Parton distribution functions from Lattice QCD

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in collaboration with:

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Aurora Scapellato (Univ. of Cyprus, Univ. of Wuppertal)
Fernanda Steffens (Univ. of Bonn)

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Outline of the talk

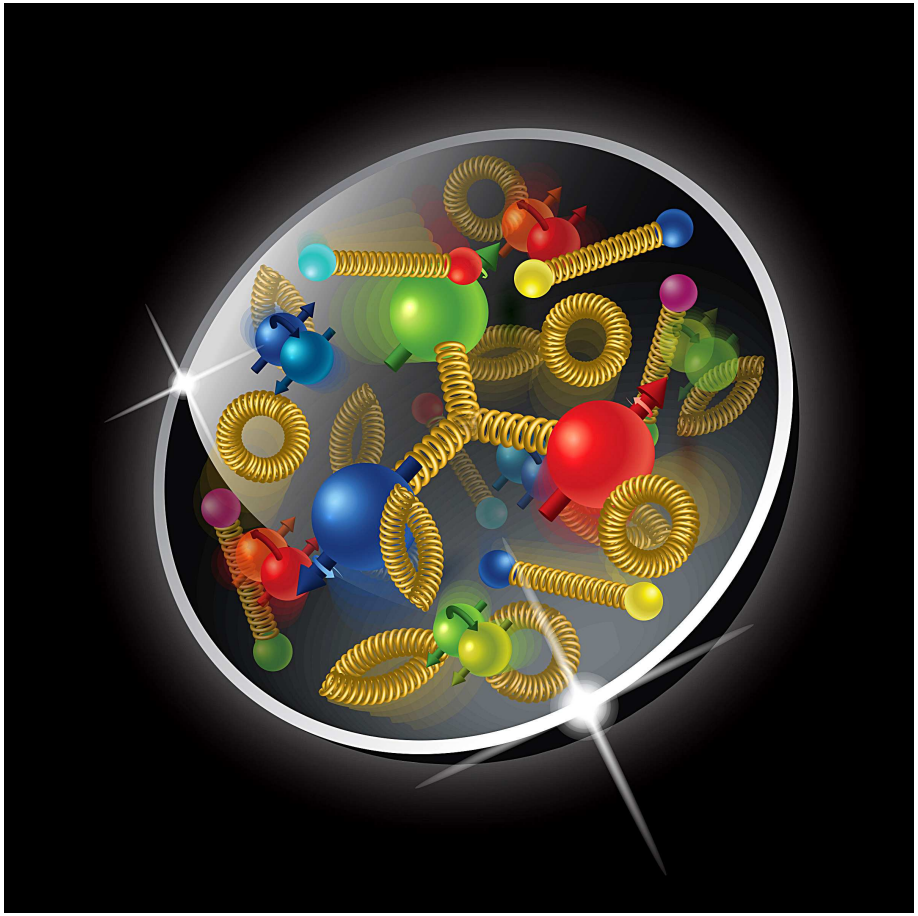
1. Lattice QCD
   - Why do we need this?
   - Lattice formulation of QCD
   - QCD simulations
   - What can we compute?
2. Parton distribution functions
   - Basics
   - Lattice calculation
   - Results
3. Conclusions and prospects

Results on PDFs based on:

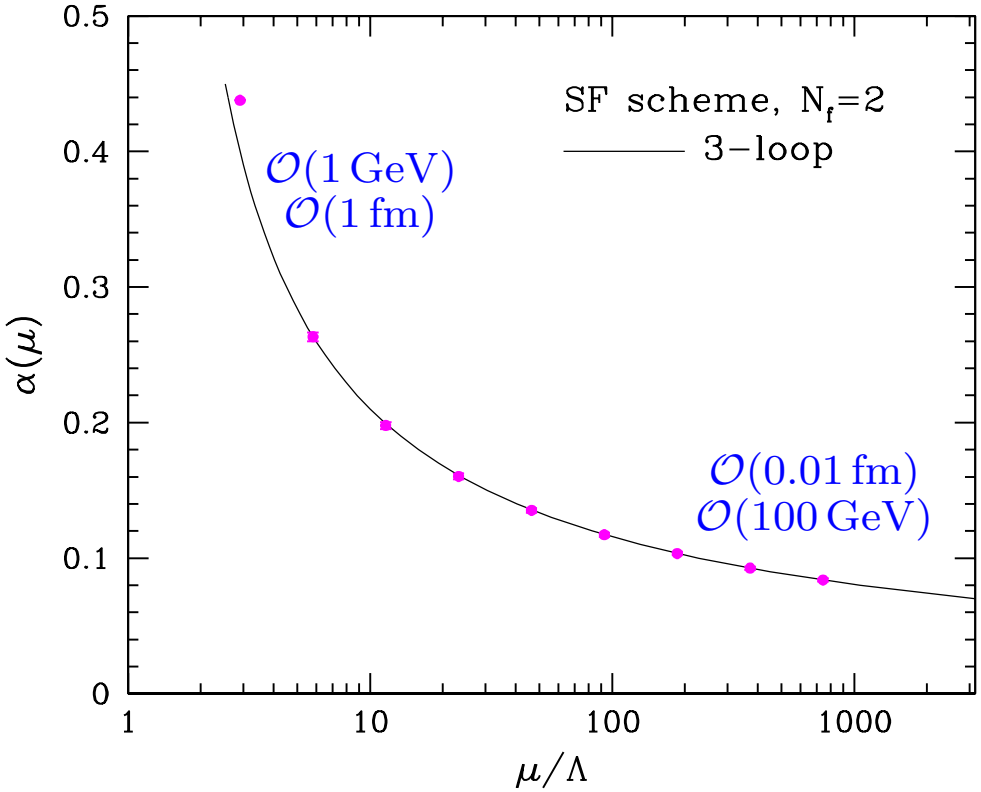
- **C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, "Reconstruction of light-cone parton distribution functions from lattice QCD simulations at the physical point", Phys. Rev. Lett. 121 (2018) 112001**
- **C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, "Transversity parton distribution functions from lattice QCD", Phys. Rev. D (Rapid Communications), in press, arXiv: 1807.00232 [hep-lat]**
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagopoulos, F. Steffens, "A complete non-perturbative renormalization prescription for quasi-PDFs", Nucl. Phys. B923 (2017) 394-415 (invited Frontiers Article)
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, F. Steffens, C. Wiese, "Updated Lattice Results for Parton Distributions", Phys. Rev. D96 (2017) 014513
- C. Alexandrou, K. Cichy, V. Drach, E. Garcia-Ramos, K. Hadjiyiannakou, K. Jansen, F. Steffens, C. Wiese, "A Lattice Calculation of Parton Distributions", Phys. Rev. D92 (2015) 014502

Lattice QCD

QCD and the need for the lattice



ALPHA Collaboration, 2004



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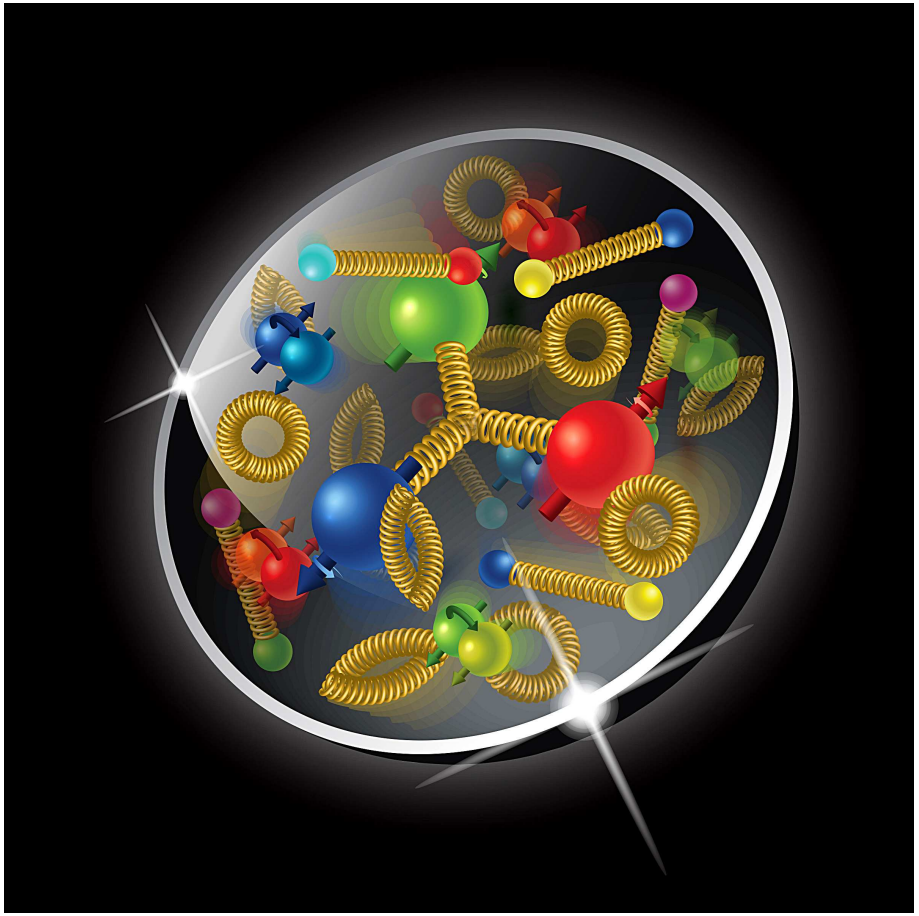

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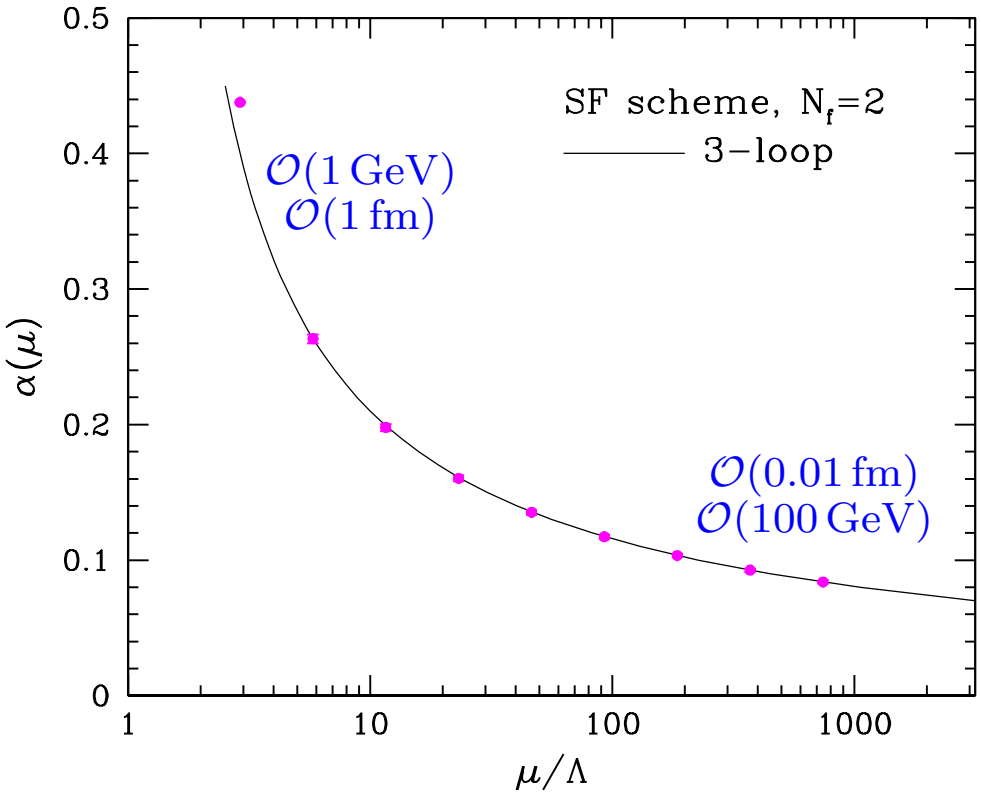

NON-PERTURBATIVE REGIME

$\Rightarrow$

quantitative study needs LATTICE

QCD and the need for the lattice

Lagrangian of QCD:

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \sum_{(f)=1}^{N_f} \bar{\psi}_{(f)} \left(i\gamma^\mu D_\mu - m_{(f)} \right) \psi_{(f)}$$

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Note:

- Minkowski path integral can not be used in practice – the phase factor e^{iS} would lead to oscillatory behaviour.
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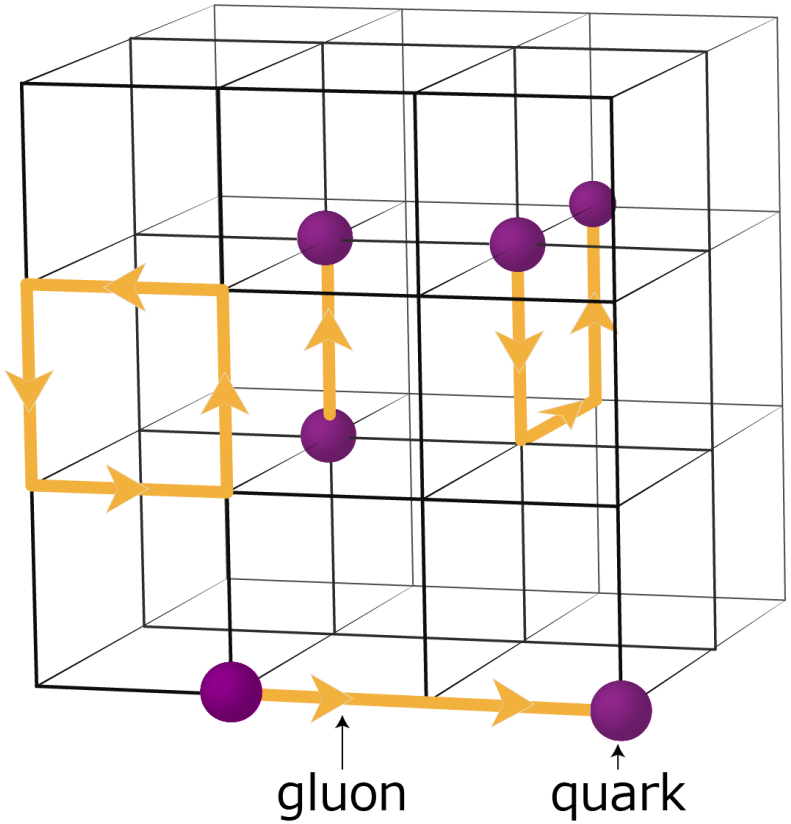
- The integral is not well-defined, unless a regulator is introduced $\rightarrow$ **finite space-time lattice** (discretization of the theory).

Lattice formulation



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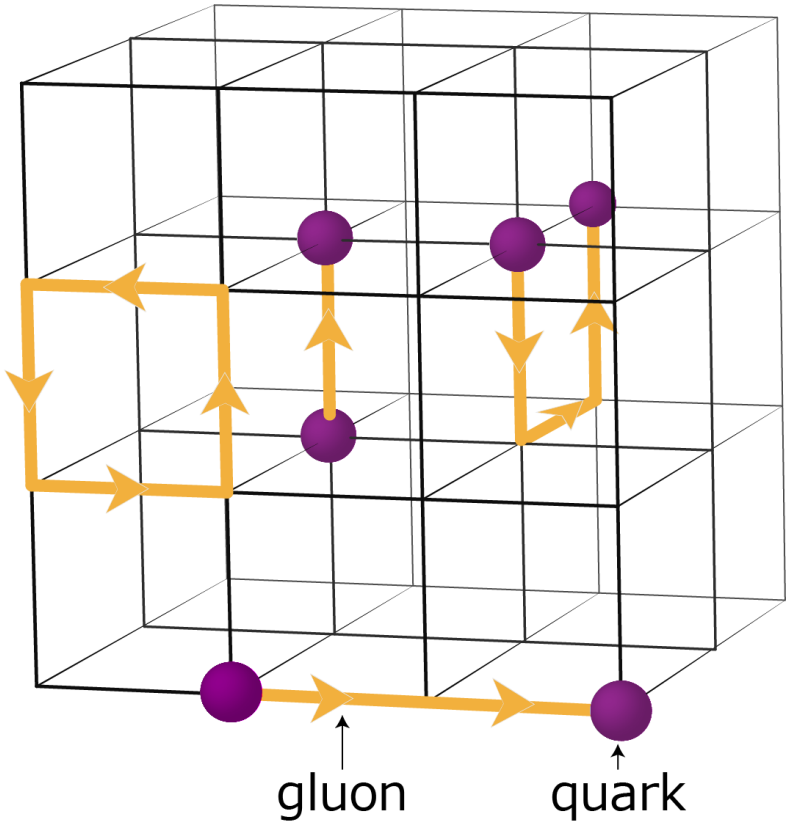

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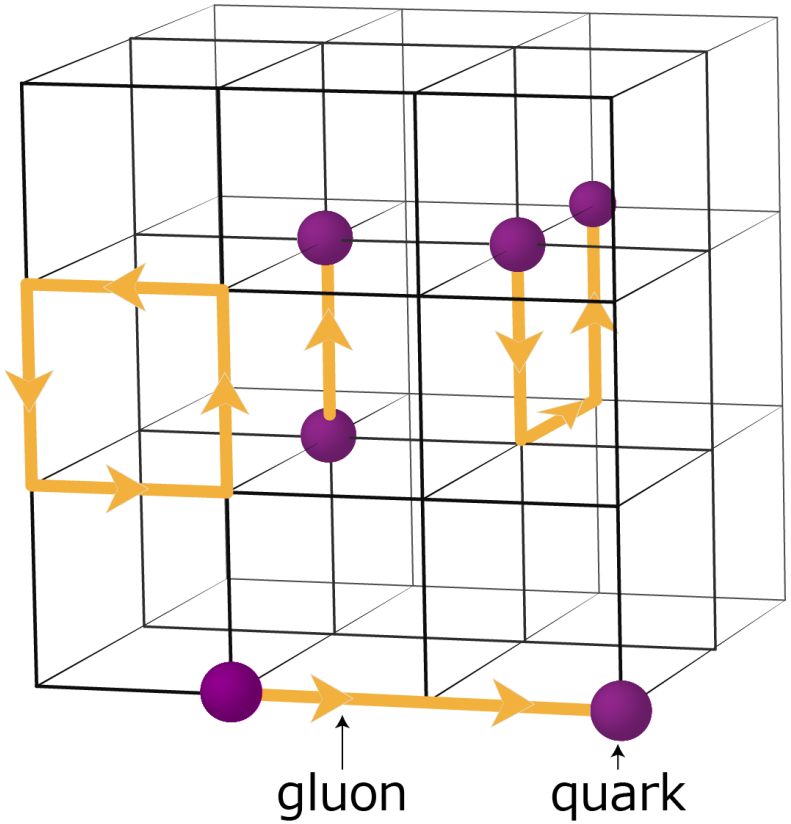

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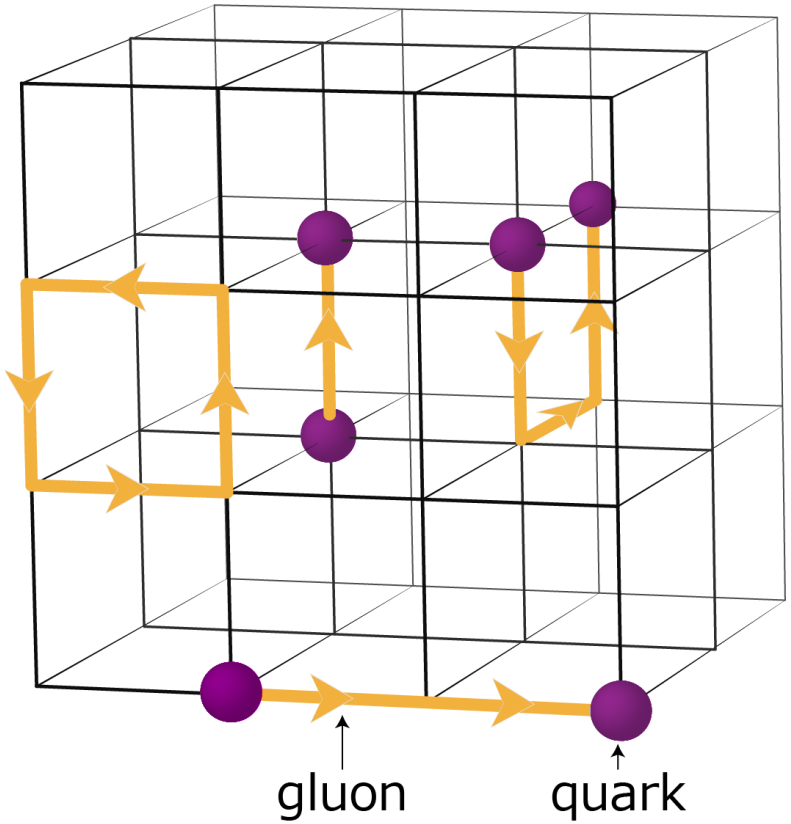

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- Remove the regulator:
  - ⋆ continuum limit $a \to 0$,
  - ⋆ infinite volume limit $L \to \infty$.

Discretization of the action

- gluonic part – "easy" – gauge action constructed from Wilson loops of size 1x1 (plaquettes) and 1x2 (rectangles):

$$S_G[U] = \frac{\beta}{3} \sum_x \Big(b_0 \sum_{\mu,\nu=1} \mathrm{Re}\,\mathrm{Tr}\big(1 - P^{1\times1}_{x;\mu,\nu}\big) + b_1 \sum_{\mu\neq\nu} \mathrm{Re}\,\mathrm{Tr}\big(1 - P^{1\times2}_{x;\mu,\nu}\big) \Big),$$

where $\beta = 6/g_0^2$, g_0 is the bare coupling and the b_0, b_1 parameters are normalized according to: $b_0 = 1 - 8b_1$.

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  - ⋆ discretizations used in practice:
    - ◇ clover fermions,
    - ◇ twisted mass (TM) fermions,
    - ◇ overlap fermions,
    - ◇ domain wall fermions,
    - ◇ staggered fermions,
    - ◇ other less popular.

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- Hence, huge computational resources needed!
- **QCD was one of the first branches of science that "asked" for such computational resources and thus inspired the development of supercomputers.**

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Lattice QCD

Some of the aspects of QCD that can be studied on the lattice:

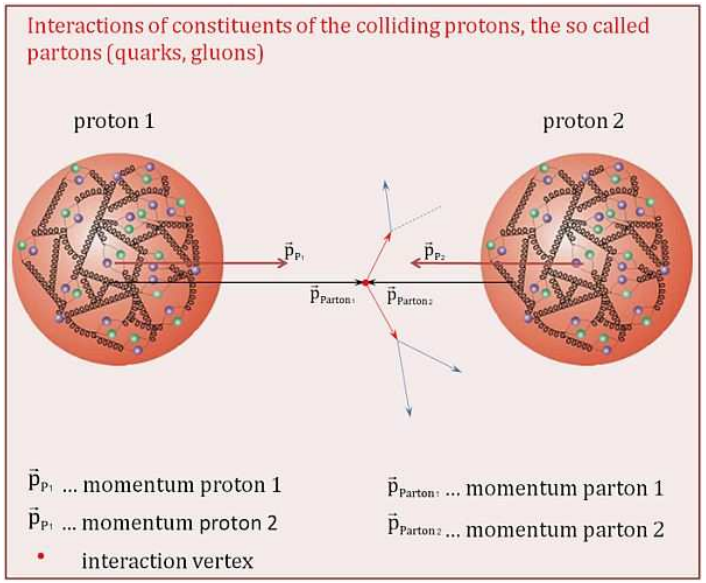
- QCD parameters: α_s, Λ_{QCD}, quark masses etc.
- hadron spectrum: meson and baryon masses, exotic hadrons
- hadron structure: nucleon charges, EM form factors, parton distribution functions, GPDs, nucleon spin content
- QCD thermodynamics: QCD phase diagram, deconfinement, chiral symmetry restoration
- Standard Model parameters: CKM matrix
- constraints on effective theories: χPT, HQET

Some collaborations in LQCD:
Alpha, BMW, CLS, CP-PACS, ETMC, HALQCD, hotQCD, JLQCD, LHC, LSD, Mainz, MILC, NME, NPLQCD, QCDSF, PNDME, RBC, RQCD, SWME, tmFT, TWQCD, UKQCD, USQCD, WHOT-QCD
in total $\approx 500 - 600$ physicists

Parton distribution functions (PDFs)

PDFs

- Hadrons are complicated systems with properties resulting from the strong dynamics of quarks and gluons inside them.



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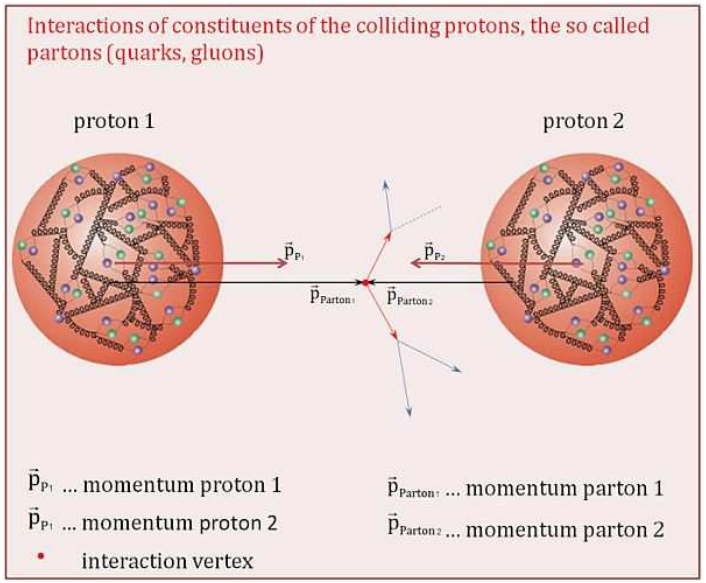

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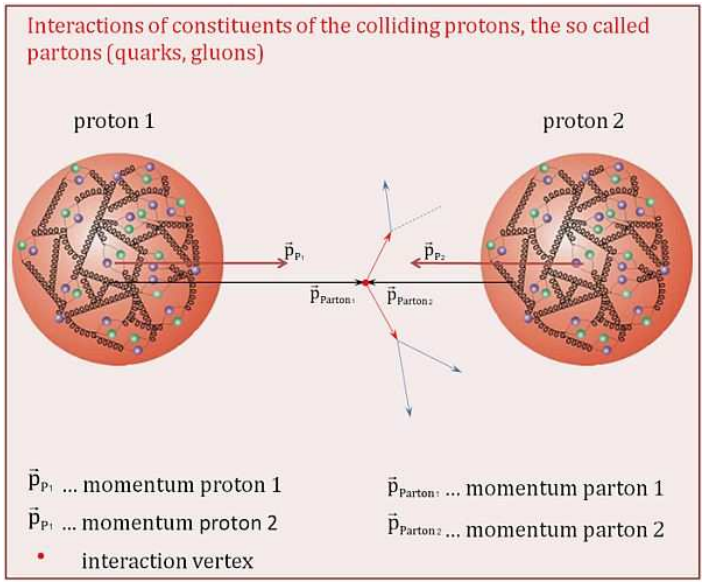

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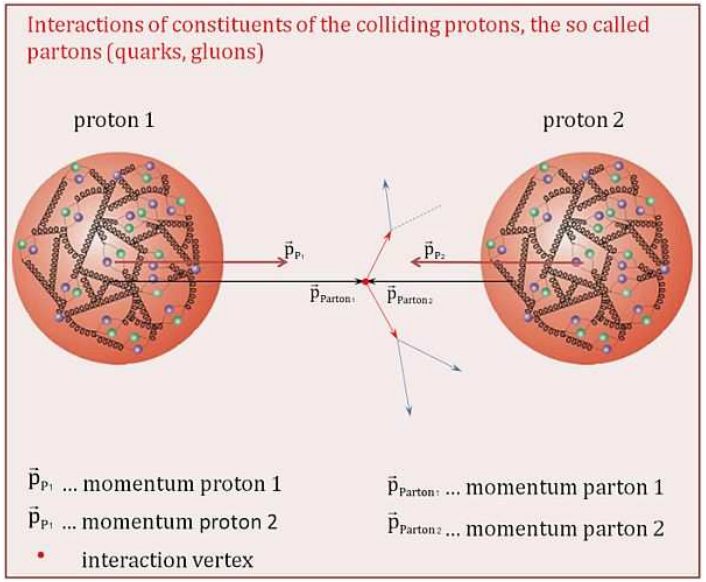

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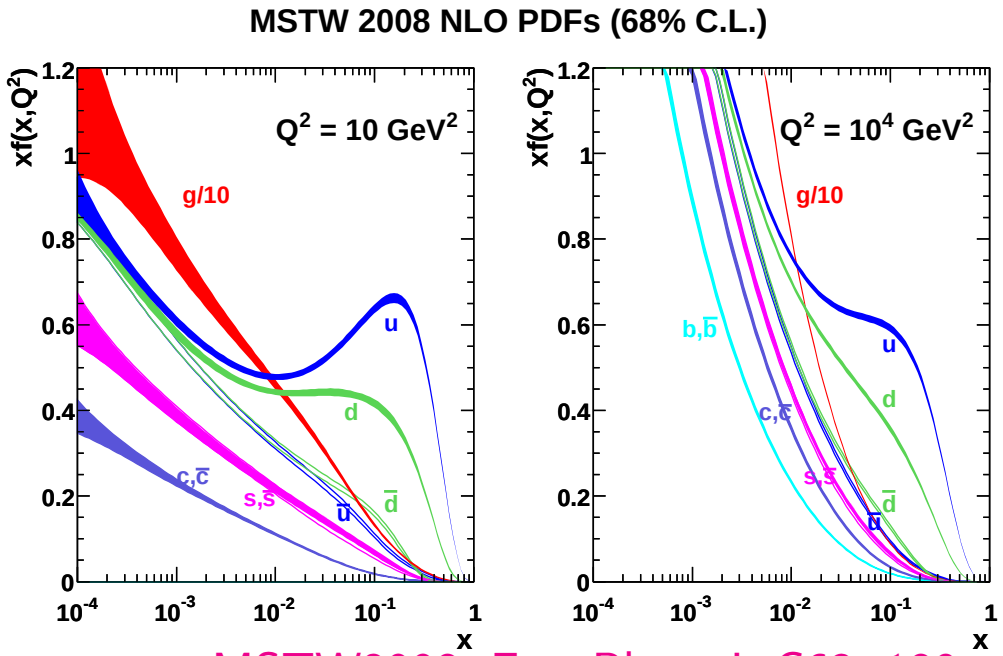

MSTW2008, Eur. Phys. J. C63, 189

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$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle,$$

where: $\xi^- = \frac{\xi^0 - \xi^3}{\sqrt{2}}$ and $\mathcal{A}(\xi^-,0)$ is the Wilson line from 0 to ξ^-.

- This expression is light-cone dominated – needs $\xi^2 = \vec{x}^2 + t^2 \sim 0$ – very hard due to non-zero lattice spacing!

PDFs – why is it difficult on the lattice?

- PDFs have non-perturbative nature $\Rightarrow$ LATTICE?
- But: PDFs given in terms of non-local light-cone correlators – intrinsically Minkowskian – problem for the lattice!

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle,$$

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- Accessible on the lattice – moments of the distributions, but ...

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  X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

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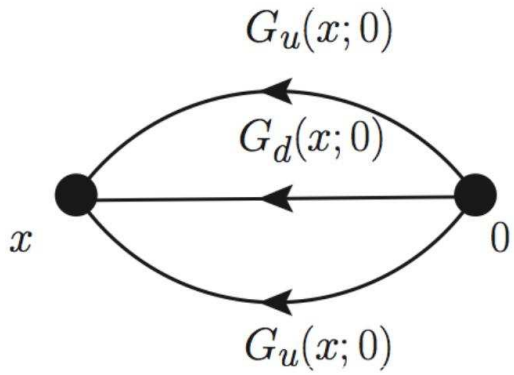

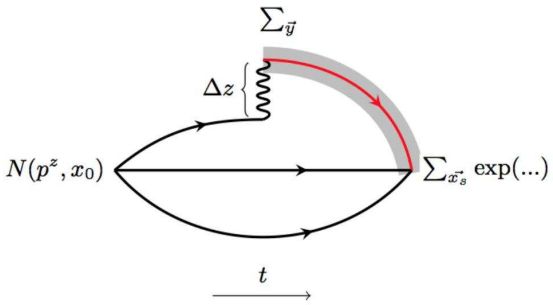

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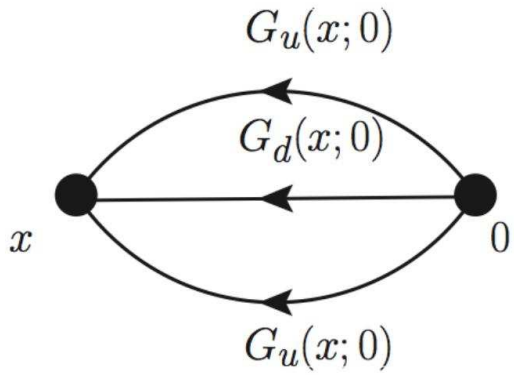

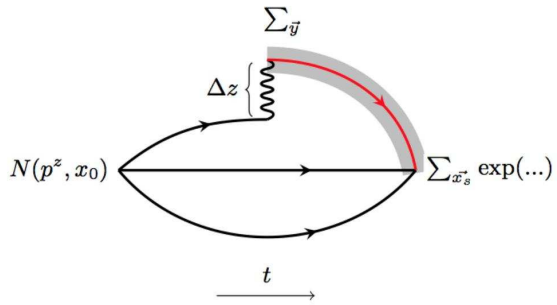

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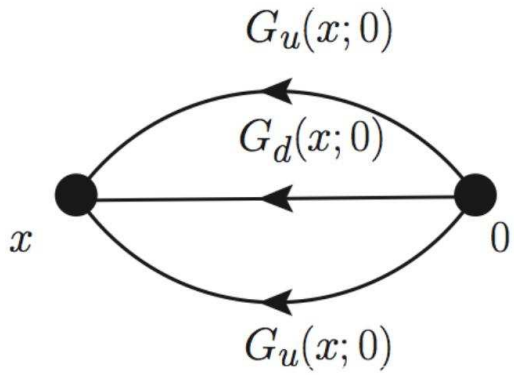

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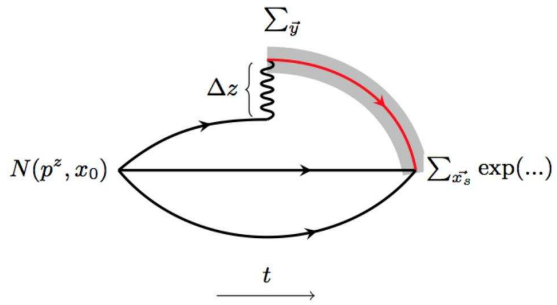
- The highly non-trivial aspect:
  how to relate $\tilde{q}(x,\mu^2,P_3)$ to the light-front PDF $q(x,\mu^2)$ (infinite momentum frame)
  $\Rightarrow$ **Large Momentum Effective Theory (LaMET)**



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Non-perturbative renormalization scheme: RI'-MOM.

G. Martinelli et al., Nucl. Phys. B445 (1995) 81

Matching of quasi-PDFs and PDFs

To relate the quasi-PDFs to the usual PDFs, one uses the fact that the IR region of the distributions is untouched when going from a finite to an infinite momentum. In other words, if $q(x, \mu)$ is the usual PDF defined through light-cone correlations, then one should have:

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Explicit formulae for 1-loop perturbative matching:

- transverse momentum cutoff scheme to $\overline{\mathrm{MS}}$ matching
  X. Xiong et al., PRD **90** (2014) 014051
- $\overline{\mathrm{MS}}$ to $\overline{\mathrm{MS}}$ matching W. Wang, S. Zhao, R. Zhu, arXiv:1708.02458 [hep-ph]
- RI to $\overline{\mathrm{MS}}$ matching I.W. Stewart, Y. Zhao, arXiv:1709.04933 [hep-ph]
- treatment of the UV log divergence in wave function corrections T. Izubuchi et al., arXiv:1801.03917 [hep-ph], C. Alexandrou et al., arXiv:1803.02685, 1807.00232 [hep-lat]

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7. Apply target mass corrections to eliminate residual m_N/P_3 effects.

Lattice setup

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 2.1$

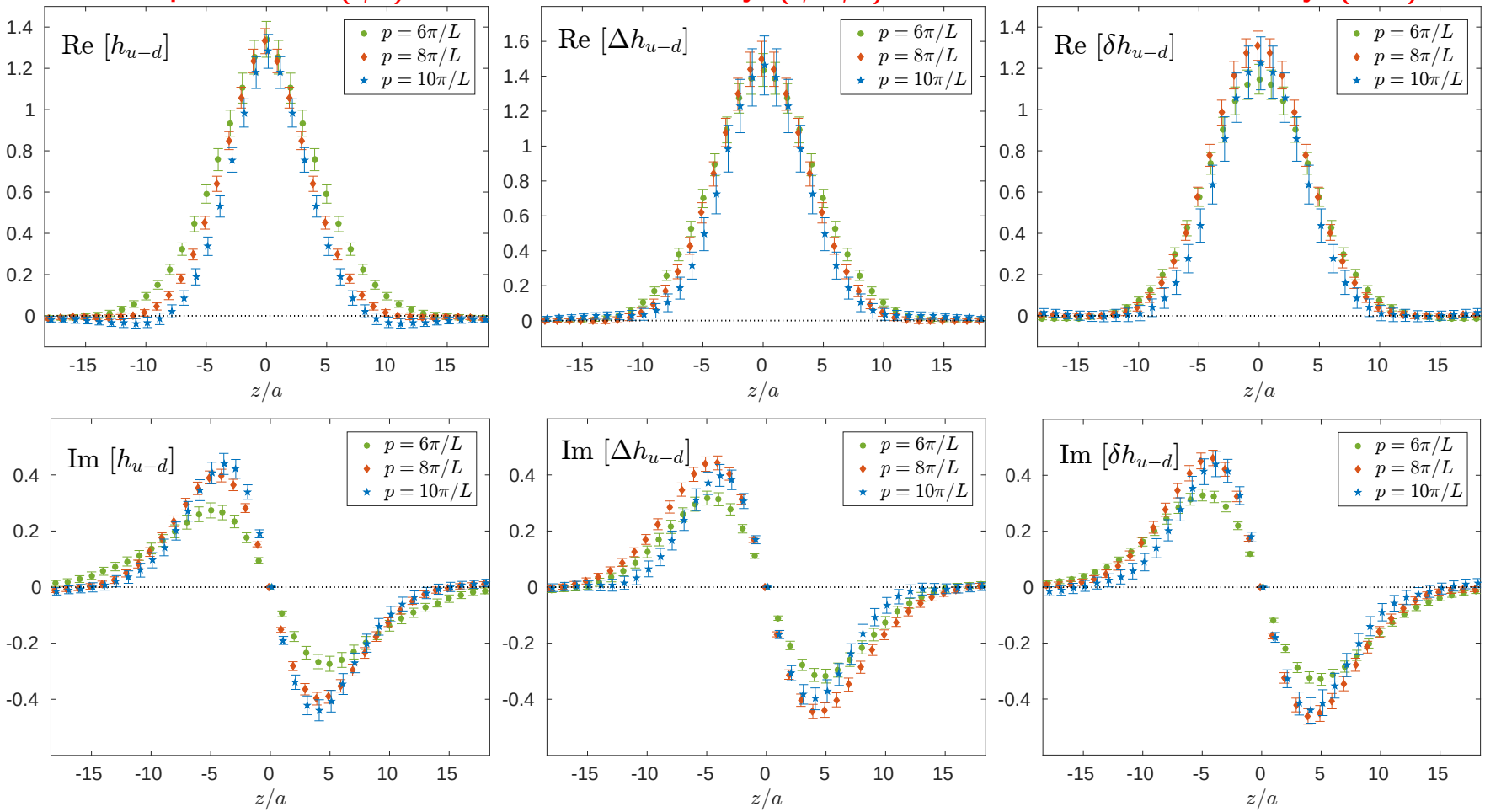
β=2.10,	$c_{\rm SW}$=1.57751,	a=0.0938(3)(2) fm
$48^3 \times 96$	$a\mu = 0.0009$	$m_N = 0.932(4)$ GeV
$L = 4.5$ fm	$m_\pi = 0.1304(4)$ GeV	$m_\pi L = 2.98(1)$

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001
C. Alexandrou et al., arXiv: 1807.00232 [hep-lat]

Results

Bare matrix elements at $t_s = 12a$

unpolarized (γ_0) helicity ($\gamma_5\gamma_3$) transversity (σ_{3i})



C. Alexandrou et al.: Phys. Rev. Lett. 121 (2018) 112001 and 1807.00232 [hep-lat]

STATISTICS:

$P_3 = \frac{6\pi}{L}$ – 4800 meas.	$P_3 = \frac{6\pi}{L}$ – 6240 meas.	$P_3 = \frac{6\pi}{L}$ – 9600 meas.
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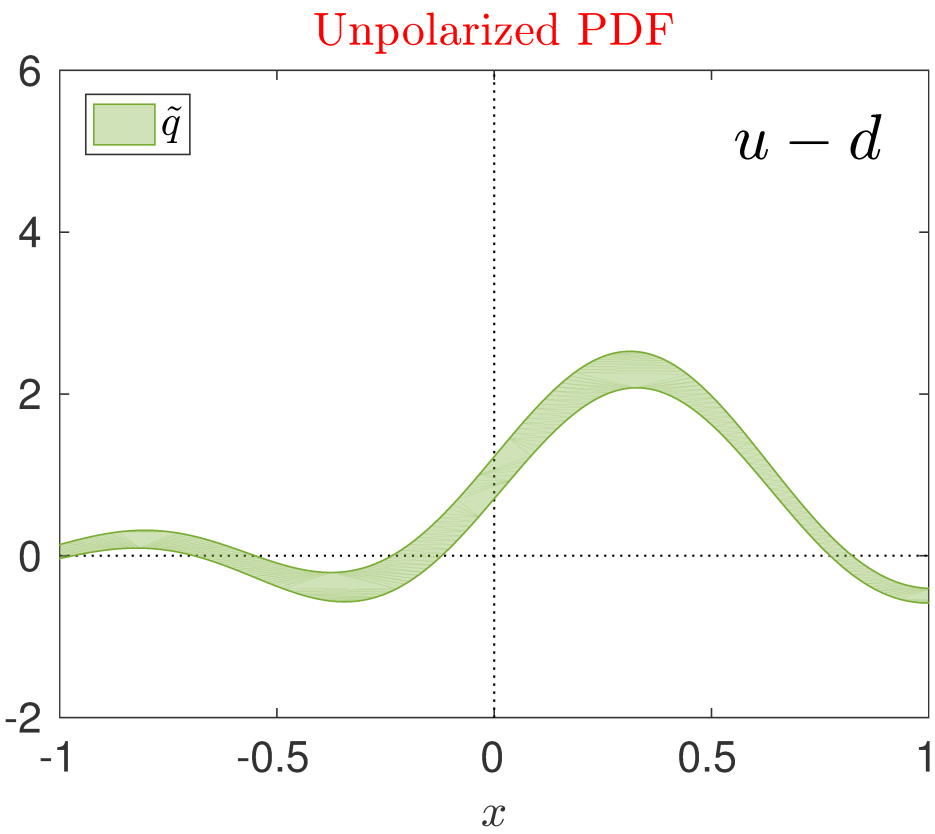

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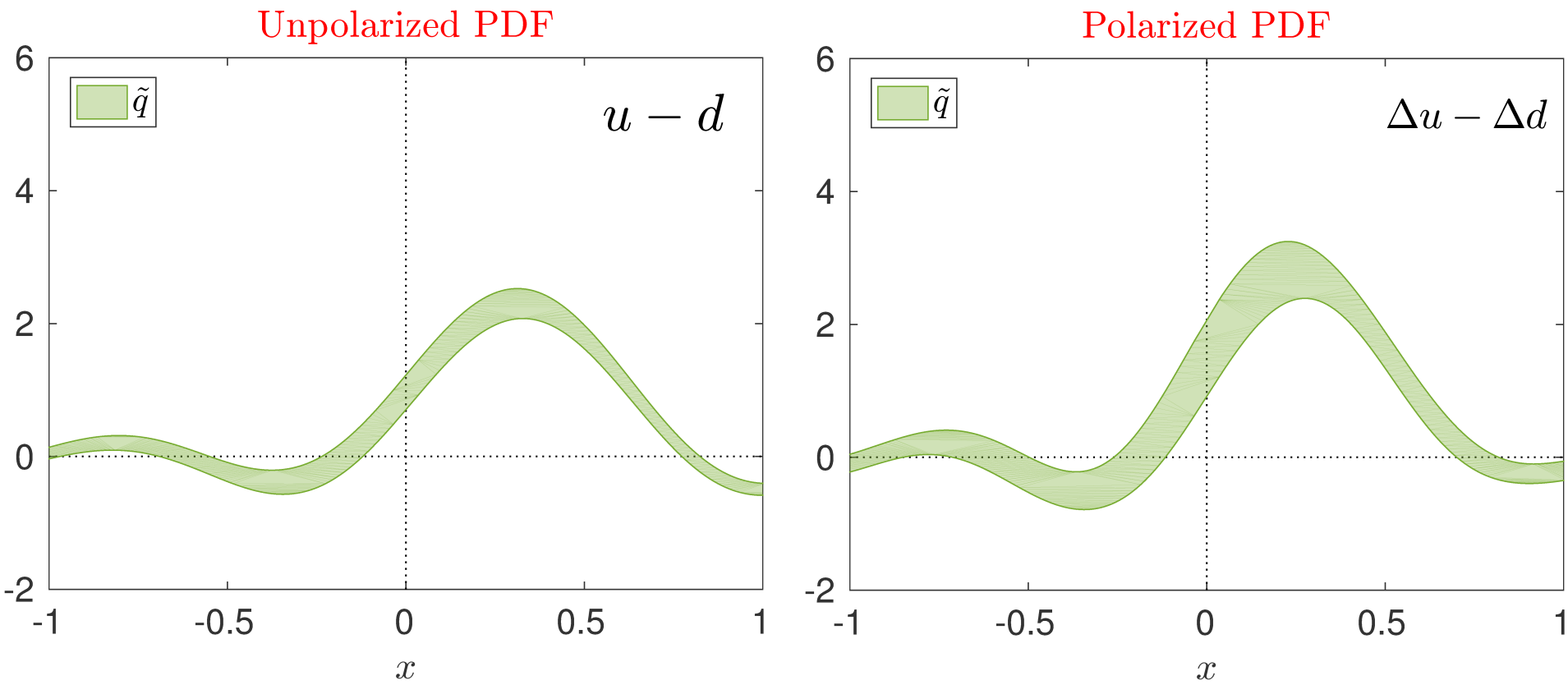

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Quasi-PDFs + pheno

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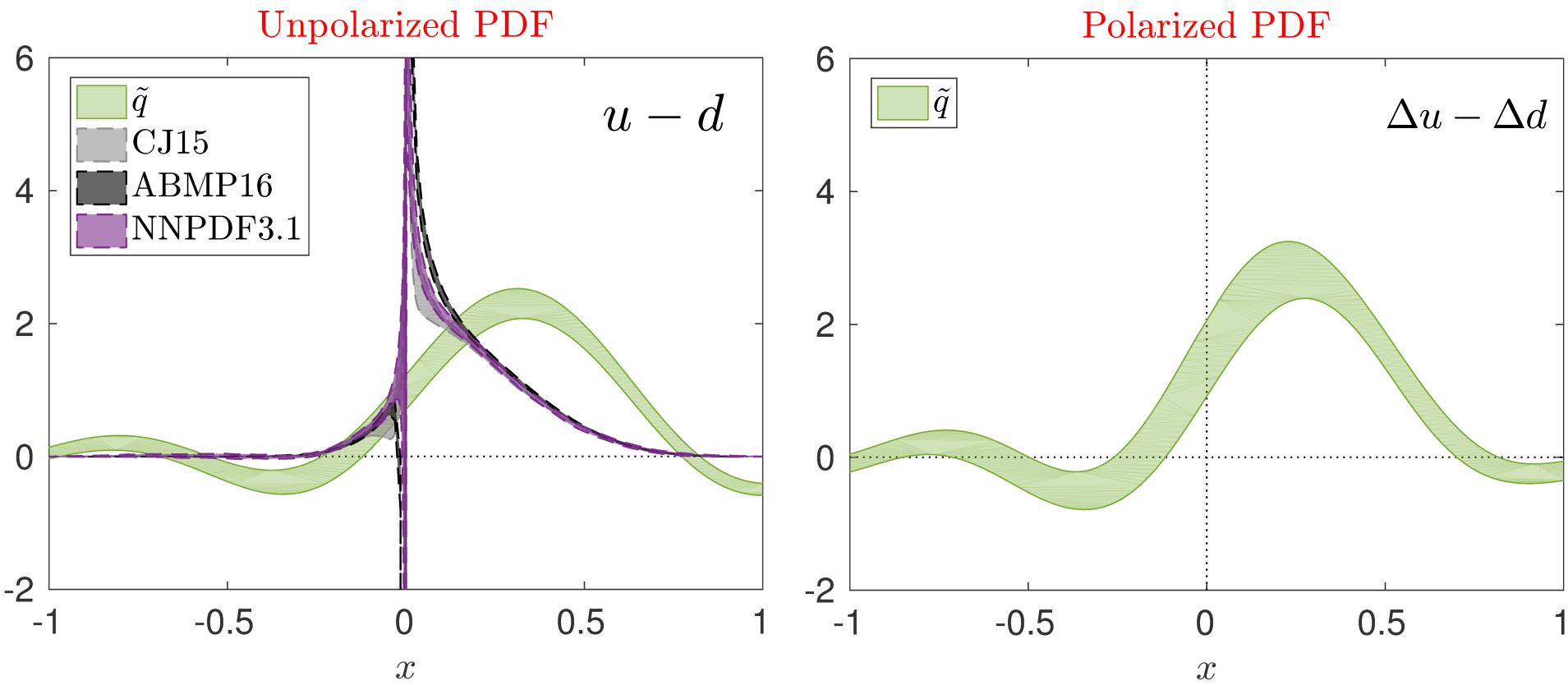

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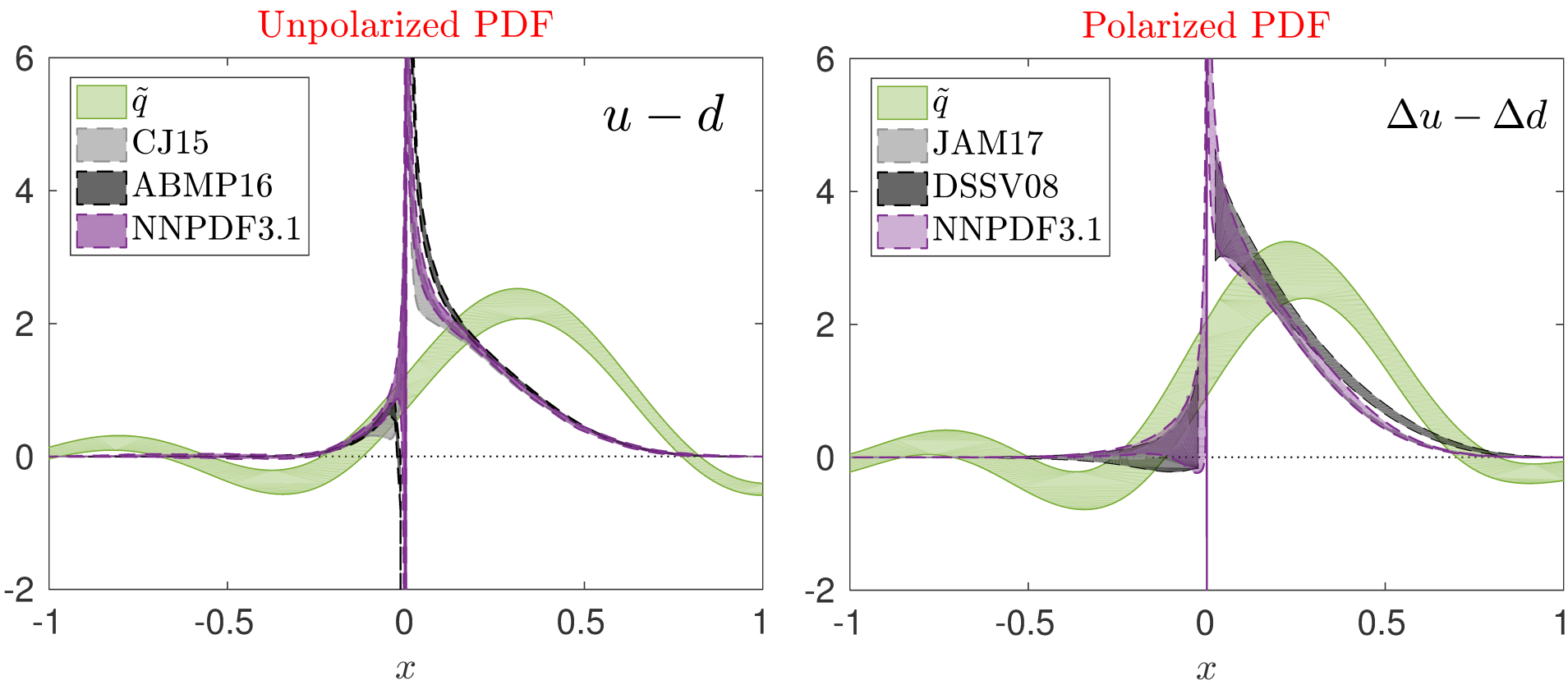

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Matching to light-front PDFs

The matching formula can be expressed as:

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$$

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C – matching kernel:

$$C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{1+\xi^2}{1-\xi} \ln \frac{\xi}{\xi-1} + 1 + \frac{3}{2\xi}\right]_+ & \xi > 1, \\ \left[\frac{1+\xi^2}{1-\xi} \ln \frac{x^2 P_3^2}{\xi^2 \mu^2} (4\xi(1-\xi)) - \frac{\xi(1+\xi)}{1-\xi} + 2\iota(1-\xi)\right]_+ & 0 < \xi < 1, \\ \left[-\frac{1+\xi^2}{1-\xi} \ln \frac{\xi}{\xi-1} - 1 + \frac{3}{2(1-\xi)}\right]_+ & \xi < 0, \end{cases}$$

[T. Izubuchi et al., arXiv:1801.03917 [hep-ph], C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001]

$\iota=0$ for γ_0 and $\iota=1$ for $\gamma_3/\gamma_5\gamma_3$.

Plus prescription at $\xi=1$:

$$\int \frac{d\xi}{|\xi|} \left[C\left(\xi, \frac{\xi\mu}{xP_3}\right)\right]_+ \tilde{q}\left(\frac{x}{\xi}\right) = \int \frac{d\xi}{|\xi|} C\left(\xi, \frac{\xi\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}\right) - \tilde{q}(x) \int d\xi\, C\left(\xi, \frac{\mu}{xP_3}\right).$$

Matched PDFs

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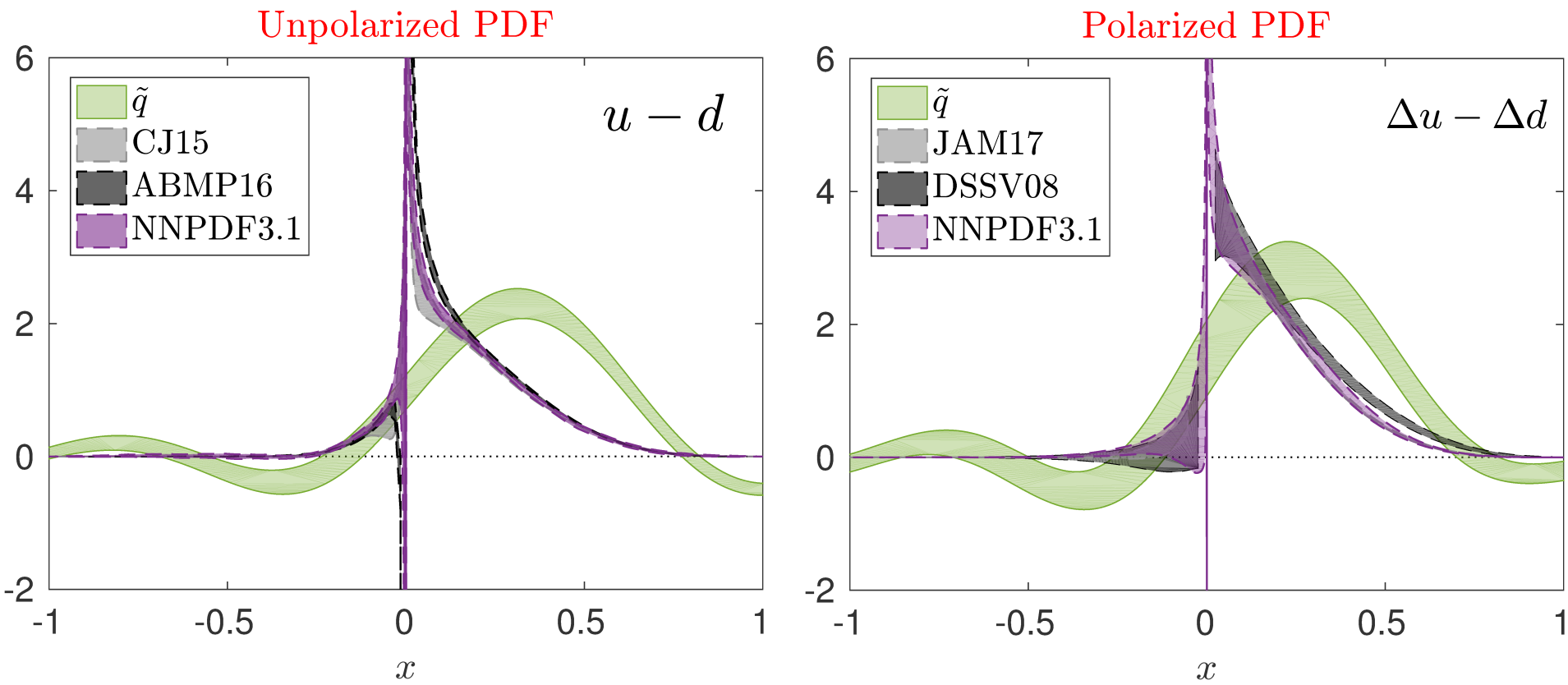

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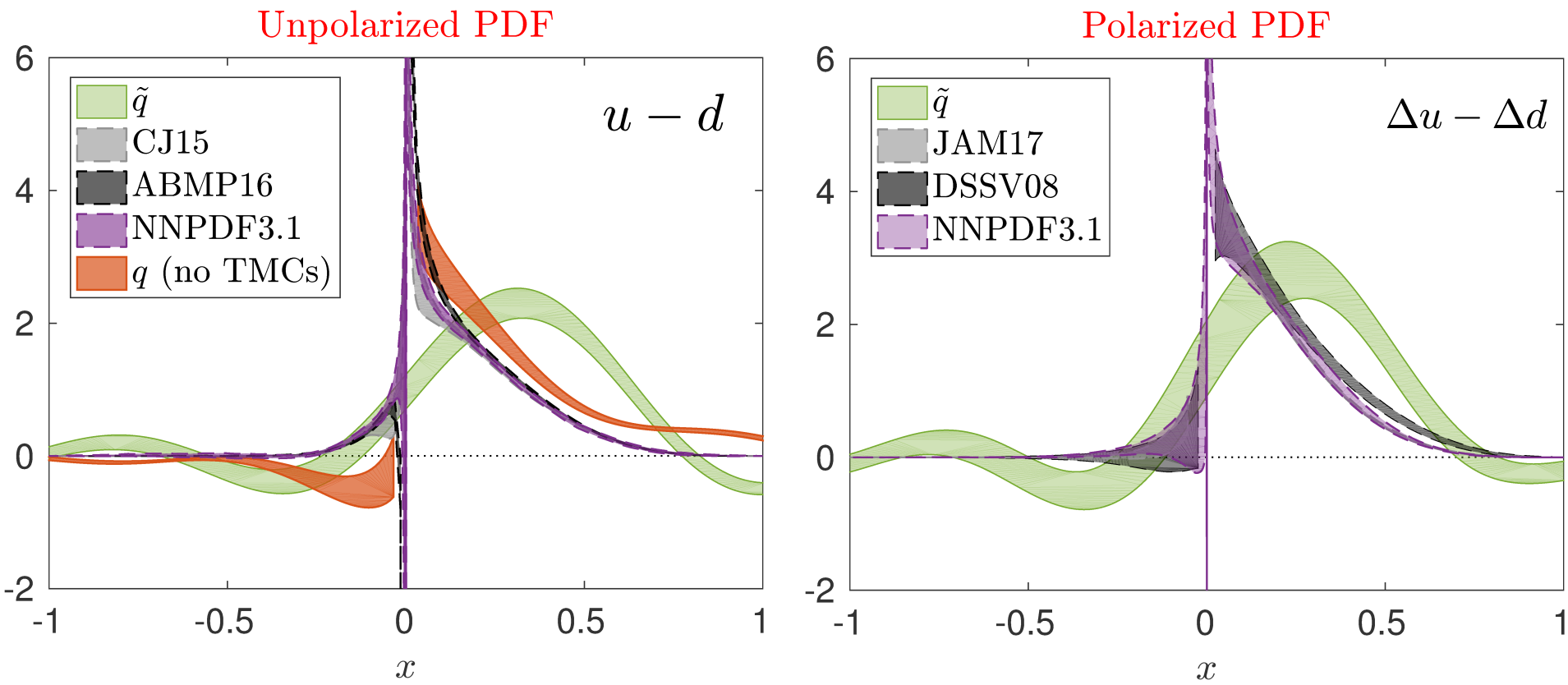

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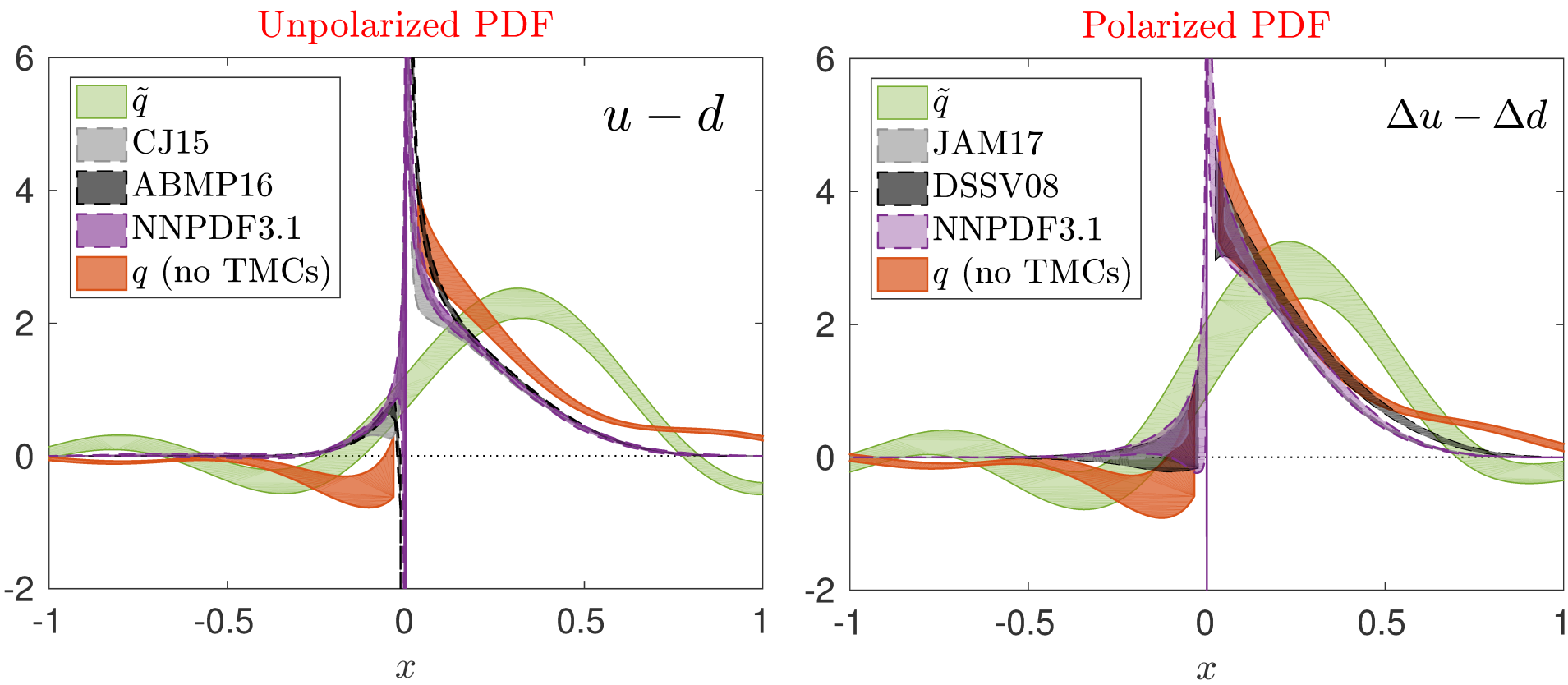

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Matched PDF + TMCs

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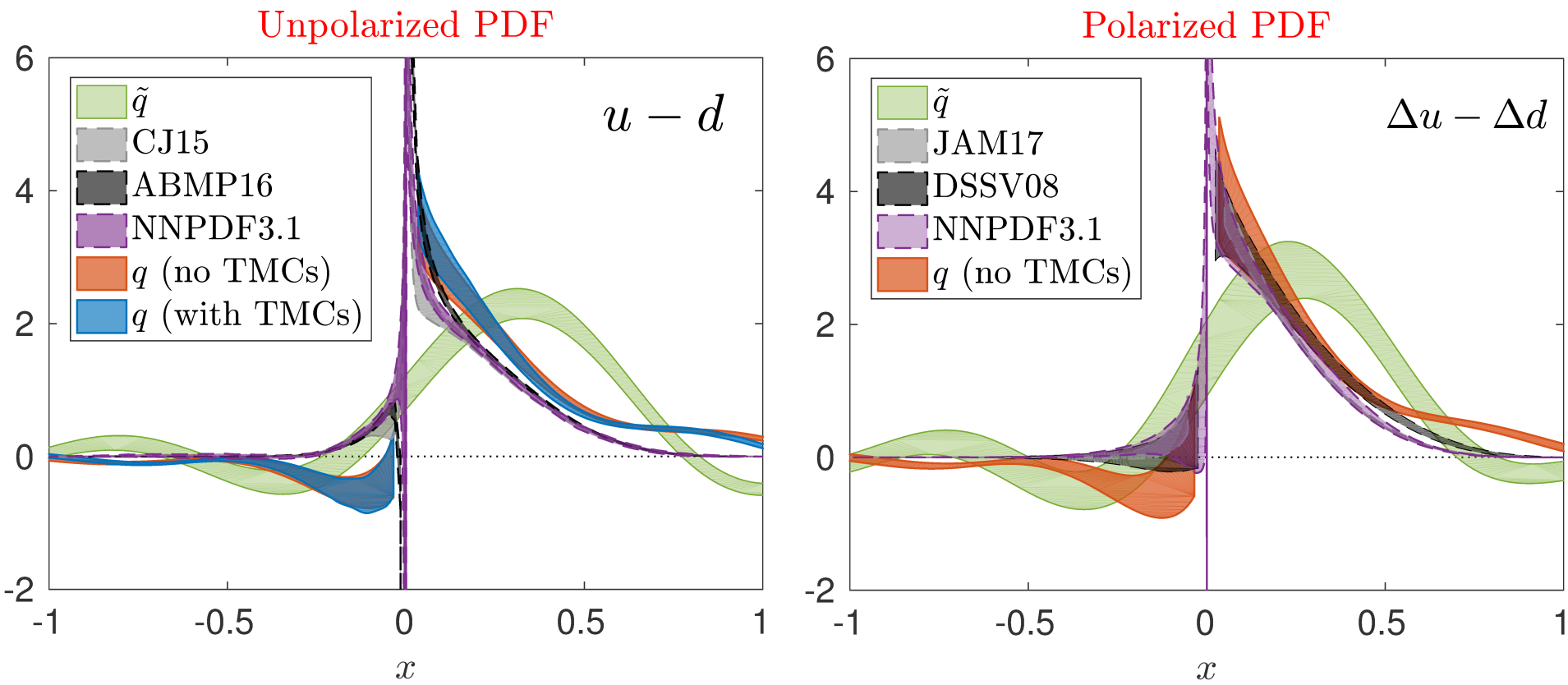

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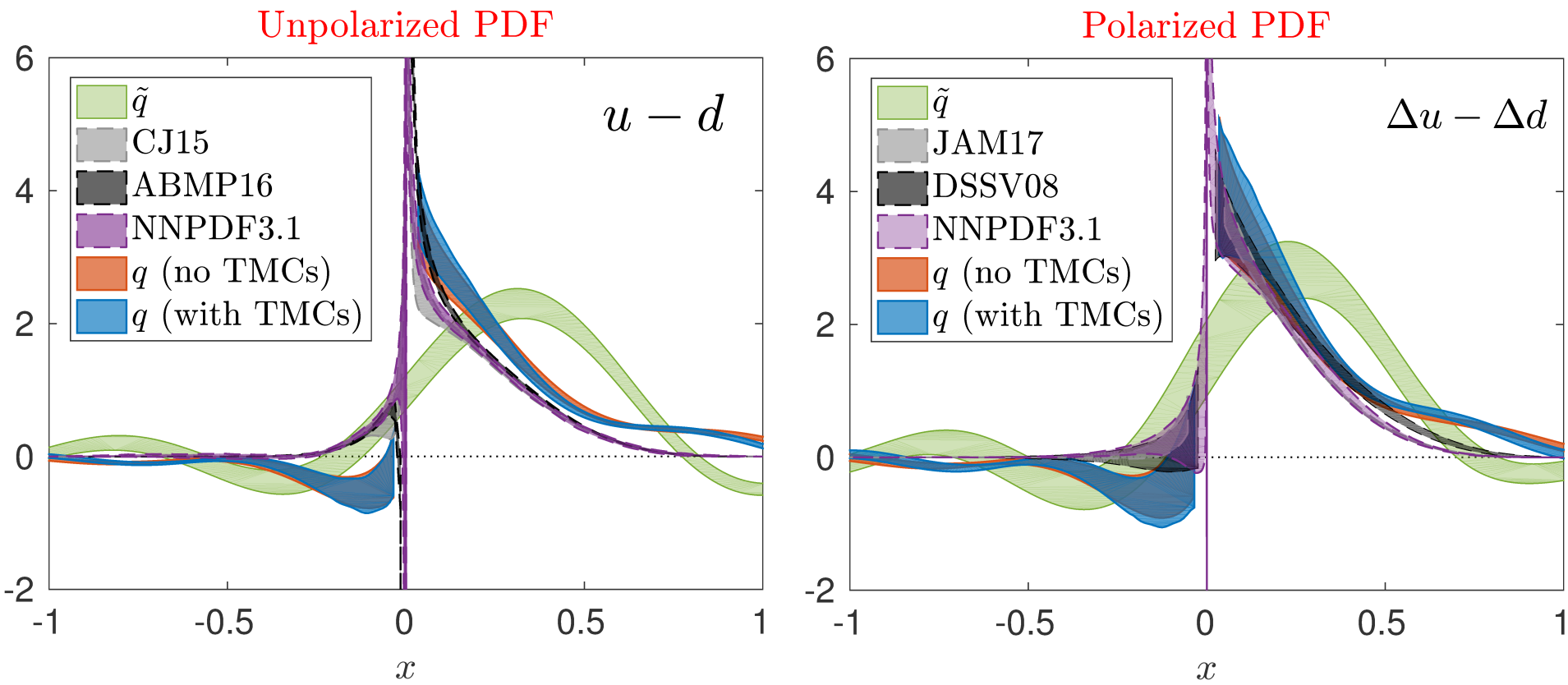

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Momentum dependence of final PDF

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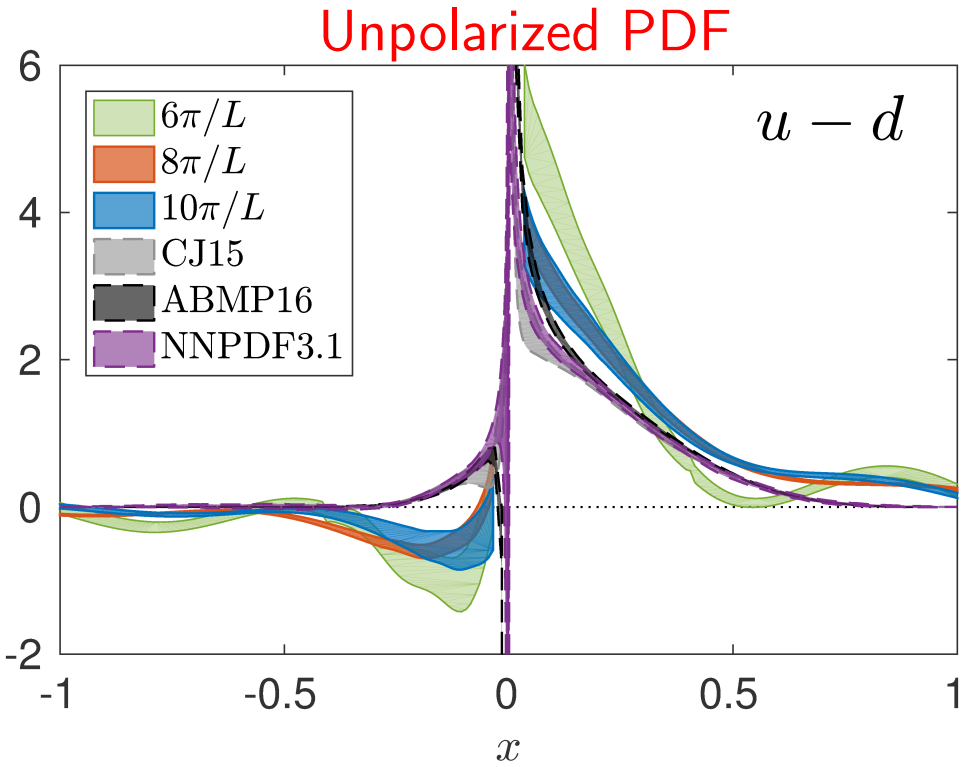

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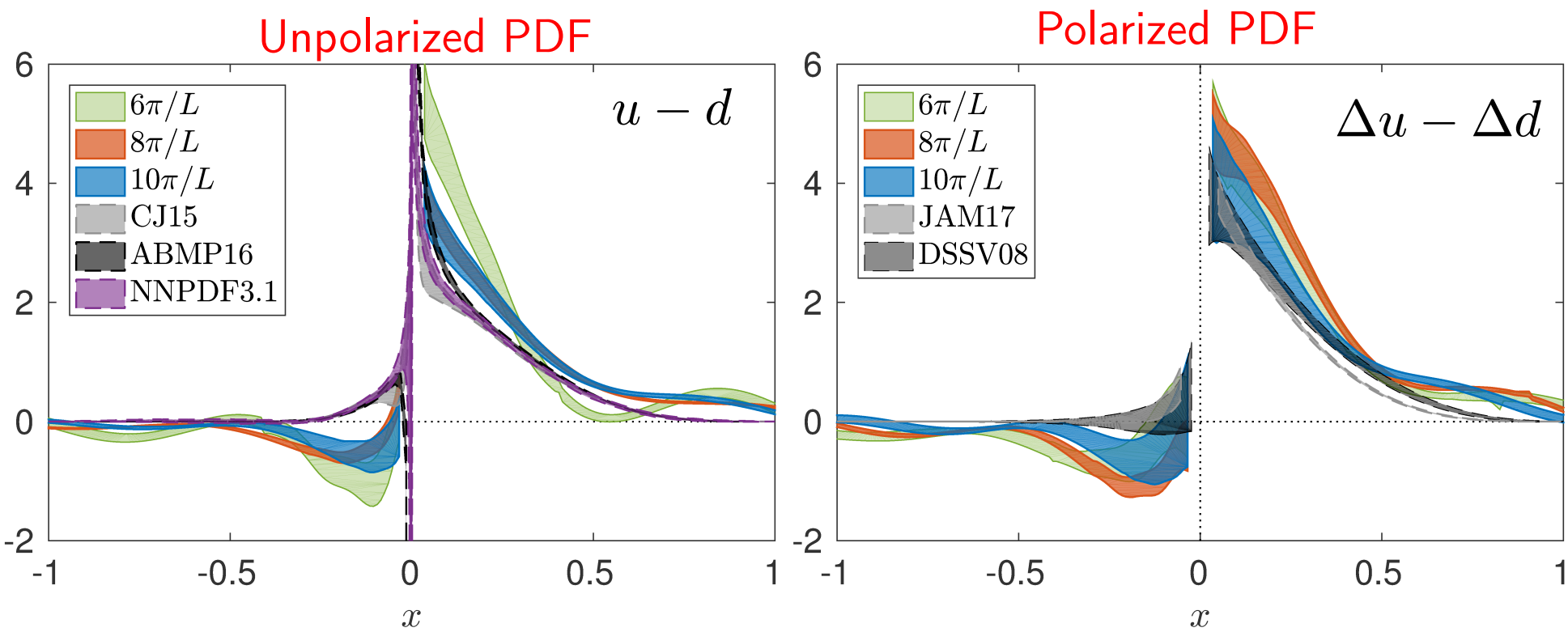

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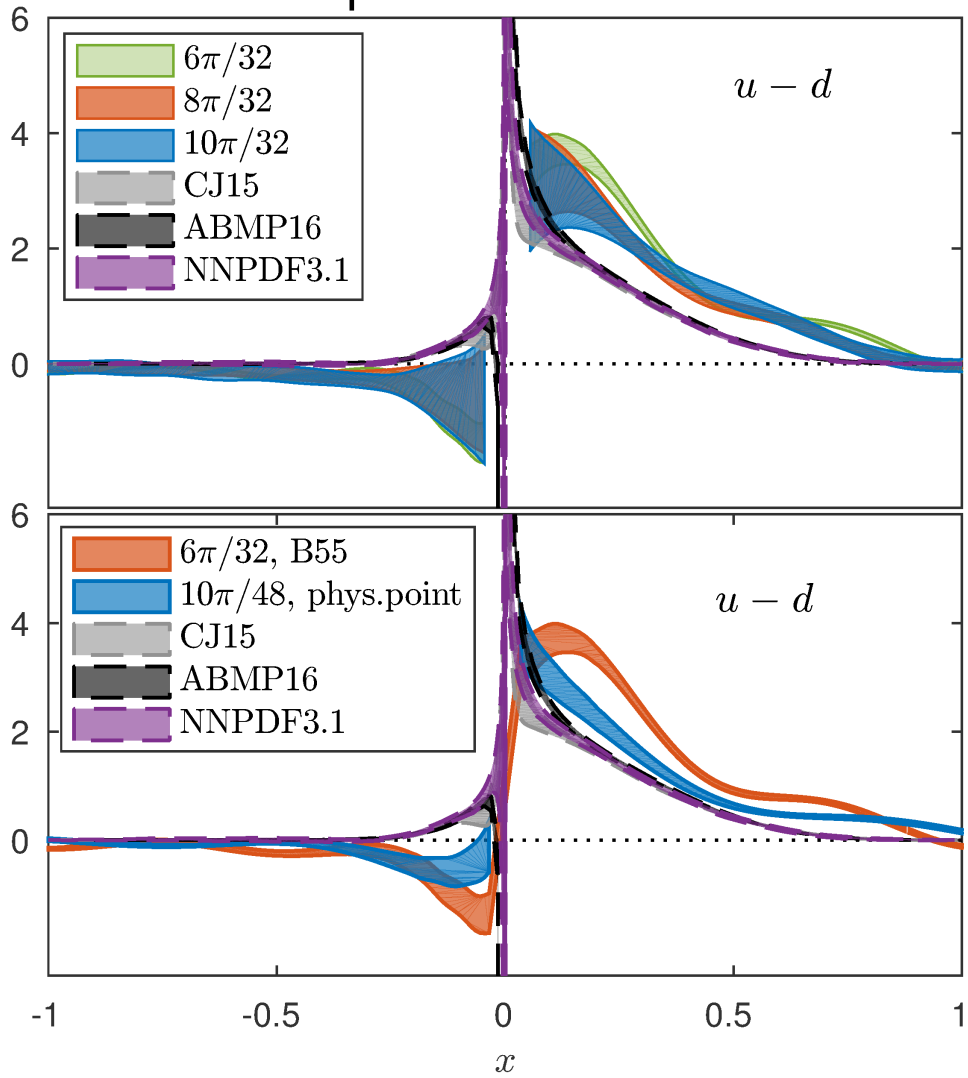
Comparison with non-physical pion mass

Physical vs. non-physical pion mass – 135 vs. 375 MeV
unpolarized PDF



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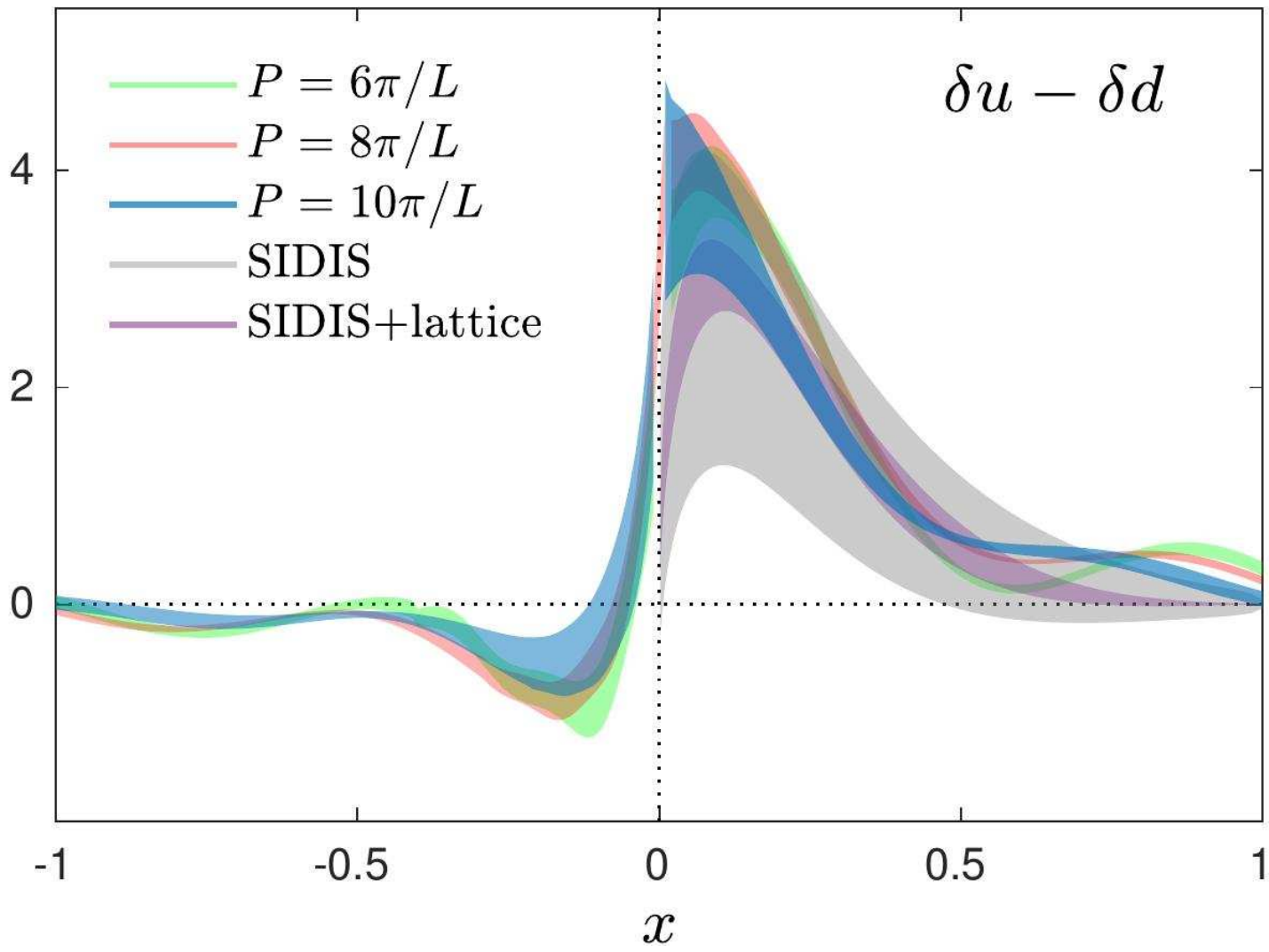
Transversity PDF



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Systematics

Different systematic effects still need to be addressed:

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Biggest challenge:
Reach large momenta at large source-sink separations

A guide to light-cone PDFs from lattice QCD: an overview of approaches, techniques and results

Krzysztof Cichy[1], Martha Constantinou[2] [a]

[1] *Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland*

[2] *Department of Physics, Temple University, Philadelphia, PA 19122 - 1801, USA*

- **97 pages, today on arXiv: 1811.07248**
- discusses in detail quasi-distributions:
  nucleon: **non-singlet quark qPDFs**, qGPDs, qTMDs, singlet qPDFs, gluon qPDFs; pion: qPDFs, qDAs
- reviews also other approaches:
  hadronic tensor, auxiliary scalar quark, auxiliary heavy quark, auxiliary light quark, pseudo-distributions, "OPE without OPE", lattice cross sections

Conclusions and prospects

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Thank you for your attention!