Solutions of the BFKL equation with higher order corrections

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Outline

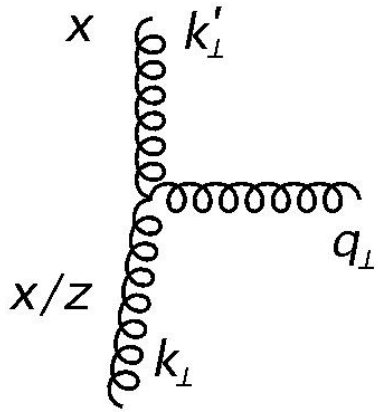
- Motivation
- Introduction
  - The BFKL equation, the BFKL equation + DGLAP terms
  - Kinematical constraint
- Semi-analytical results
  - Kinematical constraint in Mellin space
- Numerical results
- Summary

Motivation

- Obtaining new TMDs/unintegrated PDFs
- Improving previous analysis by Kimber and Stasto
- Study how the kinematical constraint generates higher order terms
- Study various evolution equations, different versions of the kinematical constraint and see the effects on the resulting TMDs

The BFKL equation

- multi-regge kinematics
- low x resummation
- unintegrated PDFs



$$\mathbf{k}_\perp'^2 = \mathbf{k}_\perp^2 - 2\cos(\phi)\,\mathbf{k}_\perp \mathbf{q}_\perp + \mathbf{q}_\perp^2$$

real term

virtual term

$$\mathcal{F}(\mathbf{x},\mathbf{k}_\perp^2) = \mathcal{F}_0(\mathbf{x},\mathbf{k}_\perp^2) + \int_{Q_0^2}^{\infty} \frac{\overline{\alpha}_s}{\pi} \frac{d\mathbf{q}_\perp^2}{\mathbf{q}_\perp^2} \int d\phi \int_{\mathbf{x}}^{1} \frac{d\mathbf{z}}{\mathbf{z}} \left[\Theta(\mathbf{k.c.})\mathcal{F}(\mathbf{x}/\mathbf{z},\mathbf{k}_\perp'^2) - \Theta(\mathbf{k}_\perp^2 - \mathbf{q}_\perp^2)\mathcal{F}(\mathbf{x}/\mathbf{z},\mathbf{k}_\perp^2)\right]$$

– Note: k.c. - kinematical constraint expression

BFKL + DGLAP terms

- Two integration regions

$$\mathcal{F}(x, k_\perp^2) = \mathcal{F}_0(x, k_\perp^2) + T_{BFKL} + T_1 + T_2$$

collinear

$$T_C = \int_{Q_0^2}^{k_\perp^2} \bar{\alpha}_s(k_\perp^2) \frac{dk_\perp'^2}{k_\perp^2} \int_x^1 \frac{dz}{z} z\tilde{P}(z) \mathcal{F}(x/z, k_\perp'^2)$$

anti-collinear

$$T_A = \int_{k_\perp^2}^{(1-z)\frac{k_\perp^2}{z}} \bar{\alpha}_s(k_\perp'^2) \frac{dk_\perp'^2}{k_\perp'^2} \int_x^1 \frac{dz}{z} z'\tilde{P}(z') \mathcal{F}(x/z, k_\perp'^2)$$

$$\tilde{P}(z) = \frac{P(z)}{2N_C} - \frac{1}{z} \qquad z' = z(k_\perp'^2/k_\perp^2 + 1)$$

Kinematical constraint

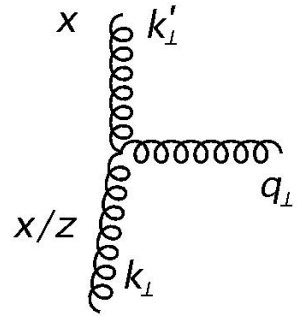
- Follows from consistency requirement for the virtuality of the *t*-channel gluon → $k^2<0$
- Effectively implements energy conservation

$$k_\perp^2 > \frac{z}{1-z} q_\perp^2$$

- Complicated expression → approximations possibly useful (used in literature):

small z $\quad k_\perp^2 > z\, q_\perp^2$

small z and $k'_T \approx q_T$ $\quad k_\perp^2 > z\, k_\perp'^2$



$$k_\perp'^2 = k_\perp^2 - 2\cos(\phi)\, k_\perp q_\perp + q_\perp^2$$

J. Kwiecinski, A. D. Martin and P. J. Sutton

Kinematical constraint

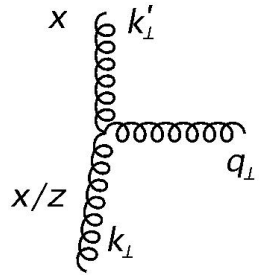
Derivation

- Follows from consistency requirement for the virtuality of the *t*-channel gluon → $k^2<0$

$$k^2 = -z\,\bar{z}\,\hat{s} - k_T^2$$

$$\downarrow \quad k^2<0$$

$$k_T^2 > z\,\bar{z}\,\hat{s}$$



$$\mathbf{k}_\perp'^2 = \mathbf{k}_\perp^2 - 2\cos(\phi)\,\mathbf{k}_\perp\mathbf{q}_\perp + \mathbf{q}_\perp^2$$

- Inserting the on-shell condition for emitted parton

$$q^2 = \bar{z}(1-z)\hat{s} - {q_T}^2 = 0 \quad \rightarrow \quad \boxed{k_T^2 > \frac{z\,{q_T}^2}{1-z}}$$

J. Kwiecinski, A. D. Martin and P. J. Sutton

Kinematical constraint in Mellin space

BFKL equation in Mellin space

- Solving the equation for eigenvalues of the BFKL kernel Mellin transform

$$\omega = \chi(\omega, \gamma)$$

$$\omega = \chi_{\mathrm{eff}}(\overline{\alpha}_s, \gamma)$$

$$\tilde{\phi}(\omega) = \int_0^1 \phi(z)\, z^{\omega-1} dz$$

- BFKL kernel without the kinematical constraint

$$\chi_{\mathrm{eff}}(\overline{\alpha}_s, \gamma) = \overline{\alpha}_s (\psi(1) - \psi(\gamma) - \psi(1-\gamma))$$

  – $1/\gamma$ pole

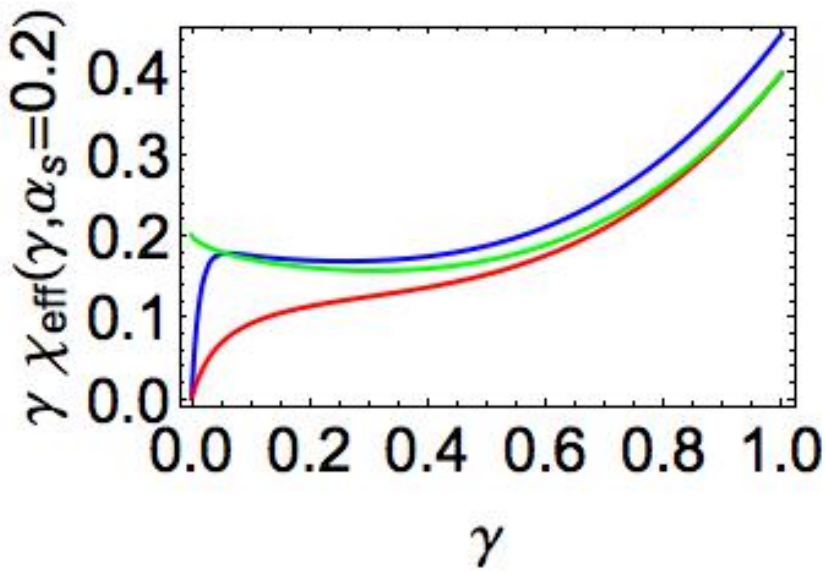
Kinematical constraint in Mellin space

- different behavior of eigen-values of the kernel

$$k_\perp^2 > \frac{z}{1-z}\, q_\perp^2$$

$$k_\perp^2 > z\, q_\perp^2$$

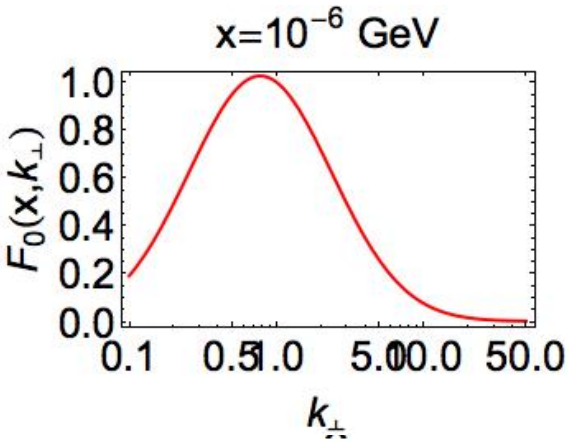
$$k_\perp^2 > z\, k_\perp'^2$$

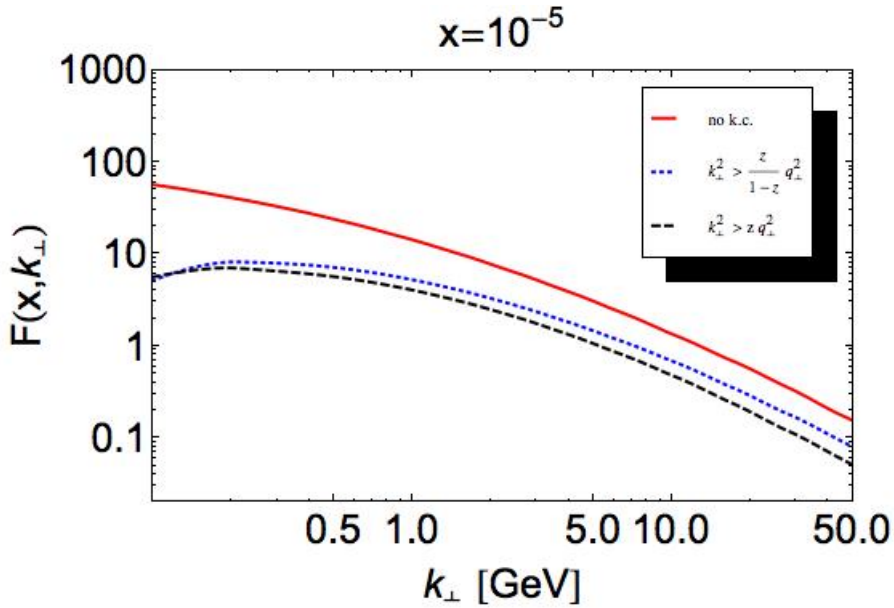


- 1. and 2. do not exhibit a $1/\gamma$ pole
- 1. and 3. match for $\gamma \to 1$

Numerical results

- significant differences



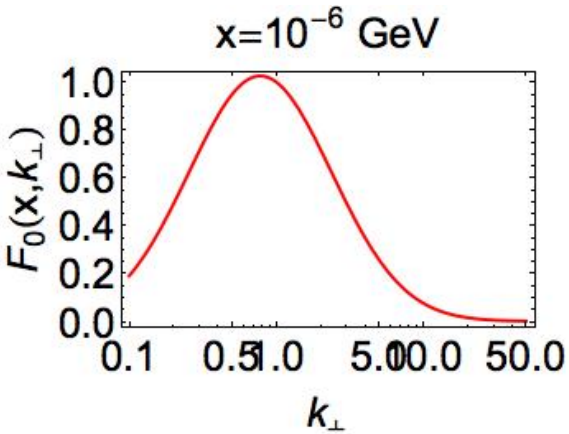


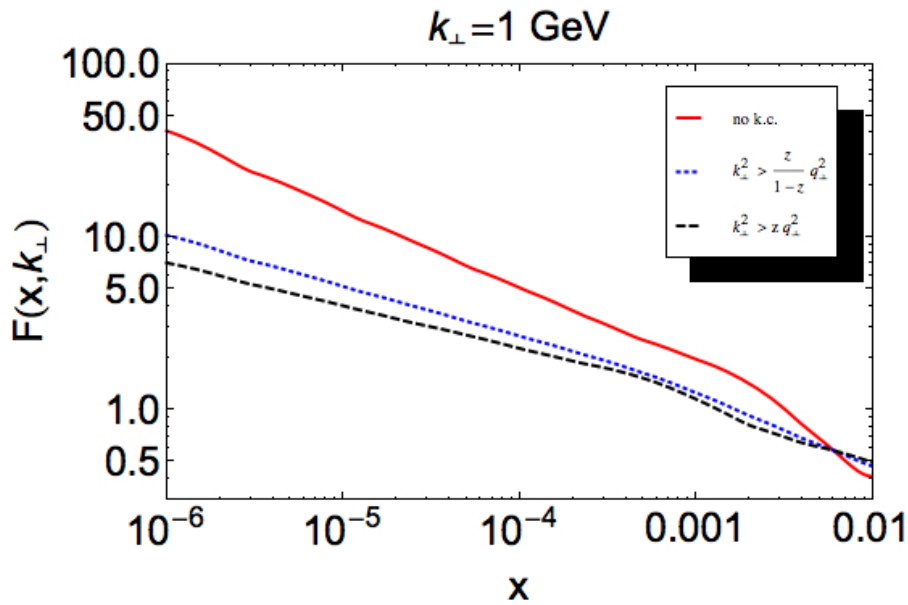
- initial condition

$$\mathcal{F}_0\left(x, k_\perp^2\right) = \exp\left[p_0 \log\left(k_\perp^2\right) + p_1 \left(\log\left(k_\perp^2\right)\right)^2\right] (1-x)^{p_2}$$

Numerical results

- significant differences





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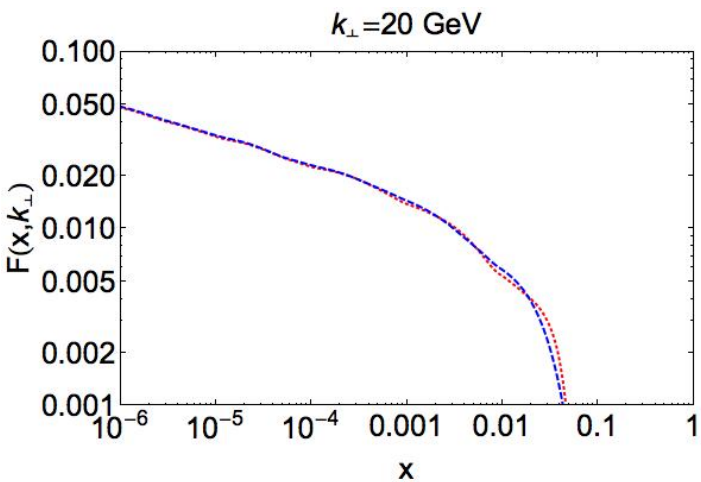
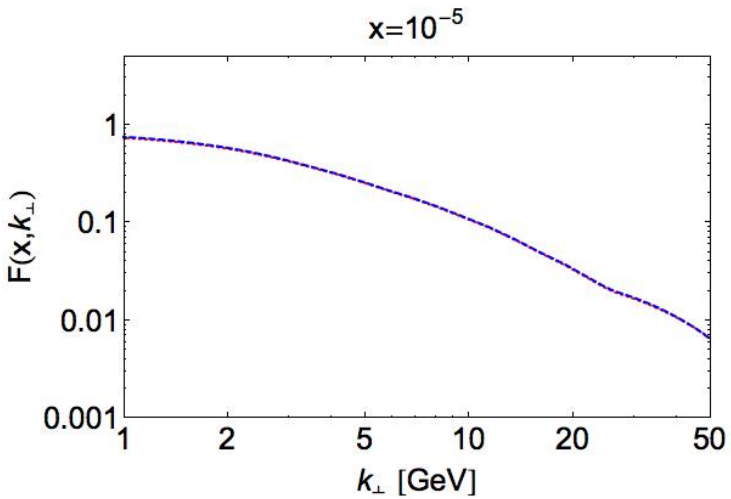
Numerical results

- BFKL with DGLAP - big corrections





- small differences between $k_\perp^2 > z\, q_\perp^2$ & $k_\perp^2 > \frac{z}{1-z}\, q_\perp^2$

Summary & Outlook

- analysis of kinematical constraint impact on solutions of the BFKL equation
  - semi-analytical
  - Numerical

- using the framework to perform fits to F_2 data and compare different evolution equations/kinematical constraint