TMD gluon distributions for multiparton processes

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Based on arXiv:1809.08968 (MB, P. Kotko and K. Kutak)

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Motivation

- Although TMD factorization fails Collins, Qiu 2007
  for processes with more than 2 colored partons, the TMDs are useful in dilute-dense collisions at small-x

Small-x generalized TMD factorization Dominguez, Marquet, Xiao, Yuan 2011

$$\frac{d\sigma^{pA\to dijets+X}}{dy_1\, dy_2\, d^2p_{T1}\, d^2p_{T2}} \propto \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_i H^{(i)}_{ag\to cd} \mathcal{F}^{(i)}_{ag}(x_2,k_T)$$

Improved small-x TMD factorization Kotko, Kutak, Marquet, Petreska, Sapeta, Hameren 2015

$$\frac{d\sigma^{pA\to dijets+X}}{dy_1\, dy_2\, d^2p_{T1}\, d^2p_{T2}} \propto \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_i K^{(i)}_{ag^*\to cd}(k_T) \Phi^{(i)}_{ag}(x_2,k_T)$$

Valid for an arbitrary value of k_T

- Such generalized factorization formulae agree with Color Glass Condensate effective theory

TMD gluon distributions

- TMDs describe the structure of hadrons in 3D momentum space
- Not universal, their operator definition depends on the process under consideration
- Fourier Transform of hadronic ME of bilocal field operators

$$\mathcal{F}\left(x,k_T\right) = 2\int \frac{d\xi^- d^2\xi_T}{\left(2\pi\right)^3 P^+}\, e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T}\, \langle P|\, \mathrm{Tr}\left\{\hat{F}^{i+}\left(0\right)\mathcal{U}_{C_1}\hat{F}^{i+}\left(\xi\right)\mathcal{U}_{C_2}\right\}|P\rangle \ ,$$

- Wilson Lines must be inserted to ensure gauge invariance

$$\mathcal{U}_C = \mathcal{P}\exp\left\{-ig\int_C dz_\mu \hat{A}^\mu\left(z\right)\right\} .$$

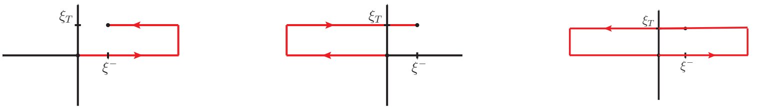
- Shape of the links is determined by the hard process Mulders et al. 2004

TMD gluon distributions

$$\mathcal{F}\left(x,k_T\right) = 2\int \frac{d\xi^- d^2\xi_T}{\left(2\pi\right)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \left\langle P\right| \mathrm{Tr}\left\{\hat{F}^{i+}\left(0\right)\mathcal{U}_{C_1}\hat{F}^{i+}\left(\xi\right)\mathcal{U}_{C_2}\right\}\left|P\right\rangle ,$$

$$\mathcal{U}_C = \mathcal{P}\exp\left\{-ig\int_C dz_\mu \hat{A}^\mu\left(z\right)\right\} .$$

- Shape of the links is determined by the hard process



$\mathcal{U}^{[+]}$ $\mathcal{U}^{[-]}$ $\mathcal{U}^{[\square]} = \mathcal{U}^{[-]\dagger}\mathcal{U}^{[+]}$

The universality of the TMD parton distribution is lost

- For arbitrary process the procedure is known Bomhof, Mulders, Pijlman 2006

Gauge-links in an arbitrary process

Procedure:

1. Assign $\mathcal{U}^{[+]}$ to each final state and $\mathcal{U}^{[-]}$ to initial state (not connected to the TMD) in fundamental or adjoint representation
2. The remaining initial state (connected to TMD) is attached to $\hat{F}^{\mu\nu}$
3. Gauge links replace deltas for color summation when the amplitude is squared

Problems:

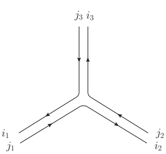
- lots of diagrams
- gauge invariance of subsets of diagrams corresponding to the same TMD operators is not evident

Improvements:

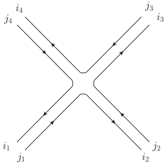
- use color-flow representation: $A_a^\mu \to (\hat{A}^\mu)^i_j = A_a^\mu (t^a)^i_j$ Maltoni 2003
- use decomposition of amplitudes into color-ordered amplitudes

Color-flow Feynman rules

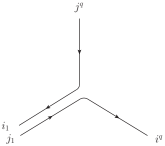
- Vertices (color part)



$\sim\delta^{i_2}_{j_1}\delta^{i_3}_{j_2}\delta^{i_1}_{j_3}$

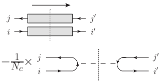


$\sim\delta^{i_2}_{j_1}\delta^{i_3}_{j_2}\delta^{i_4}_{j_3}\delta^{i_1}_{j_4}$



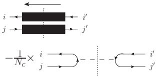
$\sim\delta^{i^q}_{j_1}\delta^{i_1}_{j^q}$

- Wilson lines



$\left(\mathcal{U}^{[+]}\right)_{i'i}\left(\mathcal{U}^{[+]\dagger}\right)^{jj'}-\frac{1}{N_c}\delta^{j}_{i}\delta^{j'}_{i'}$

Outgoing gluon



$\left(\mathcal{U}^{[-]\dagger}\right)^{ii'}\left(\mathcal{U}^{[-]}\right)_{j'j}-\frac{1}{N_c}\delta^{i}_{j}\delta^{i'}_{j'}$

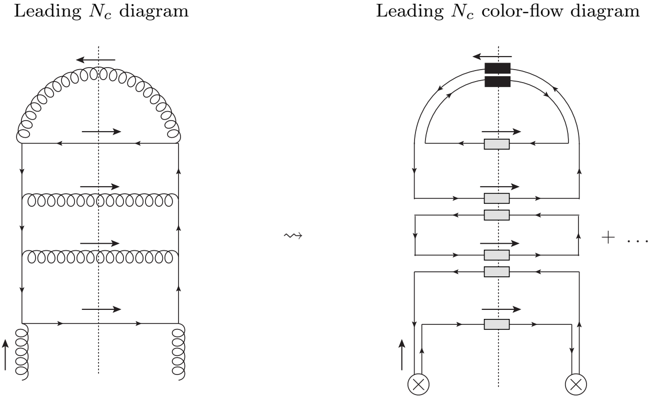
Incoming gluon

$\left(\hat{F}^{+i}(\xi)\right)^{j}_{i}\left(\hat{F}^{+i}(0)\right)^{j'}_{i'}$

Field strength operators

Example

$gg \to q\bar{q}gg$



$$N_c \mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[+]\dagger} F(0)\,\mathcal{U}^{[+]}\right\}\ \mathrm{Tr}\mathcal{U}^{[\Box]}\ \mathrm{Tr}\mathcal{U}^{[\Box]\dagger}$$

This expression enters the TMD definition

Color decompositions

- Adjoint basis (gluons only)

$$\mathcal{M}^{a_1\ldots a_n}\left(k_1,\ldots,k_n\right)=\sum_{\pi\in S_{n-2}}\left(T^{a_{\pi(2)}}\ldots T^{a_{\pi(n-1)}}\right)_{a_1 a_n}\mathcal{A}\left(1,\pi\left(2\right),\ldots,\pi\left(n-1\right),n\right)$$

$\left(T^a\right)_{bc}=-if^{abc}$
$\mathcal{A}$ - gauge invariant color-ordered amplitudes
(only planar diagrams, with fixed order of external legs contribute)

- Color-flow basis

$$\mathcal{M}^{i_1\ldots i_n}_{j_1\ldots j_n}\left(k_1,\ldots,k_n\right)=2^{-n/2}\sum_{\pi\in S_{n-1}}\delta^{i_1}_{j_{\pi(2)}}\delta^{i_{\pi(2)}}_{j_{\pi(3)}}\delta^{i_{\pi(3)}}_{j_{\pi(4)}}\ldots\delta^{i_{\pi(n)}}_{j_1}\mathcal{A}\left(1,\pi\left(2\right),\ldots,\pi\left(n\right)\right)$$

- The hard ME can be expressed as

$$|\mathcal{M}|^2=\vec{\mathcal{A}}^{\dagger}\,\mathbf{C}\,\vec{\mathcal{A}},\qquad \vec{\mathcal{A}}=\begin{pmatrix}\mathcal{A}(1,2,3,\ldots,n-1,n)\\ \vdots\\ \mathcal{A}(1,n,n-1,\ldots,3,2)\end{pmatrix},\qquad \mathbf{C}-\text{color matrix}$$

Summary of new results

To calculate TMD structure for a given process, it is enough to consider all color flows - no need to consider all diagrams

New results:

- The operator structures were calculated explicitly for hard processes with five and six colored partons
- The gluon TMD for any process can be build from 10 'basis' TMD distributions
- TMDs for pure gluonic hard process with an arbitrary number of legs were calculated in large N_c limit

Basis structures

$$\mathcal{F}_{qg}^{(1)}(x,k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{qg}^{(2)}(x,k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{qg}^{(3)}(x,k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(1)}(x,k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(2)}(x,k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \frac{1}{N_c}\left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\right]\mathrm{Tr}\left[\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(3)}(x,k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle,$$

$$\mathcal{F}_{gg}^{(4)}(x,k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[-]}\right]\right\rangle,$$

$$\mathcal{F}_{gg}^{(5)}(x,k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle,$$

$$\mathcal{F}_{gg}^{(6)}(x,k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(7)}(x,k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

Gluon TMDs for 4 gluons

- The hard ME convoluted with the TMD PDFs can be expressed as

$$|\mathcal{M}|^2 \otimes TMDs = \vec{\mathcal{A}}^\dagger \, \mathbf{\Phi}_{gg\to gg} \, \vec{\mathcal{A}}$$

where the TMD matrix reads

$$\mathbf{\Phi}_{gg\to gg} = \begin{pmatrix} \Phi_1 & \Phi_2 \\ \Phi_2 & \Phi_1 \end{pmatrix}$$

with

$$\Phi_1 = \frac{1}{2N_c^2}\left(N_c^2 \mathcal{F}_{\rm gg}^{(1)} - 2\mathcal{F}_{\rm gg}^{(3)} + \mathcal{F}_{\rm gg}^{(4)} + \mathcal{F}_{\rm gg}^{(5)} + N_c^2 \mathcal{F}_{\rm gg}^{(6)}\right) ,$$

$$\Phi_2 = \frac{1}{N_c^2}\left(N_c^2 \mathcal{F}_{\rm gg}^{(2)} - 2\mathcal{F}_{\rm gg}^{(3)} + \mathcal{F}_{\rm gg}^{(4)} + \mathcal{F}_{\rm gg}^{(5)} + N_c^2 \mathcal{F}_{\rm gg}^{(6)}\right) .$$

Gluon TMDs for 5 gluons

$$|\mathcal{M}|^2 \otimes TMDs = \vec{\mathcal{A}}^\dagger \, \mathbf{\Phi}_{gg\to ggg} \, \vec{\mathcal{A}}$$

$$\mathbf{\Phi}_{gg\to ggg} = \begin{pmatrix} \Phi_1 & \Phi_2 & \Phi_2 & \Phi_3 & \Phi_3 & \Phi_4^* \\ \Phi_2 & \Phi_1 & \Phi_3 & \Phi_4^* & \Phi_2 & \Phi_3 \\ \Phi_2 & \Phi_3 & \Phi_1 & \Phi_2 & \Phi_4^* & \Phi_3 \\ \Phi_3 & \Phi_4^* & \Phi_2 & \Phi_1 & \Phi_3 & \Phi_2 \\ \Phi_3 & \Phi_2 & \Phi_4^* & \Phi_3 & \Phi_1 & \Phi_2 \\ \Phi_4^* & \Phi_3 & \Phi_3 & \Phi_2 & \Phi_2 & \Phi_1 \end{pmatrix}$$

$$\Phi_1 = \frac{1}{4N_c^2}\left((N_c^2+2)\mathcal{F}_{\rm gg}^{(1)} - 4\mathcal{F}_{\rm gg}^{(2)} - 4\mathcal{F}_{\rm gg}^{(3)} + 3N_c^2\mathcal{F}_{\rm gg}^{(6)} + 2\mathcal{F}_{\rm gg}^{(7)}\right) ,$$

$$\Phi_2 = -\frac{1}{2N_c^2}\left(-2\mathcal{F}_{\rm gg}^{(1)} + 4\mathcal{F}_{\rm gg}^{(2)} + 2\mathcal{F}_{\rm gg}^{(3)} + \mathcal{F}_{\rm gg}^{(4)} + \mathcal{F}_{\rm gg}^{(5)} - 2N_c^2\mathcal{F}_{\rm gg}^{(6)} - 2\mathcal{F}_{\rm gg}^{(7)}\right)$$

$$\vdots$$

Gluon TMDs for 6 gluons

$$|\mathcal{M}|^2 \otimes TMDs = \vec{\mathcal{A}}^\dagger \, \mathbf{\Phi}_{gg\to gggg} \, \vec{\mathcal{A}}$$

$$\mathbf{\Phi}_{gg\to gggg} = \begin{pmatrix} T_1 & T_2 & T_3 & T_4 \\ T_2 & T_1 & T_5 & T_6 \\ T_3^\intercal & T_5 & T_1 & T_7 \\ T_4^\intercal & T_6^\intercal & T_7 & T_1 \end{pmatrix},$$

where for example

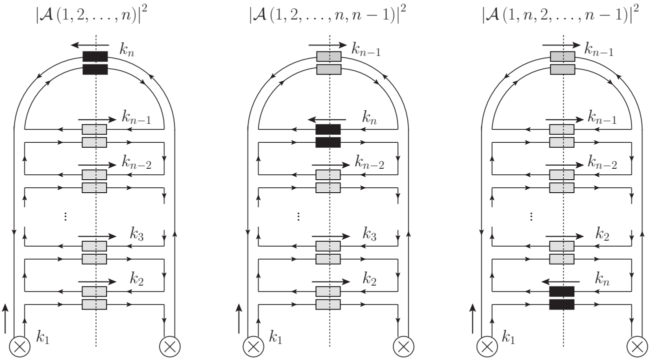
$$T_1 = \begin{pmatrix} \Phi_1 & \Phi_2 & \Phi_2 & \Phi_3 & \Phi_3 & \Phi_4^* \\ \Phi_2 & \Phi_1 & \Phi_3 & \Phi_4^* & \Phi_2 & \Phi_3 \\ \Phi_2 & \Phi_3 & \Phi_1 & \Phi_2 & \Phi_4^* & \Phi_3 \\ \Phi_3 & \Phi_4^* & \Phi_2 & \Phi_1 & \Phi_3 & \Phi_2 \\ \Phi_3 & \Phi_2 & \Phi_4^* & \Phi_3 & \Phi_1 & \Phi_2 \\ \Phi_4^* & \Phi_3 & \Phi_3 & \Phi_2 & \Phi_2 & \Phi_1 \end{pmatrix}, \quad T_2 = \begin{pmatrix} \Phi_2 & \Phi_5 & \Phi_3 & \Phi_6 & \Phi_7 & \Phi_8^* \\ \Phi_5 & \Phi_2 & \Phi_7 & \Phi_8^* & \Phi_3 & \Phi_6 \\ \Phi_3 & \Phi_7 & \Phi_4^* & \Phi_8^* & \Phi_9 & \Phi_{10} \\ \Phi_6 & \Phi_8^* & \Phi_8^* & \Phi_{10} & \Phi_{10} & \Phi_{11} \\ \Phi_7 & \Phi_3 & \Phi_9 & \Phi_{10} & \Phi_4^* & \Phi_8^* \\ \Phi_8^* & \Phi_6 & \Phi_{10} & \Phi_{11} & \Phi_8^* & \Phi_{10} \end{pmatrix},$$

with

$$\Phi_1 = \frac{2N_c^2(\mathcal{F}_{\rm gg}^{(1)} - 4\mathcal{F}_{\rm gg}^{(3)} + 2\mathcal{F}_{\rm gg}^{(6)} + \mathcal{F}_{\rm gg}^{(7)}) + N_c^4(\mathcal{F}_{\rm gg}^{(1)} + 7\mathcal{F}_{\rm gg}^{(6)}) + 4(-2\mathcal{F}_{\rm gg}^{(3)} + \mathcal{F}_{\rm gg}^{(4)} + \mathcal{F}_{\rm gg}^{(5)})}{8N_c^4}$$

$$\vdots$$

Large N_c analysis for arbitrary number of gluons

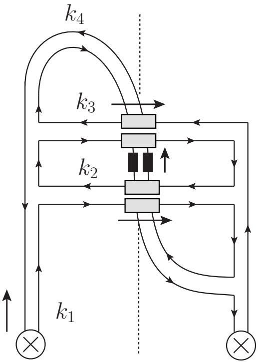


1) $\frac{N_c^{n-3}}{N_c^{n-2}} \mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[-]\dagger}F(0)\,\mathcal{U}^{[+]}\right\}\mathrm{Tr}\mathcal{U}^{[\Box]\dagger} \rightsquigarrow \mathcal{F}_{gg}^{(1)}$

2) $\frac{N_c^{n-4}}{N_c^{n-2}} \mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[+]\dagger}F(0)\,\mathcal{U}^{[+]}\right\}\mathrm{Tr}\mathcal{U}^{[\Box]\dagger}\mathrm{Tr}\mathcal{U}^{[\Box]} \rightsquigarrow \mathcal{F}_{gg}^{(6)}$

3) $\mathcal{F}_{gg}^{(1)}$

Large N_c analysis for arbitrary number of gluons



$$N_c \mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[\Box]}\right\}\mathrm{Tr}\left\{F(0)\,\mathcal{U}^{[\Box]\dagger}\right\} = N_c^2 \mathcal{F}_{gg}^{(2)}\,.$$

Summary

We studied the gauge invariant operators defining the TMD gluon distributions for multiparton processes.

- We calculated TMD gluon distributions for processes with 5 and 6 partons $\rightarrow$ sufficient to calculate 3 and 4 forward jet production, using effective small-x factorization
- We applied color-flow Feynman rules for gauge links and color decompositions
- We found new basic TMD gluon distributions
- We analysed the large N_c limit for multigluon processes

Although TMD factorization does not hold for every process, the effective factorization can be obtained from Color Glass Condensate effective theory