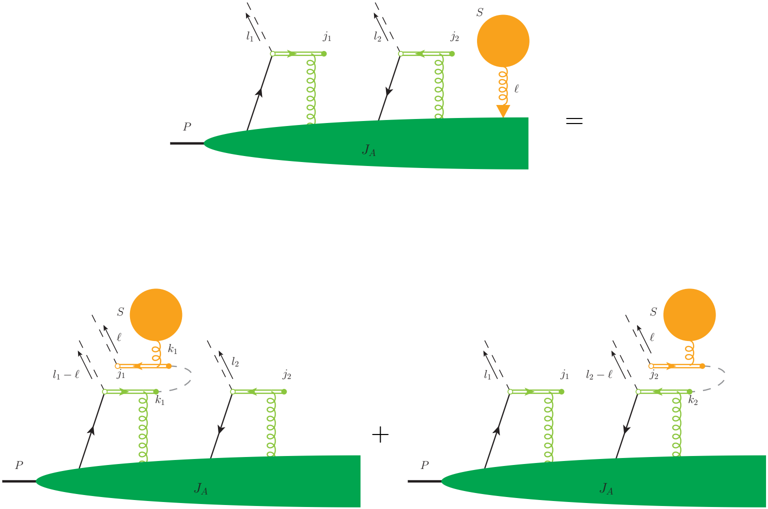
# Ward identities: one gluon attachment

The case of single-gluon attachment is the base case for the recursion.



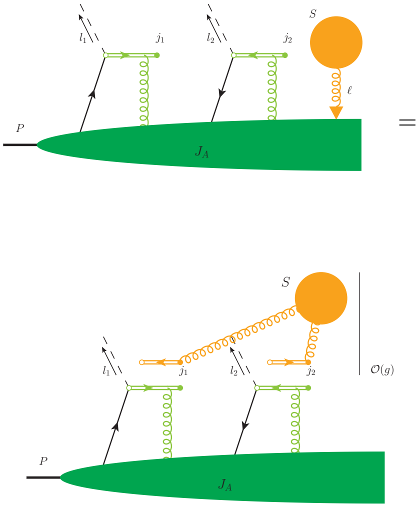
# Ward identities: one gluon attachment

The case of single-gluon attachment is the base case for the recursion.



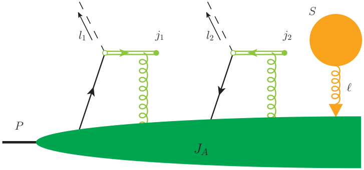
# Ward identities: one gluon attachment

The case of single-gluon attachment is the base case for the recursion.



# Ward identities: one gluon attachment

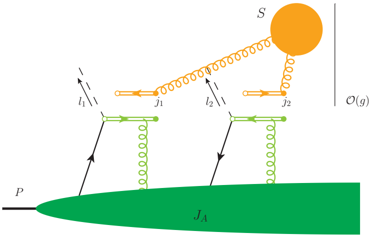
The case of single-gluon attachment is the base case for the recursion.




$$= \langle 0| \ldots A^b_\alpha(\ell) \ldots |0\rangle \, Y_R^{\alpha\beta}(\ell) \left[ J^b_{A\beta}(l_1, l_2, l_1', l_2'; \ell) \right]_{j_1 j_2 j_1' j_2'}$$

$$= \langle 0| \left[\mathcal{W}_R(\ell)\right]_{j_1 k_1} \left[\mathcal{W}_R(\ell)\right]_{j_2 k_2} \ldots |0\rangle \Big|_{\mathcal{O}(g)} \left[ J_A(l_1, l_2, l_1', l_2') \right]_{k_1 k_2 k_1' k_2'}$$

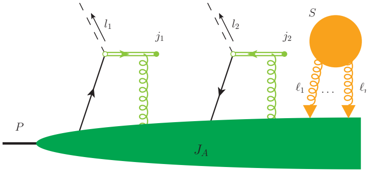
$=$



# Ward identities: recursive proof

## Sketch of the proof for $n$ soft gluon attachments

1. Ward identity to the overall diagram

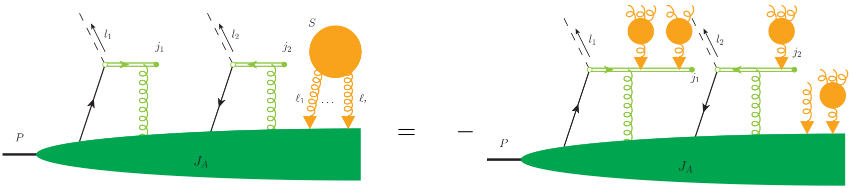



$=$

# Ward identities: recursive proof

## Sketch of the proof for *n* soft gluon attachments

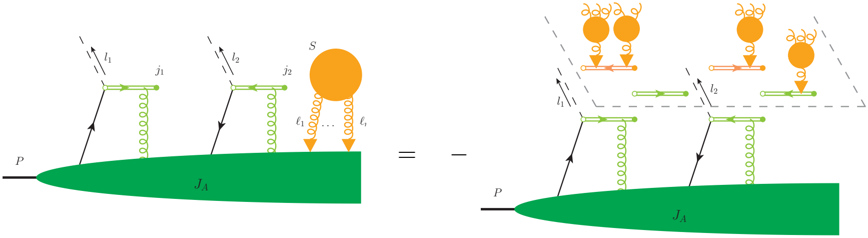
1. Ward identity to the overall diagram
2. soft approximation in Wilson lines & recursion hypothesis



# Ward identities: recursive proof

**Sketch of the proof for $n$ soft gluon attachments**

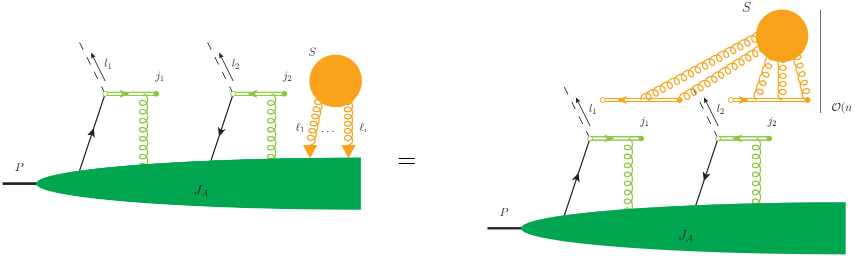
1. Ward identity to the overall diagram
2. soft approximation in Wilson lines & recursion hypothesis
3. Ward identity to matrix element of Wilson lines



# Ward identities: recursive proof

**Sketch of the proof for *n* soft gluon attachments**

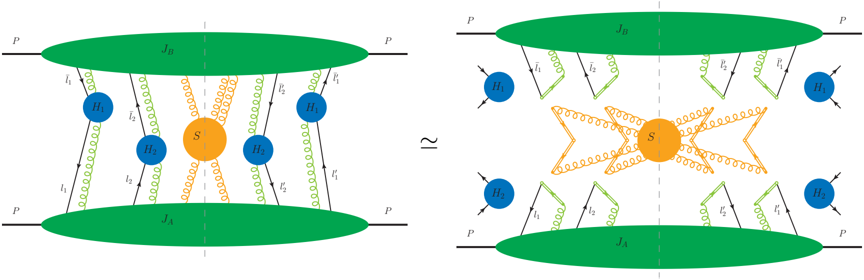
1. Ward identity to the overall diagram
2. soft approximation in Wilson lines & recursion hypothesis
3. Ward identity to matrix element of Wilson lines



# Final formula

After Fourier transforming, we obtain the final factorized formula:

$$
\begin{aligned}
\mathrm{d}\sigma = {} & \frac{1}{C}\,\frac{1}{4P\cdot\bar{P}}\left[\frac{\mathrm{d}^4 q_1}{(2\pi)^4}\,\frac{\mathrm{d}^4 q_2}{(2\pi)^4}\right]\int \mathrm{d}^2\boldsymbol{\xi}_1\,\mathrm{d}^2\boldsymbol{\xi}_2\,\mathrm{d}^2\boldsymbol{\xi}_1'\,\mathrm{d}^2\boldsymbol{\xi}_2'\int \mathrm{d}^4 l_1\,\mathrm{d}^4\bar{l}_1\,\mathrm{d}^4 l_2\,\mathrm{d}^4\bar{l}_2\int \mathrm{d}^4 l_1'\,\mathrm{d}^4\bar{l}_1'\,\mathrm{d}^4 l_2'\,\mathrm{d}^4\bar{l}_2' \\
& \times e^{i\boldsymbol{\xi}_1\cdot(\boldsymbol{q}_1-\boldsymbol{l}_1-\bar{\boldsymbol{l}}_1)}\,\delta(q_1^+ - l_1^+)\,\delta(q_1^- - \bar{l}_1^-)\,e^{i\boldsymbol{\xi}_2\cdot(\boldsymbol{q}_2-\boldsymbol{l}_2-\bar{\boldsymbol{l}}_2)}\,\delta(q_2^+ - l_2^+)\,\delta(q_2^- - \bar{l}_2^-) \\
& \times e^{i\boldsymbol{\xi}_1'\cdot(\boldsymbol{q}_1-\boldsymbol{l}_1'-\bar{\boldsymbol{l}}_1')}\,\delta(q_1^+ - l_1'^+)\,\delta(q_1^- - \bar{l}_1'^-)\,e^{i\boldsymbol{\xi}_2'\cdot(\boldsymbol{q}_2-\boldsymbol{l}_2'-\bar{\boldsymbol{l}}_2')}\,\delta(q_2^+ - l_2'^+)\,\delta(q_2^- - \bar{l}_2'^-) \\
& \times \langle 0|\left[W_R(\boldsymbol{\xi}_1')\,W_R^\dagger(\boldsymbol{\xi}_2')\,W_R(\boldsymbol{\xi}_2)\,W_R^\dagger(\boldsymbol{\xi}_1)\right]\left[W_R(\boldsymbol{\xi}_2')\,W_R^\dagger(\boldsymbol{\xi}_1')\,W_R(\boldsymbol{\xi}_1)\,W_R^\dagger(\boldsymbol{\xi}_2)\right]|0\rangle \\
& \times H_1(q_1; l_1, \bar{l}_1, l_1', \bar{l}_1')\,H_2(q_2; l_2, \bar{l}_2, l_2', \bar{l}_2')\,J_A(l_1, l_2, l_1', l_2')\,J_B(\bar{l}_1, \bar{l}_2, \bar{l}_1', \bar{l}_2') + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)
\end{aligned}
$$

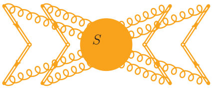


# Where does the soft factor go?

The soft subgraph has a few properties:

[Buffing, Diehl, Kasemets '18]

- hermitian in colour space: $S_{a_1 a_2} = S^\dagger_{a_1 a_2}$
- projection on representations: ${}^{RR'}S_{a_1 a_2} \propto P_R \, S_{a_1 a_2} \, P_{R'}$
- hermitian in rep. space: ${}^{RR'}S_{a_1 a_2} = ({}^{R'R}S_{a_1 a_2})^*$




## TMD factorization

$${J_B}^c \, S^{cd} \, {J_A}^d$$

In SPS and DPS, the soft factor depends on the colour structure and on the Wilson-lines rapidity $y = \log \frac{v^+}{v^-}$

$$\downarrow$$

Collins-Soper equation:

$$\frac{\partial}{\partial y} S(y) = K \, S(y)$$

## collinear factorization

$${J_B}^c \, S^{cd,ef} \, {J_A}^d \, {H_1}^e \, {H_2}^f$$

In SPS, integration over transverse momenta implies $S_q = 1$.

In DPS, ${}^{RR'}S_{a_1 a_2} \propto \delta_{RR'}$, colour singlet ${}^{11}S_{qq} = 1$, but colour non-singlets like ${}^{88}S_{qq}$ depend on rapidity (however Sudakov suppressed).

# Conclusions

- double-parton scattering contributions to double Drell–Yan can be in some specific cases as important as SPS contributions
- a factorization proof is the theoretical starting point to apply perturbative QCD
- we provided the missing piece for a complete proof of the factorization formula in double-parton scattering
- the proof is valid for any number of hard scatterers ($n$-ple Drell–Yan)

# Conclusions

- double-parton scattering contributions to double Drell–Yan can be in some specific cases as important as SPS contributions
- a factorization proof is the theoretical starting point to apply perturbative QCD
- we provided the missing piece for a complete proof of the factorization formula in double-parton scattering
- the proof is valid for any number of hard scatterers ($n$-ple Drell–Yan)

**What's next?**

- study in detail the effects of color correlations
- provide tools to implement the more complicated structure of DPS in the calculations
- use the theory-motivated factorization formula in the data analyses to estimate DPS contributions