Pion nucleus Drell-Yan process and parton transverse momentum in the pion

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Based on F.A.C., A. Courtoy, S. Noguera and S. Scopetta, EPJ C78 (2018) no.8, 644

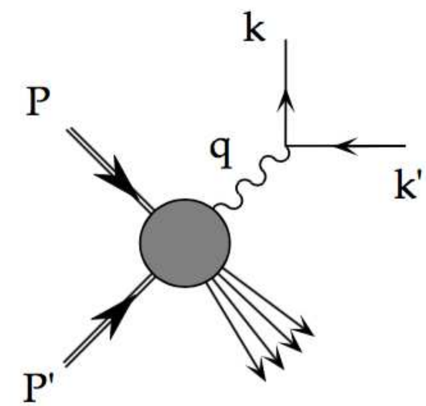
Motivation

- In hadronic collisions, Drell-Yan pair production is the only process for which factorization is theoretically proven at all orders .
- Thanks to a long theoretical effort pursued up to present days, DY is known with high accuracy in perturbation theory for a wide number of observables.
- Therefore DY production in πW collisions is an ideal prototype process to study the partonic structure of pions.
- In particular, the study of the DY q_T-spectrum at low q_T provides test:
  - resummation techniques in perturbation theory;
  - the study of test non perturbative model of the pion transverse structure.

q_T differential DY cross section (1)

- DY process proceeds via the production of a virtual photon with large invariant mass Q^2 and transverse momentum q_T in the collision of two hadrons $h_{1,2}$:

  $h_1(p_1)\ h_2(p_2) \to \gamma^*(q) + X$



- When q_T^2 becomes small compared to Q^2, large logarithmic corrections of the form of $\alpha_s^n \log^m(Q^2/q_T^2)$ with $0 \leq m \leq 2n-1$ appear at every fixed order in the perturbative expansion, making it unreliable.

- These large logarithmic corrections can be resummed to all orders by using the Collins-Soper-Sterman (CSS) formalism, giving a finite result for the cross section at any given logarithmic accuracy

q_T differential DY cross section (2)

- In the small q_T limit the cross section reads:

$$\frac{d\sigma}{dq_T^2 d\tau dy} = \sum_{a,b} \sigma_{q\bar{q}}^{(LO)} \int_0^\infty db \frac{b}{2} J_0(b\, q_T)\, S_q(Q,b)\, S_{NP}^{h_1 h_2}(b) \cdot$$

$$\cdot \Big[\left(f_{a/h_1} \otimes C_{qa}\right) \left(x_1, \frac{b_0^2}{b^2}\right) \left(f_{b/h_2} \otimes C_{\bar{q}b}\right) \left(x_2, \frac{b_0^2}{b^2}\right) + q \leftrightarrow \bar{q} \Big] + Y \,.$$

- resummation is naturally performed in the space b, Fourier conjugated to q_T
- $b_0 = 2e^{-\gamma_e}$, $\otimes$ stands for convolution
- $\sigma_{q\bar{q}}^{(LO)}$ is the leading-order total partonic cross section for producing a lepton pair, $\sigma(q\bar{q} \to l^+ l^-)$,
- $J_0(b\, q_T)$ is the Bessel function of first kind
- $f_{i/h}$ corresponds to the distribution of a parton i in a hadron h, evaluated at a scale $\mu_F = b_0/b$.
- Kinematical variables are expressed as

$$\tau = Q^2/s, \qquad x_{1(2)} = \sqrt{\tau} e^{\pm y}, \qquad y = \frac{1}{2} \ln \frac{x_1}{x_2}, \qquad x_F = x_1 - x_2 = 2q_{\|}/\sqrt{s}$$

- Y-term takes into account the large q_T constribution (neglected in this analysis)

Perturbative form factor : NLL accuracy

- The large logarithmic corrections are exponentiated in b-space in the Sudakov perturbative form factor

$$S_q(Q,b) = \exp\left\{ -\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \left[A(\alpha_s(q^2)) \ \ln\frac{Q^2}{q^2} + B(\alpha_s(q^2)) \right] \right\}$$

- The functions A, B and C_{ab} all have perturbative expansions in α_s:

$$A(\alpha_s) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^n A^{(n)}, \quad B(\alpha_s) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^n B^{(n)} \quad ,$$

$$C_{ab}(\alpha_s, z) = \delta_{ab}\,\delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^n C_{ab}^{(n)}(z)$$

- the perturbative Sudakov form factor can be evaluated at next-to-next-to-leading logarithmic (NNLL) accuracy (de Florian, Grazzini, 2000)
- In the $q\bar{q}$ annihilation channel pertinent to Drell-Yan production, the evaluation of the Sudakov form factor to next-to-leading logarithmic (NLL) accuracy, the one reached in the present analysis, involves the coefficients:

$$A^{(1)} = 2C_F \quad B^{(1)} = -3C_F$$

NLL accuracy (2)

- the latter are the coefficient of the singular $(1-z)^{-1}$ and $\delta(1-z)$ terms of the one-loop splitting function $P^{(0)}_{qq}(z)$
- to NLL one needs also

$$A^{(2)} = K A^{(1)} \quad K = C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - n_f T_R \frac{10}{9}$$

which is coefficient of the singular term of the two-loop splitting function $P^{(1)}_{qq}(z)$ in the $z \to 1$ limit (Kodaira Trentadue '82)

- The general expression for $C^{(1)}_{ab}$ are given by (Davies, Stirling '84)

$$C^{(1)}_{qa}(z) = C^{(1)}_{\bar{q}b}(z) = \delta_{qa} C_F\,(1-z) + \delta_{qa}\,\delta(1-z) C_F\left(-4 + \frac{\pi^2}{2}\right),$$

$$C^{(1)}_{qg}(z) = C^{(1)}_{\bar{q}g}(z) = 2T_R\, z(1-z)$$

- Together with the use of NLO pdfs, this guarantees the evaluation of the cross section at small q_T at NLL accuracy.

Non Perturbative form factor

- The last ingredient in the cross section is the non perturbative form factor, $S_{NP}^{h_1 h_2}(b)$

$$\frac{d\sigma}{dq_T^2 d\tau dy} = \sum_{a,b} \sigma_{q\bar{q}}^{(LO)} \int_0^\infty db \frac{b}{2} J_0(b\, q_T)\, S_q(Q,b)\, S_{NP}^{h_1 h_2}(b) \cdot$$
$$\cdot \Big[\Big(f_{a/h_1} \otimes C_{qa} \Big) \left(x_1, \frac{b_0^2}{b^2} \right) \Big(f_{b/h_2} \otimes C_{\bar{q}b} \Big) \left(x_2, \frac{b_0^2}{b^2} \right) + q \leftrightarrow \bar{q} \Big] .$$

- It encodes the transverse structure of both the colliding hadrons, h_1 and h_2, respectively.
- The latter is either:
  - fixed by comparison with data or
  - parametrized with the help of hadronic models, as we shall do in this analysis.
- In our case we take the proton structure from the literature
- The pion strucutre entirely extracted from NJL model

The KN05 proton structure

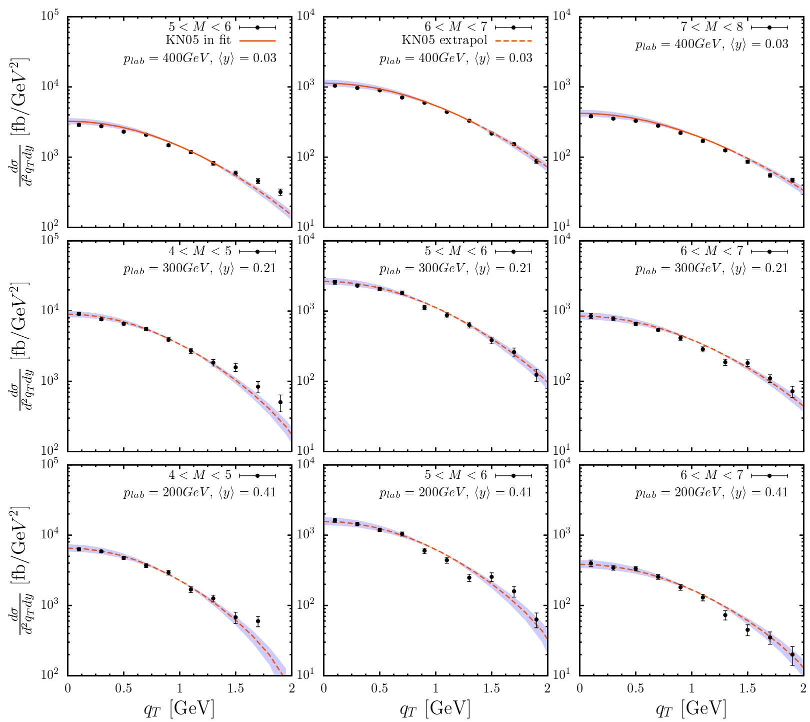
- KN05 (Konichev & Nadolsky '05) performed a fit to low q_T DY and Z-boson data, to fix $S_{NP}^{pp}(b)$ to NLL accuracy

- DY pA data (Ito 1980) at $p_{lab} = 400$ GeV $q_T < 1.4$ GeV, $5 < M/\text{GeV} < 9$ were included in the fit

- large-b treatment: $b_\star(b, b_{max}) = \dfrac{b}{\sqrt{1+\left(\frac{b}{b_{max}}\right)^2}}$

  with $b_{max} = 1.5\,\text{GeV}^{-1}$

- theory (dashed) extrapolates well at lower $\sqrt{s}$ and higher DY rapidity ($0 < x_F < 0.3$) up to $q_T \sim 2$ GeV.

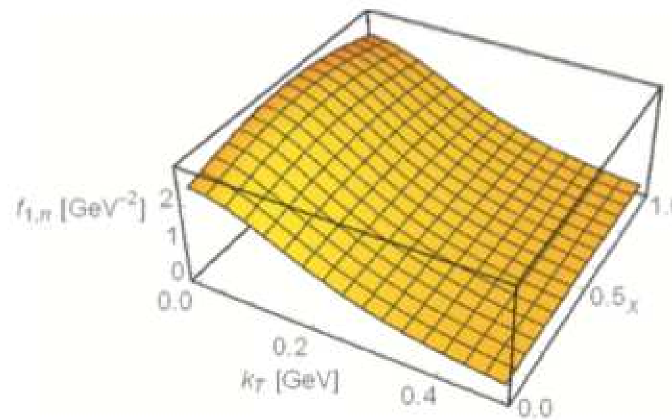


- $S_{NP}^{pp}(b) = \exp\{-[a_1 + a_2 \ln(M/(3.2\,\text{GeV})) + a_3 \ln(100 x_1 x_2)]b^2\}$
  $a_1 = 0.201 \pm 0.011, \quad a_2 = 0.184 \pm 0.018, \quad a_3 = -0.026 \pm 0.007\,.$

The NJL pion TMD (1)

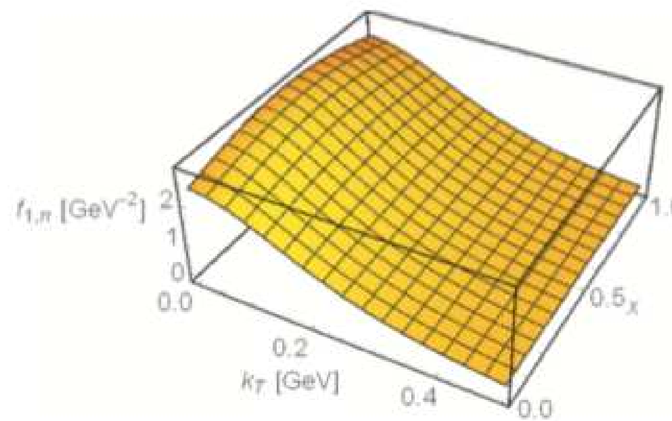
- unpolarized pion TMDs in a NJL framework, with Pauli-Villars regularisation (Noguera and Scopetta '15)
- $\int dx\; x\; q\left(x_\pi, Q_0^2\right) = 0.5$, with Q_0 hadronic scale associated to the model
- $\boldsymbol{k_T}$ dependence is generated by the NJL dynamics
- pion TMD obtained in the chiral limit:
  - excellent approximation to the NJL full result;
  - it allows for a factorisation of the x and k_T dep at Q_0:

  $f^{q/\pi}(x_\pi, \boldsymbol{k_T}, Q_0^2) = q(x_\pi, Q_0^2) T(\boldsymbol{k_T})$
- it satisfies the normalisation $\int d^2\boldsymbol{k_T}\, T(\boldsymbol{k_T}) = 1$
- $q(x_\pi, Q_0^2) = d_v(x_\pi, Q_0^2) = \bar{u}(x_\pi, Q_0^2) = 1$
- $T(\boldsymbol{k_T}) = \frac{3}{4\pi^3}\left(\frac{m}{f\pi}\right)^2 \sum_{i=0,2} \frac{c_i}{k_T^2 + m_i^2}$
- it behaves as k_T^{-6} at large k_T (very different from the frequently assumed gaussian)



The NJL pion TMD (2)

- The various parameters are given by:
  $m_0^2 = m^2 = (0.238\ \text{GeV})^2$
  $m_1^2 = m^2 + \Lambda^2$
  $m_2^2 = m^2 + 2\Lambda^2$
  $\Lambda = 0.860$ GeV
  $c_0 = 1, \quad c_1 = -2, \quad c_2 = 1,$
  $f_\pi = 0.0924 \quad$ GeV.



- its 2D Fourier Transform is given by

$$
\begin{aligned}
S_{NP}^{\pi}(b) &= \frac{3}{2\pi^2}\left(\frac{m}{f_\pi}\right)^2 \sum_{i=0,2} \int dk_T\, k_T\, J_0(bk_T) \frac{c_i}{k_T^2 + m_i^2} \\
&= \frac{3}{2\pi^2}\left(\frac{m}{f_\pi}\right)^2 \sum_{i=0,2} c_i K_0(m_i\, b)\,,
\end{aligned}
$$

where K_0 is the modified Bessel function of the second kind.

More on individuals NP b-profiles

- $S_{NP}^{\pi W}(b) = S_{NP}^{\pi}(b)\sqrt{S_{NP}^{pp}(b)}$
- Compare with fitted profile from WLS (Wang, Lu and Schmidt '17)

◇ Pion g's fitted:
$S_{NP}^{\pi}(Q,b) = \frac{g_1^{\pi}}{2}b^2 + \frac{g_2^{\pi}}{2}\ln\frac{b}{b_*}\ln\frac{Q}{Q_0}$

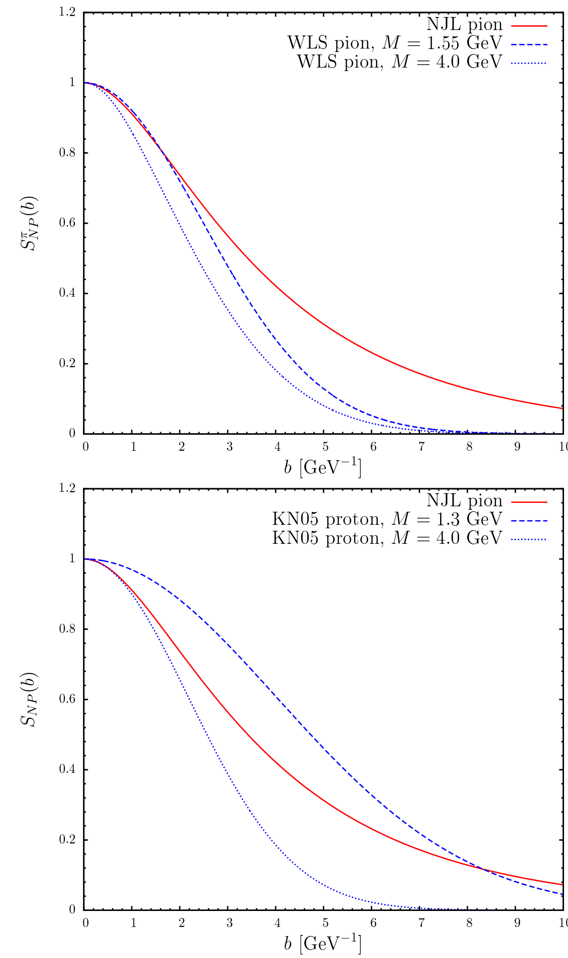
◇ proton from Sun , Isaacson, C.P. Yuan, F.Yuan '14:
$S_{NP}^{p}(Q,b) = \frac{g_1}{2}b^2 + \frac{g_2}{2}\ln\frac{b}{b_*}\ln\frac{Q}{Q_0}$

◇ NJL vs WLS: WLS falls more rapidly than NJL at large b:
it predicts a larger partonic intrinsic k_T vs NJL.

◇ WLS shows mild Q dependence

- NJL vs KN05 proton: large M dependence, absent in NJL
$S_{NP}^{pp}(b) = \exp\{-[a_1 + a_2\ln\frac{M}{3.2\,\text{GeV}} + a_3\ln(100x_1x_2)]b^2\}$

◇ KN05 vs NJL at $M = 4$ GeV: very small difference at small b



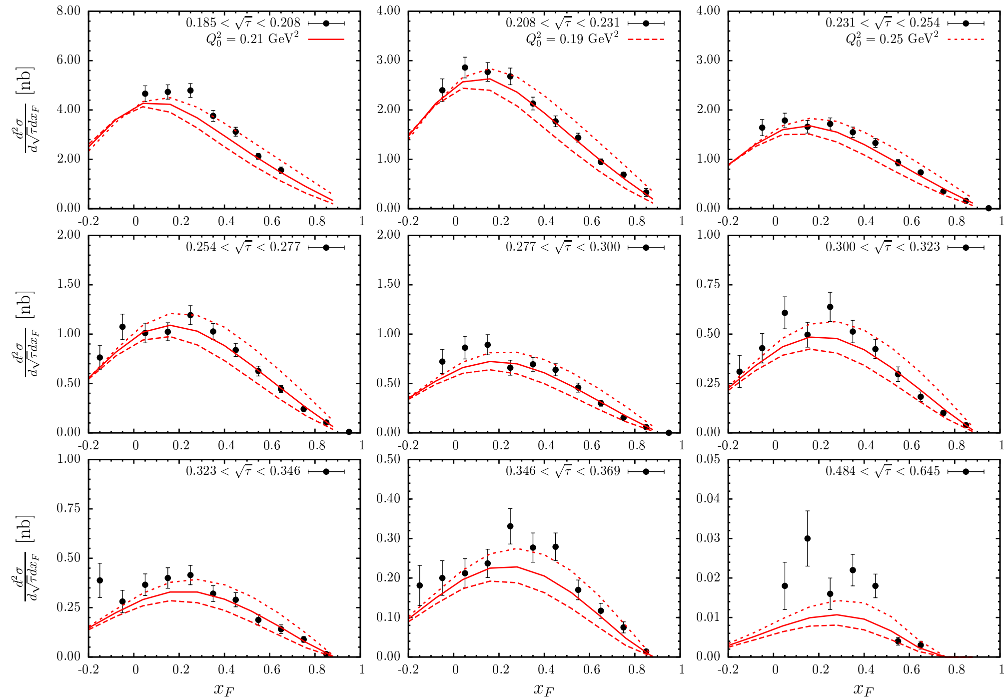
More on the perturbative Sudakov form factor

- large-b treatment as in KN05: $b_\star(b, b_{max}) = \frac{b}{\sqrt{1+\left(\frac{b}{b_{max}}\right)^2}}$
- KN05 proton : $b^p_{max} = 1.5\ \mathrm{GeV}^{-1}$
- NJL pion : $b^\pi_{max} \sim b_0/Q_0 \sim 2.44\ \mathrm{GeV}^{-1}$
- sensitivity to this choice explored by setting $b^\pi_{max} = b^p_{max} = 1.5\ \mathrm{GeV}^{-1}$
- These setting used also in pdf's calls: $\mu_F = b_0/b_*$
- In order to accomodate all these different settings, the perturbative Sudakov is evenly splitted:

$$
\begin{aligned}
S_q(Q,b) \equiv S_q(Q, b_*, b^p_{max}, b^\pi_{max}) \quad &= \quad \exp\left\{ -\frac{1}{2} \int_{\frac{b_0^2}{b_*^2(b^p_{max})}}^{Q^2} \frac{dq^2}{q^2} \left[A(\alpha_s(q^2)) \, \ln \frac{Q^2}{q^2} + B(\alpha_s(q^2)) \right] \right\} \\
\times \quad & \quad \exp\left\{ -\frac{1}{2} \int_{\frac{b_0^2}{b_*^2(b^\pi_{max})}}^{Q^2} \frac{dq^2}{q^2} \left[A(\alpha_s(q^2)) \, \ln \frac{Q^2}{q^2} + B(\alpha_s(q^2)) \right] \right\} .
\end{aligned}
$$

Determination of NJL's scale

- The hadronic scale Q_0 of the NJL model not determined $\rightarrow$ tune theory to data
- Exp : E615 $\pi^- W$ at $p_{lab} = 252$ GeV. DY x-sections differential in $\sqrt{\tau}$ and y, Conway '89
- Theo: NLO $\overline{\text{MS}}$, Sutton '92
  - NLO pion pdf's from NJL, evolved with QCDNUM, Botje '11
  - NLO nuclear pdf's from CTEQ10
- Results: we find good agreement at large x_F, *i.e.* valence quark in the pion, where NJL is expected to work better.



- The tuning gives $Q_0^2 = 0.21$ GeV^2 with $\chi^2/dof \sim 2$

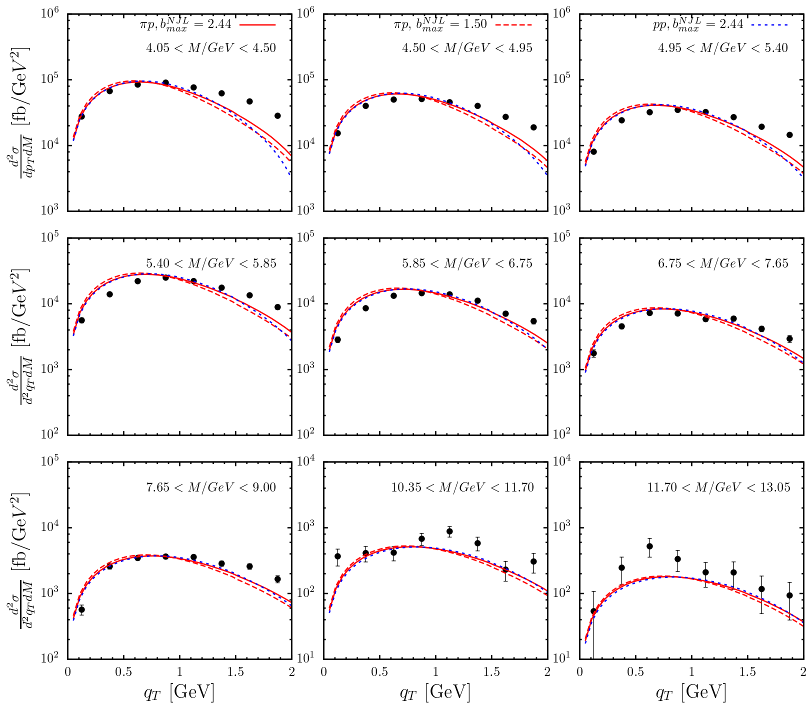
q_T spectrum vs M

- Exp : E615 $\pi^- W$ at $p_{lab} = 252$ GeV.
  DY differential q_T-sections integrated over $0 < x_F < 1$ and given mass intervals (Conway '89)

- The quality of the description slight improves moving to higher masses.
  Increased effect of PT Sudakov

- More in detail:
  - theory slight overshoots data at small q_T
  - theory undershoots data at larger q_T

- Stability:
  - Theory almost insensitive to b_{max} choices on NJL side.
  - if NJL pion substituted with KN05 proton: theory similar, b-profiles similar at small b at $M = 4$ GeV



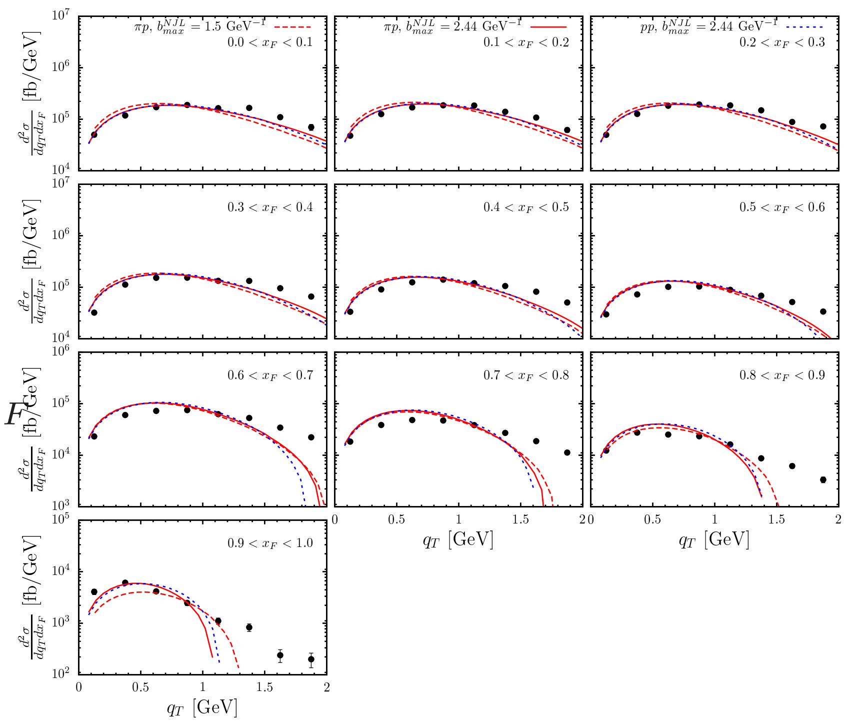
q_T spectrum vs x_F

- Exp : E615 $\pi^- W$ at $p_{lab} = 252$ GeV.
  DY differential q_T-sections integrated over $4 < M/GeV < 8.55$ and given x_F intervals (Conway '89)

- The quality of the description deteriorates with increasing x_F

- More in detail:
  - theory slight overshoots data at small q_T at all x_F
  - theory undershoots data at larger q_T, this trend is more pronounced as x_F increases : large x partons in the projectile (pion).

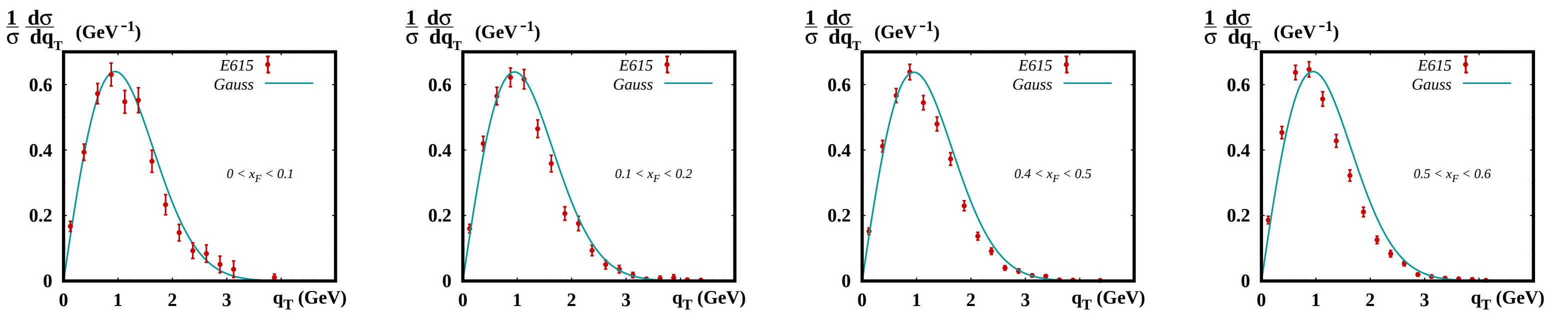
- q_T-sections in x_F-bins are extremely useful to explore eventual x-dependence of NP and PT Sudakov form factor



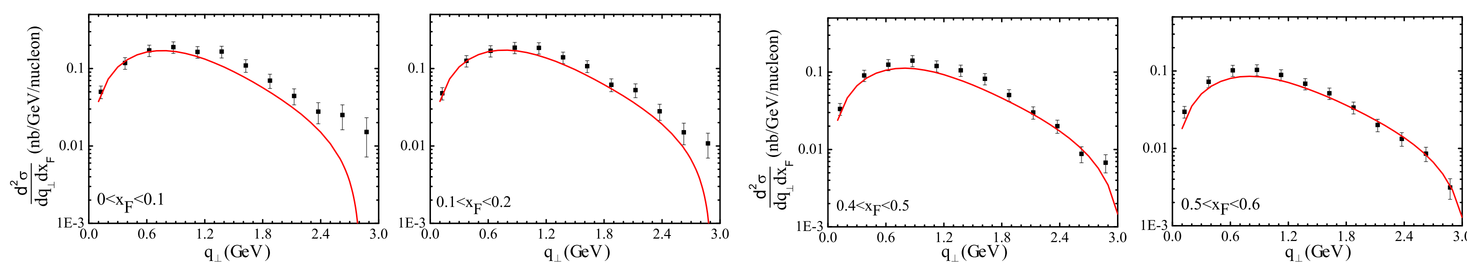
- Overall, NJL gives a reasonbly good description of data (say, up to $q_T \sim 1$ GeV)

Comparison with other studies

- Fit to E615 πW data with Gaussian models, No TMD evo (Pasquini, Schweitzer '14)

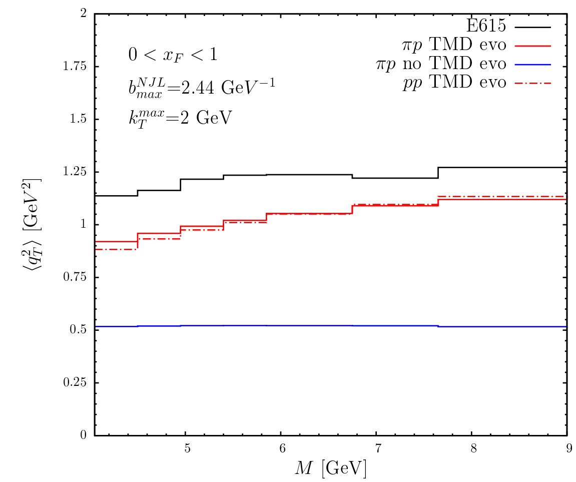


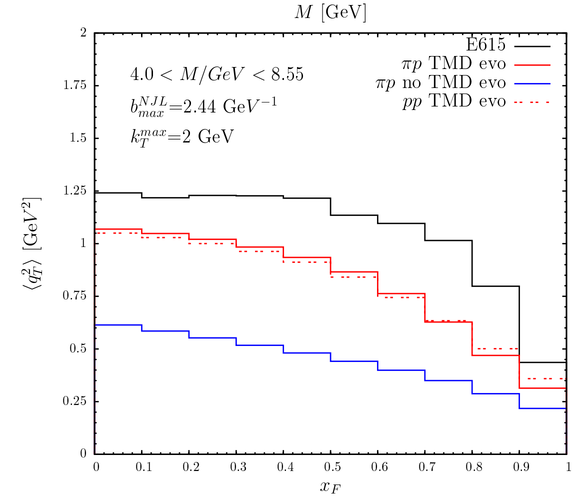
- Full TMD fit to E615 πW data at small q_T at NLL (Wang, Lu and Schmidt '17)



Averaged transverse momentum $\langle q_T^2 \rangle$

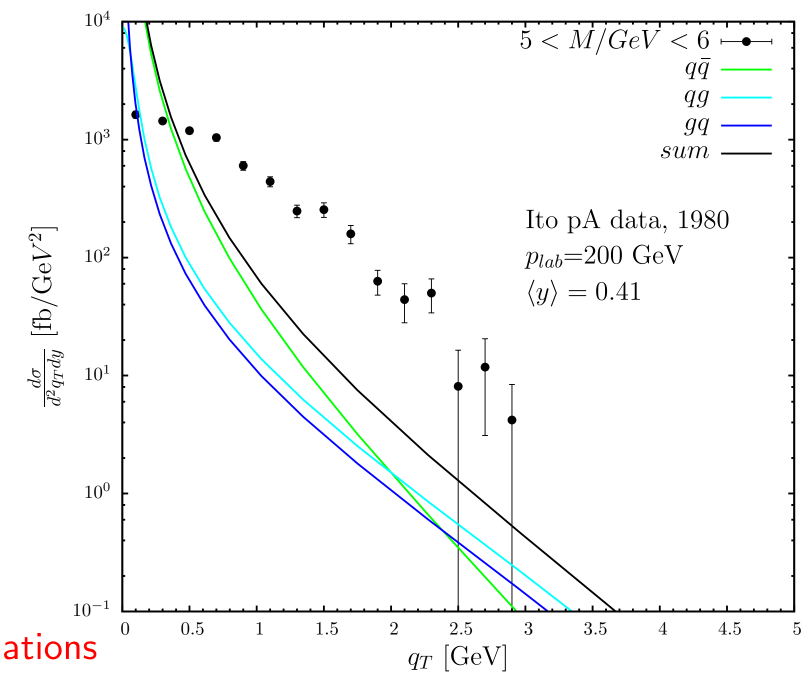
- $\langle q_T^2 \rangle$ vs x_F integrated in $4.0 < M/\text{GeV} < 8.55$.
- $\langle q_T^2 \rangle$ vs M in the range $0 < x_F < 1$.
- red $=$ with Sudakov, blue $=$ no Sudakov, black $=$ data
- Averaged values are obtained integrating both predictions and the phenomenological parametrisation of the data up to $q_T^{max} = 2$ GeV.
- mild (log?) increase as M increases (qualitatively reproduced)
- shrinkage at high x_F (qualitatively reproduced)
- black-vs-red : deficit due to missing high q_T contribution from fixed order contribution
- blue-vs-red: large contribution to $\langle q_T^2 \rangle$ from Sudakov





The large q_T problem (1)

- At large q_T, pQCD FO (collinear) calculations should reproduce the q_T-spectrum well
- This indeed happens at high centre-of-mass energy, i.e. Z-boson production at Tevatron, LHC
- at low $\sqrt{s}$, say $\sqrt{s} < 60$ GeV, theory systematically undershoots data at large q_T , say $q_T > Q$.
- without the tail, one is not able to establish where the transition between TMD and collinear regimes occurs, therefore one is unable to judge the goodness of TMD NP input.
- Including higher order corrections $\mathcal{O}(\alpha_s^2)$ or other resummation (joint $q_T - y$) helps just a little



- effect not covered neither by pdf's error nor scale variations
- This effect has been spotted by various groups see INT Workshop "Probing Nucleons and Nuclei in High Energy Collisions" in Seattle, presentations by N.Sato, F.Piacenza and G.Bozzi `http://www.int.washington.edu/talks/WorkShops/int_18_3/`
- NB: effect not visible on q_T-integrated cross sections since the integral of the tail is small

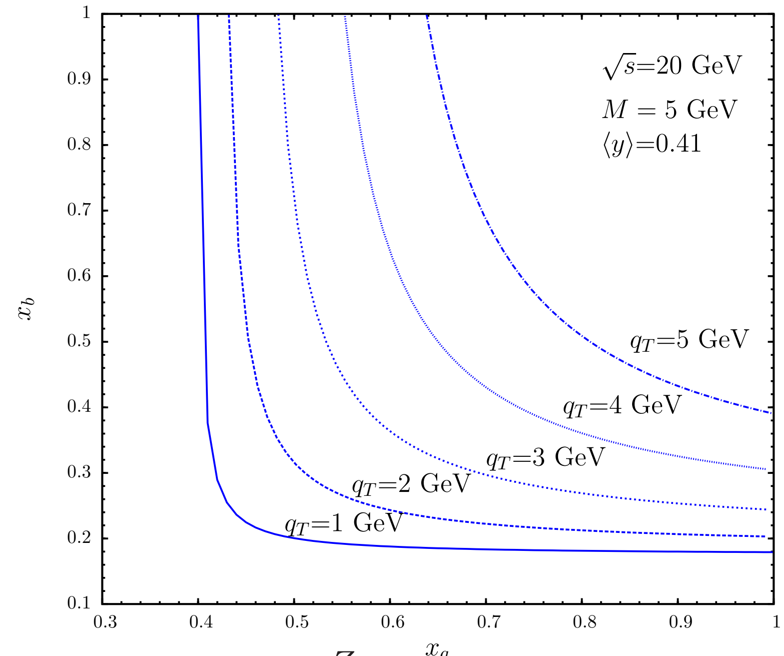
The large q_T problem (2)

- high q_T DY cross section in a nutshell:

$$\frac{d\sigma}{dM^2 dy dq_T^2} = \sum_{ab} \int_{x_{a,min}}^{1} dx_a \frac{x_a x_b}{x_a - x_1} f_a(x_a, \mu_F^2) f_b(x_b, \mu_F^2) \frac{d\hat{\sigma}}{dM^2 d\hat{t}}$$

$$x_{1,2} = \frac{\sqrt{M^2 + q_T^2}}{\sqrt{s}} e^{\pm y}, \quad x_{a,min} = \frac{x_1 - \tau}{1 - x_2}$$

$$\frac{x_2 - \tau}{1 - x_1} < x_b = \frac{x_a x_2 - \tau}{x_a - x_1} < 1$$



- as q_T increases, so do the x's
- NB: at large q_T, the q_T-spectrum is dominated by Compton qg term
- problems in low Q, large x gluons?
- Test: including these data in global PDF fits would lead to inconsistencies?
- NB : NNPDF 3.1 includes in their fit data from $pp \to ZX$ at LHC with $q_T^Z > 30$ GeV

Conclusions & Outlook

- We have presented a study of the q_T-spectrum of DY produced in πW collisions
- proton TMD from literature $+$ pion TMD from NJL
- perturbative accuracy: next-to-leading logs at small q_T
- once fixed Q_0 of NJL model, (almost) parameter free predictions for the q_T spectrum of DY pairs produced in πW collisions
- Good results are obtained at low q_T, NJL successful, however ...
- fixed order calculations gives very poor description of the high q_T tail:
  - matching with resummed prediction at small q_T is precluded
  - no conclusive statement on the goodness of NP models can be given