Parton Branching method and parton shower

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with contributions from
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L. Banos Estevez, D. Dominguez Damiani, J. Lidrych

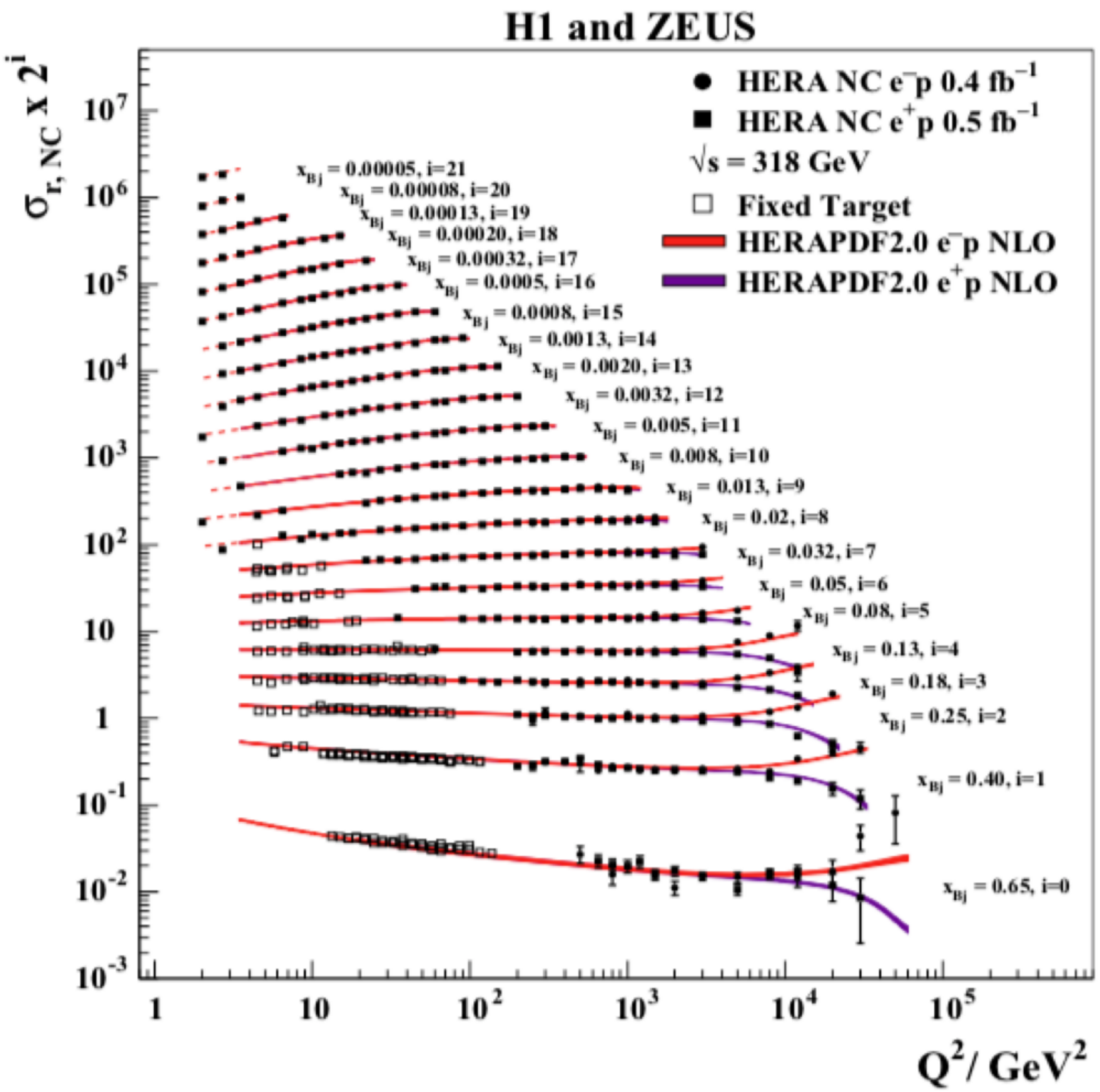
- From inclusive to exclusive distributions
  - Parton Branching method for TMDs

- Parton Shower and TMDs
  - NLO parton shower
  - Matching with hard process calculations at LO and NLO

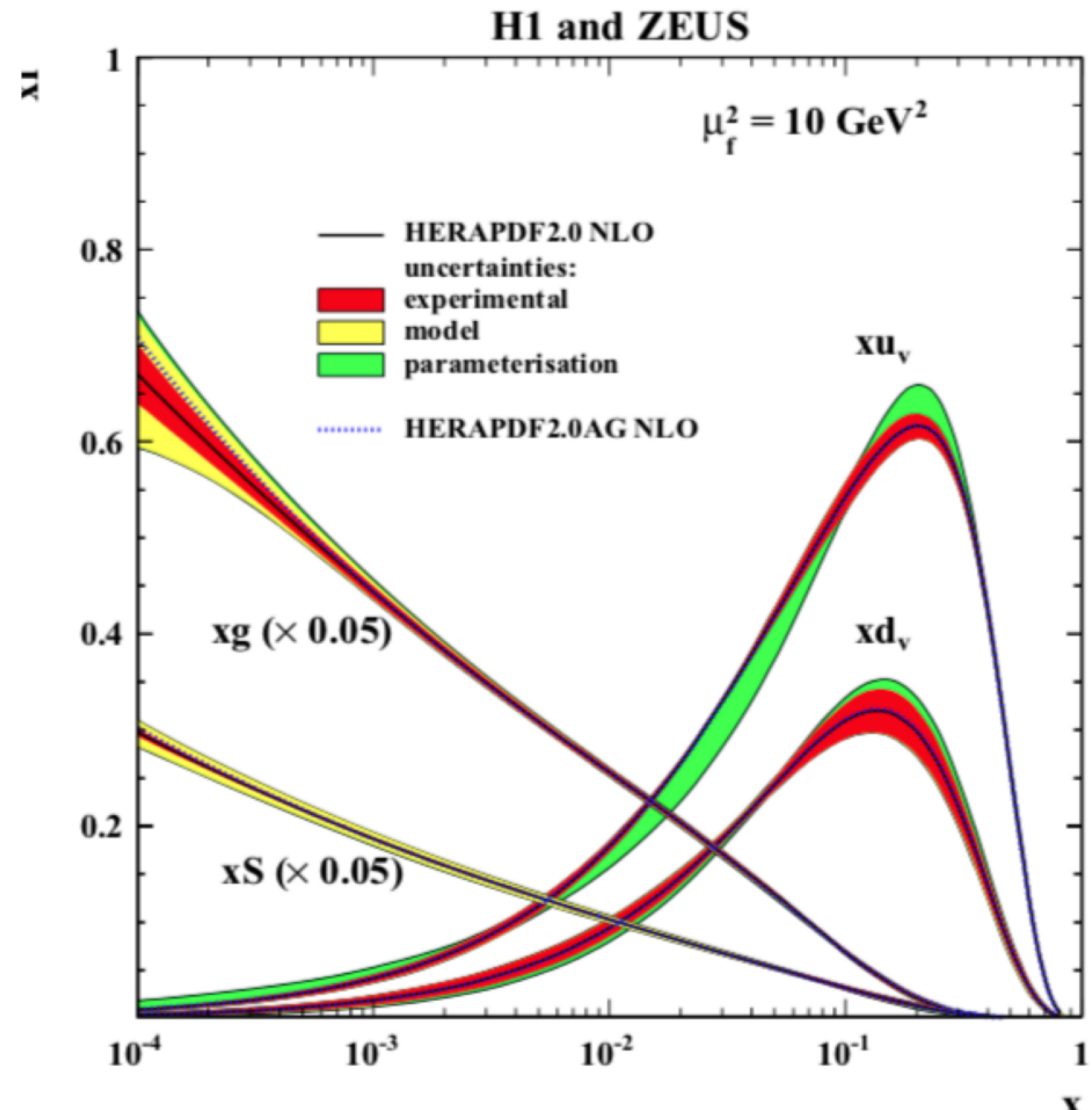
Inclusive cross section and inclusive PDFs

Abramowicz, H. et al Eur. Phys. J., C75(12), 580

- Inclusive DIS cross section



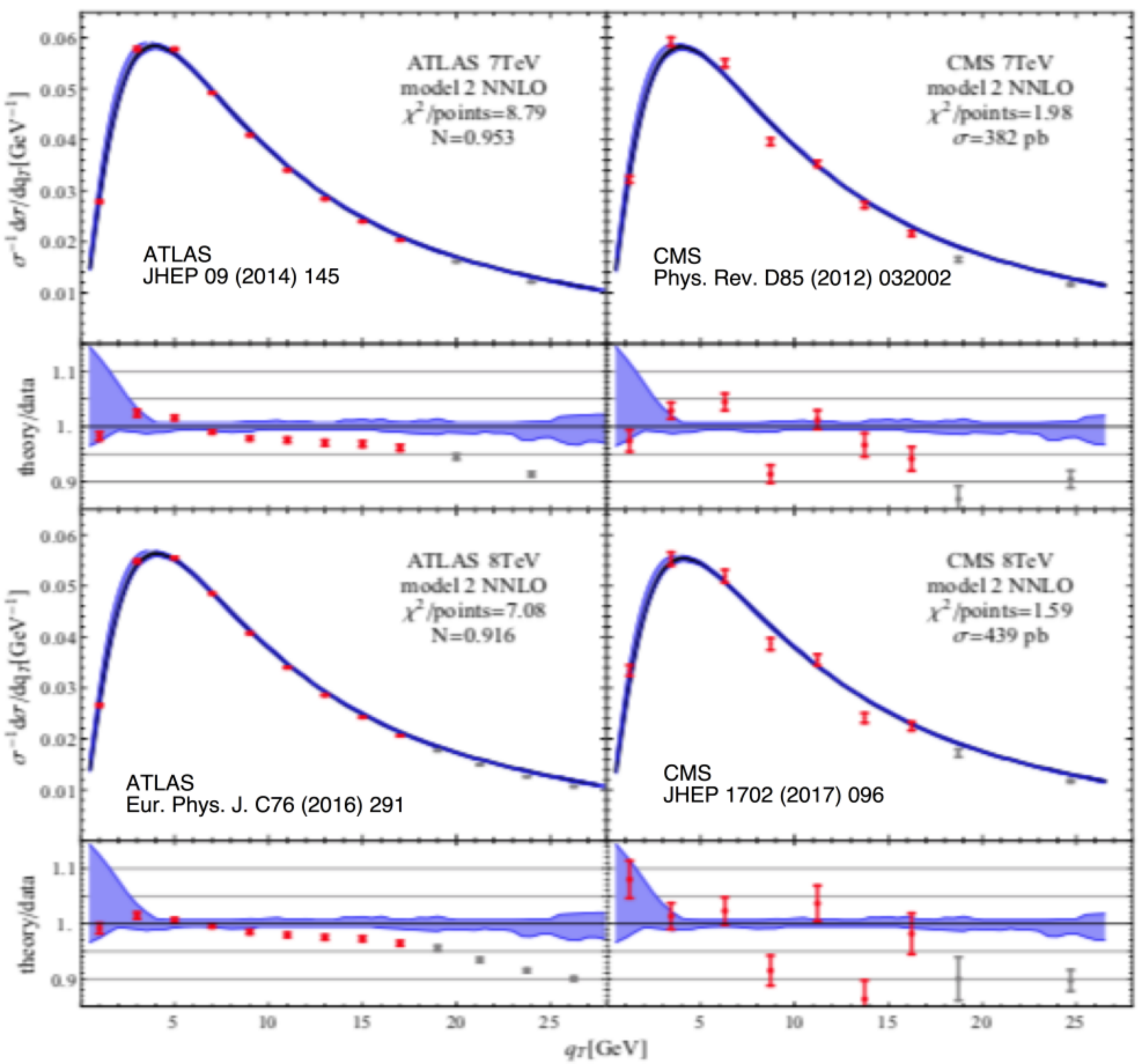
- Inclusive integrated PDFs



Inclusive cross section and inclusive TMDs

Scimemi, I. and Vladimirov, A. (2018).
Eur. Phys. J., C78(2), 89

- Inclusive DY cross section

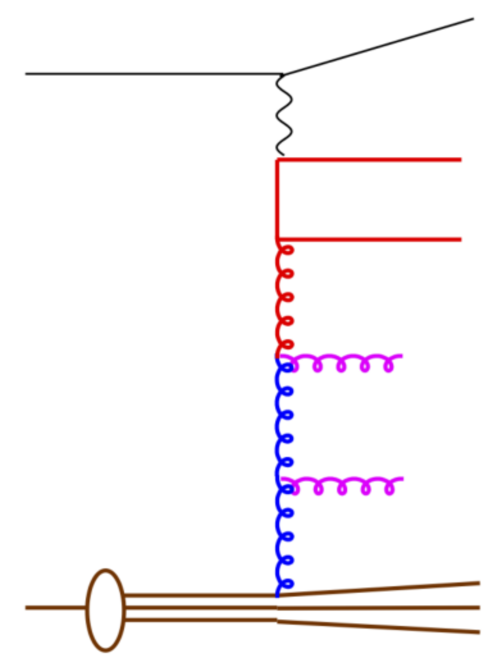


- Inclusive TMD

?

From inclusive to exclusive parton densities

- Standard parton density evolution is inclusive:
  - $f(x,\mu^2)$ gives probability (at LO) to find parton at x and μ^2
  - nothing is said about history of evolution
  - nothing is said about parton emissions at p_T and y
    - is enough for inclusive calculations
- But our physics picture and intuition is not inclusive:
  - parton evolution proceeds via real parton emissions
- Formulate exclusive evolution equation for parton densities → Parton Branching Method



[1] F. Hautmann, H. Jung, A. Lelek, V. Radescu, and R. Zlebcik. Phys. Lett., B772:446, 2017.

[2] F. Hautmann, H. Jung, A. Lelek, V. Radescu, and R. Zlebcik. JHEP, 01:070, 2018.

[3] A. Bermudez Martinez, P. Connor, F. Hautmann, H. Jung, A. Lelek, V. Radescu, and R. Zlebcik. arXiv 1804.11152

see also eg Jadach, S., Placzek, W., Skrzypek, M., and Stoklosa, P. (2010). Comput. Phys. Commun., 181(2010), 393

DGLAP evolution – exclusive description

- differential form:

$$\mu^2 \frac{\partial}{\partial \mu^2} f(x,\mu^2) = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} P_+(z)\, f\left(\frac{x}{z},\mu^2\right)$$

$$\Delta_s(\mu^2) = \exp\left(- \int^{z_M} dz \int_{\mu_0^2}^{\mu^2} \frac{\alpha_s}{2\pi} \frac{d\mu'^2}{\mu'^2} P^{(R)}(z) \right)$$

- differential form using f/Δ_s with

$$\mu^2 \frac{\partial}{\partial \mu^2} \frac{f(x,\mu^2)}{\Delta_s(\mu^2)} = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} \frac{P^{(R)}(z)}{\Delta_s(\mu^2)} f\left(\frac{x}{z},\mu^2\right)$$

- integral form

$$f(x,\mu^2) = f(x,\mu_0^2)\Delta_s(\mu^2) + \int \frac{dz}{z} \int \frac{d\mu'^2}{\mu'^2} \cdot \frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)} P^{(R)}(z) f\left(\frac{x}{z},\mu'^2\right)$$

no – branching probability from $\mu^2{}_0$ to μ^2

DGLAP evolution: Parton Branching method

$$f(x,\mu^2) = f(x,\mu_0^2)\Delta_s(\mu^2) + \int \frac{dz}{z} \int \frac{d\mu'^2}{\mu'^2} \cdot \frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)} P^{(R)}(z) f\left(\frac{x}{z},\mu'^2\right)$$

- solve integral equation via iteration:

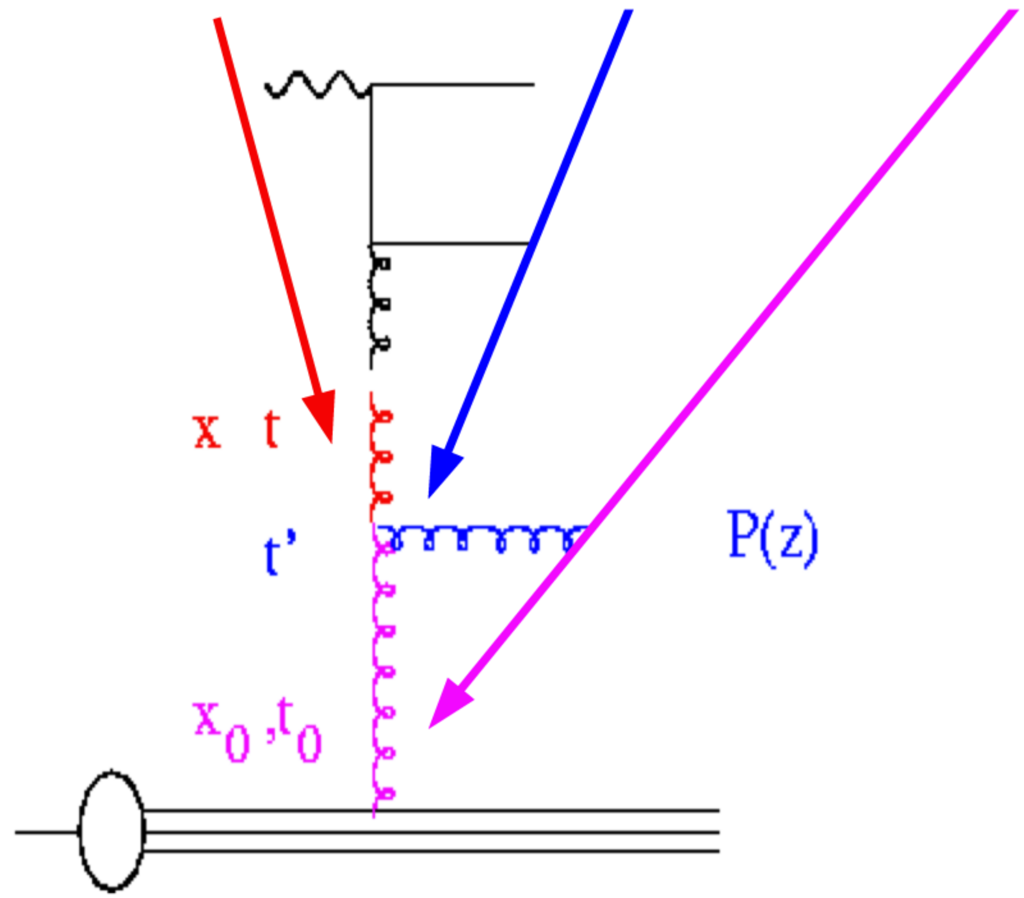
from t' to t w/o branching

branching at t'

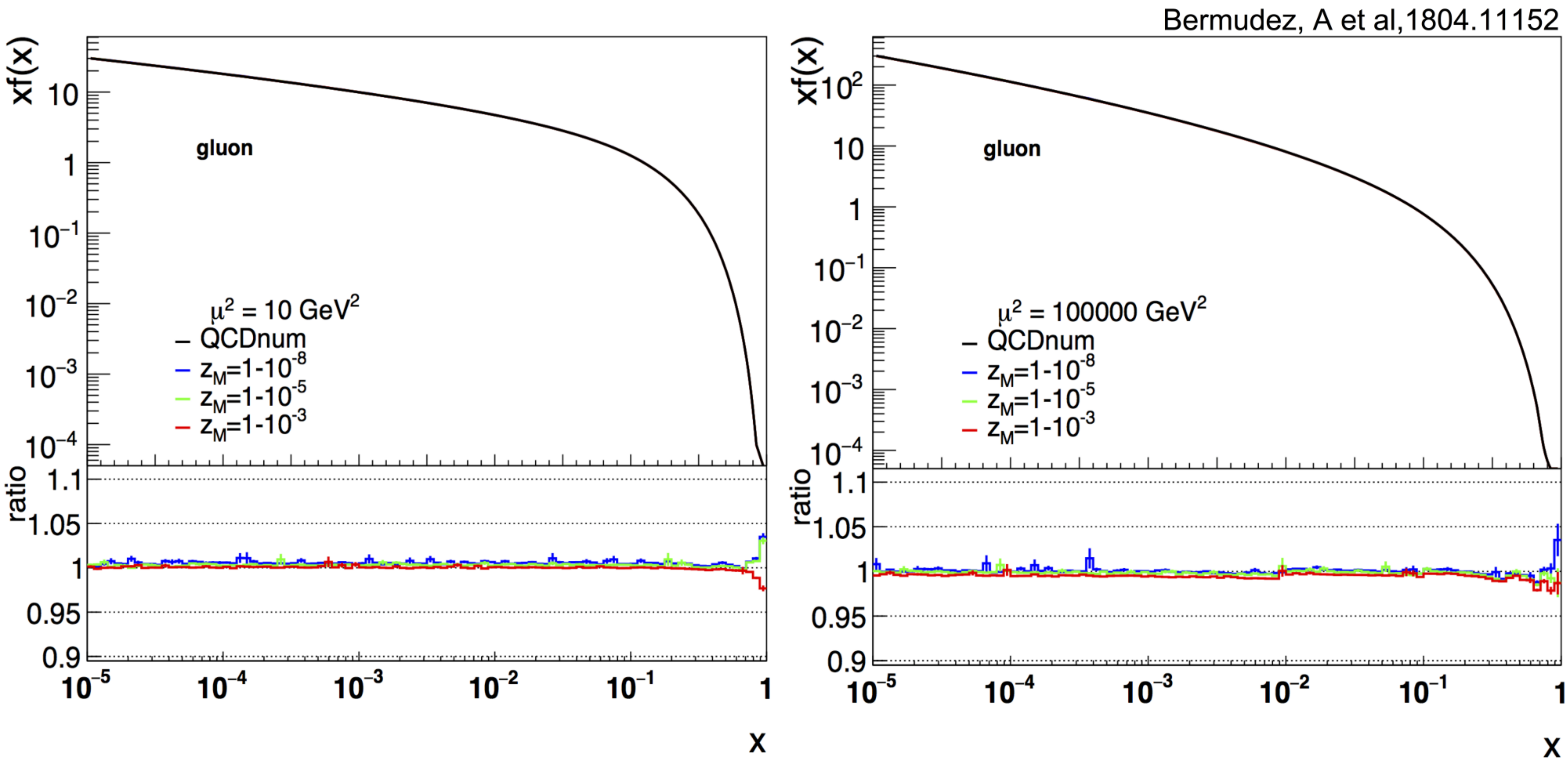
from t_0 to t' w/o branching

$$f_0(x,\mu^2) = f(x,\mu_0^2)\Delta(\mu^2)$$

$$f_1(x,\mu^2) = f(x,\mu_0^2)\Delta(\mu^2) + \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta(\mu^2)}{\Delta(\mu'^2)} \int \frac{dz}{z} P^{(R)}(z) f(x/z,\mu_0^2)\Delta(\mu'^2)$$



Validation of PB method at NLO



- Comparison of exclusive solution at NLO with inclusive calculation at NLO
  - ➔ Very good agreement with NLO - QCDnum
  - No dependence on z_M if z_M is large enough (details in talk A. Lelek)

Why does this work at NLO ?

- essential for evolution is Sudakov:

$$\Delta_a(z_M, \mu^2, \mu_0^2) = \exp\left(- \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz \; z \; P_{ba}^{(R)}(\alpha_s, z) \right)$$

- Sudakov describes probability for no emission between μ_0 and μ
  - → this interpretation works as long as

$$\int dz z P_{ba} > 0$$

  - → checked explicitly:

A. Lelek, PhD thesis 2018, http://bib-pubdb1.desy.de/record/408557

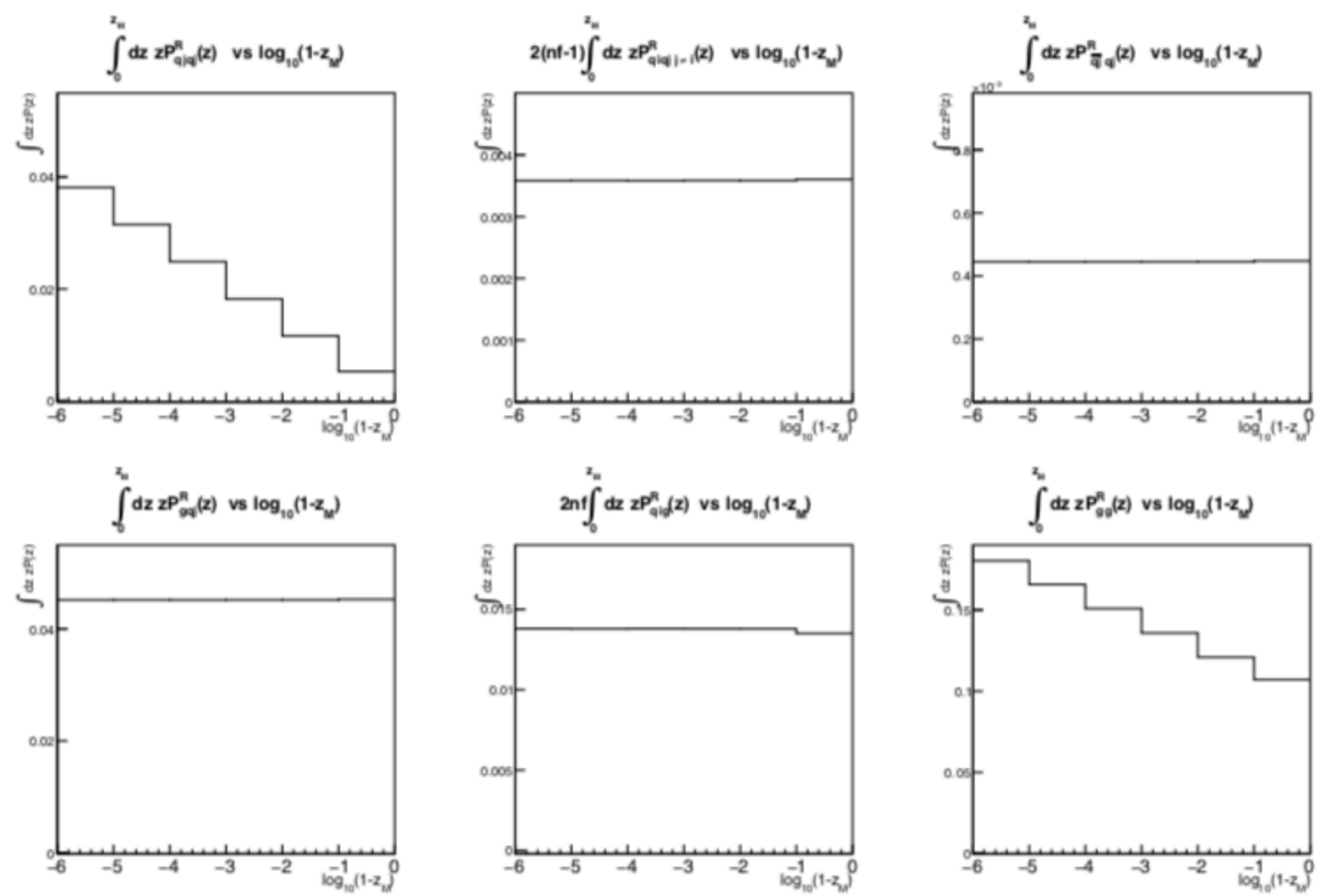


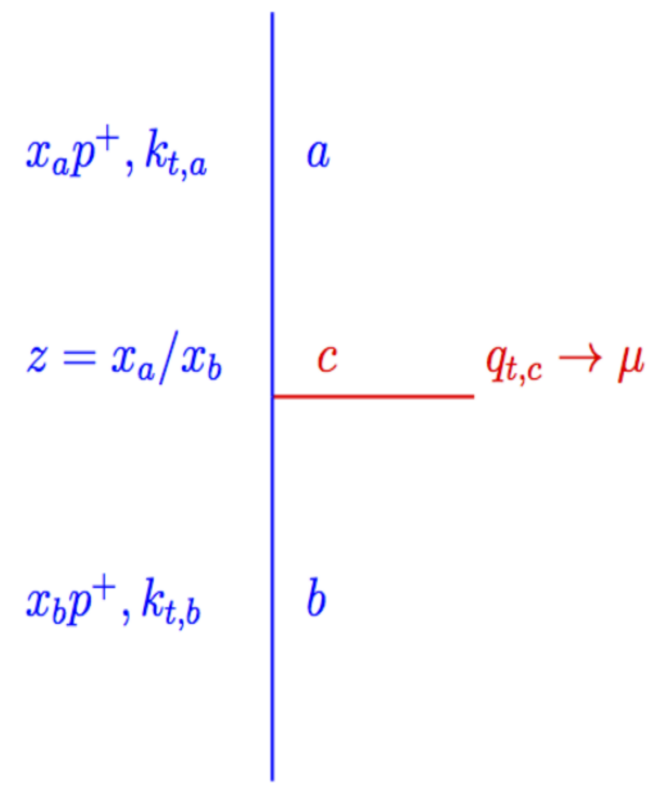
Figure 6.17: The integrals of the real parts of the splitting functions multiplied by z over z vs $\log_{10}(1-z_M)$ at NLO.

What is the gain with exclusive evolution ?

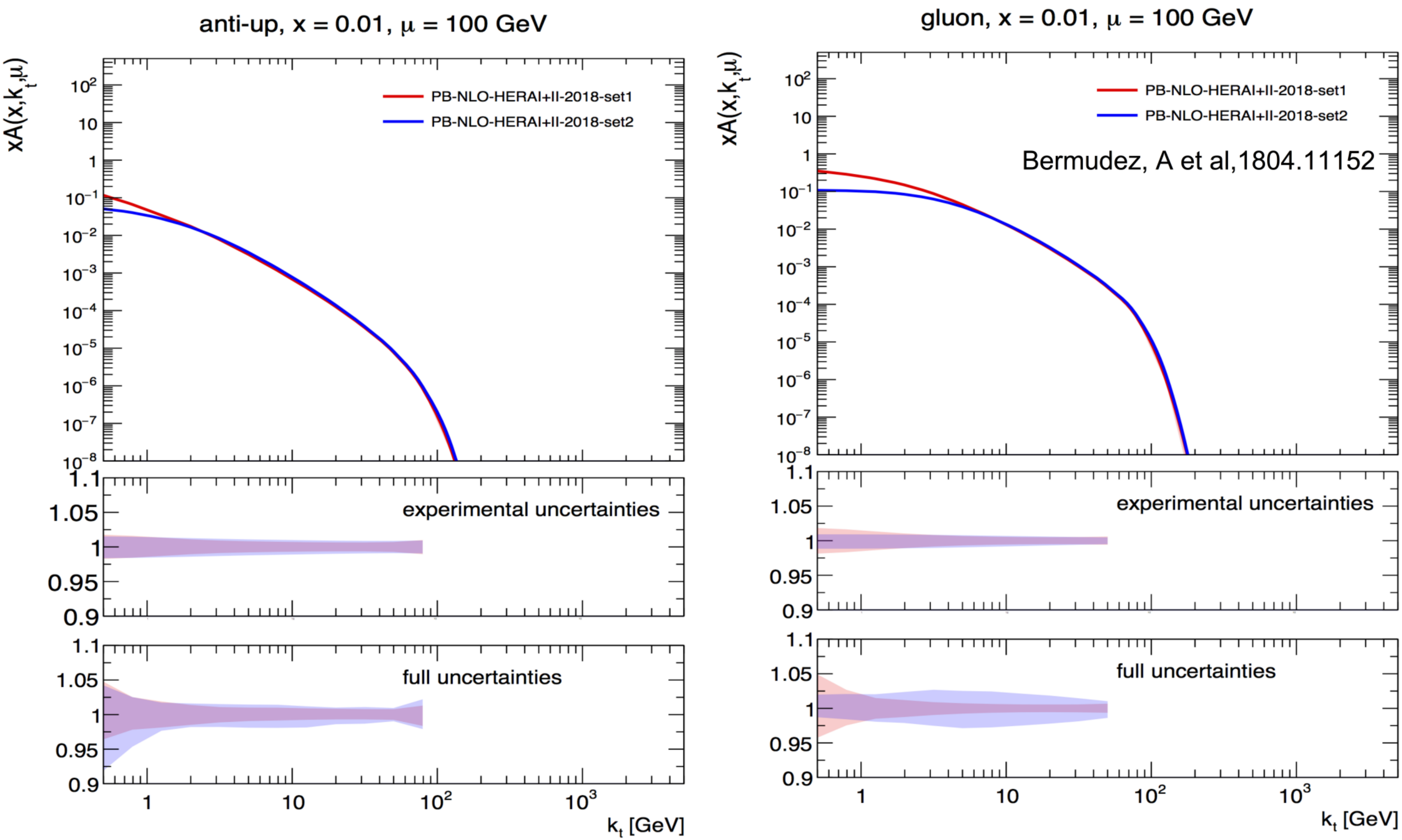
Transverse Momentum Dependence

- Parton Branching evolution generates every single branching:
  - kinematics can be calculated at every step
- Give physics interpretation of evolution scale:
  - angular ordering:

    $\mu = q_T/(1-z)$



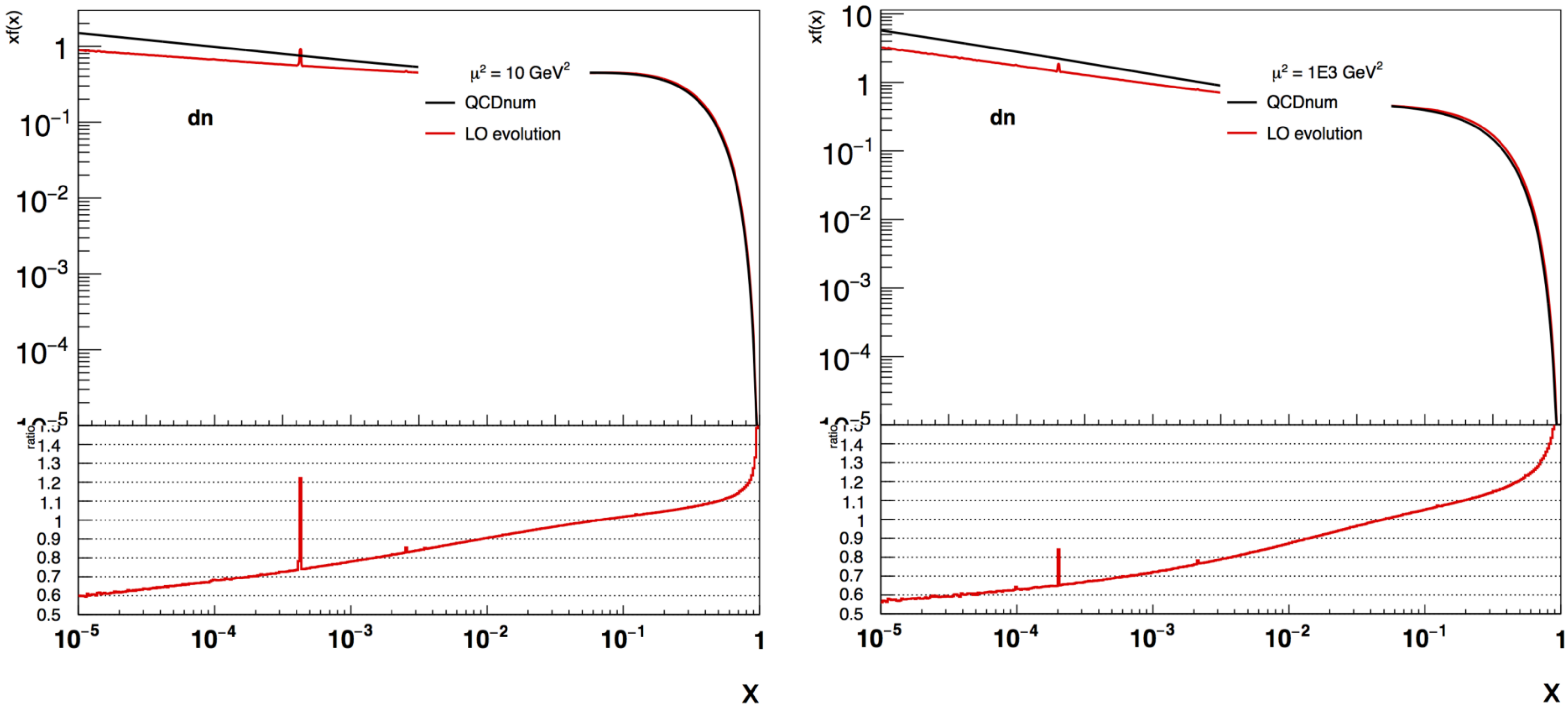
TMD distributions from fit to HERA data



- model and experimental uncertainties determined (details in talk A. Bermudez Martinez)

Effect of LO vrs NLO evolution

- using same starting distribution, but LO or NLO splitting functions



- effect of NLO evolution (α_s and P_{ij}) is very large: ~ 50 % for quarks

NB: spikes in plots come from stat fluctuations in MC solution

LO and NLO splitting functions

- Splitting functions at LO:

$$P_{gg}^{(0)} = 6\left(\frac{\alpha_s}{2\pi}\right)\left(\frac{1-z}{z} + \frac{z}{1-z} + \ldots\right)$$

$$P_{gq}^{(0)} = \frac{4}{3}\left(\frac{\alpha_s}{2\pi}\right)\left(\frac{1+(1-z)^2}{z}\right)$$

$$P_{qg}^{(0)} = \frac{1}{2}\left(\frac{\alpha_s}{2\pi}\right)\left(z^2+(1-z)^2\right)$$

$$P_{q_iq_i}^{(0)} = \frac{4}{3}\left(\frac{\alpha_s}{2\pi}\right)\left(\frac{1+z^2}{1-z}\right)$$

$$P_{q_jq_i}^{(0)} = 0$$

$$P_{\bar{q}_iq_i}^{(0)} = 0$$

- Splitting functions at NLO

$$P_{gg}^{(NLO)} = \left(\frac{\alpha_s}{2\pi}\right)P_{gg}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{gg}^{(1)}$$

$$P_{gq}^{(NLO)} = \left(\frac{\alpha_s}{2\pi}\right)P_{gq}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{gq}^{(1)}$$

$$P_{qg}^{(NLO)} = \left(\frac{\alpha_s}{2\pi}\right)P_{qg}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{qg}^{(1)}$$

$$P_{q_iq_i}^{(NLO)} = \left(\frac{\alpha_s}{2\pi}\right)P_{q_iq_i}^{(0)} \left(\frac{\alpha_s}{2\pi}\right)^2 P_{q_iq_i}^{(1)}$$

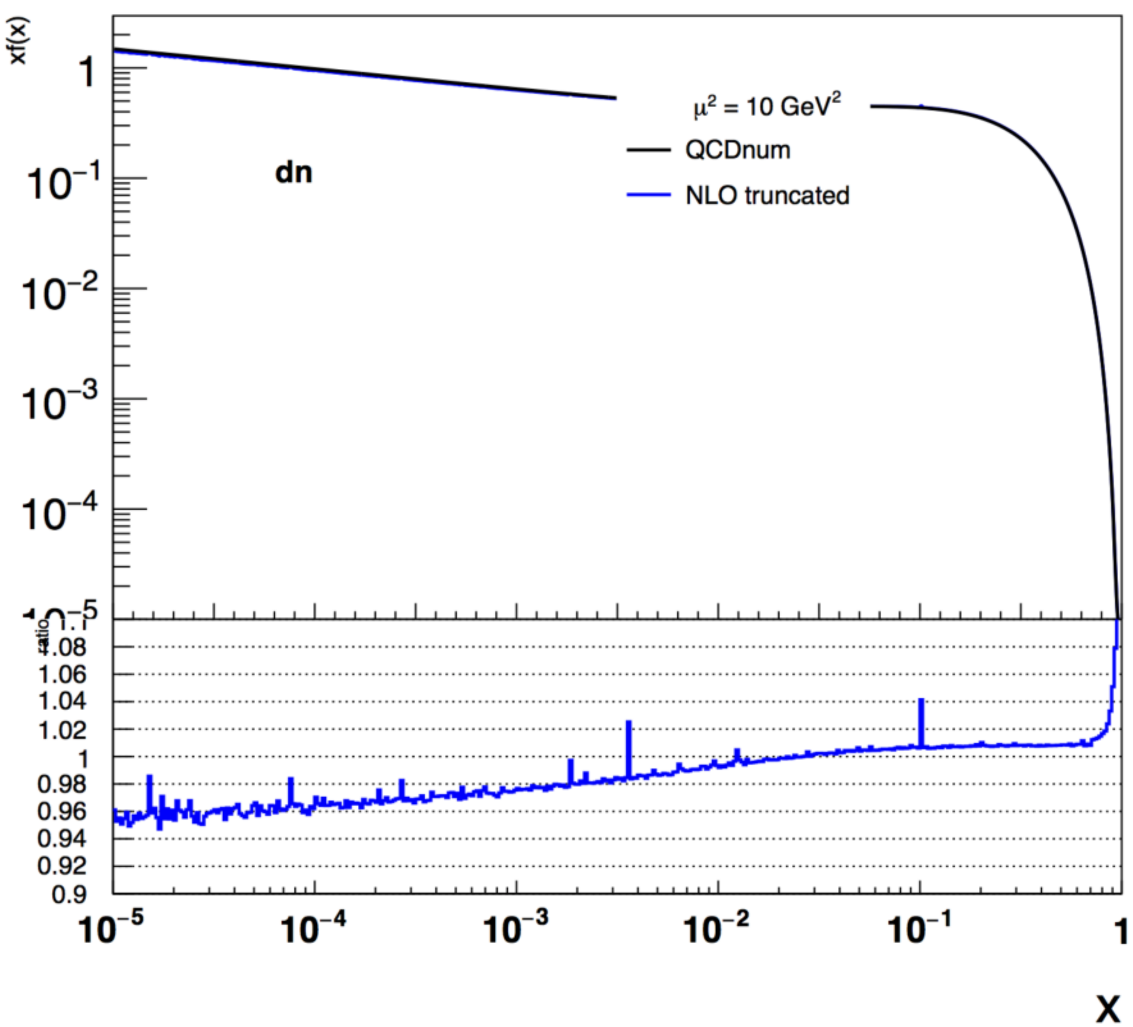
$$P_{q_jq_i}^{(NLO)} = \left(\frac{\alpha_s}{2\pi}\right)^2 P_{q_jq_i}^{(1)}$$

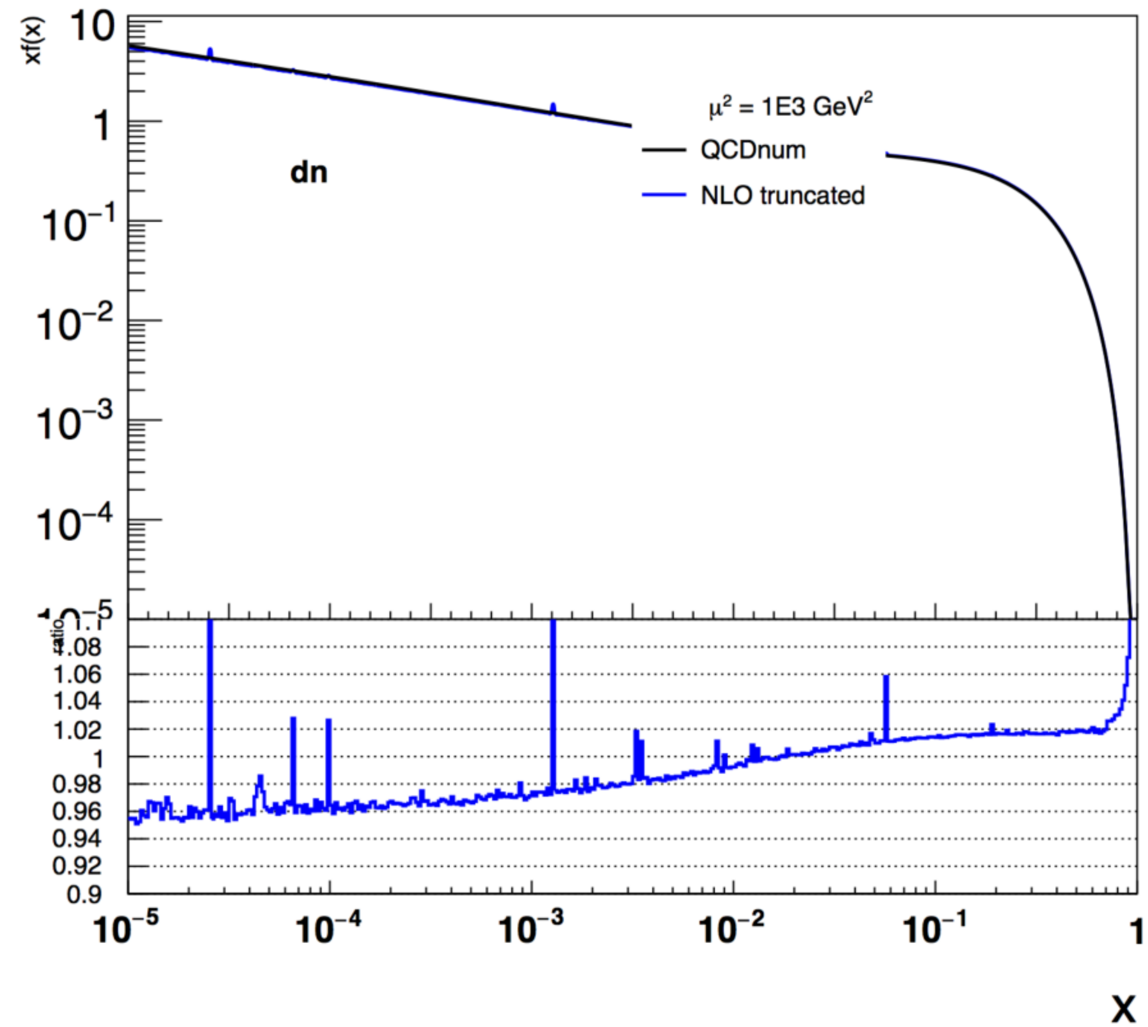
$$P_{\bar{q}_iq_i}^{(NLO)} = \left(\frac{\alpha_s}{2\pi}\right)^2 P_{\bar{q}_iq_i}^{(1)}$$

At NLO new channels are opened

Effect of truncation of NLO phase space: quarks

- using truncated NLO splitting functions (neglecting additional channels)



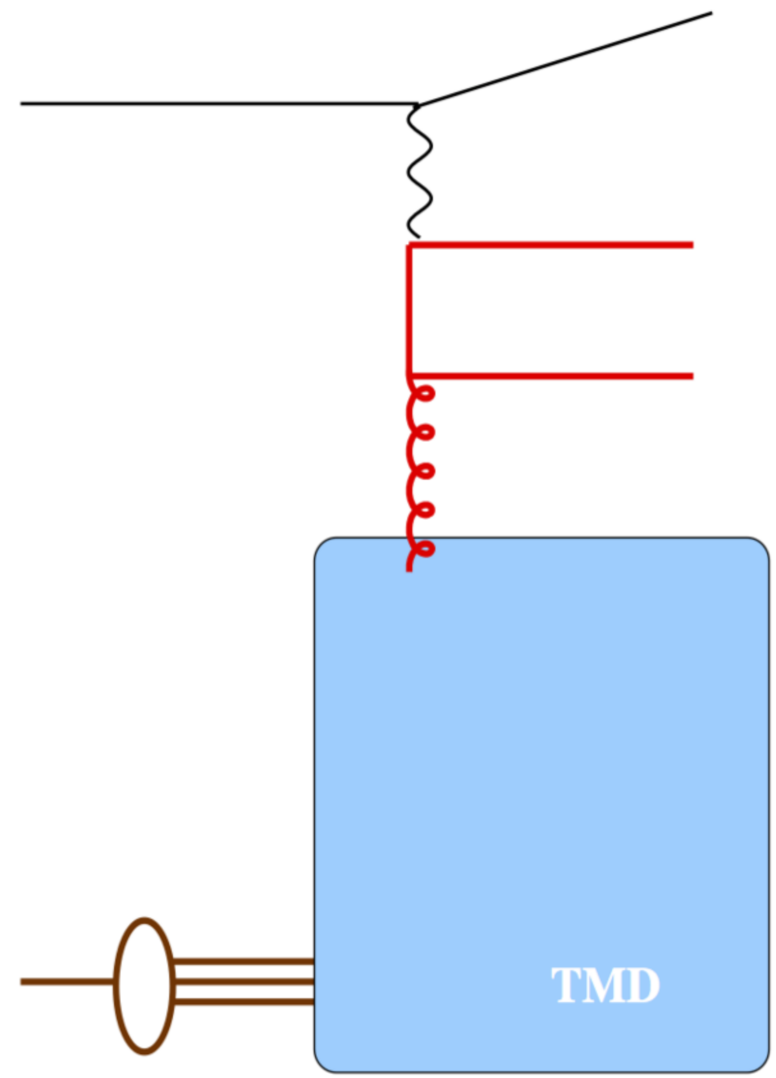


- effect of neglecting “new” quark channels at NLO is significant: ~ 5 %

NB: spikes in plots come from stat fluctuations in MC solution

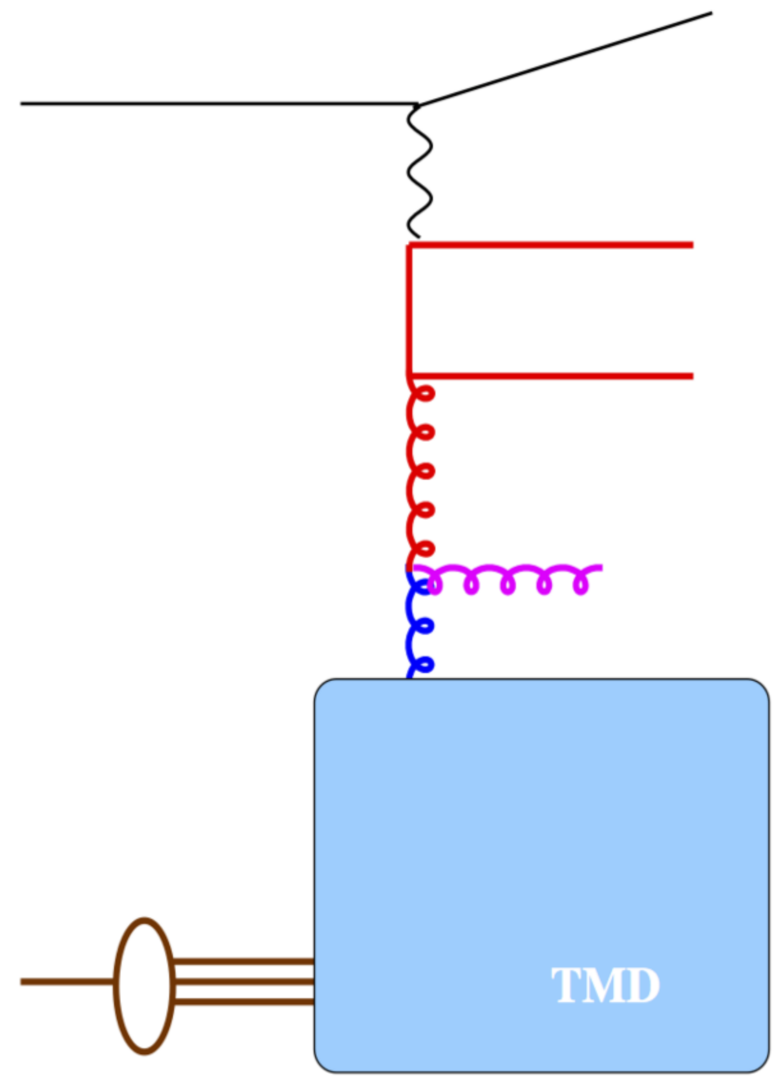
PB - TMDs and Parton Shower

- basic elements are:
  - Matrix Elements:
    - ➔ on shell/off shell
  - PDFs
    - ➔ PB - TMDs



TMDs and parton shower

- basic elements are:
  - Matrix Elements:
    - ➔ on shell/off shell
  - PDFs
    - ➔ PB - TMDs
  - Parton Shower
    - ➔ backward evolution
      - ➔ from hard scattering towards hadrons
    - ➔ reverse of PB evolution
    - ➔ following PB -TMDs for initial state



TMDs and parton shower

- basic elements are:
  - Matrix Elements:
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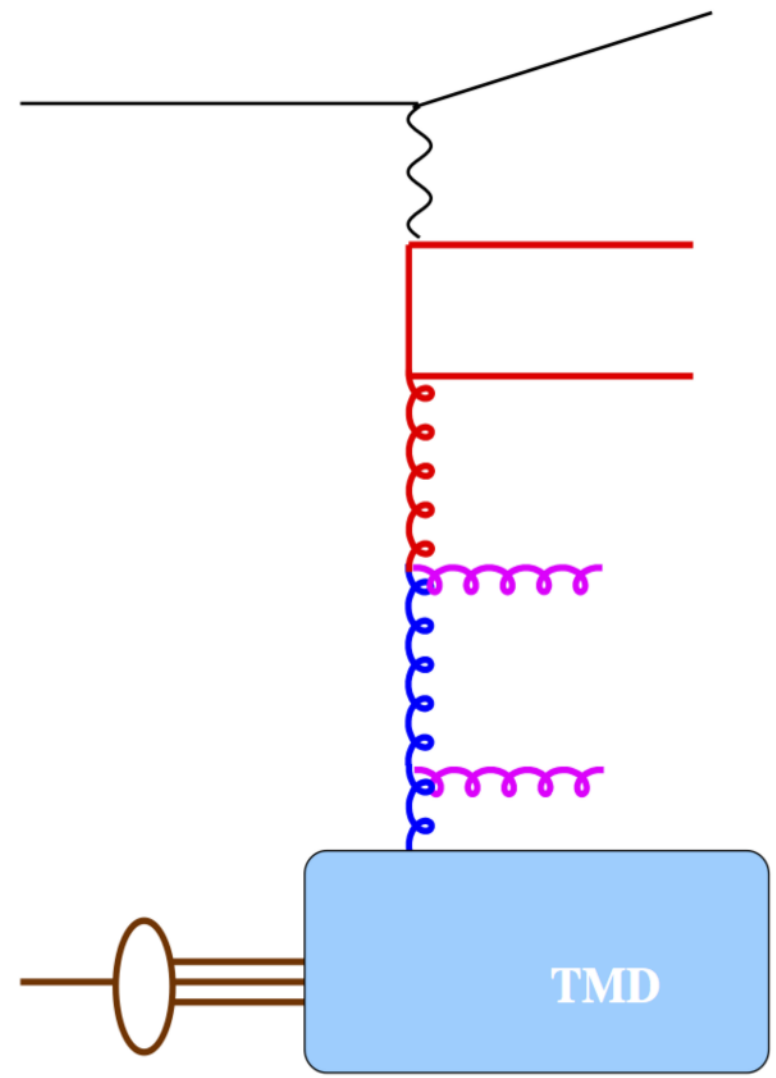
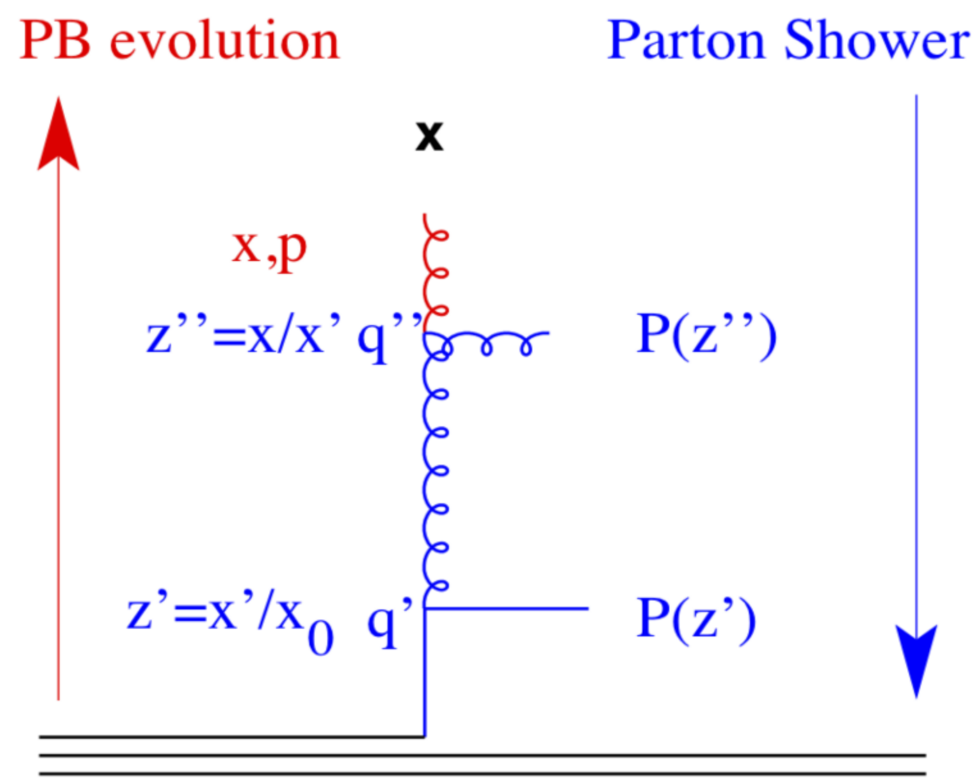
      - ➔ from hard scattering towards hadrons
    - ➔ reverse of PB evolution
    - ➔ following PB -TMDs for initial state !



Parton Branching evolution and Parton Shower

- **Parton Branching evolution**
  - start from hadron side and evolve from small to large scale μ^2

$$\Delta_s = \exp\left(- \int^{z_M} dz \int_{\mu_0^2}^{\mu^2} \frac{\alpha_s}{2\pi} \frac{d\mu'^2}{\mu'^2} P(z) \right)$$



- **Parton Shower**
  - backward evolution from hard scale μ^2 to hadron scale $\mu^2{}_0$ (for efficiency reasons)

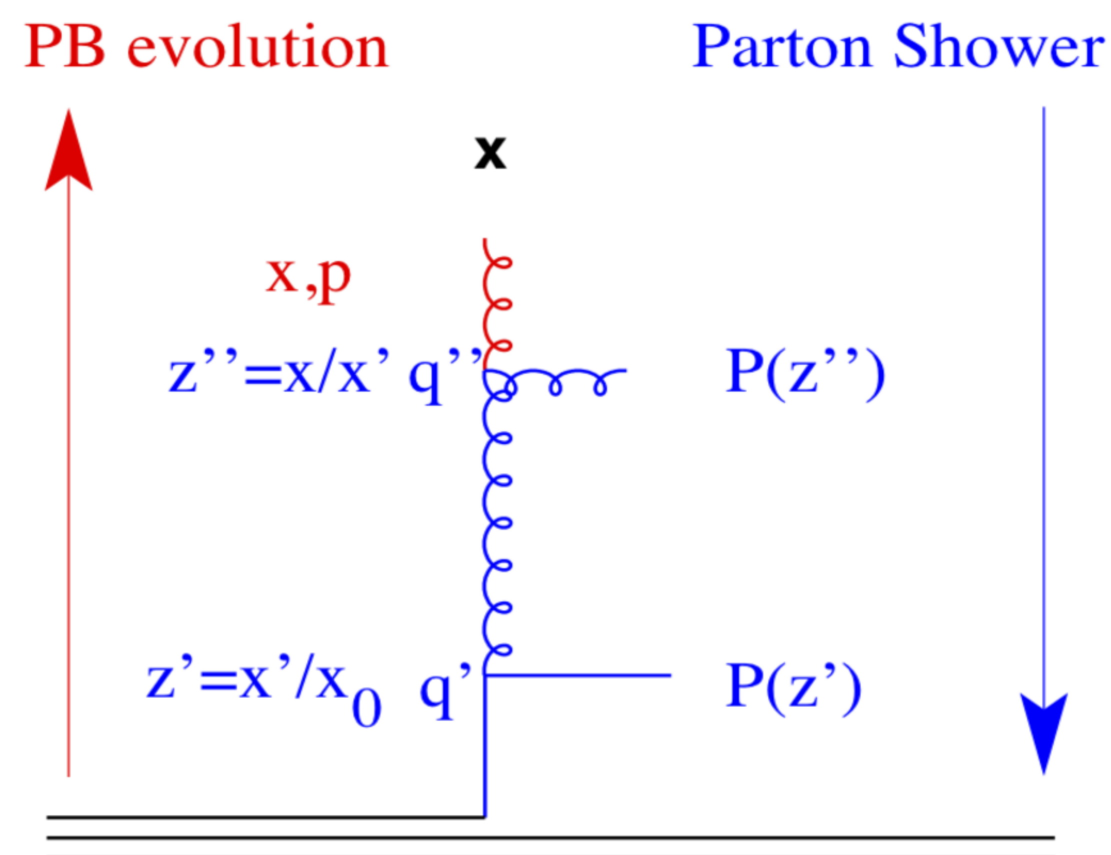
$$\Delta_s = \exp\left(- \int^{z_M} dz \int_{\mu_0^2}^{\mu^2} \frac{\alpha_s}{2\pi} \frac{d\mu'^2}{\mu'^2} P(z) \frac{\frac{x}{z}\mathcal{A}\left(\frac{x}{z}, k'_\perp, \mu'\right)}{x\mathcal{A}(x, k_\perp, \mu')} \right)$$

➔ in backward evolution, parton density (TMD) imposed further constraint !

Parton Shower at NLO: does this work ?

- **NLO splitting functions:**
  - truncated at z_M
  - as in PB method,

if $\int dz z P_{ba} > 0$

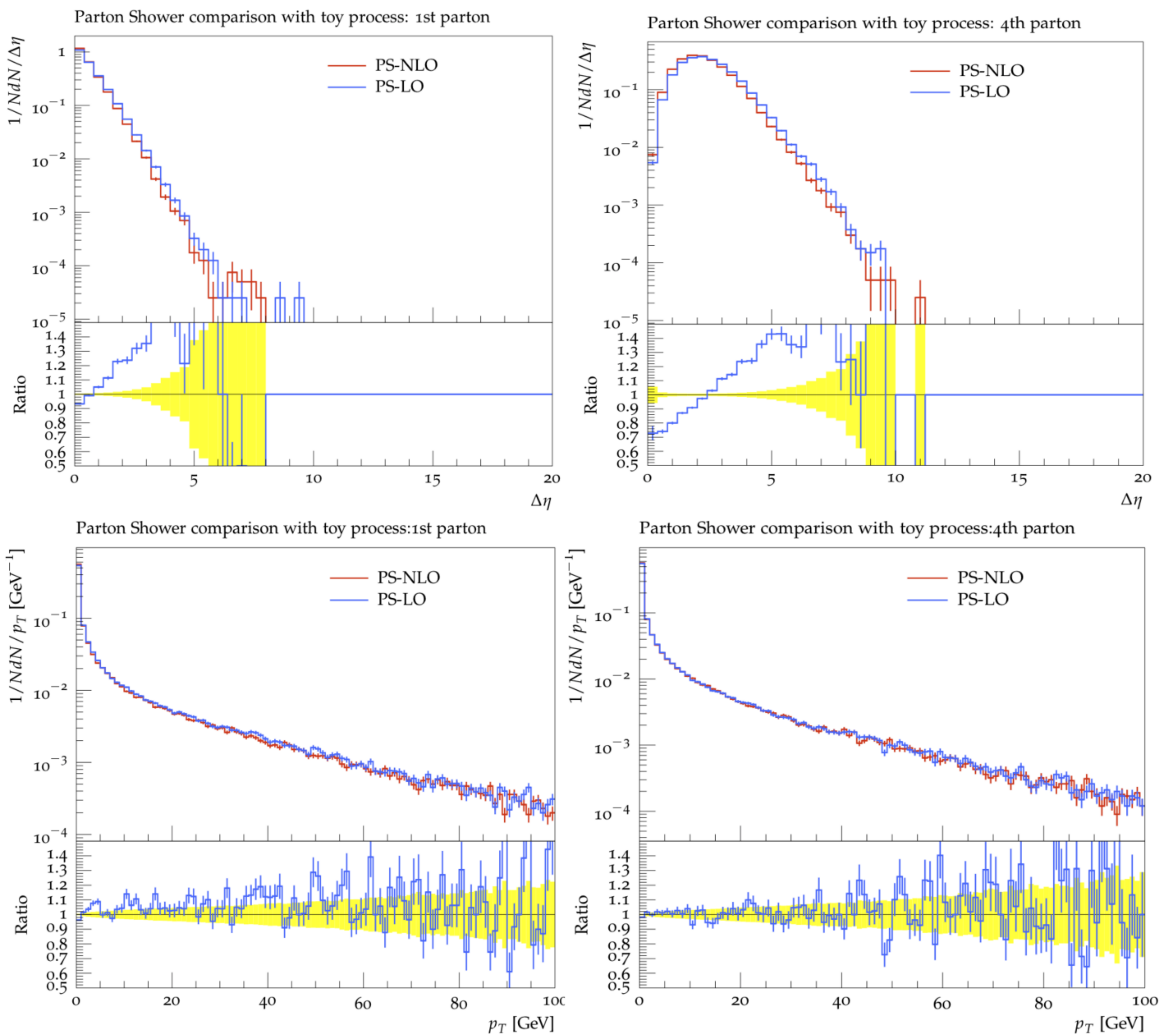


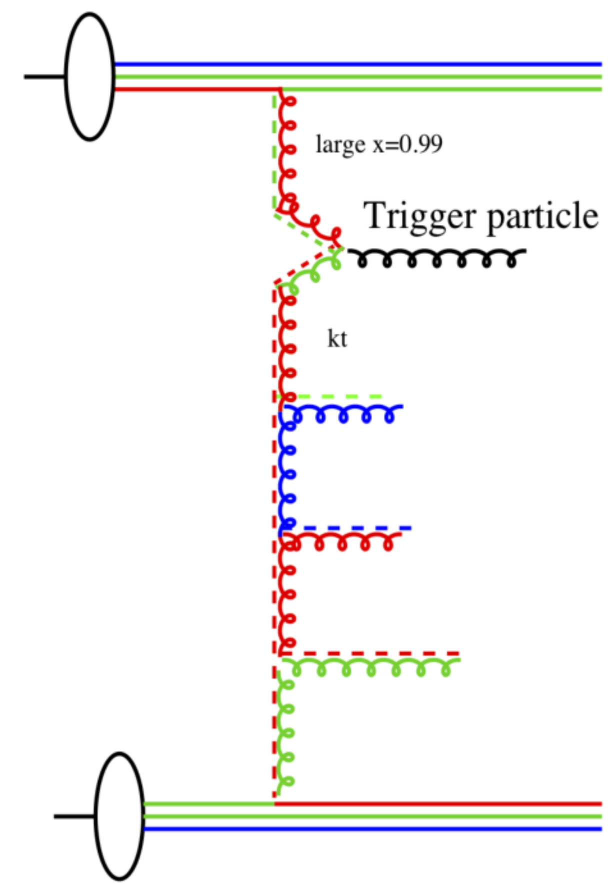
➔ Sudakov Δ_S of backward evolution can be interpreted as probability

$$\Delta_s = \exp\left(-\int^{z_M} dz \int_{\mu_0^2}^{\mu^2} \frac{\alpha_s}{2\pi} \frac{d\mu'^2}{\mu'^2} P(z) \frac{\frac{x}{z}\mathcal{A}\left(\frac{x}{z}, k'_\perp, \mu'\right)}{x\mathcal{A}(x, k_\perp, \mu')}\right)$$

- P_{ba} can be still negative, but in practical applications, this does happen only rarely ($< 10^{-5}$)

NLO parton showers (compared to LO)





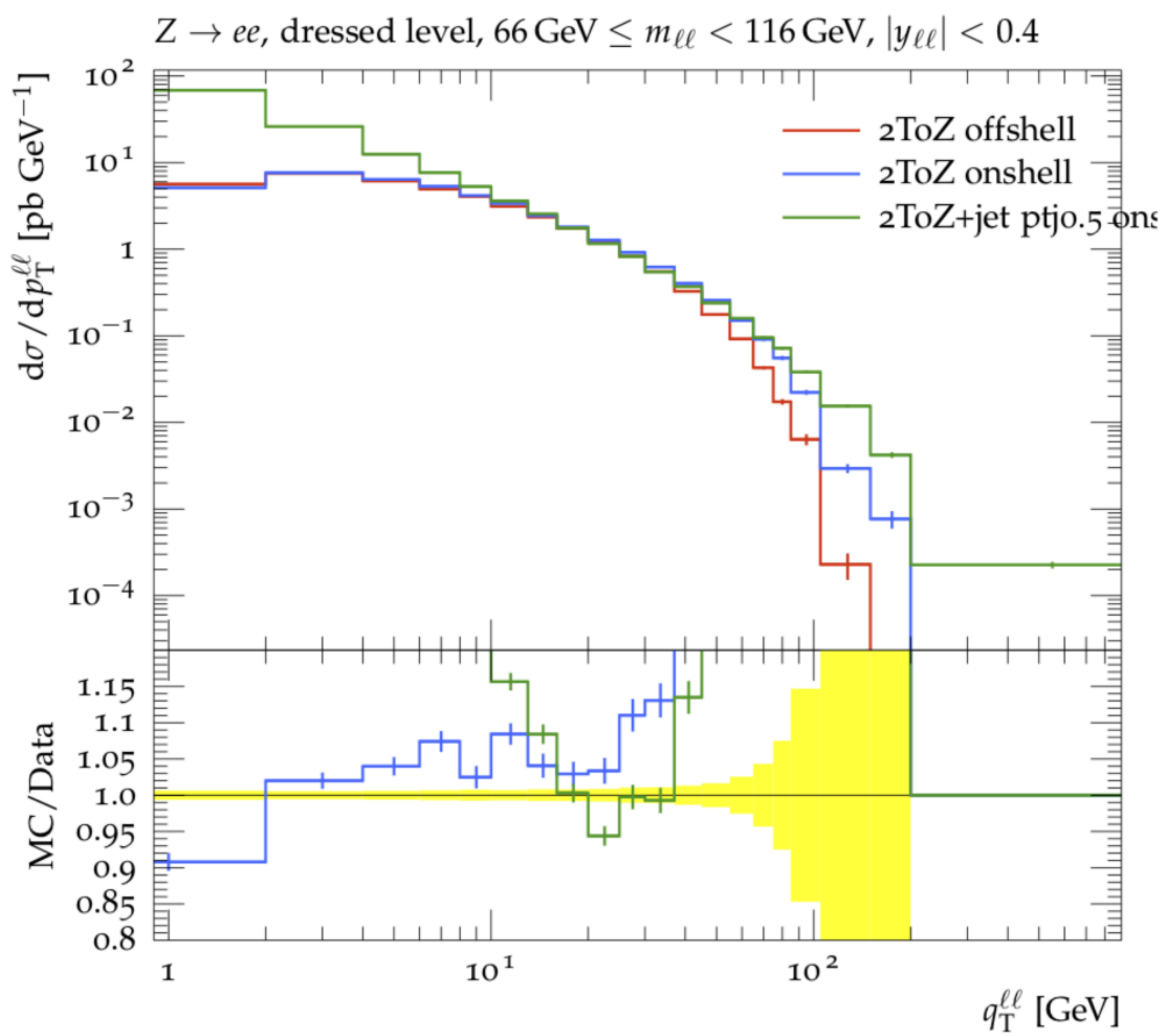
Parton shower LO – NLO at fixed x=0.01 and μ =100 GeV

- differences in $\Delta\ y$
- no difference in pT

Matching to hard process: off-shell ME with KaTie

van Hameren, A. CPC, 224, 371, 2018, arXiv 1611.00680

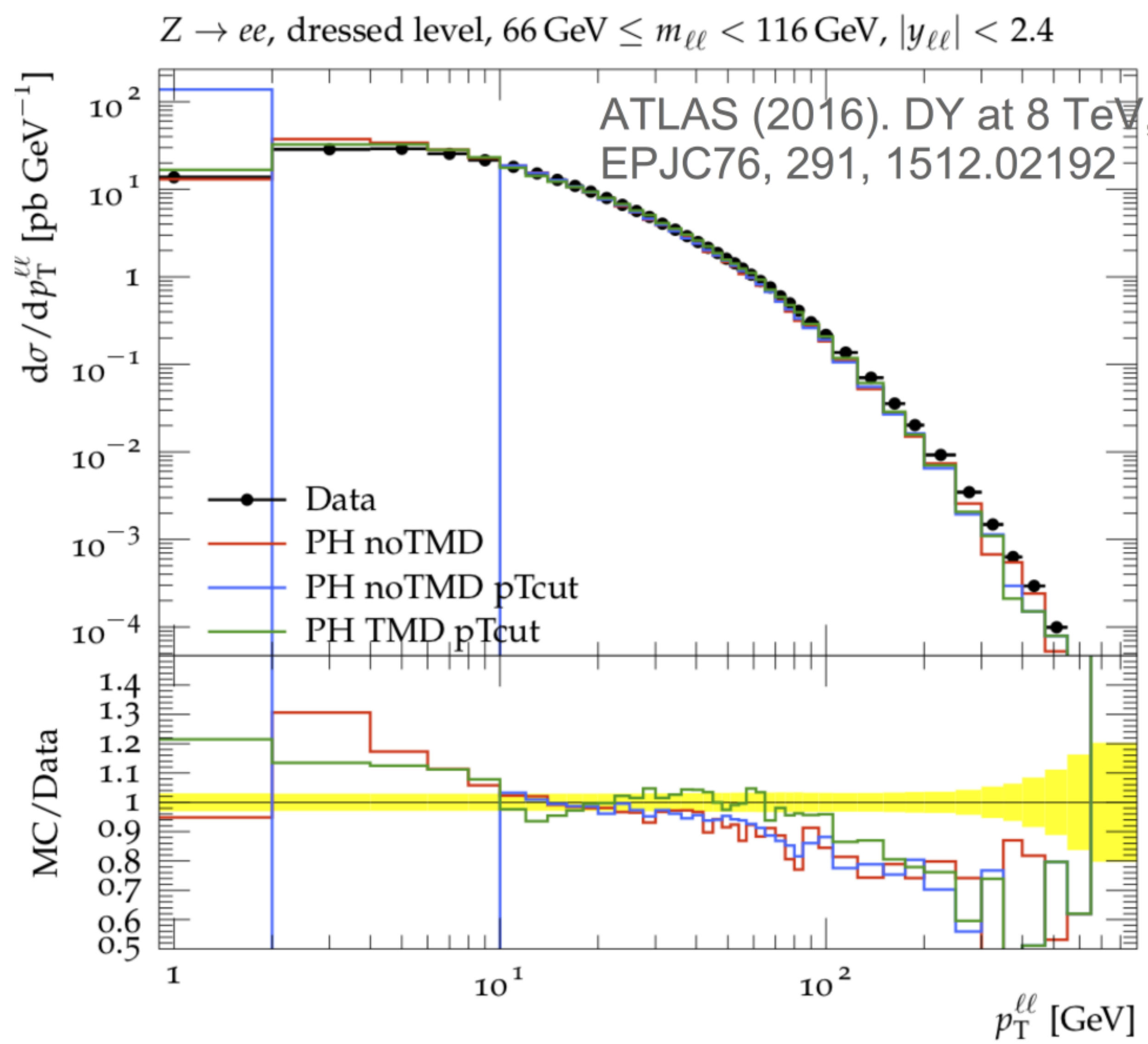
- KaTie (see talk by A. Kusina on Z+jet)
  - off-shell kinematics with TMDs used to calculate hard process
    - no kinematic corrections needed
    - parton shower below scale μ
  - off-shell agrees with on-shell with TMD added (and keeping mass fixed) at small q_T
    - → important check for application with collinear NLO calculation
  - off-shell agrees with $2 \to 2$ on-shell at medium q_T
    - → important check for merging different parton multiplicities



Matching to hard process: POWHEG method

Frixione, S., Nason, P., and Ridolfi, G. (2007). JHEP, 09, 126 arXiv 0707.3088
Frixione, S., Nason, P., and Oleari, C. JHEP, 0711(), 070 arXiv 0709.2092

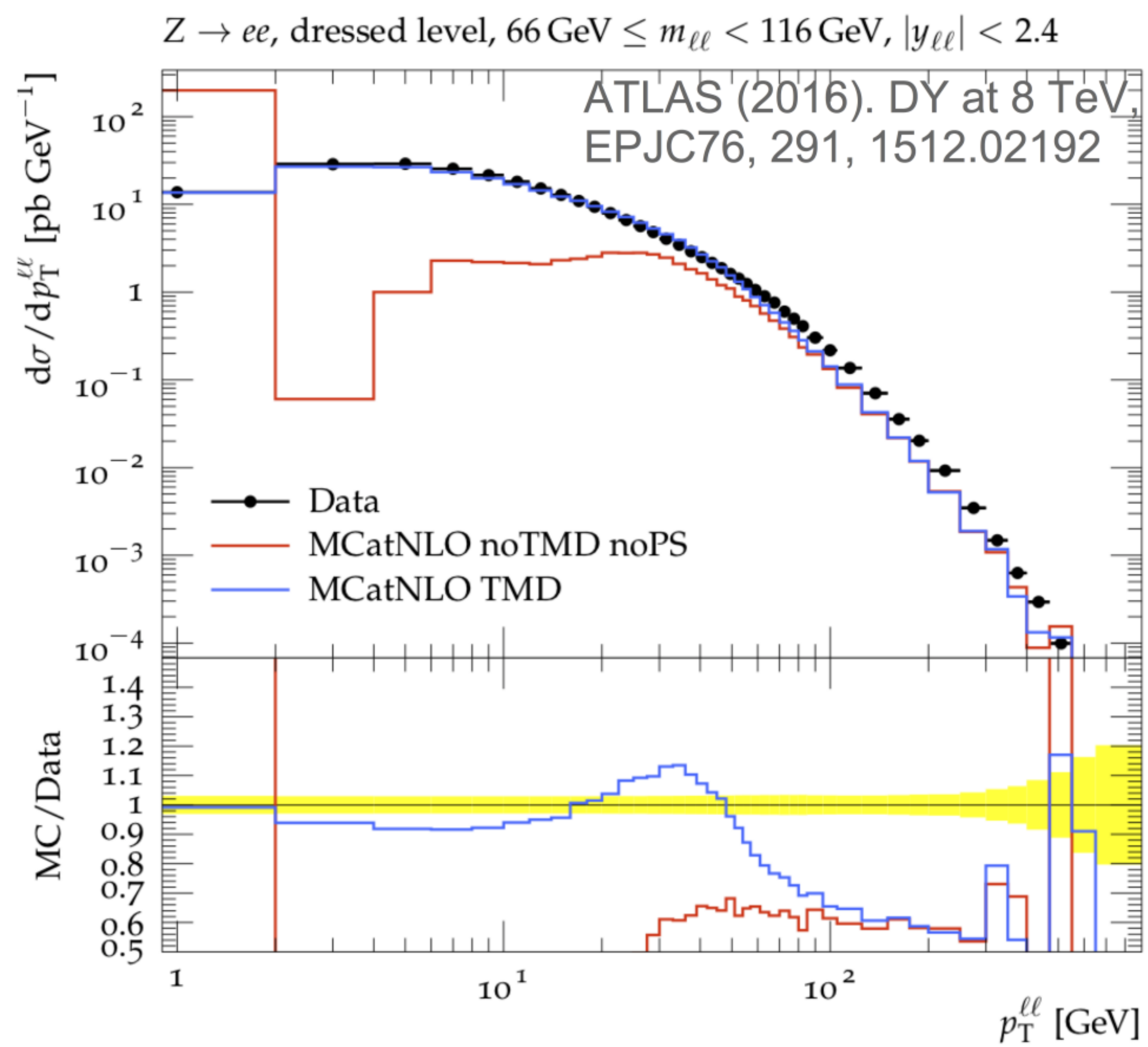
- POWHEG exponentiates real emission (soft part): Sudakov for 1st emission
  - DY-process as example
  - q_T cut applied (`ptsqmin`) to allow for contribution from TMD (and PS)
    - low q_T region filled by TMD + PS
    - large q_T by real emission
  - DY production described reasonably well with TMD + POWHEG with q_T cut
    - TMD fills low q_T part



Matching to hard process: MC@NLO method

Frixione, S. and Webber, B. JHEP, 0206, 029, arXiv hep-ph/0204244
Alwall, J., et al JHEP, 1407, 079 arXiv 1405.0301

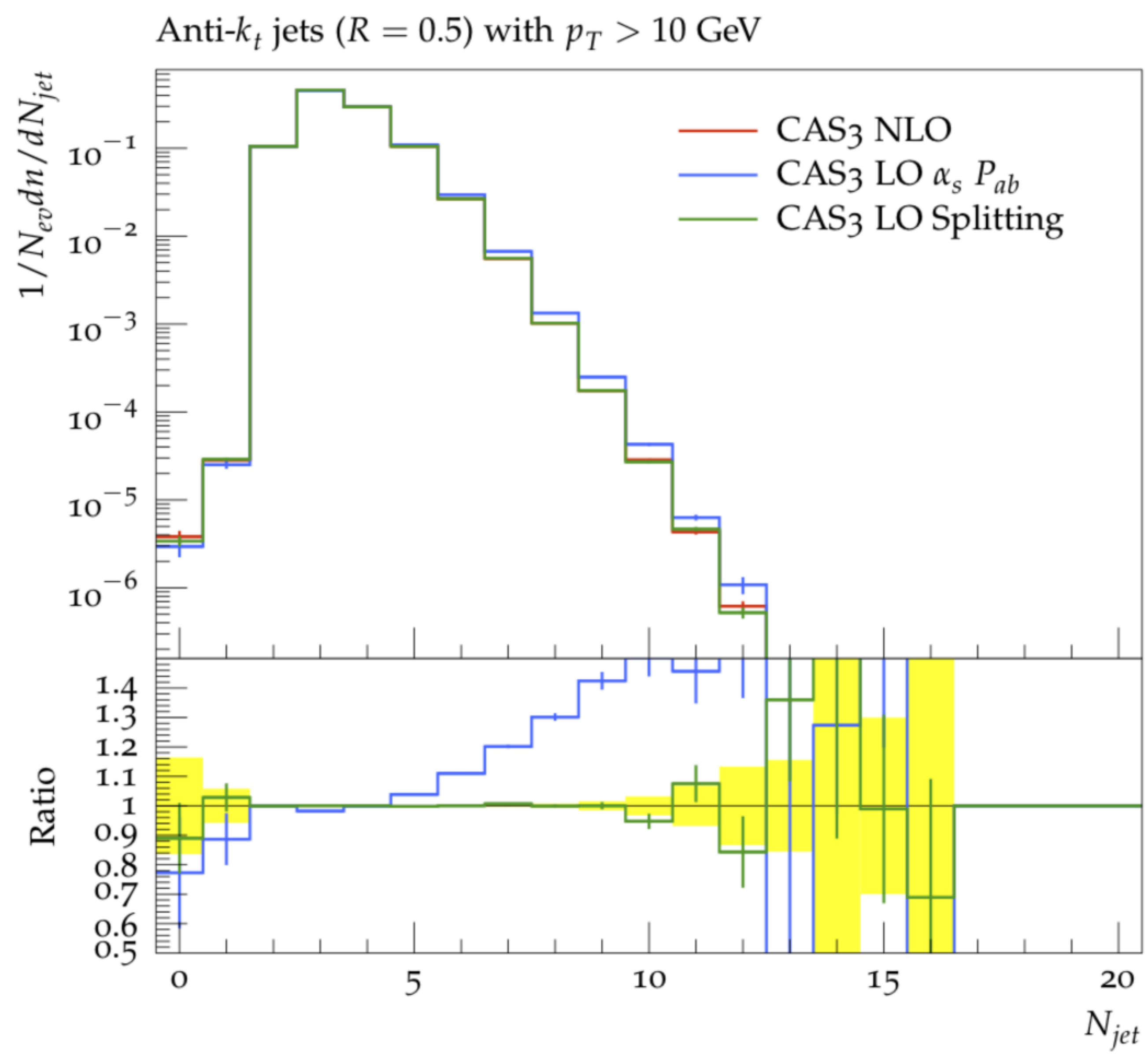
- MC@NLO subtracts soft & collinear parts from NLO (added by TMD and shower)
- MC@NLO without shower unphysical
  - DY-process as example
  - low q_T region affected by subtraction of soft & collinear parts
    - to be filled by TMD (+ PS)
  - DY production very well described by TMD with MC@NLO
    - TMD fills low q_T part



Matching to hard process: POWHEG + TMD + PS

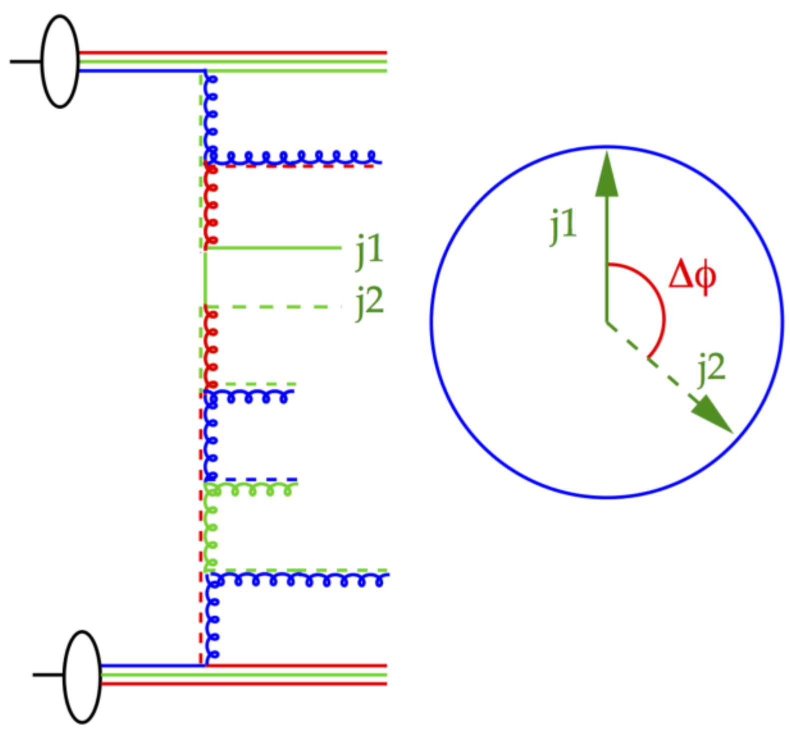
- POWHEG dijets NLO
  - k_t included from TMD
  - initial state parton shower included
    - LO splitting and LO α_s
    - NLO splitting and NLO α_s

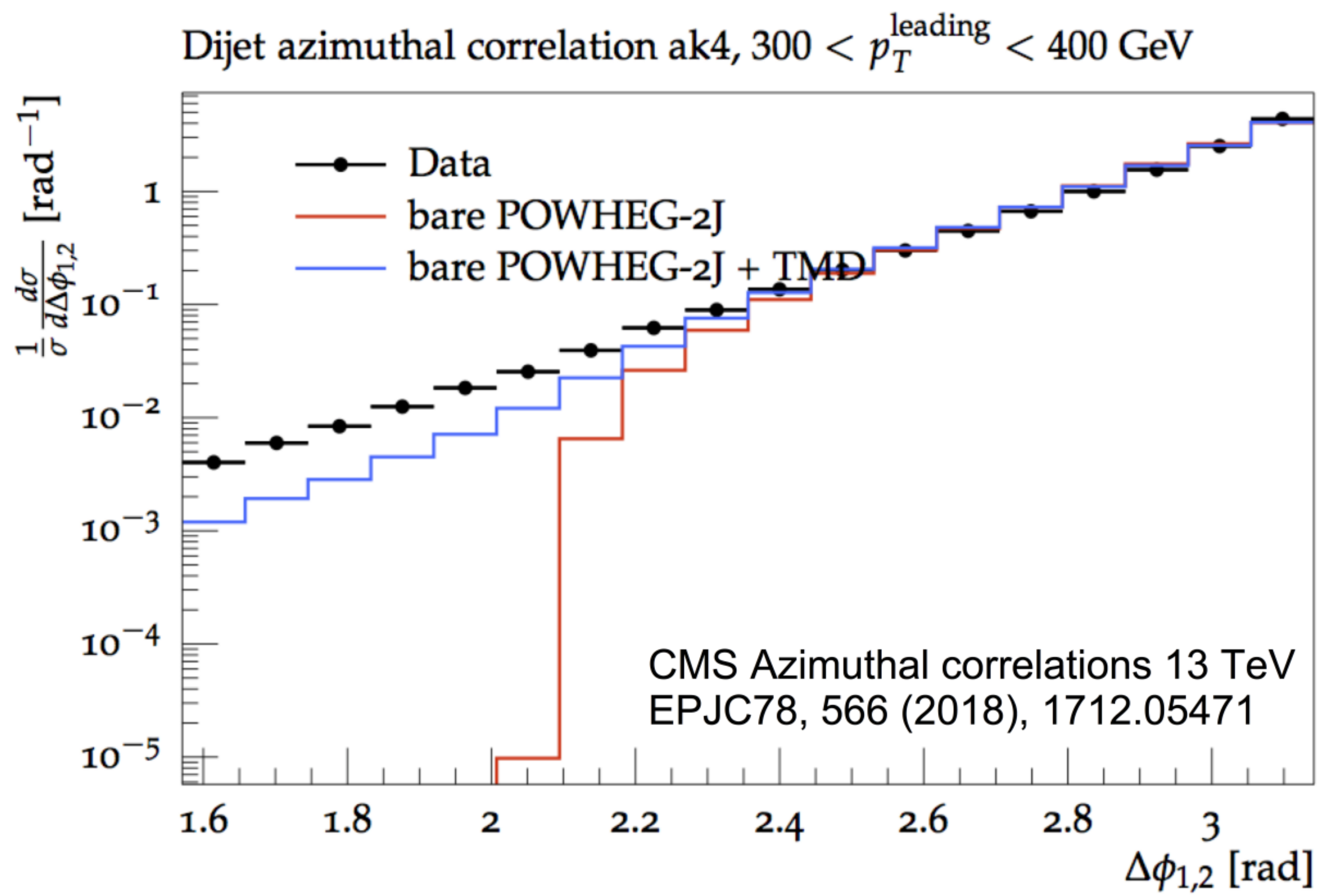
- Due to constraint from TMD, little difference of LO and NLO splitting fcts.
- LO coupling α_s changes jet multiplicity.



Matching to hard process: POWHEG + TMD + PS

- Dijet production at in pp, a test for TMDs and PS :





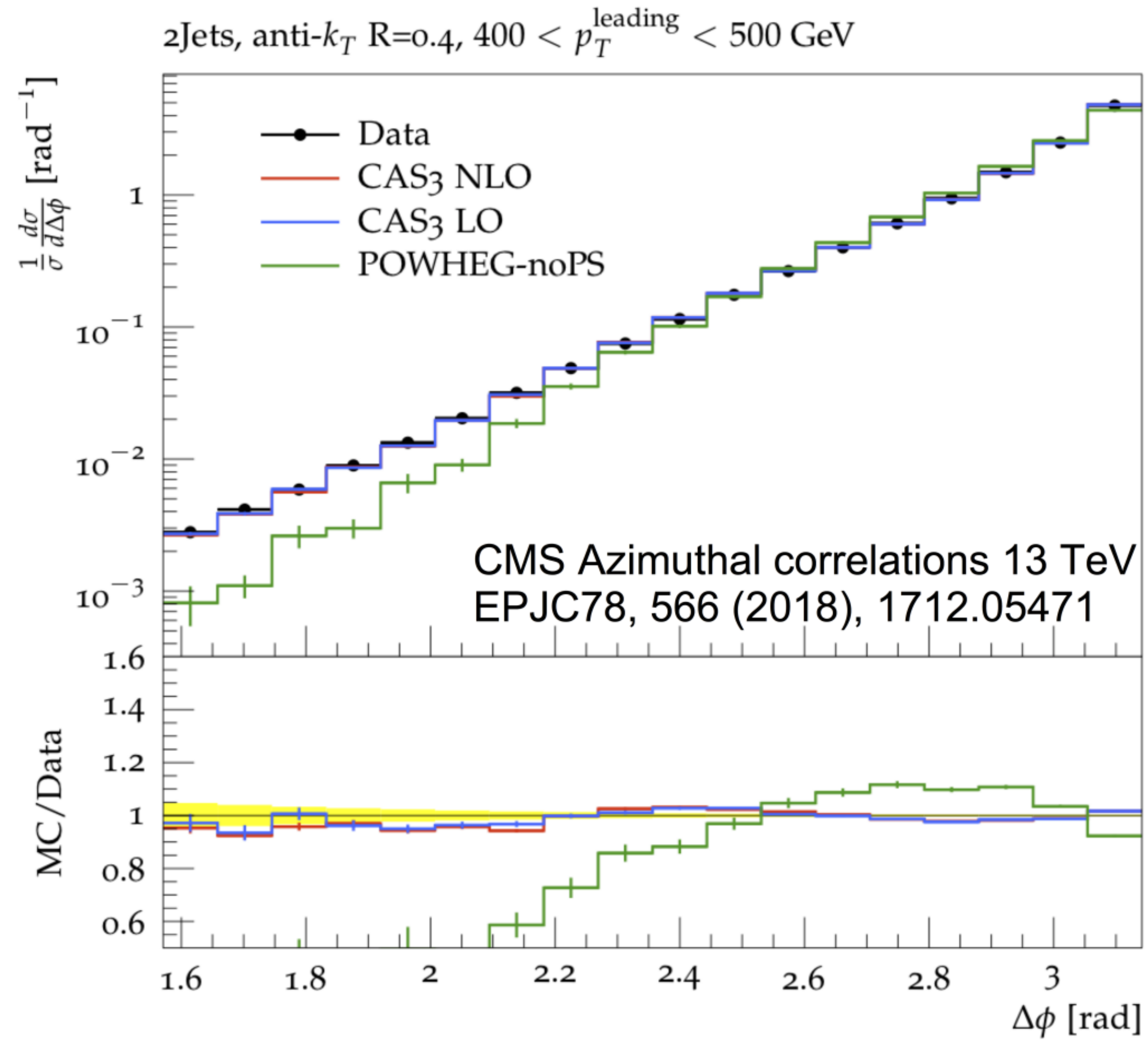
- TMDs with NLO dijets get closer to data !

Matching to hard process: POWHEG + TMD + PS

- POWHEG dijets NLO
  - k_t included from TMD
  - initial state parton shower included
    - LO splitting fct and LO α_s
    - NLO splitting fct and NLO α_s

- Effect of NLO shower on observables

- TMD + PS gives very good description of measurement
  - Due to constraint from TMD, little difference of LO and NLO splitting fcts are observed !



Conclusion

- Parton Branching method to solve DGLAP equation at LO, NLO and NNLO
  - ➔ consistence for collinear (integrated) PDFs shown
  - ➔ advantages of Parton Branching method !
  - ➔ TMD distributions for all flavors determined at LO and NLO, without free parameters

- First complete NLO TMD parton shower
  - ➔ TMD initial parton shower:
    - ➔ backward evolution following exactly the TMD density
    - ➔ matching with k_T off-shell calculations: KaTie
    - ➔ matching with NLO collinear ME calculations: MC@NLO and POWHEG
    - ➔ available as CASCADE-lhe
      - ➔ exercises and advices in tutorial on Friday

Conclusion

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This completes a major step in calculations using LO and NLO hard processes with TMDs and matched with parton showers !

Thanks

- This work would not have been possible without the scholarship of FNP and Humboldt foundation for extensive visits to IFJ PAN Cracow
  - and due to the many discussions and common work and studies with the members of the department here in Cracow of the group of K. Kutak.

    Special thanks to
    - K. Kutak
    - M. Bury
    - P. Kotko
    - O. Kusina
    - S. Sapeta
    - M. Serino
    - A. van Hameren

Appendix

Evolution equation and parton branching method

- use momentum weighted PDFs: $xf(x,t)$

$$
\begin{aligned}
xf_a(x,\mu^2) \quad = \quad & \Delta_a(\mu^2)\; xf_a(x,\mu_0^2) \\
& + \sum_b \int_{\mu_0}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \int_x^{z_M} dz P_{ab}^{(R)}(\alpha_s, z)\; \frac{x}{z} f_b\left(\frac{x}{z}, \mu'^2\right)
\end{aligned}
$$

  - with $P_{ab}{}^{(R)}\,(\alpha_s(t'),z)$ real emission probability (without virtual terms)
    - z_M introduced to separate real from virtual and non-emission probability
    - reproduces DGLAP up to $\mathcal{O}(1 - z_M)$

- make use of momentum sum rule to treat virtual corrections
  - use Sudakov form factor for non-resolvable and virtual corrections

$$
\Delta_a(z_M, \mu^2, \mu_0^2) = \exp\left(- \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\; z\; P_{ba}^{(R)}(\alpha_s, z) \right)
$$

The limit z_M

- Investigating the large *z* part:

$$\sum_b \int_x^1 dz\, K_{ab}(\alpha_s)\, \frac{1}{(1-z)_+}\, \widetilde{f}_b(x/z,\mu^2)$$

$$= \sum_b \int_x^1 dz\, K_{ab}(\alpha_s)\, \frac{1}{1-z}\, \widetilde{f}_b(x/z,\mu^2) - \sum_b \int_0^1 dz\, K_{ab}(\alpha_s)\, \frac{1}{1-z}\, \widetilde{f}_b(x,\mu^2)$$

- in the region $1 > z > z_M$ expand:

$$\widetilde{f}_b(x/z,\mu^2) = \widetilde{f}_b(x,\mu^2) + (1-z)\frac{\partial \widetilde{f}_b}{\partial \ln x}(x,\mu^2) + \mathcal{O}(1-z)^2$$

- up to $\mathcal{O}(1-z_M)$:

$$\sum_b \int_x^1 dz\, K_{ab}(\alpha_s)\, \frac{1}{(1-z)_+}\, \widetilde{f}_b(x/z,\mu^2)$$

$$= \sum_b \int_x^{z_M} dz\, K_{ab}(\alpha_s)\, \frac{1}{1-z}\, \widetilde{f}_b(x/z,\mu^2) - \sum_b \int_0^{z_M} dz\, K_{ab}(\alpha_s)\, \frac{1}{1-z}\, \widetilde{f}_b(x,\mu^2)$$

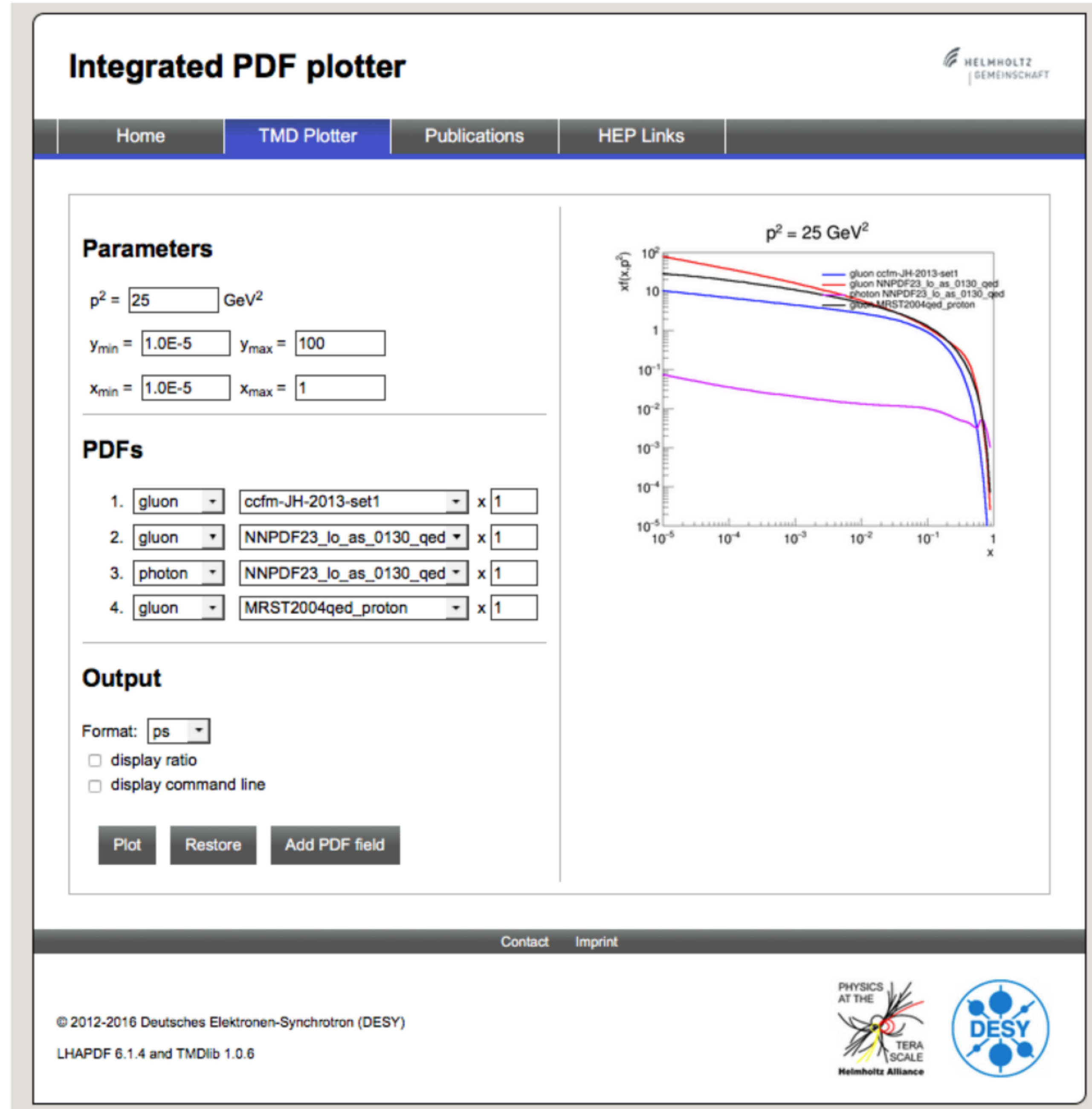
Where to find TMDs ? TMDlib and TMDplotter

- TMDlib proposed in 2014 as part of REF workshop and developed since
- combine and collect different ansaetze and approaches:

  http://tmd.hepforge.org/ and http://tmdplotter.desy.de

→ TMDlib: a library of parametrization of different TMDs and uPDFs (similar to LHApdf)

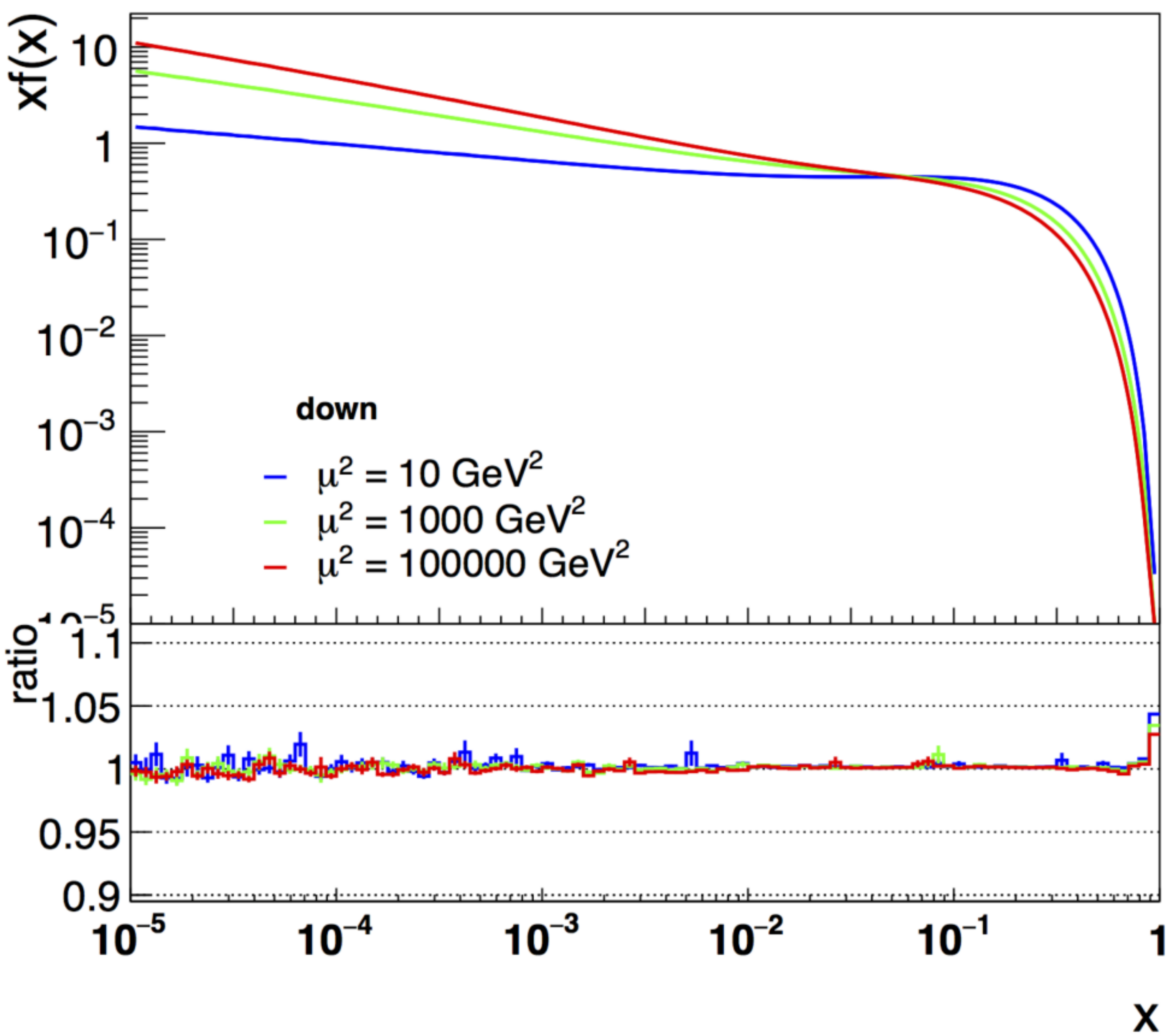
TMDlib and TMDplotter: library and plotting tools for transverse-momentum-dependent parton distributions, *F. Hautmann et al.* arXiv 1408.3015, **Eur. Phys. J., C 74(12):3220, 2014.**

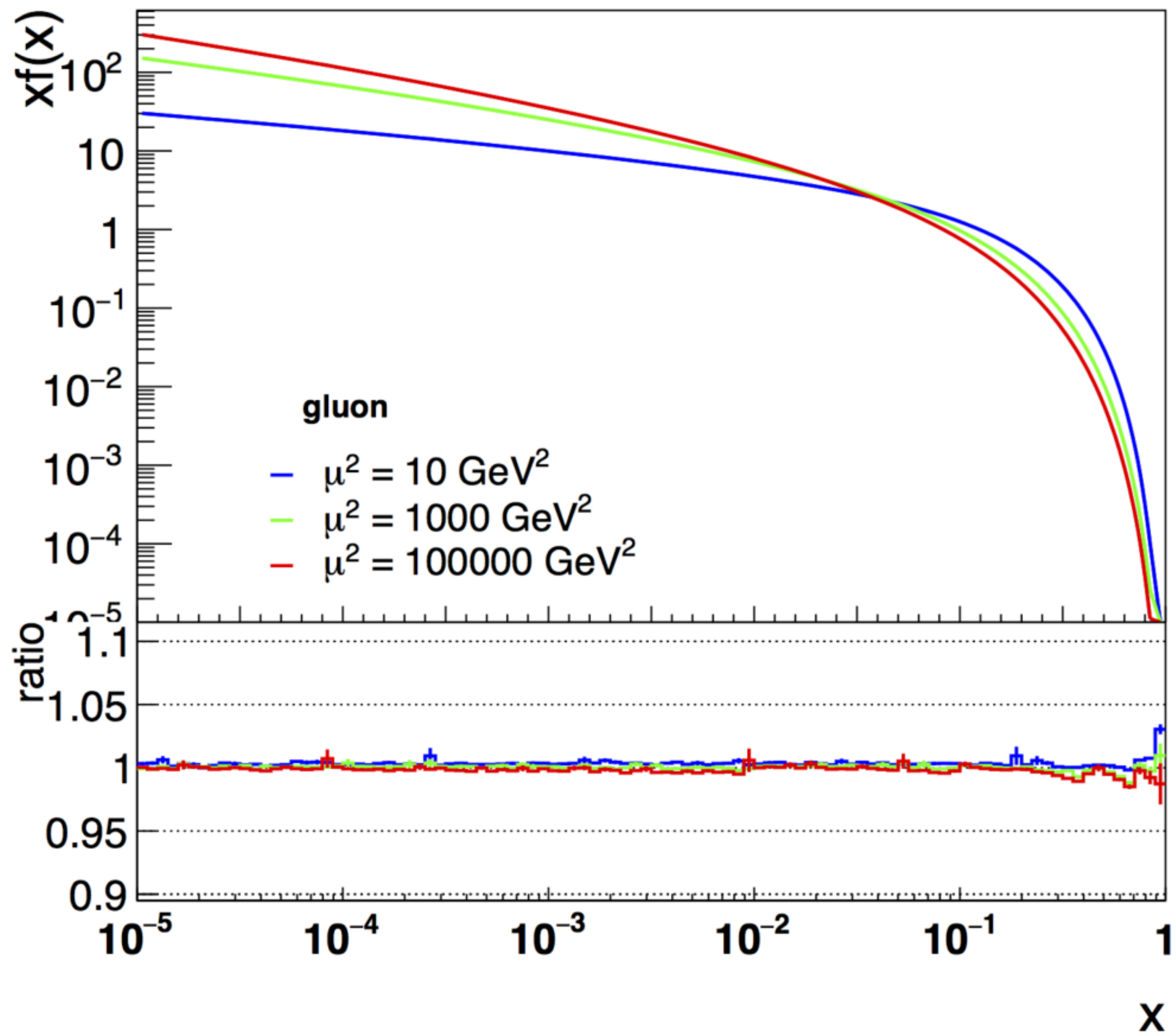


→ Also integrated pdfs (including photon pdf are available via LHAPDF)

- Feedback and comments from community is needed – just use it !

Validation of method with QCDnum at NLO





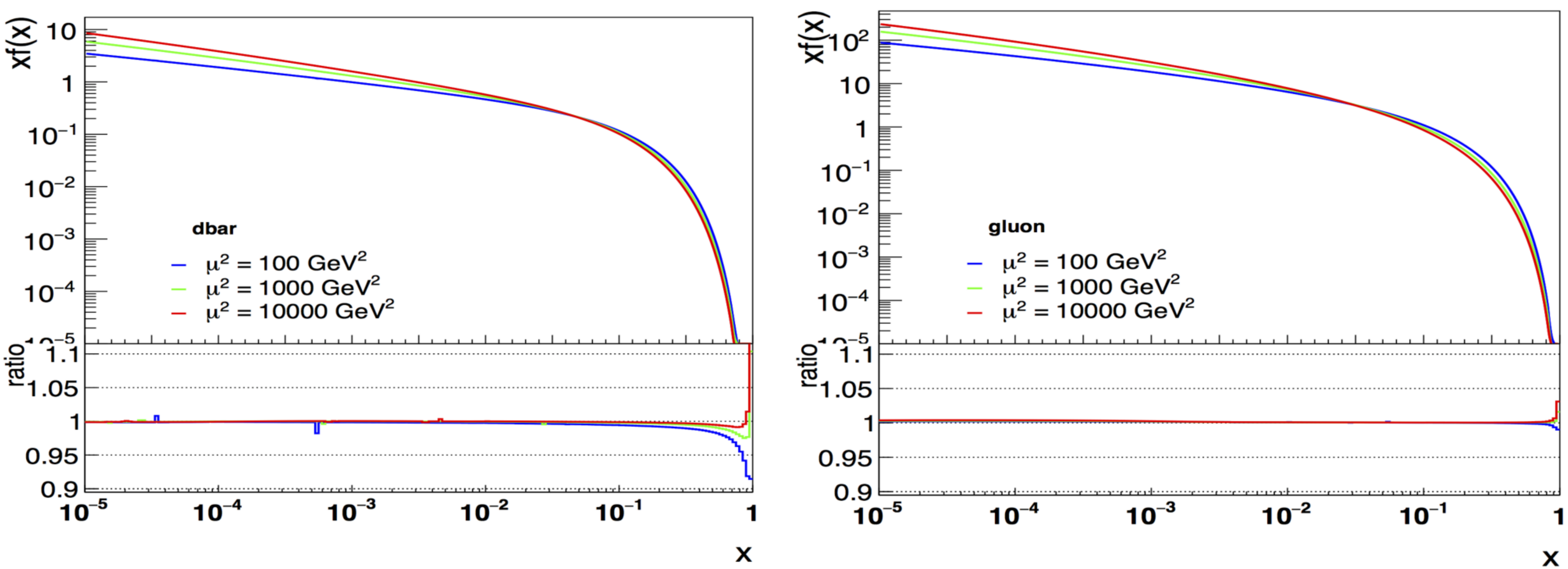
- Very good agreement with NLO - QCDnum over all x and μ^2
  - the same approach works also at NNLO !

Parton branching method in xFitter

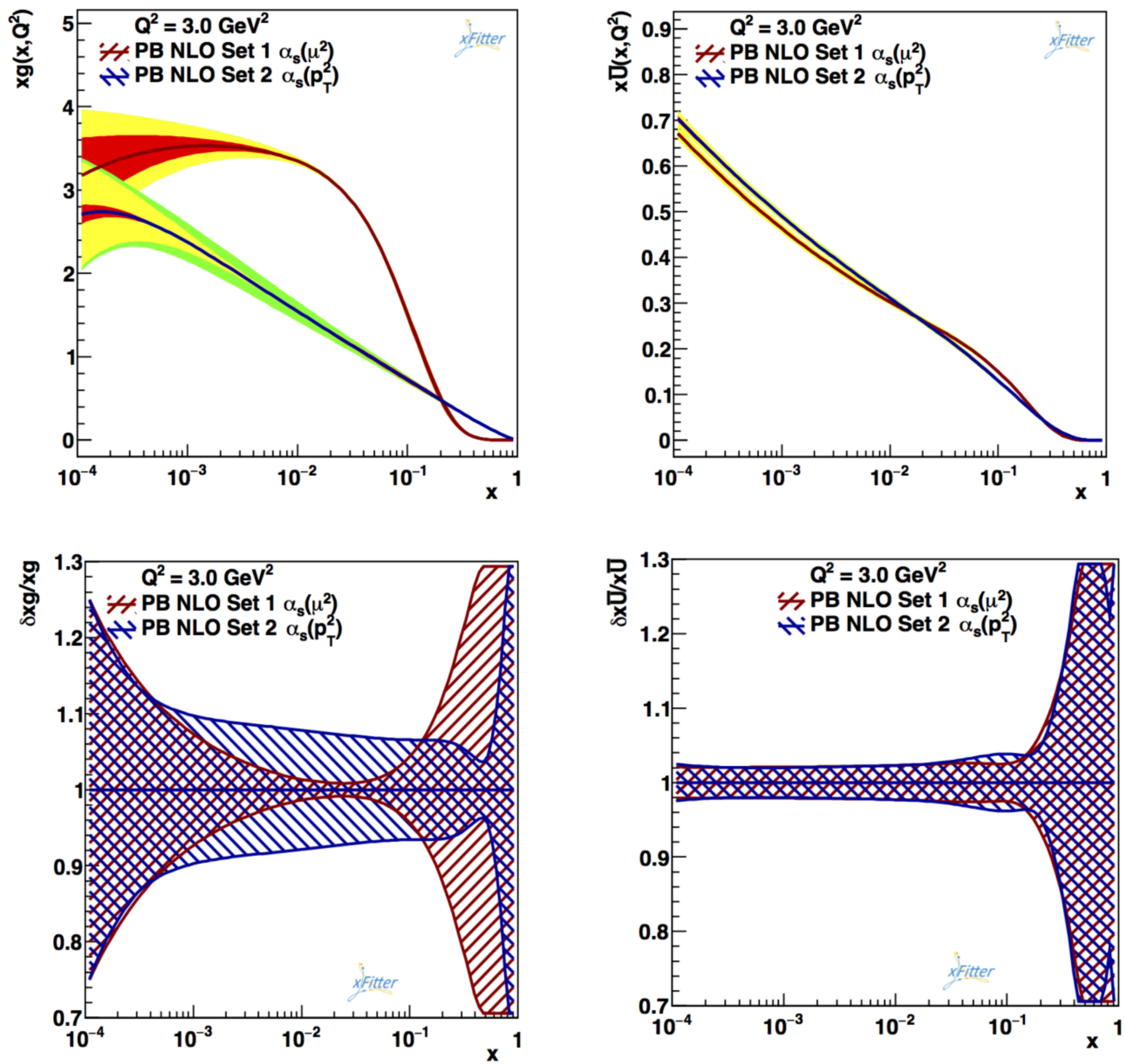
- Convolution of kernel with starting distribution

$$
\begin{aligned}
x f_a(x,\mu^2) &= x \int dx' \int dx'' \mathcal{A}_{0,b}(x') \tilde{\mathcal{A}}_a^b \left(x'', \mu^2\right) \delta(x'x'' - x) \\
&= \int dx' \mathcal{A}_{0,b}(x') \cdot \frac{x}{x'} \; \tilde{\mathcal{A}}_a^b \left(\frac{x}{x'}, \mu^2\right)
\end{aligned}
$$

  - kernel defined on grid (for integrated and TMD distribution)
  - validation of method:



Fit with different scale in α_s: at small Q^2



- fit 1 with $\alpha_s(q)$
  - as good as HERAPDF2.0 $\chi^2/ndf = 1.2$
- fit 2 with $\alpha_s(q(1-z))$
  - $\chi^2/ndf = 1.21$
- very different gluon distribution obtained at small Q^2

Fit with different scale in α_s: at large Q^2