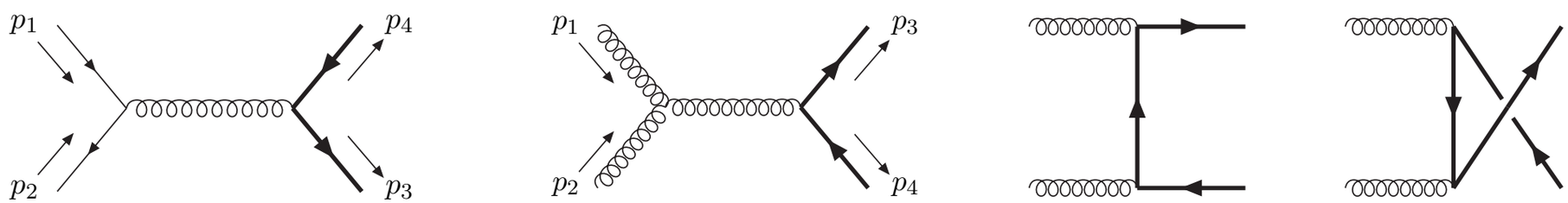
Top quark pair production at NNLO+NNLL′

Ben Pecjak
IPPP Durham

In collaboration with: M. Czakon, A. Ferroglia, D. Heymes, A. Mitov, D. J. Scott, X. Wang, and L.L.Yang

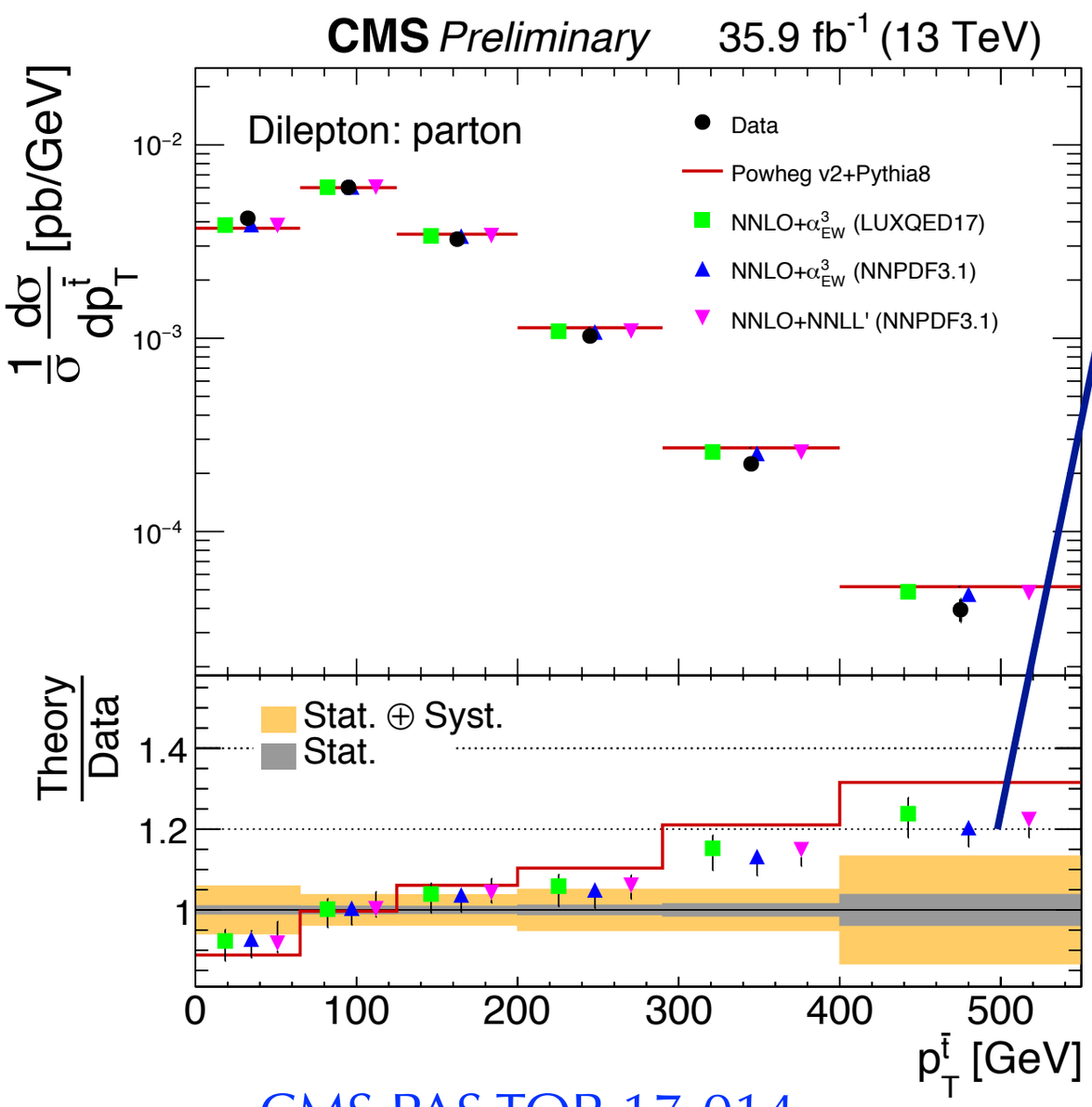
Top quark pair production



A standard candle for the LHC and future colliders

* Test of the SM at the energy frontier
* Possible signals of new physics
* Major background to many searches
* **Precise theoretical and experimental results have already enabled us to gain useful information!**

Deviation in pT spectrum?



CMS PAS TOP-17-014

Persistent shape difference in the transverse momentum spectrum

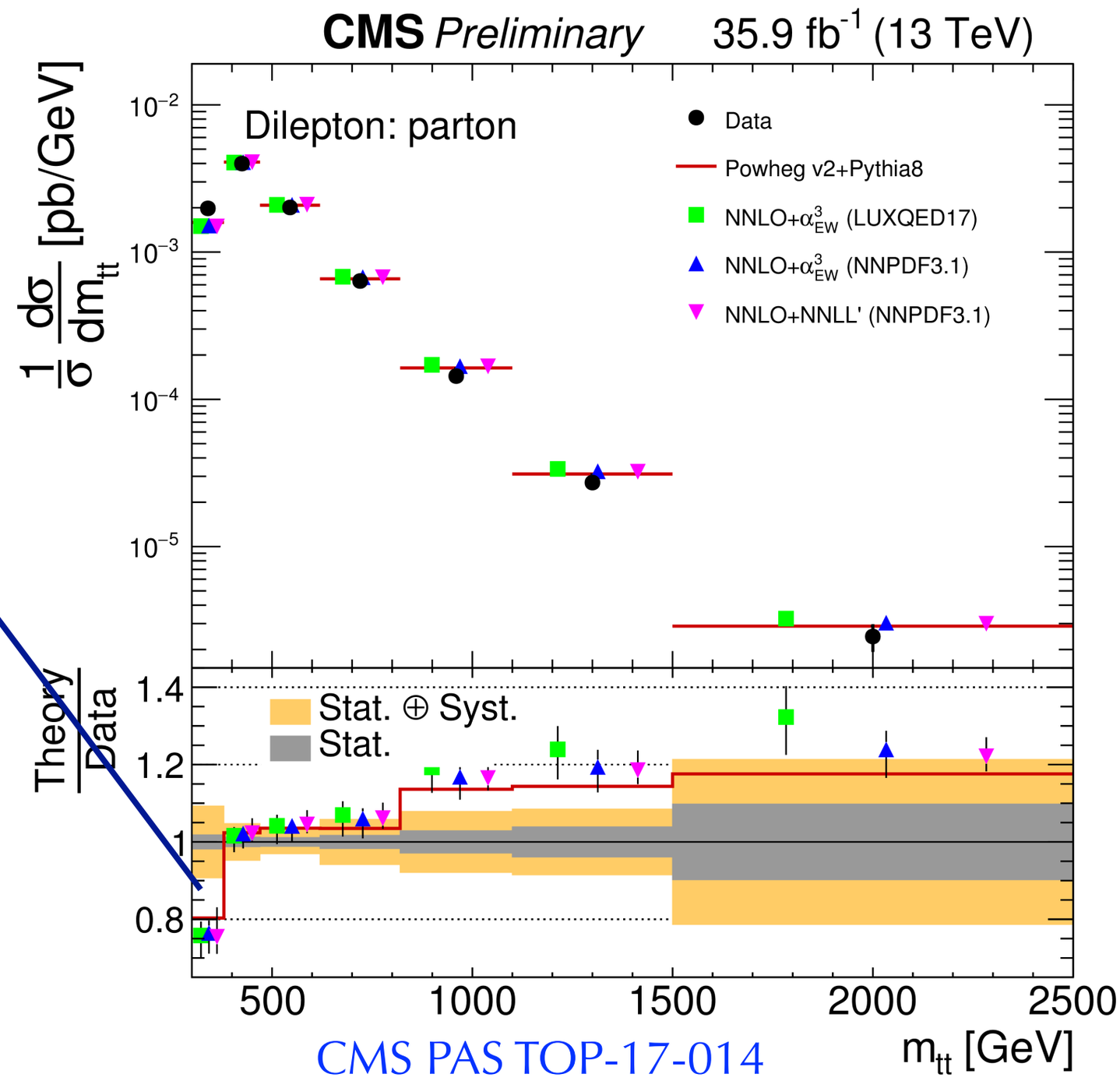
The high precision of the theoretical calculations and the experimental measurements allows to see this difference clearly!

Deviation in M_{tt} spectrum?

What's going on here?

Top quark mass?

Threshold effect?



CMS PAS TOP-17-014

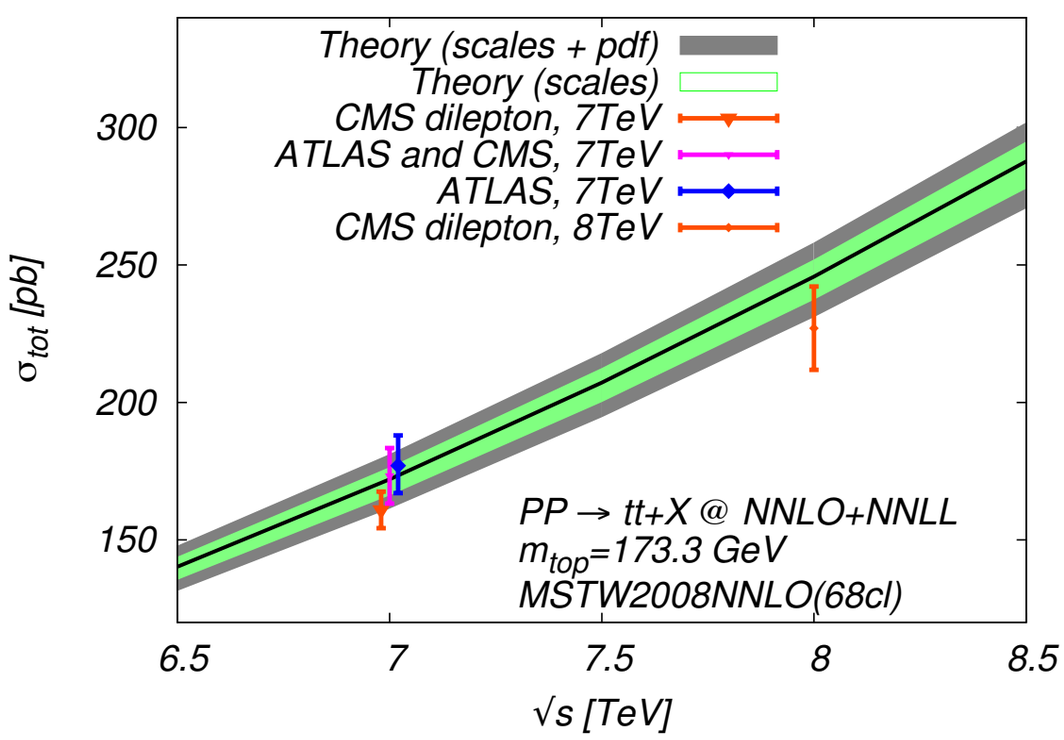
QCD for top-pair production

* Baseline is NNLO…
* and beyond NNLO resummation corrections (kinematics dependent, analytic resummation)
* **Today's Talk:** State-of-the-art for differential Mtt and pT distributions: NNLO+NNLL′: Czakon, Ferroglia, Heymes, Mitov, BP, Scott, Wang, Yang: 1803.07623

NNLO QCD for top pair

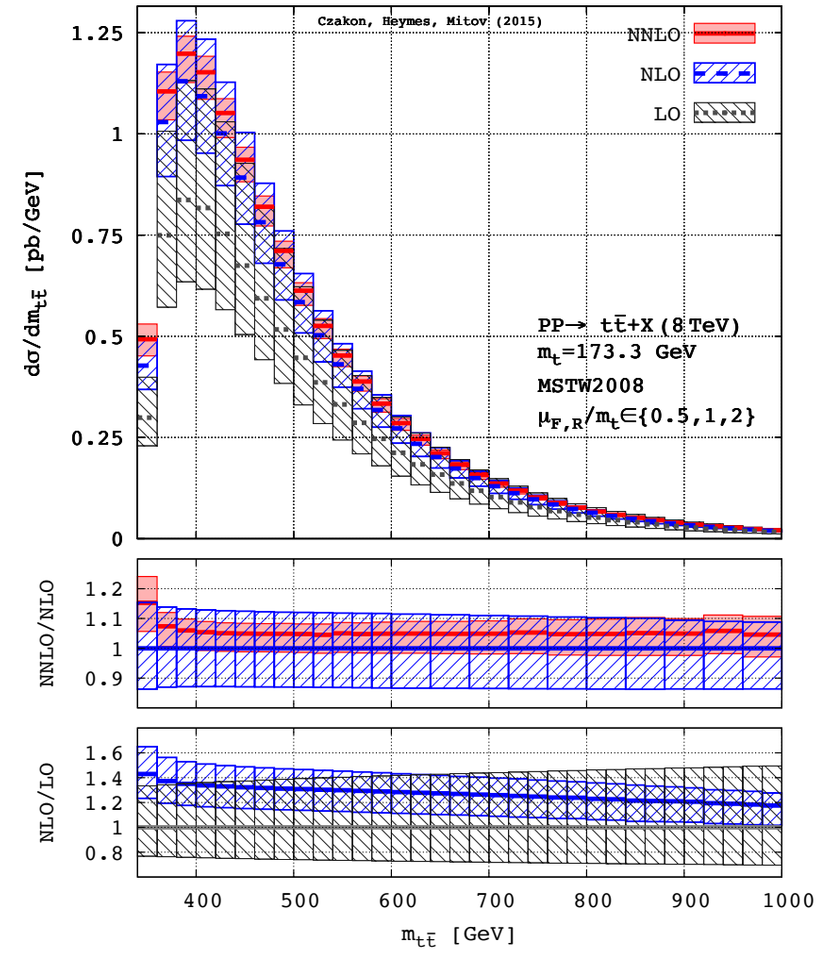
Total cross section

Baernreuther, Czakon, Mitov: 1204.5201;
Czakon, Fiedler, Mitov: 1303.6254



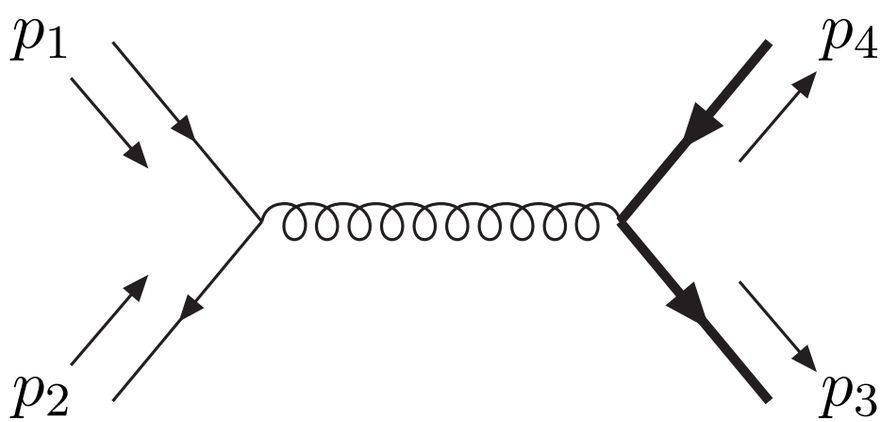
Differential distributions

Czakon, Heymes, Mitov: 1511.00549



Scale choices for distributions

The difficulty for fixed-order calculations: multiple-scale process with complicated kinematics!



Many kinematic variables:

top quark mass
p_T of top
p_T of anti-top
rapidity of top
rapidity of anti-top
Invariant mass M_{tt}
…

Which (combination) should be used for the renormalization/factorization scales?

NNLO with dynamic scales

Czakon, Heymes, Mitov: 1606.03350

Determine optimal "scale scheme" by minimizing higher order corrections

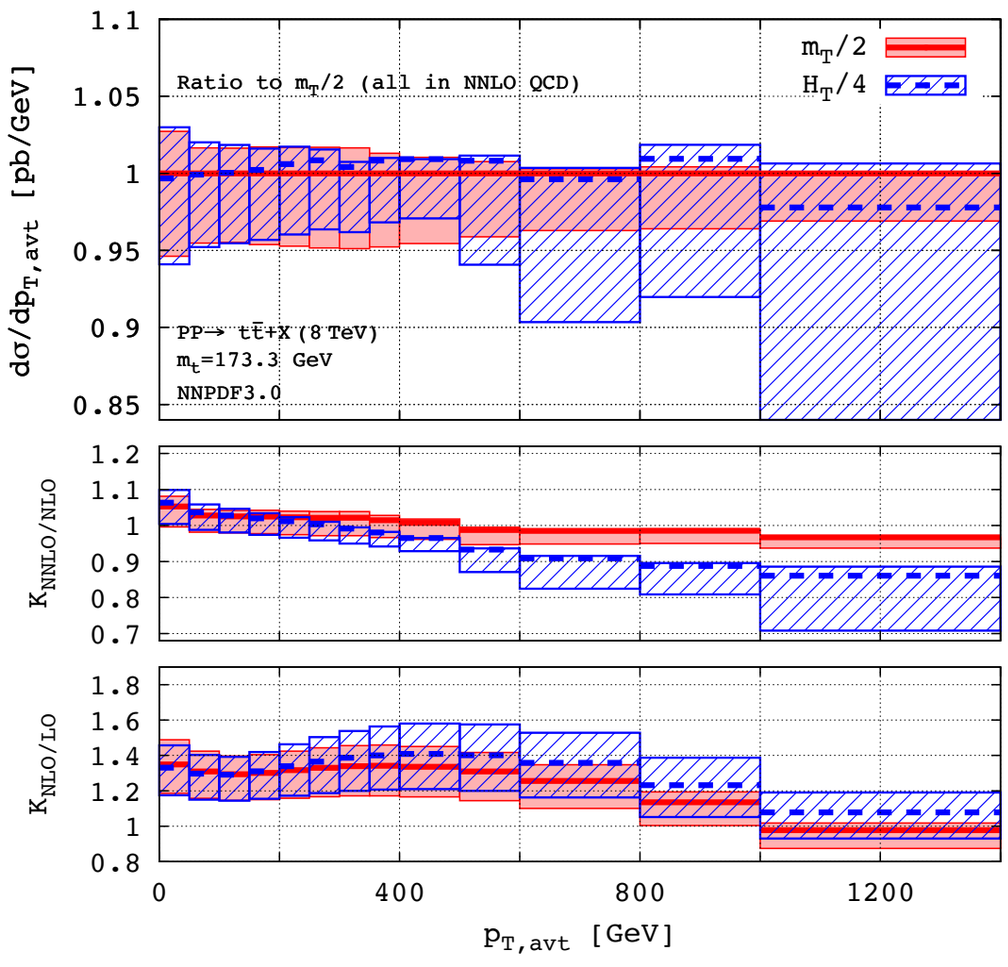
$$\mu_0 \sim m_t \,,$$
$$\mu_0 \sim m_T = \sqrt{m_t^2 + p_T^2} \,,$$
$$\mu_0 \sim H_T = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2} \,,$$
$$\mu_0 \sim H_T' = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2} + \sum_i p_{T,i} \,,$$
$$\mu_0 \sim E_T = \sqrt{\sqrt{m_t^2 + p_{T,t}^2}\sqrt{m_t^2 + p_{T,\bar{t}}^2}} \,,$$
$$\mu_0 \sim H_{T,\mathrm{int}} = \sqrt{(m_t/2)^2 + p_{T,t}^2} + \sqrt{(m_t/2)^2 + p_{T,\bar{t}}^2} \,,$$
$$\mu_0 \sim m_{t\bar{t}} \,,$$

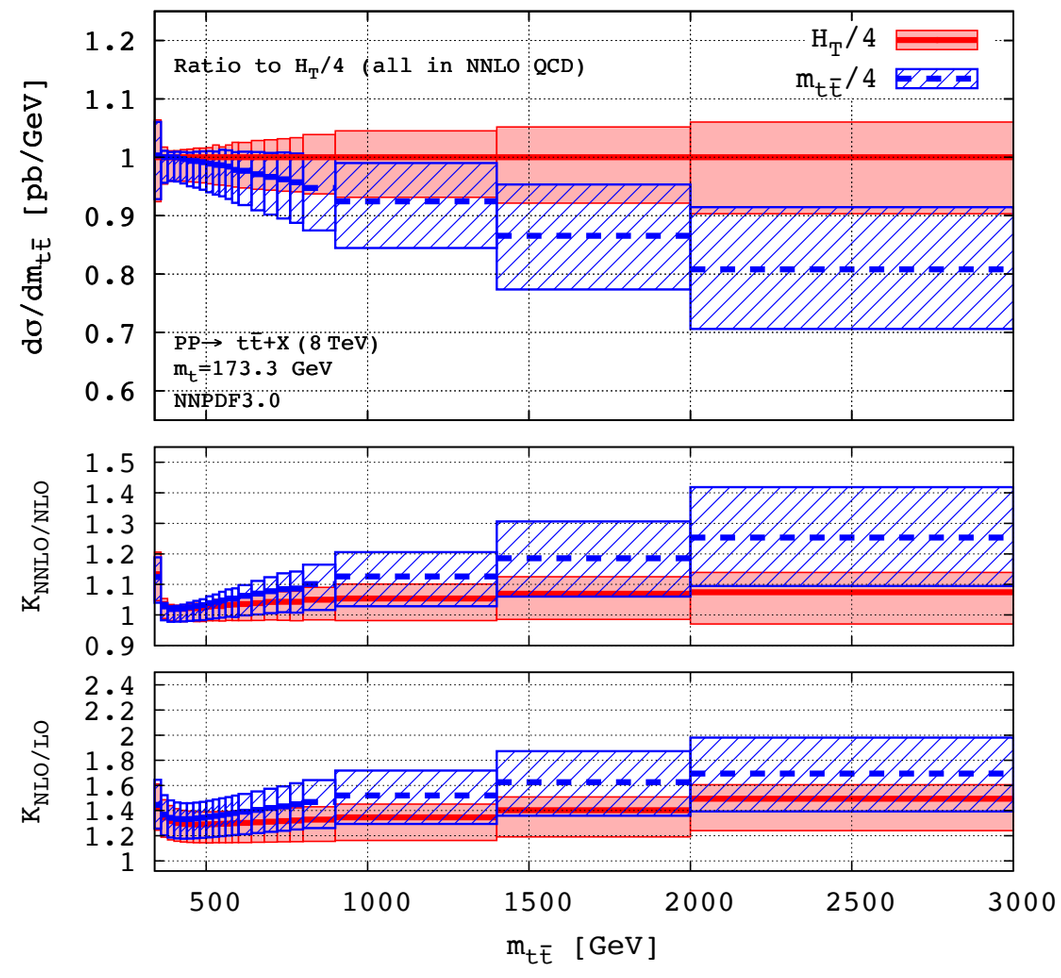
$$\mu_0 = \begin{cases} \frac{m_T}{2} & \text{for :} \quad p_{T,t},\ p_{T,\bar{t}} \text{ and } p_{T,t/\bar{t}} \,, \\ \frac{H_T}{4} & \text{for :} \quad \text{all other distributions} \,. \end{cases}$$

mu ~ H_T scale for M_{tt} distribution!

NNLO with dynamic scales

Czakon, Heymes, Mitov: 1606.03350

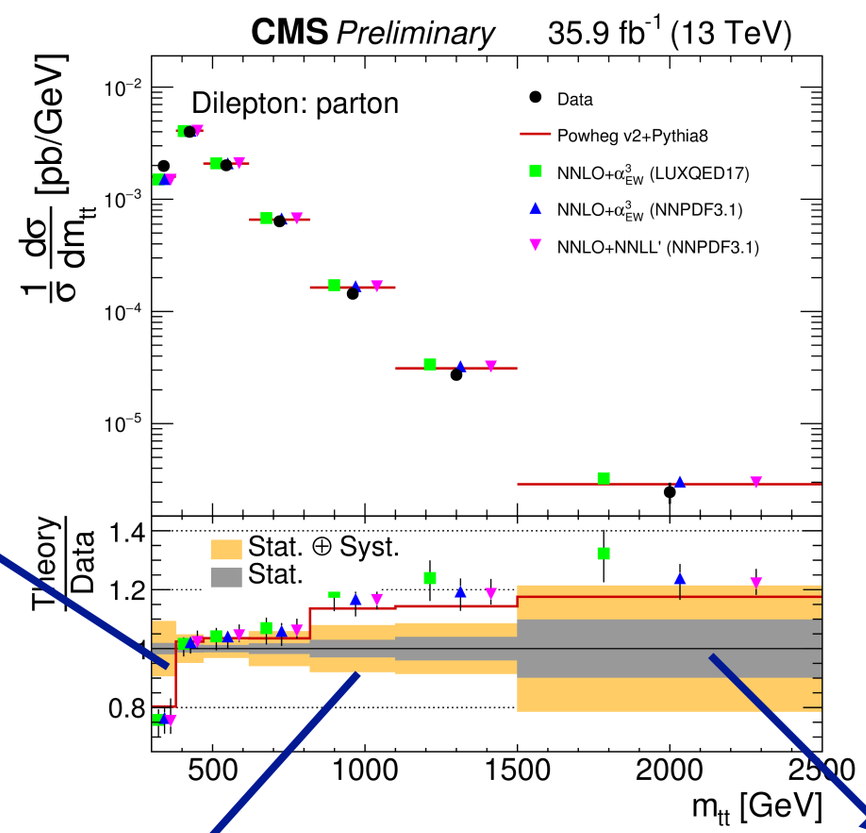




Vastly different behaviors with different scales, even at NNLO and especially for p_T , M_{tt} >> m_t (boosted regime)

Beyond NNLO: resummation

Threshold region:
$M_{tt} \sim 2m_t$
soft+Coulomb



Boosted region:
p_T, $M_{tt} >> m_t$
soft+quasi-collinear
emissions to all orders

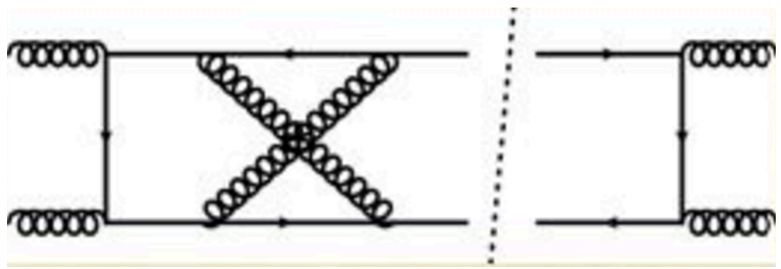
BP, Scott, Wang, Yang: 1601.07020
Czakon, Ferroglia, Heymes, Mitov, BP,
Scott, Wang, Yang : 1803.07623

Whole region:
$s \sim M_{tt}^2$
soft emissions to all orders

Ahrens, Ferroglia, Neubert,
BP, Yang: 1003.5827

Soft gluon resummation

Hard function

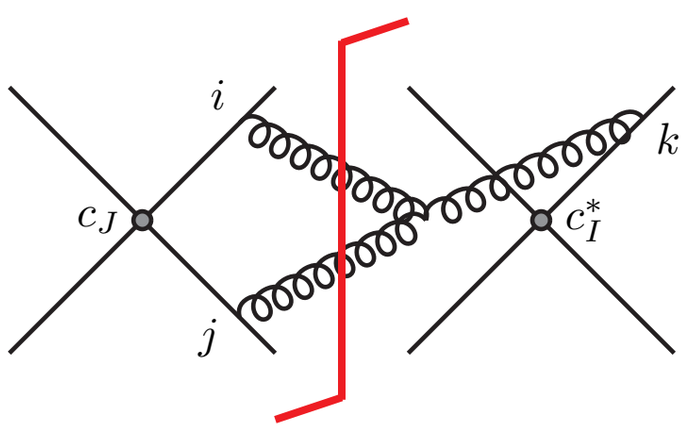


Kidonakis, Sterman: hep-ph/9705234

Ahrens, Ferroglia, Neubert,
BP, Yang: 1003.5827

RG evolution from hard to soft

Governed by IR structure



Soft function

IR anomalous dimension

Becher, Neubert: 0904.1021

Ferroglia, Neubert, BP, Yang: 0907.4791; 0908.3676

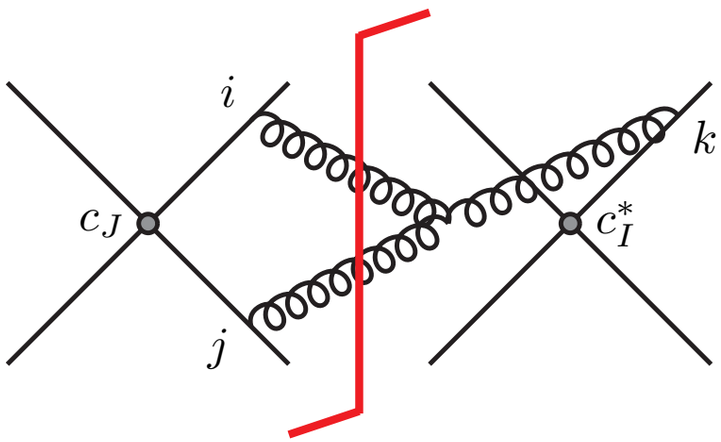
$$\begin{aligned}\boldsymbol{\Gamma} = &\sum_{(i,j)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2}\, \gamma_{\rm cusp}(\alpha_s)\, \ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) \\ &- \sum_{(I,J)} \frac{\boldsymbol{T}_I \cdot \boldsymbol{T}_J}{2}\, \gamma_{\rm cusp}(\beta_{IJ}, \alpha_s) + \sum_I \gamma^I(\alpha_s) \\ &+ \sum_{I,j} \boldsymbol{T}_I \cdot \boldsymbol{T}_j\, \gamma_{\rm cusp}(\alpha_s)\, \ln\frac{m_I \mu}{-s_{Ij}} \qquad (2 \\ &+ \sum_{(I,J,K)} i f^{abc}\, \boldsymbol{T}_I^a\, \boldsymbol{T}_J^b\, \boldsymbol{T}_K^c\, F_1(\beta_{IJ}, \beta_{JK}, \beta_{KI}) \\ &+ \sum_{(I,J)} \sum_k i f^{abc}\, \boldsymbol{T}_I^a\, \boldsymbol{T}_J^b\, \boldsymbol{T}_k^c\, f_2\Big(\beta_{IJ}, \ln\frac{-\sigma_{Jk}\, v_J \cdot p_k}{-\sigma_{Ik}\, v_I \cdot p_k}\Big)\end{aligned}$$

$$F_1(\beta_{12}, \beta_{23}, \beta_{31}) = \frac{\alpha_s^2}{12\pi^2} \sum_{i,j,k} \epsilon_{ijk}\, g(\beta_{ij})\, r(\beta_{ki})$$

$$\begin{aligned} r(\beta) &= \beta \coth\beta\,, \\ g(\beta) &= \coth\beta \left[\beta^2 + 2\beta \ln(1 - e^{-2\beta}) - {\rm Li}_2(e^{-2\beta}) + \frac{\pi^2}{6}\right] \\ &\quad - \beta^2 - \frac{\pi^2}{6}\,. \end{aligned} \qquad (5)$$

3-parton correlations

The soft function



Known at NLO

Ahrens, Ferroglia, Neubert, BP, Yang: 1003.5827

Known at NNLO in the massless limit (except an off-diagonal 3-parton piece)

Ferroglia, BP, Yang: 1207.4798

Recent calculation at NNLO with massive tops

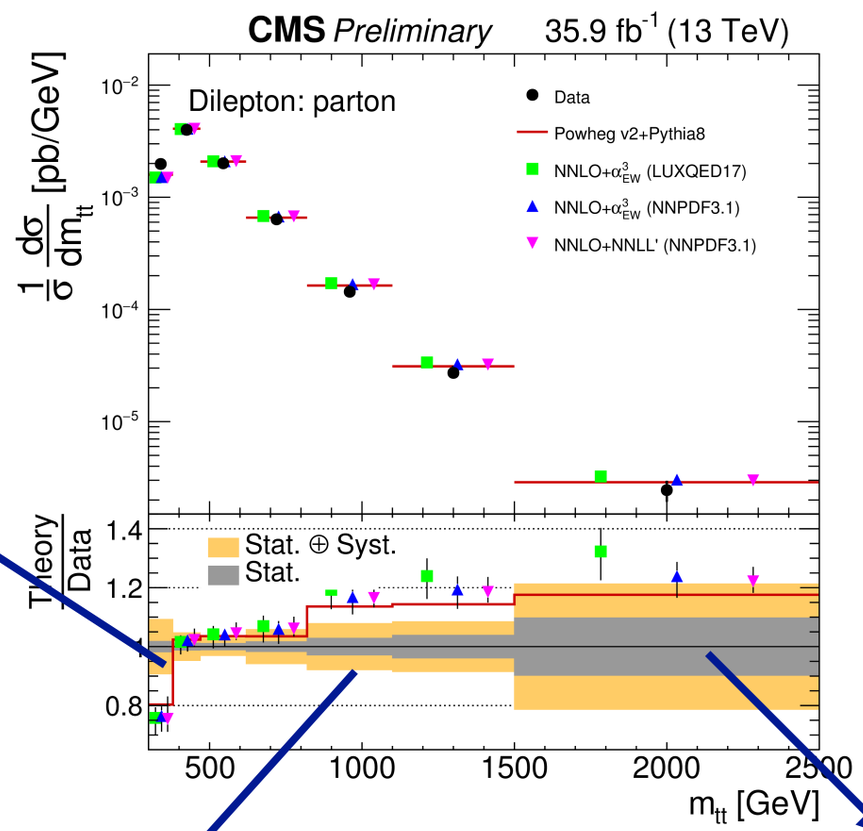
Wang, Xu, Yang , Zhu: 1804.05218

$$
\begin{aligned}
\tilde{s}_{22}^{q\bar{q},(2)}(0,\beta,y)\Big|_{T_F N_l} &= \frac{16(7\beta^2-126\beta+127)}{243\beta}G_1 + \frac{8(5\beta^2+90\beta+53)}{81\beta}\big(G_{-1,-1}-G_{-1,1}-2G_{0,-1}\big) \\
&- \frac{16(7\beta^2+126\beta+127)}{243\beta}G_{-1} + \frac{8(5\beta^2-90\beta+53)}{81\beta}\big(G_{1,-1}-G_{1,1}+2G_{0,1}\big) \\
&+ \frac{8(\beta^2+18\beta+1)}{27\beta}\big(-G_{-1,-1,-1}+G_{-1,-1,1}+2G_{-1,0,-1}-2G_{-1,0,1}-G_{-1,1,-1}+G_{-1,1,1} \\
&+ 2G_{0,-1,-1}-2G_{0,-1,1}-4G_{0,0,-1}\big) + \frac{8(\beta^2-18\beta+1)}{27\beta}\big(4G_{0,0,1}+2G_{0,1,-1}-2G_{0,1,1} \\
&- G_{1,-1,-1}+G_{1,-1,1}+2G_{1,0,-1}-2G_{1,0,1}-G_{1,1,-1}+G_{1,1,1}\big) \\
&+ \frac{32}{243}\Big[28G_{-1/y}+98G_{1/y}+30\big(2G_{0,-1/y}+G_{-1/y,-1}+G_{-1/y,1}-2G_{-1/y,-1/y}\big) \\
&+ 105\big(2G_{0,1/y}+G_{1/y,-1}+G_{1/y,1}-2G_{1/y,1/y}\big)+18\big(4G_{0,0,-1/y}+2G_{0,-1/y,-1}+2G_{0,-1/y,1} \\
&- 4G_{0,-1/y,-1/y}-G_{-1/y,-1,-1}+G_{-1/y,-1,1}+2G_{-1/y,0,-1}+2G_{-1/y,0,1}-4G_{-1/y,0,-1/y} \\
&+ G_{-1/y,1,-1}-G_{-1/y,1,1}-2G_{-1/y,-1/y,-1}-2G_{-1/y,-1/y,1}+4G_{-1/y,-1/y,-1/y}\big) \\
&+ 63\big(4G_{0,0,1/y}+2G_{0,1/y,-1}+2G_{0,1/y,1}-4G_{0,1/y,1/y}-G_{1/y,-1,-1}+G_{1/y,-1,1}+2G_{1/y,0,-1} \\
&+ 2G_{1/y,0,1}-4G_{1/y,0,1/y}+G_{1/y,1,-1}-G_{1/y,1,1}-2G_{1/y,1/y,-1}-2G_{1/y,1/y,1}+4G_{1/y,1/y,1/y}\big) \\
&- \frac{332}{3}-\frac{5\pi^2}{2}+6\zeta_3\Big], \qquad (84)
\end{aligned}
$$

Fully analytic in terms of MPLs; fast numerics

Beyond NNLO: resummation

Threshold region:
$M_{tt} \sim 2m_t$
soft+Coulomb?



Boosted region:
p_T, $M_{tt} >> m_t$
soft+quasi-collinear
emissions to all orders

Whole region:
soft emissions to all orders

Ahrens, Ferroglia, Neubert,
BP, Yang: 1003.5827

BP, Scott, Wang, Yang: 1601.07020
Czakon, Ferroglia, Heymes, Mitov, BP,
Scott, Wang, Yang : 1803.07623

Boosted top production

Hard extra emissions suppressed

Top quark nearly massless

soft gluons

quasi-collinear gluons

$$\ln \frac{\hat{s} - M_{t\bar{t}}^2}{M_{t\bar{t}}^2}$$

Need to resum both!

Ferroglia, BP, Yang : 1205.3662

$$\ln \frac{m_t^2}{M_{t\bar{t}}^2}$$

Soft and small-mass factorization

Ferroglia, BP, Yang: 1205.3662

In Mellin space: $Q \sim \sqrt{s}, \sqrt{-t} \gg Q/N \gg m_t \gg m_t/N$

$$\hat{\sigma}(N,\mu_f) \sim \mathrm{Tr}\big[\boldsymbol{H}(L_h,\mu_f)\,\boldsymbol{S}(L_s,\mu_f)\big]\,C_D^2(L_c,\mu_f)\,S_D^2(L_{sc},\mu_f)$$

$\ln \dfrac{Q^2}{\mu_f^2}$

hard log

$\ln \dfrac{Q^2}{\bar{N}^2\mu_f^2}$

soft log

$\ln \dfrac{m_t^2}{\mu_f^2}$

collinear log
(small-mass)

$\ln \dfrac{m_t^2}{\bar{N}^2\mu_f^2}$

soft-collinear log
(emergent)

Soft and small-mass resummation

Massless hard function

$$\boldsymbol{H}(L_h, \mu_h \sim Q)$$

Massless soft function

$$\boldsymbol{S}(L_s, \mu_s \sim Q/\bar{N})$$

$$\mu_f$$

RG flow

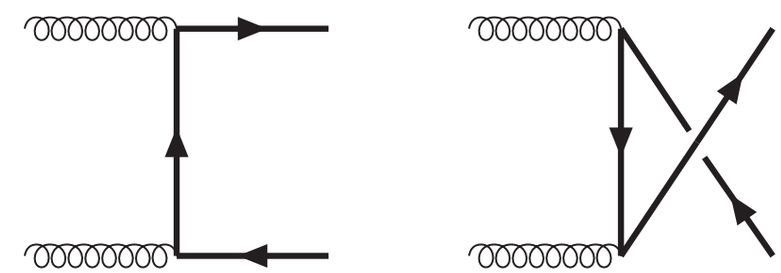
$$C_D(L_c, \mu_c \sim m_t)$$

$$S_D(L_{sc}, \mu_{sc} \sim m_t/\bar{N})$$

All ingredients known at NNLO (for NNLL′ resummation)

H_T and the large-M_{tt} region

In the boosted limit, t- and u-channel propagators push the effective hard scale to



$$-t_1\big|_{m_t\to 0} \approx \frac{M_{t\bar{t}}^2}{2}(1-\cos\theta) + m_t^2\cos\theta \xrightarrow{\cos\theta\to 1} p_T^2 + m_t^2 \equiv m_T^2 = H_T^2/4\,,$$

$$-u_1\big|_{m_t\to 0} \approx \frac{M_{t\bar{t}}^2}{2}(1+\cos\theta) - m_t^2\cos\theta \xrightarrow{\cos\theta\to -1} m_T^2 = H_T^2/4\,.$$

$$\frac{\mathcal{H}_{gg}^{\rm NLO}(\mu_h)}{\mathcal{H}_{gg}^{\rm LO}(\mu_h)}\bigg|_{t_1\to 0} = 1 + \frac{\alpha_s(\mu_h)}{36\pi}\left[-78\ln^2\left(\frac{-t_1}{\mu_h^2}\right) + 24\ln\left(\frac{-t_1}{\mu_h^2}\right)(3+2\ln x_t) + 37\pi^2 - 168\right]$$

➡ H_T is emergent scale in large Mtt region!

supports **fixed-order** scale choice Czakon, Heymes, Mitov: 1606.03350

Matching with NNLO

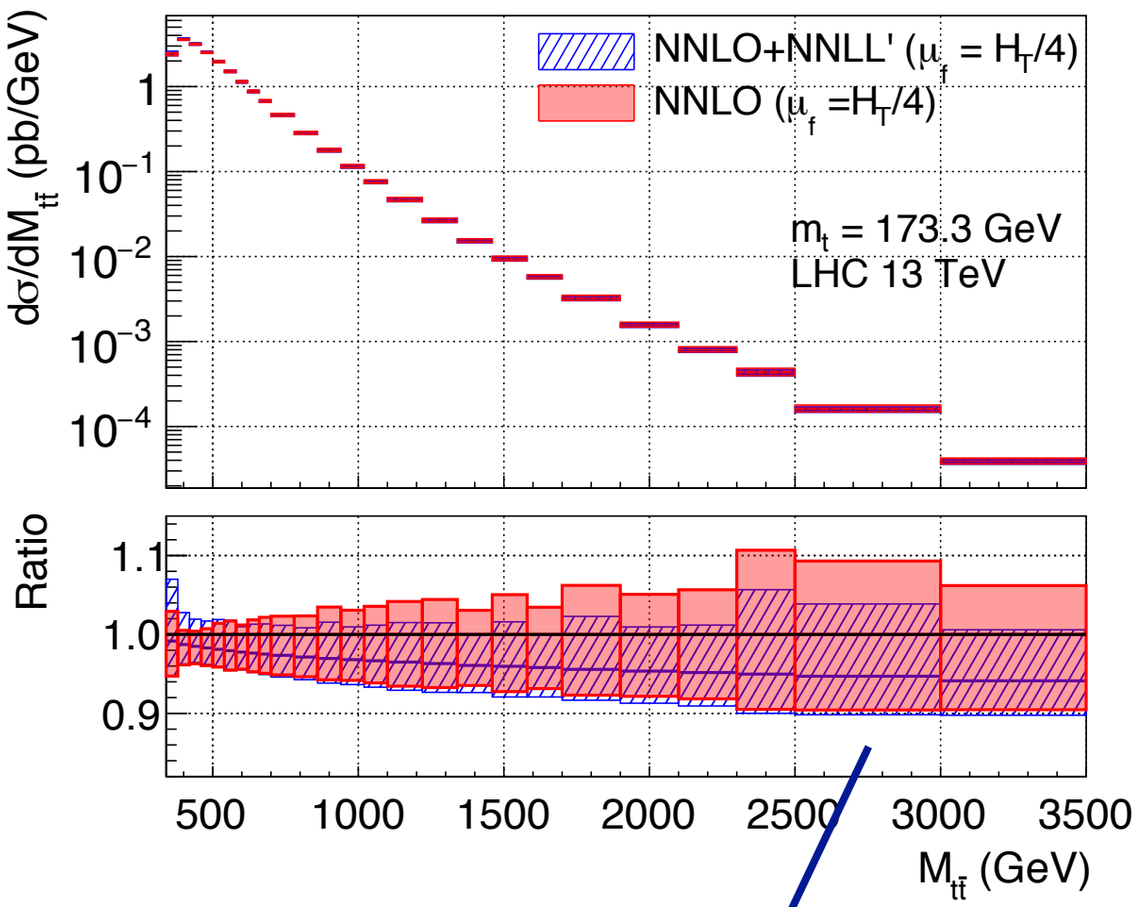
We have 3 different calculations

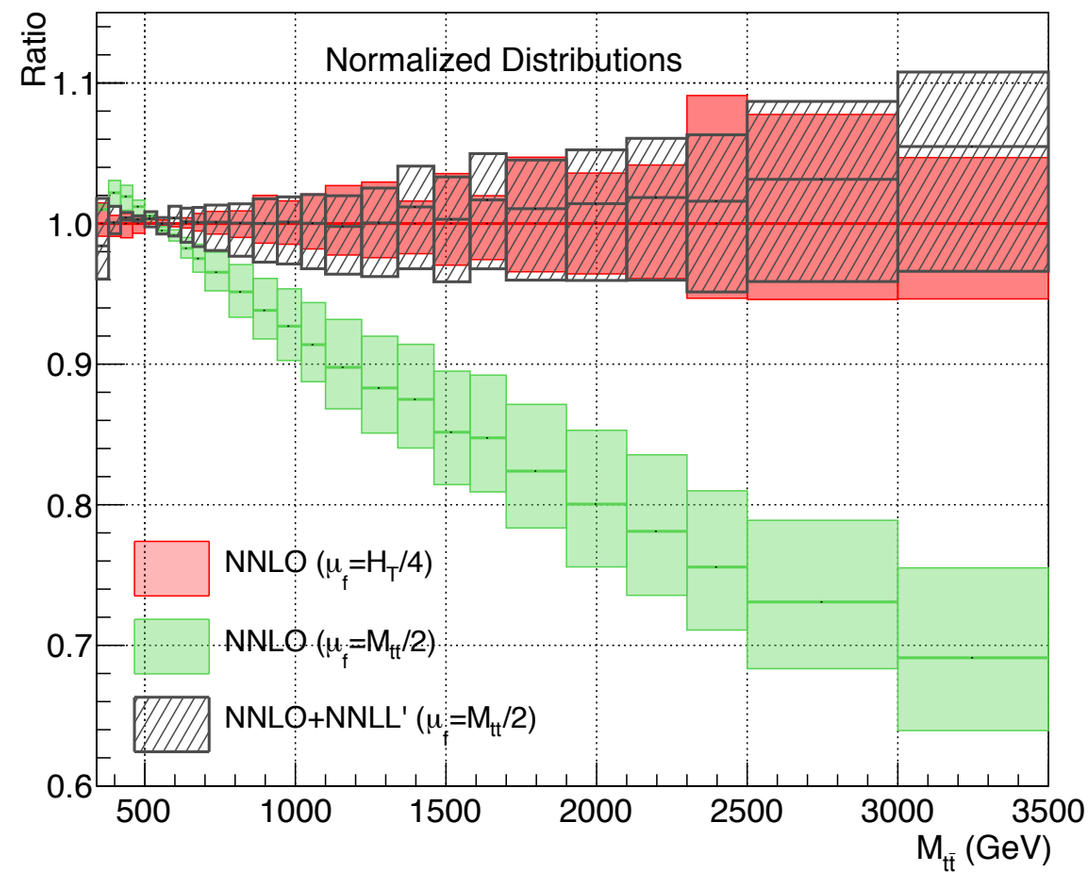
	NNLO	Soft resummation	Soft+small-mass resummation
✓	$\alpha_s^n (n \leq 2)$	$\alpha_s^n \ln^m N$	$\alpha_s^n \ln^m N$; $\alpha_s^n \ln^m (m_t/M_{t\bar{t}})$
✗	$\alpha_s^n (n > 2)$	1/N suppressed terms	1/N suppressed terms; m_t/M_{tt} suppressed terms

Need to remove double- and triple-counting when combining them!

NNLO+NNLL′ for M_{tt} spectrum

Czakon, Ferroglia, Heymes, Mitov, BP, Scott, Wang, Yang: 1803.07623



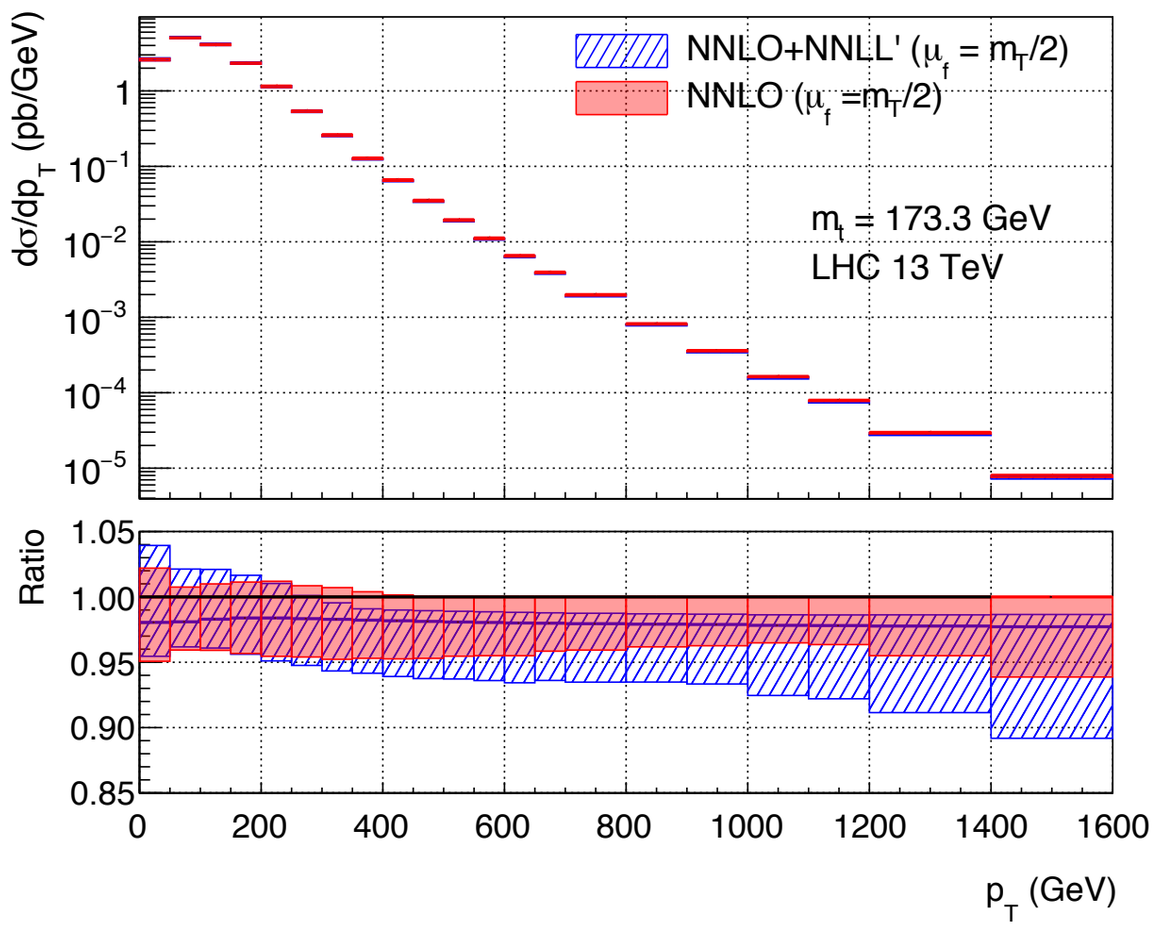


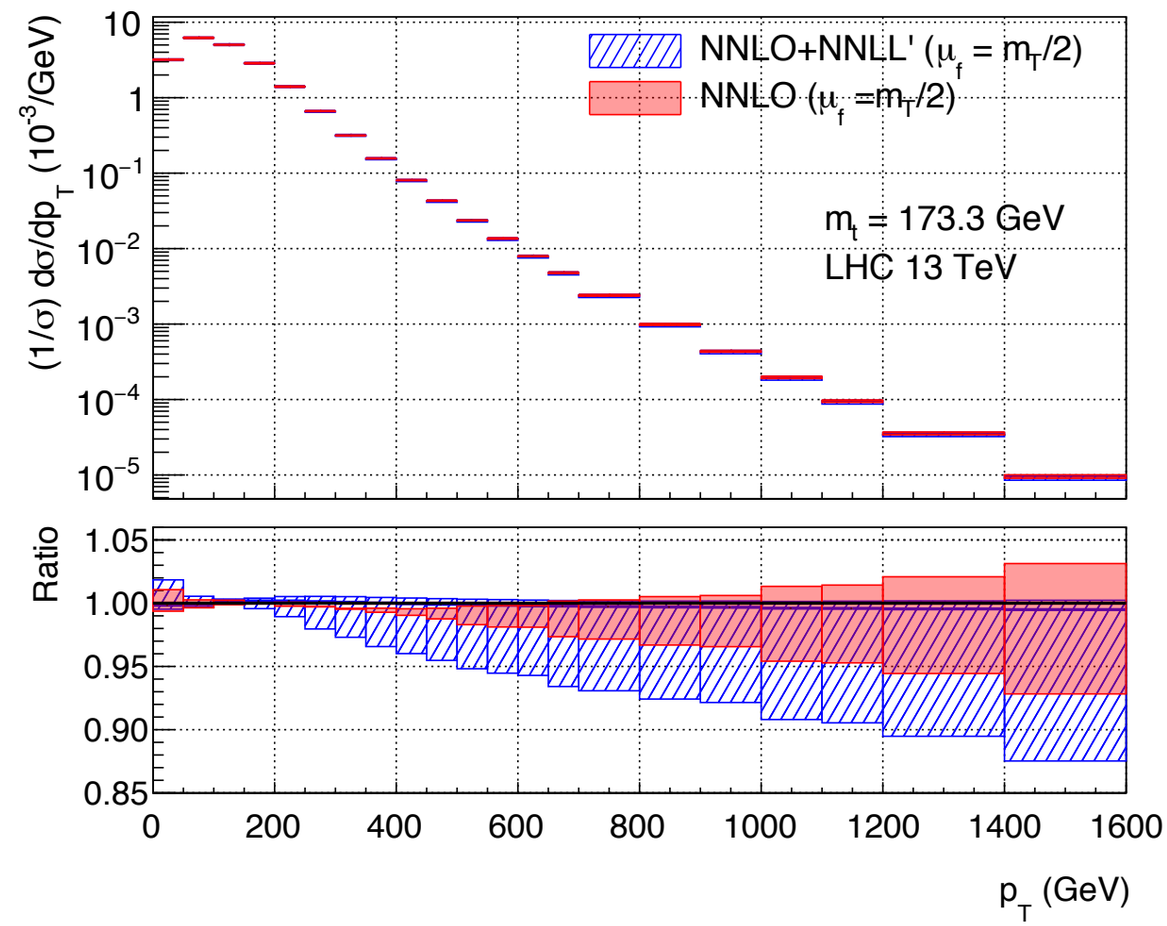
NNLL′ slightly reduces scale uncertainty for mu ~ HT

NNLL′ eliminates large difference between mu ~ HT vs. mu ~ Mtt

NNLO+NNLL′ for p_T spectrum

Czakon, Ferroglia, Heymes, Mitov, BP, Scott, Wang, Yang: 1803.07623



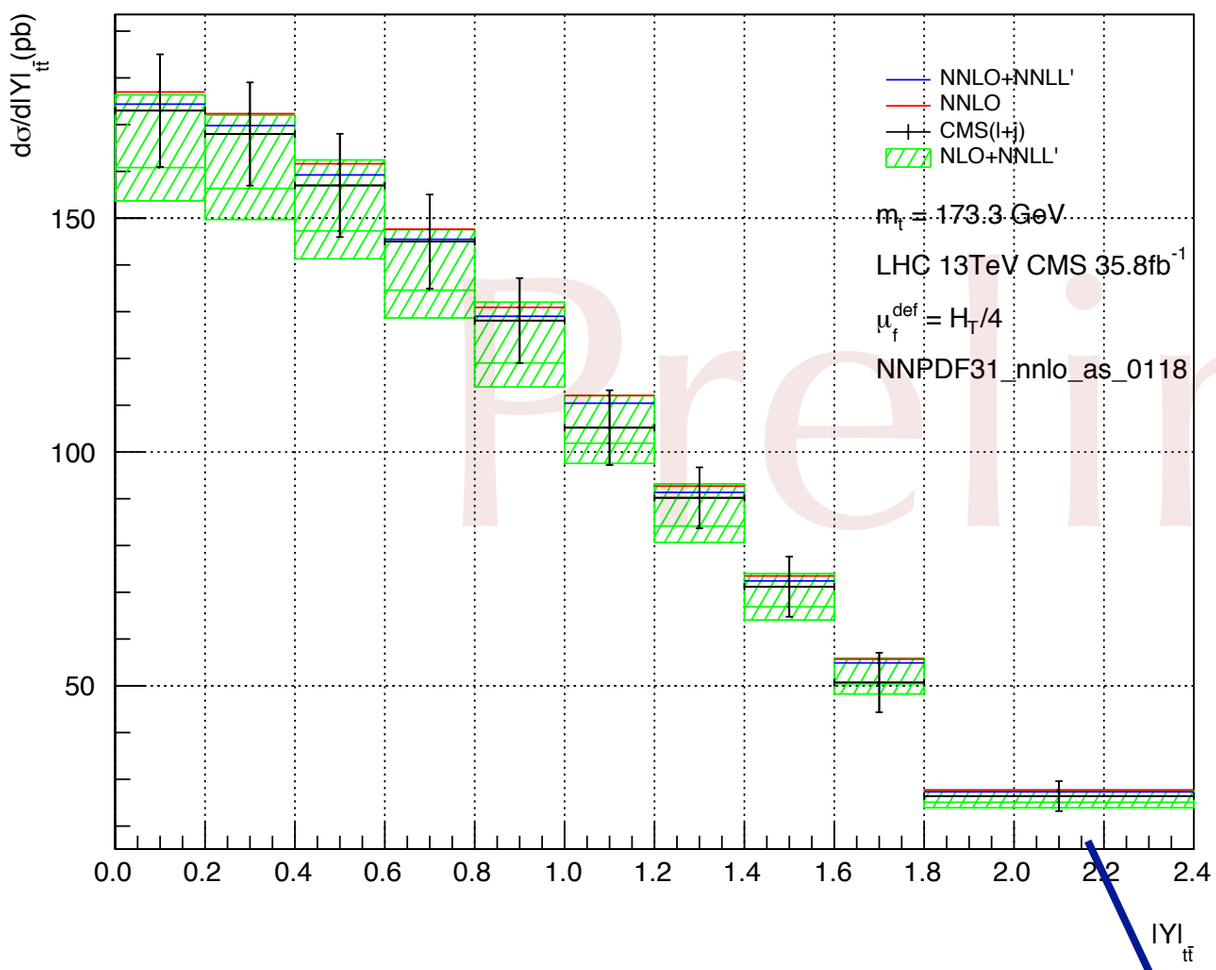


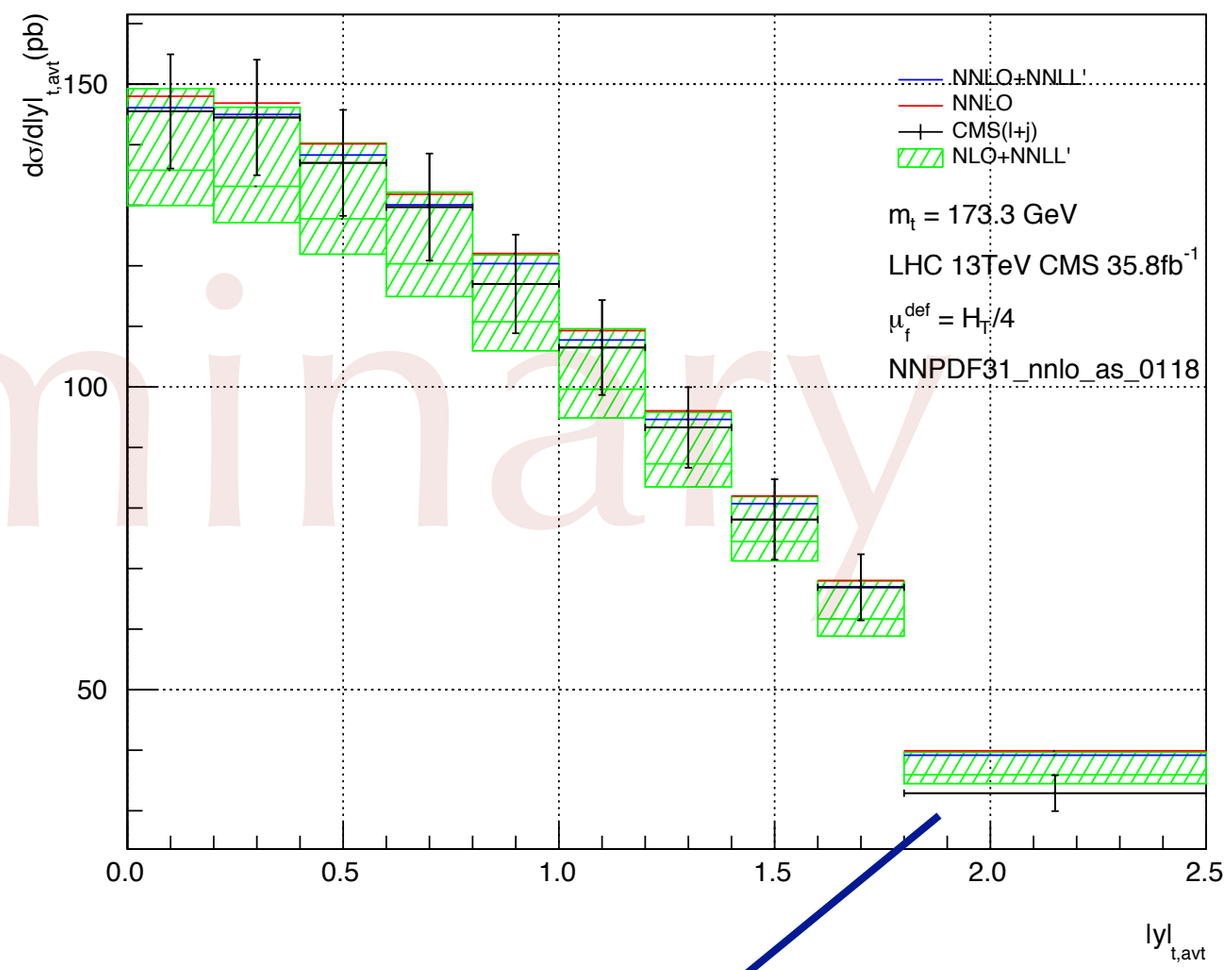
NNLL′ softens **absolute pT spectrum**

NNLL′ softens **normalised pT spectrum**

Ongoing: rapidity distributions

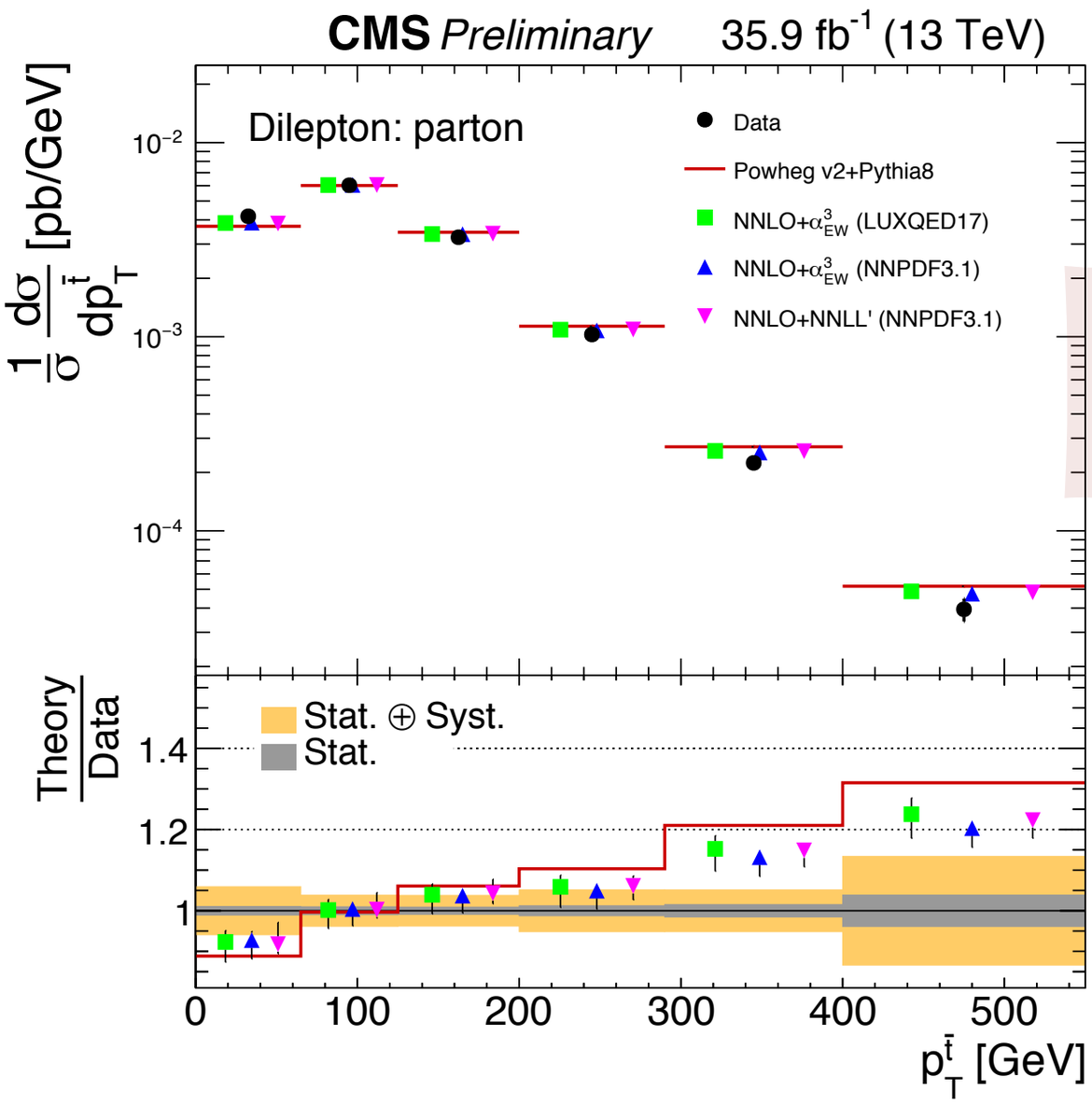
BP, Scott, Wang, Yang: to appear

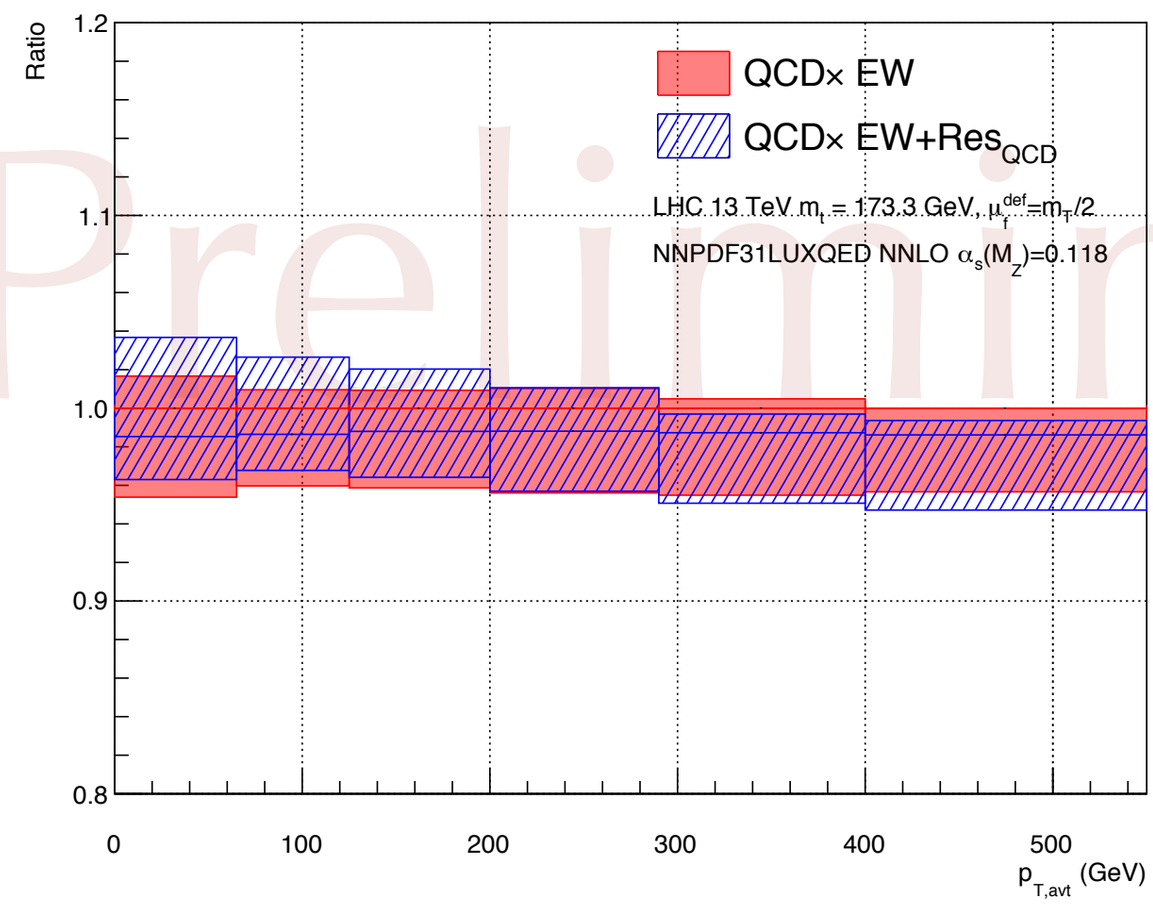




Sensitive to gluon PDF at large x

Ongoing: combination with electroweak corrections





Summary and outlook

- Top quark pair production is important
- The most precise QCD calculation: NNLO+NNLL′
- Ongoing:
  - Rapidity distributions
  - Combination with NLO electroweak corrections
- Outlook:
  - Combination with Coulomb resummation (first bin)
  - Top quark decays