On the one-loop calculations of multiscale quantities in Lipatov's EFT

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Outline.

1. Motivation
2. Introduction to Lipatov's EFT
3. One-loop rapidity-divergent integrals
4. Comparison with QCD

Motivation

- The gauge-invariant EFT for Multi-Regge processes in QCD, which includes *Reggeized gluons* [Lipatov; 1995] and *Reggeized quarks* [Lipatov, Vyazovsky; 2001] has been introduced as a systematic tool to *compute and resum* the higher-order corrections in QCD, enhanced by $\log(s/(-t))$, with the arbitrary N^kLL accuracy.
- Another motivation is the *unitarization program* for high-energy scattering. The BFKL equation at the fixed logarithmic accuracy predicts *power-like* growth of the cross-section with s, which violates Froissart bound ($\Leftarrow$ Unitarity). The basic idea is to write-down the *Hermitian* effective Lagrangian for QCD at high energies, so that Unitarity will hold automatically.

Motivation

- Currently, a number of approaches is developed with the aim of taking into account both DGLAP and BFKL effects. Many of them try to generalize the amplitudes from the Lipatov's EFT to the Soft and Collinear regions (e.g. PRA [M.N., V.A.S., *et. al.*] or HEJ [J. Andersen, *et. al.*] approaches, `KaTie` [A. van Hameren, *et. al.*] Monte-Carlo code) or incorporate BFKL effects into the framework of SCET (e.g. [I. Stewart, I. Rothstein, 2016]). Going beyond tree level is an important part of this activity.
- In the talk I would like to describe the one-loop structiure of Lipatov's EFT. The complete picture, similar to one in ordinary QCD, emerges.

Introduction to Lipatov's EFT

Sudakov (light-cone) decomposition of momenta.

It is convenient to relate the basis vectors of Sudakov decomposition with (almost) light-like momenta of colliding highly energetic particles ($P_{1,2}^2 = 0$):

$$n_-^\mu = \frac{2P_1^\mu}{\sqrt{S}},\ n_+^\mu = \frac{2P_2^\mu}{\sqrt{S}},\ S = 2P_1P_2 \Rightarrow n_+n_- = 2.$$

Then for any four-vector k^μ one has:

$$\boxed{k^\mu = \frac{1}{2}\left(k_+n_-^\mu + k_-n_+^\mu\right) + k_T^\mu,}$$

where $k_\pm = k^\pm = n_\pm k$, $n_\pm k_T = 0$. For the dot-product one has:

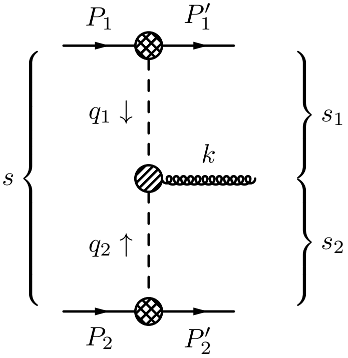
$$kq = \frac{1}{2}\left(k_+q_- + k_-q_+\right) - \mathbf{k}_T\mathbf{q}_T,\ \boxed{k^2 = k_+k_- - \mathbf{k}_T^2.}$$

Rapidity:

$$\boxed{\boxed{y = \frac{1}{2}\log\left(\frac{q^+}{q^-}\right).}}$$

Multi-Regge Kinematics.

At high energies, t-channel exchange diagrams with Multi-Regge(MRK) or Quasi-Multi Regge(QMRK) Kinematics of the final-state dominate in the $2 \to 2 + n$ amplitude.



Double Regge limit (MRK):

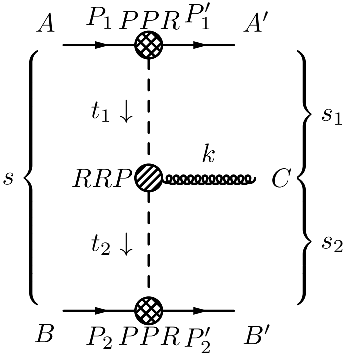
$$s_1 \gg -q_1^2, \; s_2 \gg -q_2^2,$$

momentum fractions $z_1 = q_1^+/P_1^+$, $z_2 = q_2^-/P_2^-$.

Properties of MRK:

- $y(P_1') \to +\infty$, $y(P_2') \to -\infty$, $y(k)$ – finite,
- $\boxed{z_1 \sim z_2 \sim z \ll 1}$, $|\mathbf{k}_T| \ll \sqrt{s}$,
- $q_1^+ \sim |\mathbf{q}_{T1}| \sim O(z) \gg q_1^- \sim O(z^2)$, $q_2^- \sim |\mathbf{q}_{T2}| \sim O(z) \gg q_2^+ \sim O(z^2)$.

Reggeization of amplitudes in QCD.



In MRK asymptotics, $2 \to 3$-amplitude factorizes (up to $O(z^{\#}_{1,2})$):

$$\mathcal{A}^{A'B'C}_{AB} = \gamma^{R_1}_{A'A} \cdot \left(\frac{s_1}{s_0}\right)^{\omega(t_1)} \frac{-i}{2t_1} \times$$

$$\Gamma^{C}_{R_1 R_2}(q_1, q_2) \cdot \frac{-i}{2t_2} \left(\frac{s_2}{s_0}\right)^{\omega(t_2)} \cdot \gamma^{R_2}_{B'B}$$

$\Gamma^{C}_{R_1 R_2}(q_1, q_2)$ – RRP production vertex,

$\gamma^{R}_{A'A}$ – PPR-scattering vertex,

$\omega(t)$ - Regge trajectory.

Two ways to obtain this asymptotics:

- BFKL-approach (Unitarity, renormalizability and gauge invariance), see. [Ioffe, Fadin, Lipatov, 2010].
- Effective action approach [Lipatov, 1995; Lipatov, Vyazovsky, 2001].

Structure of the EFT.

Light-cone derivatives:

$$\partial_\pm = n_\pm^\mu \partial_\mu = 2\frac{\partial}{\partial x^\mp}$$

EFT Lagrangian [Lipatov, 1995]:

$$L = L_{\rm kin} + \sum_i \left[L_{QCD}^{(y_i \leq y \leq y_{i+1})} + L_R^{(y_i \leq y \leq y_{i+1})} \right],$$

the separate copy of $L_{QCD}^{(y_i \leq y \leq y_{i+1})}$ lives in each interval in rapidity $y_i \leq y \leq y_{i+1}$. Different intervals interact via Reggeon exchanges ($R_\pm^a = R_\pm^a T_a$):

$$L_{\rm kin} = 2\partial_\mu R_+^a \partial^\mu R_-^a,$$

kinematic constraints on Reggeon-fields ($\Leftrightarrow$ QMRK):

$$\partial_- R_+ = \partial_+ R_- = 0 \Rightarrow$$

$$R_+ \text{ carries } (k_+, \mathbf{k}_T) \text{ and } R_- \text{ carries } (k_-, \mathbf{k}_T).$$

(Semi-)Infinite light-like Wilson lines

Particles highly separated in rapidity "perceive" each-other as light-like Wilson lines.

$$W_{x^\mp}[A_\pm] = \quad P \exp\left[\frac{-ig_s}{2}\int\limits_{-\infty}^{x_\mp} dx'_\mp A_\pm(x_\pm, x'_\mp, \mathbf{x}_T)\right] = \left(1 + ig_s\partial_\pm^{-1}A_\pm\right)^{-1},$$

$$W^\dagger_{x^\mp}[A_\pm] = \quad \overline{P} \exp\left[\frac{ig_s}{2}\int\limits_{-\infty}^{x_\mp} dx'_\mp A_\pm(x_\pm, x'_\mp, \mathbf{x}_T)\right] = \overline{P}\left(1 - ig_s\partial_\pm^{-1}A_\pm\right)^{-1},$$

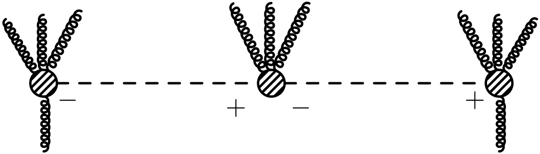
Notation for ordered integrals:

$$\frac{1}{2^n}\int\limits_{-\infty}^{x^\mp} dx_1^\mp f_1(x_1^\mp)\int\limits_{-\infty}^{x_1^\mp} dx_2^\mp f_2(x_2^\mp)\ldots\int\limits_{-\infty}^{x_{n-1}^\mp} dx_n^\mp f_n(x_n^\mp) = \underbrace{\partial_\pm^{-1}f\ldots\partial_\pm^{-1}f}_{n}.$$

In the Feynman rules:

$$\boxed{\partial_\pm^{-1} \to \frac{-i}{k^\pm + i\varepsilon}.}$$

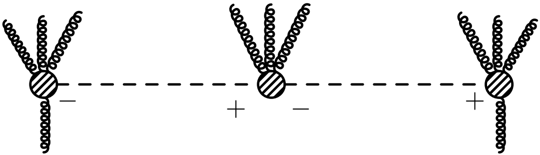
Basic structure of Induced interactions.



Induced interactions of particles and Reggeons [Lipatov, 1995]:

$$L_R^{(y_1<y<y_2)}(x) \supset \frac{i}{g_s}\,\mathrm{tr}\Big(R_+(x)\partial_\rho^2\partial_- W_x\left[A_-^{(y_1<y<y_2)}\right] + R_-(x)\partial_\rho^2\partial_+ W_x\left[A_+^{(y_1<y<y_2)}\right]\Big)\,.$$

Basic structure of Induced interactions.



Induced interactions of particles and Reggeons:

$$L_R^{(y_1<y<y_2)} \supset \frac{i}{g_s}\,\mathrm{tr}\left[R_+\partial_\rho^2\partial_- W\left[A_-^{(y_1<y<y_2)}\right] + R_-\partial_\rho^2\partial_+ W\left[A_+^{(y_1<y<y_2)}\right]\right],$$

expansion of P-exponent generaties induced vertices:

$$\begin{aligned} L_R \supset\ & \mathrm{tr}\Big[\big(R_+\partial_\sigma^2 A_- + R_-\partial_\sigma^2 A_+\big) + \\ & (-ig_s)(\partial_\sigma^2 R_+)(A_-\partial_-^{-1}A_-) + (-ig_s)^2(\partial_\sigma^2 R_+)(A_-\partial_-^{-1}A_-\partial_-^{-1}A_-) + \\ & (-ig_s)(\partial_\sigma^2 R_-)(A_+\partial_+^{-1}A_+) + (-ig_s)^2(\partial_\sigma^2 R_-)(A_+\partial_+^{-1}A_+\partial_+^{-1}A_+) \\ & +O(g_s^3)\Big], \end{aligned}$$

but this structure is non-Hermitian: R_+ – Hermitian, W – Unitary!

Hermitian effective action and pole prescription

Recently the new derivation of effective action has been proposed [Bondarenko, Zubkov, 2018] which fixes the **Hermitian** form of Reggeon-gluon interaction:

$$\frac{i}{g_s}\,\mathrm{tr}\left[R_+\partial_\rho^2\partial_-\left(W\left[A_-\right]-W^\dagger\left[A_-\right]\right)\right],$$

E.g. Rgg-vertex:

$$\frac{-ig_s}{2}\left(\partial_\sigma^2 R_+^a(x)\right)\left(A_-^{b_1}(x)\int\limits_{-\infty}^{x_-}dx_1^-A_-^{b_2}(x_1)\right)\mathrm{tr}\left[T^a\left[T^{b_1},T^{b_2}\right]\right],$$

$\Rightarrow$ Feynman rule:

$$g_s(-q^2)f^{ab_1b_2}(n_-^{\mu_1}n_-^{\mu_2})\frac{1}{2}\left[\frac{1}{k_1^-+i\varepsilon}+\frac{1}{k_1^--i\varepsilon}\right]=g_s(-q^2)(n_-^{\mu_1}n_-^{\mu_2})\frac{f^{ab_1b_2}}{[k_1^-]},$$

i.e. the PV-prescription for the $1/k^\pm$ poles for simplest induced vertices [Hentschinski, 2013].

Higher-order induced vertices

For higher-order induced vertices the $i(\partial^2 R_\pm)\partial_\mp \left(W[A_\mp] - W^\dagger[A_\mp]\right)$ interaction leads to the $i\varepsilon$ prescription proposed **independently** in [Hentschinski, 2013] (based on argments from Regge theory):

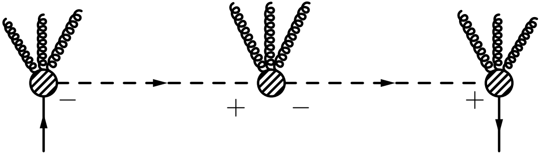
- The induced vertex is written according to $i(\partial^2 R_\pm)\partial_\mp W[A_\mp]$ interaction with $1/(k^\pm + i\varepsilon)$ prescription for all poles,
- The color structure $\mathrm{tr}\left(T^a T^{b_1} \ldots T^{b_n}\right)$ is projected on subspace, spanned by:

$$\mathrm{tr}\left(T^a \left[\left[\left[T^{b_{i_1}}, T^{b_{i_2}}\right], T^{b_{i_3}}\right], \ldots T^{b_{i_n}}\right]\right).$$

This pole prescription is very well tested: leads to the correct results for 1-loop amplitudes with Reggeized gluons and quarks and corrct 2-loop gluon Regge trajectory [Chachamis, Hentschinski, Sabio-Vera, 2012-2013; M.N., V.A.S., 2017].

The formalism is well-defined at all orders now!

EFT for QMRK-processes with quark exchange.



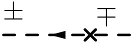
EFT for Reggeized quarks [Lipatov, Vyazovsky, 2001]:

$$L_Q = \bar{Q}_- i\hat{\partial}\left(Q_+ - W^\dagger\left[A_+\right]\psi\right) + \bar{Q}_+ i\hat{\partial}\left(Q_- - W^\dagger\left[A_-\right]\psi\right) + \text{h.c.},$$

where $\hat{p} = p_\mu \gamma^\mu$, QMRK kinematic constraints:

$$\partial_\pm Q_\mp = \partial_\pm \bar{Q}_\mp = 0,$$
$$\hat{n}^\pm Q_\mp = 0,\ \bar{Q}_\mp \hat{n}^\pm = 0. \Rightarrow$$

Reggeized quark propagator ($\hat{P}_\pm = \hat{n}_\mp \hat{n}_\pm / 4$):

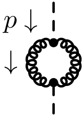
$= \hat{P}_\pm \frac{i\hat{q}}{q^2},$


$= \frac{i\hat{q}}{q^2}\hat{P}_\pm.$

Rapidity divergences and regularization.

Due to the presence of the $1/q^{\pm}$-factors in the induced vertices, loop integrals in EFT contain the light-cone (Rapidity) divergences:

$$\Sigma^{(1)}_{ab} = q\downarrow$$



$$= g_s^2 C_A \delta_{ab} \int \frac{d^d q}{(2\pi)^D} \frac{\left(\mathbf{p}_T^2 (n_+ n_-)\right)^2}{q^2 (p-q)^2 q^+ q^-}$$

The regularization by explicit cutoff in rapidity was proposed by Lipatov [Lipatov, 1995] ($q^{\pm} = \sqrt{q^2 + \mathbf{q}_T^2} e^{\pm y}$):

$$\int \frac{dq^+ dq^-}{q^+ q^-} = \int_{y_1}^{y_2} dy \int \frac{dq^2}{q^2 + \mathbf{q}_T^2},$$

then

$$\Sigma^{(1)}_{ab} \sim \delta_{ab} \mathbf{p}_T^2 \times \underbrace{C_A g_s^2 \int \frac{\mathbf{p}_T^2 d^{D-2}\mathbf{q}_T}{\mathbf{q}_T^2 (\mathbf{p}_T - \mathbf{q}_T)^2}}_{\omega^{(1)}(\mathbf{p}_T^2)} \times (y_2 - y_1) + \text{finite terms}$$

Rapidity divergent one-loop integrals

Covariant regularization.

The regularization and **pole prescription** was introduced in a series of papers [Hentschinski, Sabio Vera, Chachamis *et. al.*, 2012-2013], also known in TMD factorization as "tilted Wilson lines" [Collins, 2011].
Regularization of the light-cone divergences is achieved by shifting $n^{\pm}$ vectors from the light-cone:

$$\boxed{\tilde{n}^{\pm} = n^{\pm} + r \cdot n^{\mp},\ \tilde{k}^{\pm} = k^{\pm} + r \cdot k^{\mp},\ r \to 0,}$$

and for the lowest-order(Rgg, Qqg) induced vertices the PV prescription is at work:

$$I^{[\pm]}:\ \frac{1}{[\tilde{k}^{\pm}]} = \frac{1}{2}\left(\frac{1}{\tilde{k}^{\pm} + i\varepsilon} + \frac{1}{\tilde{k}^{\pm} - i\varepsilon}\right),$$

Regularization and gauge-invariance

Regularization should preserve the gauge-invariance of Reggeon-gluon interactions:

$$S_{Rg}^{(-)} = \int d^2\mathbf{x}_T \int\limits_{-\infty}^{+\infty} \frac{dx_+ dx_-}{2}\,\mathrm{tr}\left[R^- \tilde{\partial}_+ \partial_\sigma^2 W_{\tilde{x}_-}[\tilde{A}_+]\right]$$

$$= \int d^2\mathbf{x}_T \int\limits_{-\infty}^{+\infty} \frac{d\tilde{x}_+ d\tilde{x}_-}{1-r^2}\,\mathrm{tr}\left[R^- \frac{\partial}{\partial \tilde{x}_-} \partial_\sigma^2 W_{\tilde{x}_-}[\tilde{A}_+]\right] =$$

$$= \int d^2\mathbf{x}_T \int\limits_{-\infty}^{+\infty} \frac{d\tilde{x}_+ d\tilde{x}_-}{1-r^2} \left\{ \frac{\partial}{\partial \tilde{x}_-}\,\mathrm{tr}\left[R^- \partial_\sigma^2 W_{\tilde{x}_-}[\tilde{A}_+]\right] - \frac{1}{2}\,\mathrm{tr}\left[\left(\tilde{\partial}_+ R_-\right) \partial_\sigma^2 W_{\tilde{x}_-}[\tilde{A}_+]\right] \right\}.$$

First term – *infinite Wilson line* is gauge invariant (w.r.t. gauge transformations trivial at ∞) $\Rightarrow$ **new kinematic constraint**:

$$\boxed{\tilde{\partial}_+ R_- = \tilde{\partial}_- R_+ = 0,}$$

or $\tilde{p}^+ = 0$ for R_- and $\tilde{p}^- = 0$ for R_+.

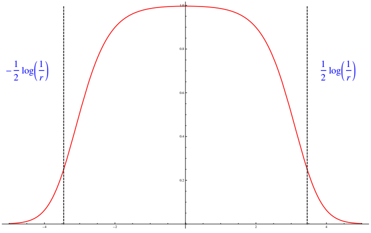
Rapidity divergences in real corrections

New constraint allows to use same regularization for RDs in *virtual and real corrections*. Lipatov's vertex ($k = q_1 + q_2$, $k^2 = 0$):

$$\Gamma_{+\mu-} = 2\left[(q_2 - q_1)_\mu + \left(\frac{q_1^2}{\tilde{k}_-} + \tilde{q}_1^+\right)\tilde{n}_\mu^- - \left(\frac{q_2^2}{\tilde{k}_+} + \tilde{q}_2^-\right)\tilde{n}_\mu^+\right],$$

without modified constraint, the Slavnov-Taylor identity $\boxed{k^\mu \Gamma_{+\mu-} = 0}$ is broken by terms $O(r)$.

The square of regularized LV:



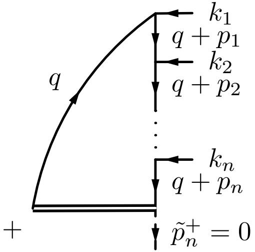
$$\Gamma_{+\mu-}\Gamma_{+\nu-}P^{\mu\nu} = \frac{16\mathbf{q}_{T1}^2\mathbf{q}_{T2}^2}{\mathbf{k}_T^2} f(y),$$

$$\longleftarrow f(y) = \frac{1}{(re^{-y} + e^{y})^2(re^{y} + e^{-y})^2},$$

$$\int\limits_{-\infty}^{+\infty} dy\; f(y) = -1 - \log r + O(r)$$

RDs in 1-loop, 1-Reggeon amplitude

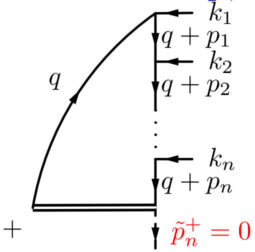
$$p_i = \sum_{j=1}^{i} k_j, \quad p_0 = 0, \quad d = 4 - 2\epsilon.$$



"Mixed" Feynman parametrization:

$$\begin{aligned} I &= \int \frac{d^d q}{q^2 (q+p_1)^2 \ldots (q+p_n)^2 (\tilde{n}^+ q)} \\ &\sim \int_0^1 da_1 \ldots da_{n+1} \int_0^\infty dx_1 \; \delta\left(1 - \sum_{j=1}^{n+1} a_j\right) \\ &\times \int d^d q \left[x_1 (\tilde{n}^+ q) + \sum_{i=1}^{n+1} a_i (q + p_{i-1})^2 \right]^{n+2} \end{aligned}$$

RDs in 1-loop, 1-Reggeon amplitude



$$I \sim \int_0^1 da_1 \dots da_{n+1} \int_0^\infty dx_1 \; \delta\left(1 - \sum_{j=1}^{n+1} a_j\right) \times \left[\mathcal{D} + x_1 \sum_{j=1}^{n-1} \tilde{p}_j^+ a_{j+1} + r x_1^2\right]^{-n-\epsilon},$$

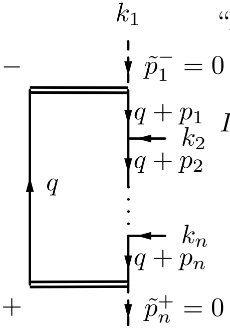
where $\mathcal{D} = -\frac{1}{2} \sum_{i,j=0}^{n} a_{i+1} a_{j+1} (p_i - p_j)^2$.

Let's put $r = 0 \Rightarrow$ after integration over x_1:

$$I \sim \int_0^1 da_1 \dots da_{n+1} \; \delta(\dots) \left(\sum_{j=1}^{n-1} p_j^+ a_{j+1}\right)^{-1} \mathcal{D}^{-n-\epsilon+1},$$

- **Log-divergent** for $\boxed{n=2}$ as $\int_0 \frac{da_2}{a_2}$, for $n > 2$ – **finite**.
- For $n = 2$, divergence can be **removed** by differentiating $\partial I / \partial k_1^2$ or $\partial I / \partial k_2^2$.

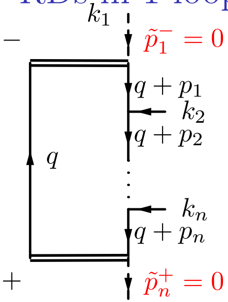
RDs in 1-loop, 2-Reggeon amplitude



"Mixed" Feynman parametrization:

$$
\begin{aligned}
I &= \int \frac{d^d q}{q^2 (q+p_1)^2 \dots (q+p_n)^2 (\tilde{n}^+ q)(\tilde{n}^- q)} \\
&\sim \int_0^1 da_1 \dots da_{n+1} \int_0^\infty dx_1 dx_2 \; \delta\left(1 - \sum_{j=1}^{n+1} a_j\right) \\
&\times \int d^d q \left[x_1 (\tilde{n}^+ q) + x_2 (\tilde{n}^- q) + \sum_{i=1}^{n+1} a_i (q + p_{i-1})^2 \right]^{n+3}
\end{aligned}
$$

RDs in 1-loop, 2-Reggeon amplitude



$$I \sim \int_0^1 da_1 \ldots da_{n+1} \int_0^\infty dx_1 dx_2 \; \delta \left(1 - \sum_{j=1}^{n+1} a_j \right)$$

$$\times \left[\mathcal{D} + \sum_{j=1}^{n} a_{j+1} \left(x_1 \tilde{p}_j^+ + x_2 \tilde{p}_j^- \right) + x_1 x_2 + r(x_1^2 + x_2^2) \right]^{-n-1-\epsilon}$$

For $r = 0$, after integration over x_2:

$$I \sim \int_0^1 da_1 \ldots da_{n+1} \int_0^\infty dx_1 \; \delta(\ldots) \left(x_1 + \sum_{j=2}^{n} a_{j+1} p_j^- \right)^{-1} \left[\mathcal{D} + x_1 \sum_{j=1}^{n-1} a_{j+1} p_j^+ \right]^{-n-\epsilon}$$

- **Log-divergence** for $\boxed{n = 1}$ $(p_1^- = p_1^+ = 0)$ as $\int_0 \frac{dx_1}{x_1}$.
- For $n > 1$ – **no divergence** if $p_2^- \neq 0, \ldots, p_n^- \neq 0$ and $p_1^+ \neq 0, \ldots, p_{n-1}^+ \neq 0$.

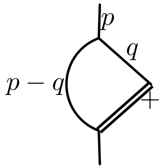
“Tadpoles” and “Bubbles”.

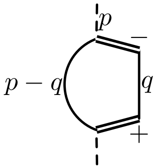
“Tadpoles” (one quadratic propagator):

$$A_0^{[+]}(p) = \int \frac{[d^d q]}{(p-q)^2[\tilde{q}^+]},\ A_0^{[+-]}(p) = \int \frac{[d^d q]}{(p-q)^2[\tilde{q}^+][\tilde{q}^-]}$$

where $[d^D q] = \frac{(\mu^2)^\epsilon d^d q}{i\pi^{D/2} r_\Gamma}$, $r_\Gamma = \Gamma^2(1-\epsilon)\Gamma(1+\epsilon)/\Gamma(1-2\epsilon)$.

“Bubbles” (two quadratic propagators):





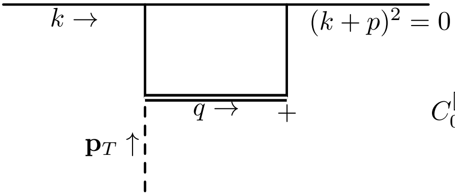
$$B_0^{[+]}(p) = \int \frac{[d^d q]}{q^2(p-q)^2[\tilde{q}^+]},$$

$$B_0^{[+-]}(\mathbf{p}_T) = \int \frac{[d^d q]}{q^2(p-q)^2[\tilde{q}^+][\tilde{q}^-]},$$

where $p^+ = p^- = 0$ for the last integral.

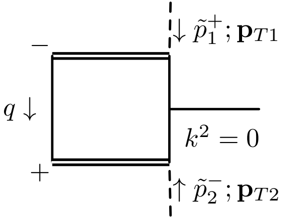
"Triangle" integrals

One light-cone propagator:



$$C_0^{[+]} = \int \frac{[d^D q]}{q^2 (p-q)^2 (p+k-q)^2 [\tilde{q}^+]}.$$

Two light-cone propagators:



$$C_0^{[+-]} = \int \frac{[d^D q]}{q^2 (p_1-q)^2 (p_2+q)^2 [\tilde{q}^+][\tilde{q}^-]}.$$

Rapidity divergences at one loop

Only log-divergence $\sim \log r$ (Blue cells in the table) is related with Reggeization of particles in t-channel.

Integrals which **do not** have log-divergence may still contain the power-dependence on r:

- $r^{-\epsilon} \to 0$ for $r \to 0$ and $\epsilon < 0$.
- $r^{+\epsilon} \to \infty$ for $r \to 0$ and $\epsilon < 0$ – **weak-power divergence** (Pink cells in the table)
- $r^{-1+\epsilon} \to \infty$ – **power divergence.** (Red)

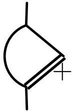
(# LC prop.) \ (# quadr. prop.)	1	2	3	4
1	$A_0^{[+]}$	$B_0^{[+]}$	$C_0^{[+]}$	...
2	$A_0^{[+-]}$	$B_0^{[+-]}$	$C_0^{[+-]}$	...
3	...	...	...	...

The **weak-power** and **power-divergences** cancel between Feynman diagrams describing one region in rapidity, so only log-divergences are left.

Results for scalar integrals.

Notation: $\left\{\frac{\mu}{k}\right\}^{2\epsilon} = \frac{1}{2}\left[\left(\frac{\mu}{k-i\varepsilon}\right)^{2\epsilon} + \left(\frac{\mu}{-k-i\varepsilon}\right)^{2\epsilon}\right].$

- [+]-bubble in general kinematics (leading term of the Mellin-Barnes representation):



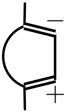
$$B_0^{[+]}(p) = \frac{1}{\tilde{p}^+}\frac{r^\epsilon}{\cos(\pi\epsilon)}\frac{1}{2\epsilon^2}\left\{\frac{\mu}{\tilde{p}^+}\right\}^{2\epsilon} + O(r^{1/2}),$$

- Tadpoles (direct integration):

$$A_0^{[+]}(p) = \frac{\epsilon\tilde{p}_+^2 r^{-1}}{(1-2\epsilon)} B_0^{[+]}(p),$$

$$A_0^{[+-]}(p) = \tilde{p}^+ B_0^{[+]}(p) + \tilde{p}^- B_0^{[-]}(p) - \left\{\frac{\mu}{\tilde{p}^+}\right\}^{\epsilon}\left\{\frac{\mu}{\tilde{p}^-}\right\}^{\epsilon}\frac{1}{\epsilon^2}\frac{\sin(\pi\epsilon)\Gamma(1-2\epsilon)\Gamma^2(1+\epsilon)}{\pi\epsilon}$$

- [+−]-bubble in transverse kinematics $p^- = p^+ = 0$ (direct integration):



$$B_0^{[+-]}(\mathbf{p}_T) = \frac{1}{\mathbf{p}_T^2}\left(\frac{\mu^2}{\mathbf{p}_T^2}\right)^{\epsilon}\frac{i\pi + 2\log r}{\epsilon},$$

- [+−]-bubble in $p^- = 0$ kinematics (leading term of MB expansion):

$$\begin{aligned} B_0^{[+-]}(\mathbf{p}_T, p^+) &= \frac{1}{\mathbf{p}_T^2}\left(\frac{\mu^2}{\mathbf{p}_T^2}\right)^{\epsilon}\frac{\Gamma^2(1+\epsilon)\Gamma(2+\epsilon)\sin(\pi\epsilon)}{\pi\epsilon^2} \\ &\times \left[i\pi + \log r - \log\frac{p_+^2}{\mathbf{p}_T^2} - \psi(1+\epsilon) + \psi(1)\right] + O(r^{1/2}) \end{aligned}$$

- [+−]-bubble in light-like kinematics $p^2 = 0$:

$$B_0^{[+-]}(\mathbf{p}_T^2, p^2 = 0) = \int \frac{[d^d q]}{q^2(q-p)^2[q^+][q^- - p^-]} = \frac{-2\Gamma(1-\epsilon)\Gamma(1+\epsilon)}{\mathbf{p}_T^2\epsilon^2}\left(\frac{\mu^2}{\mathbf{p}_T^2}\right)^{\epsilon}.$$

Single-scale triangle.

Calculation of the single-scale triangle integral:

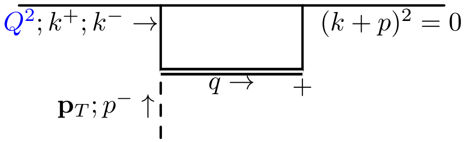
$$C_0^{[+]}(\mathbf{p}_T^2, k^+) = \int \frac{[d^D q]}{q^2 (p-q)^2 (p+k-q)^2 [\tilde{q}^+]},$$

is *significantly simplified by the new kinematic constraint* $\tilde{p}^+ = 0$.
The final result is obtained using one-fold Mellin-Barnes representation:

$$C_0^{[+]} = \frac{1}{k^+ \mathbf{p}_T^2} \left(\frac{\mu^2}{\mathbf{p}_T^2}\right)^{\epsilon} \frac{1}{\epsilon} \left[-\log r - i\pi + \log \frac{(k^+)^2}{\mathbf{p}_T^2} + \psi(1+\epsilon) + \psi(1) - 2\psi(-\epsilon) \right],$$

coincides with the result of [G. Chachamis, *et. al.*, 2012].

Triangle with two scales.



where now $k^2 = k^+k^- = -Q^2$, $X = Q^2/\mathbf{p}_T^2$.
Apply "Rudimentary DE-method". The integral:

$$\left.\frac{\partial C_0^{(+)}}{\partial X}\right|_{r=0} = -\frac{\mathbf{p}_T^2 \mu^{2\epsilon}\Gamma(3+\epsilon)}{r_\Gamma} \int\limits_0^\infty dx_1 dx_2 dx_3 \; x_1(1+x_1+x_2)^{2\epsilon}$$

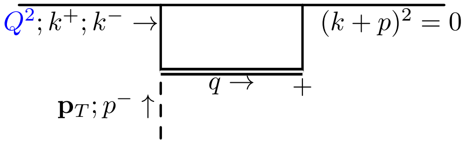
$$\times \left[\mathbf{p}_T^2 x_1(x_2+X) + k_+ x_3\right]^{-3-\epsilon},$$

is **finite** and can be calculated analytically. The answer is:

$$\left.\frac{\partial I}{\partial X}\right|_{r=0} = \frac{2X^{-1-\epsilon}}{\epsilon} - \frac{2}{\epsilon}\frac{1-X^{-\epsilon}}{1-X},$$

where $I(X) = \mathbf{p}_T^2 k_+ \left(\frac{\mu^2}{\mathbf{p}_T^2}\right)^{-\epsilon} \left[C_0^{[+]}(X) - C_0^{[+]}(X=0)\right]_{r=0}$.

Triangle with two scales.



The final answer:

$$C_0^{[+]}(X) = C_0^{[+]}(X=0) + \left(\frac{\mu^2}{\mathbf{p}_T^2}\right)^{\epsilon} \frac{I(X)}{k_+ \mathbf{p}_T^2},$$

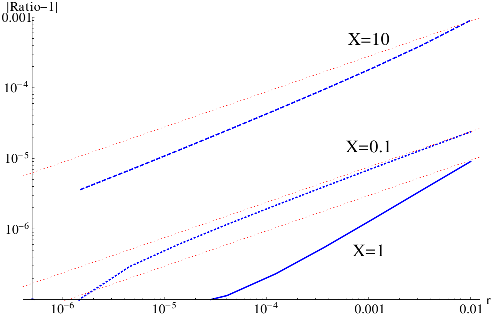
where

$$\begin{aligned} I(X) &= -\frac{2X^{-\epsilon}}{\epsilon^2} - \frac{2}{\epsilon}\int_0^X \frac{(1-x^{-\epsilon})dx}{1-x} \\ &= -\frac{2X^{-\epsilon}}{\epsilon^2} + 2\left[\mathrm{Li}_2(X) + \log(1-X)\log X\right] + O(\epsilon). \end{aligned}$$

Numerical cross-check

The results for $C_0^{(+)}$ integrals with 1 and 2 scales has been cross-checked numerically, using **sector decomposition** algorithm.

The r-dependence of the ratio ($\epsilon = -0.01$, rel.acc.$=10^{-7}$):



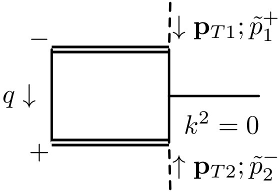
Remaining r-dependence is $O(r^{\geq 0.5})$.

- For $C_0^{(+)}$ with 2 scales: 3D integral (`cuhre` algorithm of `CUBA` was used), 8 sectors, up to 4 subsectors in some of them.
- For numerical comparison, the $1/\epsilon^2$ pole is subtracted.
- The leading r-dependent term is calculated using Mellin-Barnes:

$$\left(\frac{k_+^2}{\mathbf{p}_T^2}\right)^{\epsilon} \frac{r^{-\epsilon} X^{-2\epsilon}}{2\epsilon^2} \frac{\Gamma^2(1-2\epsilon)\Gamma(1+2\epsilon)}{\Gamma^2(1-\epsilon)}.$$

Triangle with two light-cone propagators

Usual one-loop Feynman integrals with more than 4 propagators are reducible to more simple integrals up to terms $O(\epsilon)$.



We apply method of [Bern, Dixon, Kosower, 1992]. The $O(\epsilon)$ remnant is proportional to $(d-4)I^{(d+2)}$ and integral $I^{(6)}$ is finite.
The resilt in Euclidean region ($p_1^+ > 0$, $-p_2^- > 0$, $\mathbf{p}_{T1,2}^2 > 0$):

$$C_0^{[+-]}(\mathbf{p}_{T1}^2, \mathbf{p}_{T2}^2, p_1^+, -p_2^-) = \frac{(-1)}{2\mathbf{p}_{T1}^2\mathbf{p}_{T2}^2\mathbf{k}_T^2} \times$$

$$\Big\{\mathbf{p}_{T1}^2(\mathbf{p}_{T2}^2 - \mathbf{p}_{T1}^2 + \mathbf{k}_T^2)\Big[B_0^{[+-]}(\mathbf{p}_{T1}^2, p_1^+) + (-p_2^-)C_0^{[-]}(\mathbf{p}_{T1}^2, \mathbf{p}_{T2}^2, -p_2^-)\Big]$$

$$+\mathbf{p}_{T2}^2(\mathbf{p}_{T1}^2 - \mathbf{p}_{T2}^2 + \mathbf{k}_T^2)\Big[B_0^{[+-]}(\mathbf{p}_{T2}^2, -p_2^-) + p_1^+ C_0^{[+]}(\mathbf{p}_{T2}^2, \mathbf{p}_{T1}^2, p_1^+)\Big]$$

$$-\mathbf{k}_T^2(\mathbf{p}_{T1}^2 + \mathbf{p}_{T2}^2 - \mathbf{k}_T^2)B_0^{[+-]}(\mathbf{k}_T^2, k^2 = 0)\Big\},$$

where $\mathbf{k}_T^2 = p_1^+(-p_2^-)$.
The $\log r$-divergence cancels within square brackets, as expected.

Comparison with QCD

Test process: DIS on the on-shell photon target

To perform the comaprison with QCD we consider the process:

$$\gamma^*(q) + \gamma(P) \to X,$$

where P has large P^+ momentum component and $P^2 = 0$. The LO subprocess is:

$$\gamma^*(q) + \gamma(P) \to q(k_1) + \bar{q}(k_2),$$

we introduce the usual variables:$Q^2 = -q^2,\ x_B = \frac{Q^2}{2(qP)}$, and work in the (q, P) center of mass frame, where $q^+ = -x_B P^+$, $q^- = Q^2/(x_B P^+)$, $\mathbf{q}_T = 0$.
We parametrize final-state momenta as:

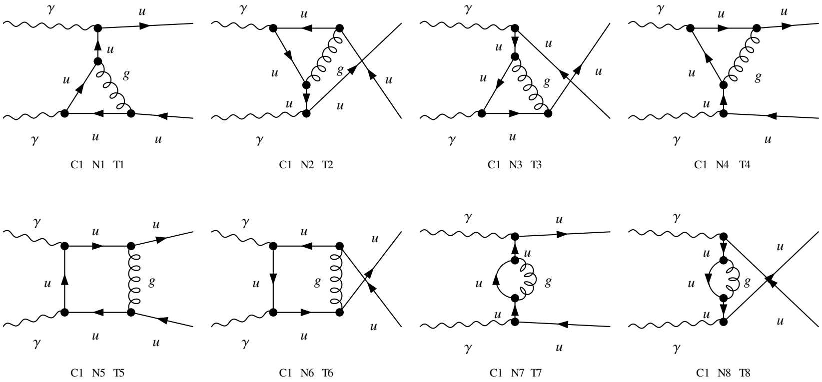
$$k_1 = q + q_1,\ k_2 = P - q_1,$$
$$t_1 = -\mathbf{q}_{T1}^2,\ x_1 = \frac{q_1^+}{P^+} = x_B \frac{Q^2 + t_1}{Q^2} \text{ for } x_B \ll 1.$$

And will study the squared amplitude projected on the F_2 structure function:

$$F_2(x_B, Q^2, t_1) \text{ in the limit } x_B \ll 1.$$

QCD at 1 loop.

$\gamma \quad \gamma \quad \rightarrow \quad u \quad u$



QCD at 1 loop.

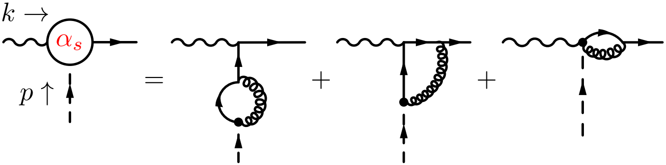
The QCD result (leading power in x_B, but exact in Q^2 and t_1):

$$\frac{F_2^{\text{QCD, 1-loop}}(x_B, Q^2, t_1)}{F_2^{\text{QCD, Born}}(x_B, Q^2, t_1)} = \frac{\bar{\alpha}_s C_F}{4\pi}\left\{-\frac{2}{\epsilon^2} + \left(\frac{2\pi^2}{3} - 7 - \log^2\frac{\mu^2}{Q^2} - 3\log\frac{\mu^2}{Q^2}\right) + \right.$$
$$+2\left(\frac{1}{\epsilon} + \log\frac{\mu^2}{t_1}\right)\log\frac{1}{x_B} + \log^2\frac{Q^2}{t_1} + 2\text{Li}_2\left(1 - \frac{Q^2}{t_1}\right)$$
$$\left. - \frac{1}{(Q^2 - t_1)^2}\left[Q^2(Q^2 - t_1) + (3t_1^2 - 4Q^2 t_1)\log\frac{Q^2}{t_1}\right]\right\} + O(x_B),$$

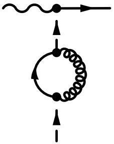
where $\bar{\alpha}_s = \frac{(\mu^2)^{-\epsilon} g_s^2}{(4\pi)^{1-\epsilon}} e^{-\epsilon\gamma_E}$, contains:

- The $1/\epsilon^2$ IR-divergence,
- Only single-log part in $\log x_B^{-1}$,
- The complicated dependence on Q^2/t_1.

EFT diagrams for $Q\gamma q$ vertex correction

$\hat{\Gamma}_1^\mu =$ 

The subtraction term (real part):

$\delta\hat{\Gamma}_1^\mu =$  $= \frac{\bar{\alpha}_s C_F}{4\pi} \hat{\Gamma}_0^\mu \left[(-2\log r + 1 + 3\epsilon) \left(\frac{1}{\epsilon} + L_1 \right) \right] + O(\epsilon),$

where $L_1 = \log\left(\frac{\mu^2}{t_1}\right)$,

$$\hat{\Gamma}_0^\mu = e e_q \bar{u}(p+k) \left(\gamma^\mu + \hat{p} \frac{n_+^\mu}{k^+} \right) \hat{n}^+,$$

is the Born $Q\gamma q$-vertex.

On-shell photon vertex @ NLO

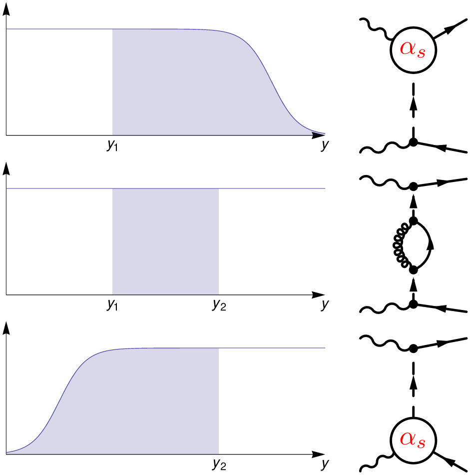
The un-subtracted $O(\alpha_s)$ correction to the Born $Q\gamma q$-vertex with the **on-shell** photon is [M. A. Nefedov, V. A. Saleev (2017)] (real part):

$$\hat{\Gamma}_1^\mu = \frac{\bar{\alpha}_s C_F}{4\pi}\left\{\frac{2}{t_1}\hat{\Delta}_0^\mu + \hat{\Gamma}_0^\mu\left[-\frac{1}{\epsilon^2} - \frac{L_1}{\epsilon} + (-\log r)\left(\frac{1}{\epsilon} + L_1\right) + \frac{2L_2}{\epsilon} - \right.\right.$$
$$\left.\left. - \left(\frac{1}{\epsilon} + L_1 + 3\right) + 2L_1L_2 - \frac{L_1^2}{2} + \frac{\pi^2}{2}\right]\right\},$$

where, $L_2 = \log\left(\frac{k^+}{\sqrt{t_1}}\right)$, and $\hat{\Delta}_0^\mu = ee_q\left(p^\mu - \frac{t_1}{2k^+}n_+^\mu\right)\left(\bar{u}(p+k)\hat{k}\hat{n}^+\right)$.

Origin of the subtraction term

The **covariant regularization** of $1/q^+$ pole corresponds to the (smooth) cutoff at $y_1 \sim -\log r^{-1}$, and regularization for $1/q^-$ pole corresponds to the cutoff $y_2 \sim \log r^{-1}$.



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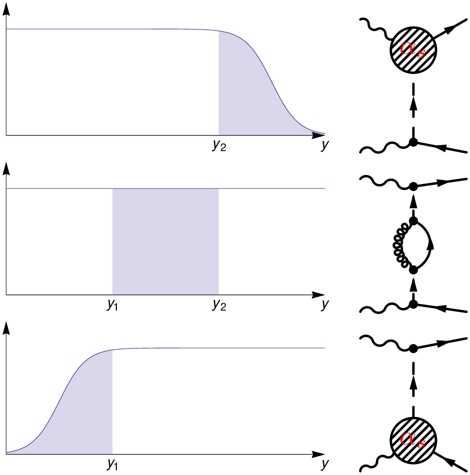


Off-shell photon vertex @ NLO

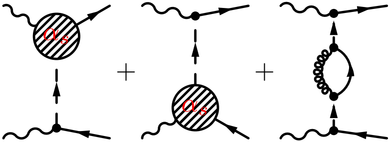
We considered the F_2-projection of the $\gamma^*(q) + Q(q_1) \to q$ squared amplitude at one loop. The ratio to Born amplitude is the un-subtracted $O(\alpha_s)$ correction factor:

$$C_V^{(-)}(Q^2, t_1, \bar{r}) = \frac{\bar{\alpha}_s C_F}{4\pi} \left\{ -\frac{1}{\epsilon} - \left(\frac{1}{\epsilon} + \log \frac{\mu^2}{t_1} \right) \log \bar{r} + \left(\frac{\pi^2}{6} - 2 - \log \frac{\mu^2}{Q^2} \right) + \frac{1}{2} \log^2 \frac{Q^2}{t_1} + 2\mathrm{Li}_2 \left(1 - \frac{Q^2}{t_1} \right) - \frac{1}{(Q^2 - t_1)^2} \left[Q^2 (Q^2 - t_1) + (t_1^2 - 2Q^4) \log \frac{Q^2}{t_1} \right] \right\},$$

where $\bar{r} = r \cdot (P^+ x_B)^2 / Q^2$. To which we apply **the same** procedure of localization in rapidity as in the on-shell case.
The result contains familiarly-looking Q^2/t_1-dependence, but no $1/\epsilon^2$ pole!

Comparison with QCD

The EFT result is given by



and it (**almost!**) reproduces the QCD-result:

$$\frac{F_2^{\text{QCD, 1-loop}}(x_B, Q^2, t_1) - F_2^{\text{EFT, 1-loop}}(x_B, Q^2, t_1)}{F_2^{\text{QCD, Born}}(x_B, Q^2, t_1)} = \frac{\bar{\alpha}_s C_F}{4\pi}\left[\frac{1}{\epsilon^2}\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} + O(\epsilon)\right] + O(x_B).$$

What could this be? Hints:

- In the calculation of massive quark impact factor [M. Ciafaloni, G. Rodrigo, 2000] it was found that procedure of "localization in rapidity" should be modified because of the additional scale m_q. This leads to the terms $\sim 1/\epsilon^2$!
- Should real and virtual corrections reproduce QCD separately?
- What is s_0 in $(s/s_0)^{\omega(t)}$? Could $\log s_0 \sim 1/\epsilon$?

Conclusions

- The consistent procedure of rapidity regularization is proposed. One should modify not only Wilson lines, but also kinematic constraints.
- One-loop integrals with log-RDs are identified. The power-RDs seem to be contained just in a few simplest integrals.
- Triangle integrals with 1 and 2 scales are calculated.
- Reduction of one-loop integrals with more than four propagators (quadratic or light-cone) seems to work similar to the case of ordinary loop integrals.
- Comparison with QCD for the DIS on a photon target is a nontrivial test of the formalism. The $(Q^2)^{-\epsilon}/\epsilon^2$-remnant is found.
- *Procedure of localization in rapidity should be modified for multiscale quantities?*

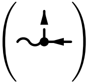
Thank you for your attention!

Backup: Infra-red structure of real corrections

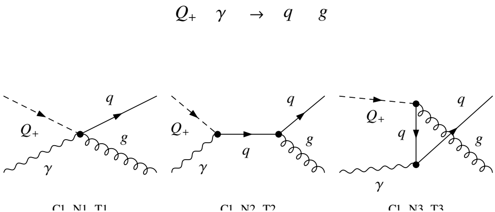
Real NLO correction to the off-shell photon vertex

To study real corrections to the $\gamma^* Q$-vertex, let's rewrite the structure function in k_T-factorized form:

$$F_2(x_B, Q^2) = \int_0^1 \frac{dx_1}{x_1^2} \int d^{D-2}\mathbf{q}_{T1}\ \Phi_{Q/\gamma}(x_1, t_1, \mu^2) \cdot \hat{F}_2(x_1, Q^2, t_1, \mu^2),$$

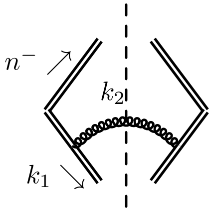
where $\Phi_{q/\gamma}$ is the *unintegrated PDF*, which is just -vertex at LO. The real NLO corrections to $\hat{F}_2$ are given by the subprocess:

$$\gamma^*(q) + Q(q_1) \to q(k_1) + g(k_2).$$



Collinear and soft limits

Diagram for the soft limit:



Convenient parametrization of momenta ($z = x_B/x_1$):

$$k_1^+ = q_1^+(1-z)x,\ k_2^+ = q_1^+(1-z)(1-x),$$
$$\mathbf{k}_{T1} = x\mathbf{q}_{T1} + \mathbf{\Delta},\ \mathbf{k}_{T2} = (1-x)\mathbf{q}_{T1} - \mathbf{\Delta},$$

where $0 \le x \le 1$. The invariant $\hat{s} = (k_1+k_2)^2 = \mathbf{\Delta}^2/x(1-x)$.

Two limits:

- Final-state collinear limit ($|\mathbf{\Delta}| \ll \min(|\mathbf{k}_{T1}|, |\mathbf{k}_{T2}|)$):

$$\hat{F}_2^{\text{coll.}} = \frac{4Q^2 e_q^2 x_B}{z} \frac{C_F g_s^2}{\hat{s}} \left[\frac{1+x^2}{1-x} - \epsilon(1-x) \right],$$

- "Soft" limit ($\mathbf{k}_{T2}^2 \ll Q^2$, no r-regularization):

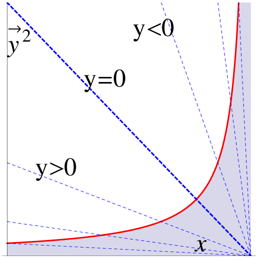
$$\hat{F}_2^{\text{soft.}} = \frac{8e_q^2 x_B}{1-z} \frac{C_F g_s^2}{\mathbf{y}^2 \left[\mathbf{y} - \mathbf{n}\sqrt{1-x}\right]^2},$$

where $\mathbf{y} = \mathbf{k}_{T2} \left(Q^2 \frac{1-z}{z}(1-x)\right)^{-1/2}$ and $\mathbf{n}$ is the direction of $\mathbf{q}_{T1}$.

"Soft" kinematic region

Definition: $\boxed{\mathbf{k}_{T2}^2 < \delta_T^2 Q^2 \frac{1-z}{z}}$, $\delta_T \ll 1 \Rightarrow$

$$\boxed{\mathbf{y}^2 < \frac{\delta_T^2}{1-x}.}$$



Soft integral with r-regularization ($\rho = \frac{Q^2 z}{(1-z)(x_B P^+)^2} \gg 1$):

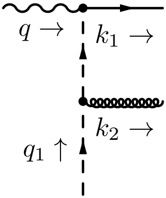
$$I_{\text{soft}} = \frac{2}{\Omega_{D-2}} \int\limits_{1-\delta_s}^{1} \frac{dx}{(1-x)^\epsilon} \int \frac{d^{D-2}\mathbf{y}\ \theta\left(\delta_T^2(1-x)^{-1} - \mathbf{y}^2\right)}{(\mathbf{y}^2 + r\rho^{-1}(1-x))\left[\mathbf{y} - \mathbf{n}\sqrt{1-x}\right]^2},$$

using the representation for θ-function in terms of Mellin integral $\theta(a-b) = \lim\limits_{\alpha\to 0^+} \int\limits_{-i\infty}^{+i\infty} \frac{dz}{2\pi i} \frac{1}{z+\alpha} \left(\frac{a}{b}\right)^z$. The result is:

$$\boxed{I_{\text{soft}} = \frac{(\delta_T^2)^{-\epsilon}}{2\epsilon}\left[\psi(-\epsilon) - \psi(1-\epsilon) + \log(r\rho^{-1})\right] + O(\delta_T^2).}$$

Localization in rapidity

To localize the soft correction in rapidity one have to subtract (r-regularization included):



$$\Delta F_2^{\text{soft}} = \frac{8e_q^2 x_B}{1-z} \frac{C_F g_s^2}{[(1-x) + r\rho\mathbf{y}^2][\mathbf{y}^2 + r\rho^{-1}(1-x)]},$$

Integral over the soft region:

$$\boxed{\Delta I_{\text{soft}}^{\text{MRK}} = \frac{(\delta_T^2)^{-\epsilon}}{2\epsilon}[2\log r] + O(\delta_T^2),}$$

contains only $1/\epsilon$.

The expected cancellation ($\log r - 2\log r$) happens:

$$I_{\text{soft}} - \Delta I_{\text{soft}}^{\text{MRK}} = \frac{1}{2\epsilon^2} - \frac{1}{2\epsilon}\log\left(r\rho\delta_T^2\right) + ...,$$

but $1/\epsilon^2$ pole is left.