QED evolution of TMDs

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1. ***Motivation***
2. ***TMDs in QCD: summary***
3. ***QED corrections to TMD evolution***
4. ***Conclusions & Outlook***

[Bacchetta, MGE 1810.02297 (PLB)]

1. Motivation

Why care about QED corrections?

- Push theoretical precision to **better constrain nonperturbative Physics**
- Currently: **(N)NNLL TMD evolution** and **NNLO Wilson coefficients** for various TMDs
- Several recent theory/pheno improvements regarding **QED corrections to DGLAP evolution of integrated PDFs and photon PDF**
- **Why not?** Formally it's in any case an interesting issue

QED corrections to DGLAP kernels

Extending DGLAP equations

6 **Introducing QED corrections**

- *DGLAP equations dictate the evolution of PDFs*
- **EW** interactions connects **QCD** partons with **photons and leptons.**

New PDFs (leptons & photon) + *New splitting functions* → ***Modified DGLAP equations***

- **Extend original DGLAP equations to deal with new *objects*:**

$$\frac{dg}{dt} = \sum_f P_{gf} \otimes f + \sum_f P_{g\bar{f}} \otimes \bar{f} + P_{gg} \otimes g + P_{g\gamma} \otimes \gamma$$

$$\frac{d\gamma}{dt} = \sum_f P_{\gamma f} \otimes f + \sum_f P_{\gamma\bar{f}} \otimes \bar{f} + P_{\gamma g} \otimes g + P_{\gamma\gamma} \otimes \gamma$$

$$\frac{dq_i}{dt} = \sum_f P_{q_i f} \otimes f + \sum_f P_{q_i\bar{f}} \otimes \bar{f} + P_{q_i g} \otimes g + P_{q_i\gamma} \otimes \gamma$$

$$\frac{dl_i}{dt} = \sum_f P_{l_i f} \otimes f + \sum_f P_{l_i\bar{f}} \otimes \bar{f} + P_{l_i g} \otimes g + P_{l_i\gamma} \otimes \gamma$$

Kernels with fermions

Kernels with photons

Photon distributions

Kernels with leptons

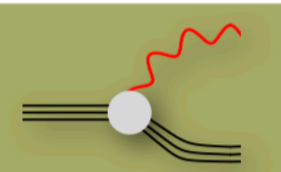
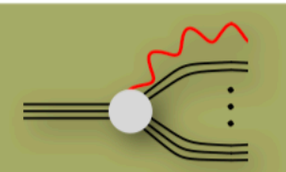
Lepton distributions

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282 and arXiv:1606.02887 [hep-ph]

From a talk by GFR Sborlini

- They calculated **QED corrections** up to $\mathbf{O}(\alpha^2)$ and $\mathbf{O}(\alpha_s\alpha)$
- Several PDF sets already incorporate them

Photon PDF from DIS structure functions (1/3)

	 elastic	 inelastic	LHAPDF public computer-readable form?
Gluck Pisano Reya 2002	dipole	model	✘
MRST2004qed	✘	model	✓
CT14qed_inc	dipole	model (data-constrained)	✓
Martin Ryskin 2014	dipole (only electric part)	model	✘
Harland-Lang, Khoze Ryskin 2016	dipole	model	✘
NNPDF23qed (& NNPDF30qed)	no separation; fit to data		

From a talk by G. Salam

- Increasing interest on photon PDF
- **Several groups already included it in their standard fits**

Photon PDF from DIS structure functions (2/3)

LUXqed approach

[Manohar, Nason, Salam, Zanderighi 1708.01256 (JHEP), 1607.04266 (PRL)]

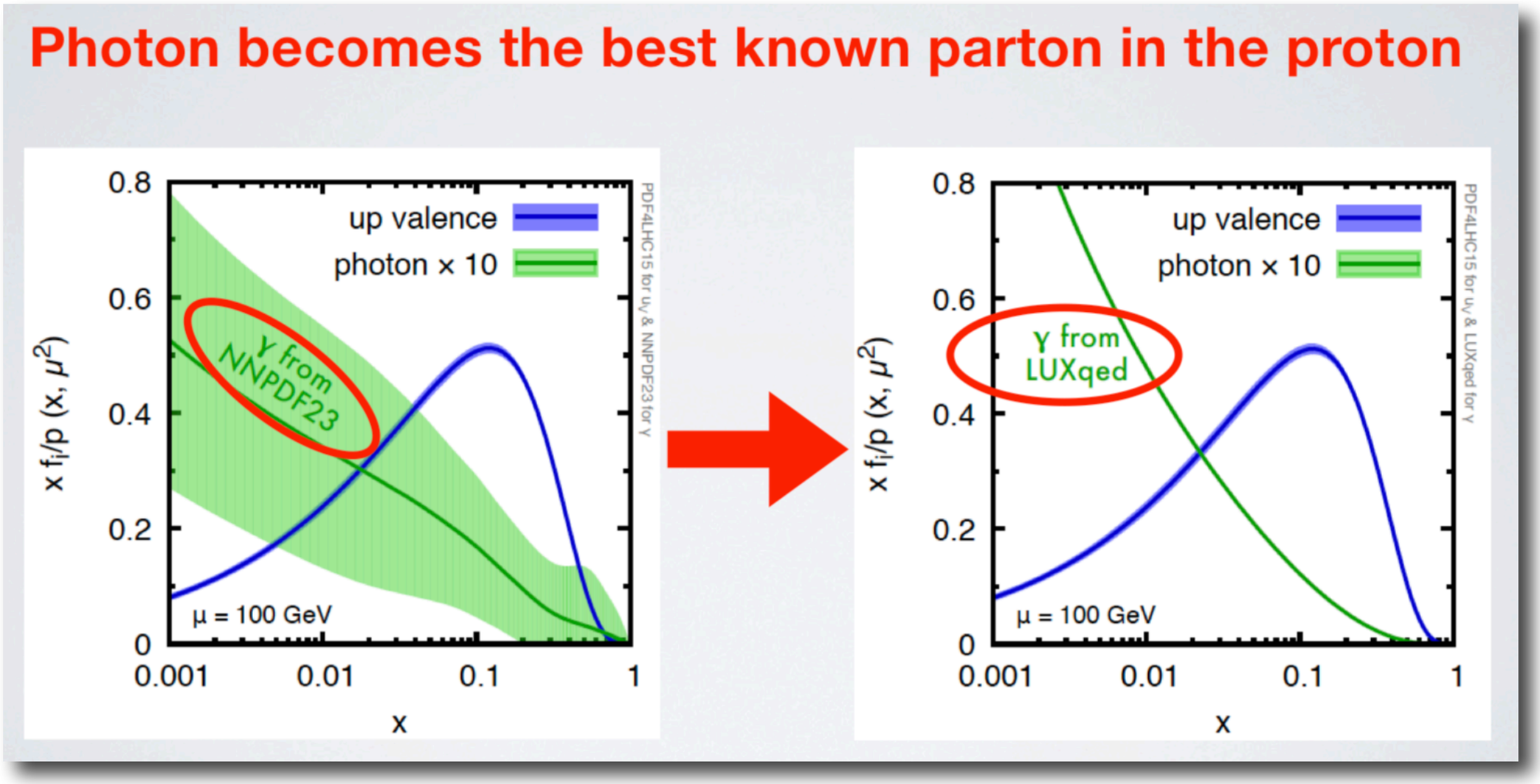
- Main idea: write the cross section for an **imaginary BSM heavy-lepton production** process (which couples to SM electron and photon) **in two ways:**
  - in terms of **DIS structure functions F_2 and F_L**
  - in terms of a **photon PDF in collinear factorization**
- **Photon PDF can then be written in terms of F_2 and F_L** (model independent):

$$xf_{\gamma/p}(x,\mu^2) = \frac{1}{2\pi\alpha(\mu^2)}\int_x^1 \frac{dz}{z}\left\{\int_{\frac{x^2 m_p^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2}\alpha^2(Q^2)\right.$$

$$\left[\left(zp_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2}\right)F_2(x/z,Q^2) - z^2 F_L\left(\frac{x}{z},Q^2\right)\right]$$

$$\left. - \alpha^2(\mu^2)z^2 F_2\left(\frac{x}{z},\mu^2\right)\right\}$$

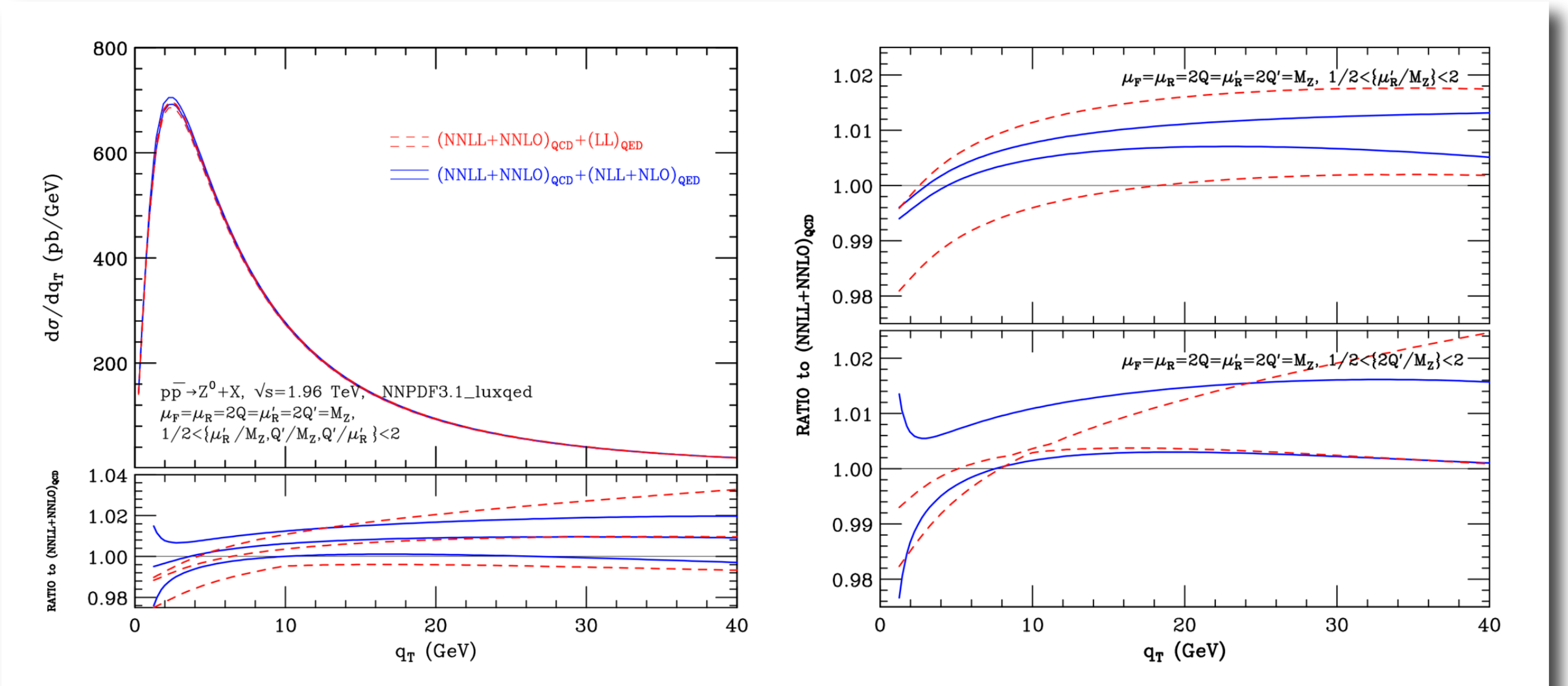
Photon PDF from DIS structure functions (3/3)



From a talk by G. Zanderighi

- Similar comparisons with other sets (e.g. CT14, MRST2004)
- EIC will potentially better measure F2 and FL, thus better constrain photon PDF

Z boson p_T distribution



[Cieri, Ferrera, Sborlini 1805.11948 (JHEP)]

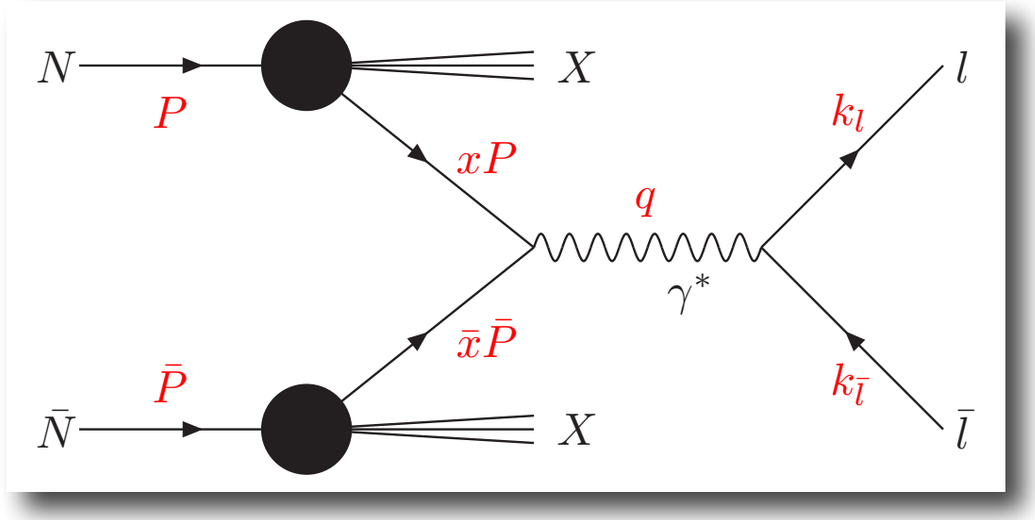
- They include QED corrections in their approach (corrections and photon PDF)
- They find a ~ few % impact on Z production at Tevatron/LHC

We do it **systematically**, **by considering the operator definition of TMDs** and obtaining the new pieces for all (un)polarized quark/gluon TMD PDFs and FFs

2. TMDs in QCD: summary

Definition of TMDs in QCD (1/2)

I take Drell-Yan production as an example



$$d\sigma = \sigma_0(\mu)\, H(Q^2,\mu)\, dy\, \frac{d^2 q_\perp}{(2\pi)^2} \int d^2 y_\perp\, e^{-i q_\perp \cdot y_\perp}\, J_n(x_A, y_\perp, \mu)\, S(y_\perp,\mu)\, J_{\bar{n}}(x_B, y_\perp, \mu)$$

$$J_n(0^+, y^-, \vec{y}_\perp) = \frac{1}{2}\sum_{\sigma_1} \langle N_1(P,\sigma_1)|\, \bar{\chi}_n(0^+, y^-, \vec{y}_\perp)\, \frac{\bar{\not{n}}}{2}\, \chi_n(0)\, |N_1(P,\sigma_1)\rangle|_{\text{zb subtracted}}$$

$$J_{\bar{n}}(y^+, 0^-, \vec{y}_\perp) = \frac{1}{2}\sum_{\sigma_2} \langle N_2(\bar{P},\sigma_2)|\, \bar{\chi}_{\bar{n}}(0)\, \frac{\not{n}}{2}\, \chi_{\bar{n}}(y^+, 0^-, \vec{y}_\perp)\, |N_2(\bar{P},\sigma_2)\rangle|_{\text{zb subtracted}}$$

$$S(0^+, 0^-, \vec{y}_\perp) = \frac{Tr}{N_c}\, \langle 0|\, [S_n^{T\dagger} S_{\bar{n}}^T](0^+, 0^-, \vec{y}_\perp)[S_{\bar{n}}^{T\dagger} S_n^T](0)\, |0\rangle$$

$$\chi_n = W_n^\dagger \xi_n$$

$$W_n(x) = \bar{P}\exp\left[\int_{-\infty}^{0} ds\, \bar{n}\cdot A_n^a(x+\bar{n}s)t^a\right]$$

$$S_n(x) = P\exp\left[\int_{-\infty}^{0} ds\, n\cdot A_s^a(x+ns)t^a\right]$$

Proper definition of TMDs in QCD (2/2)

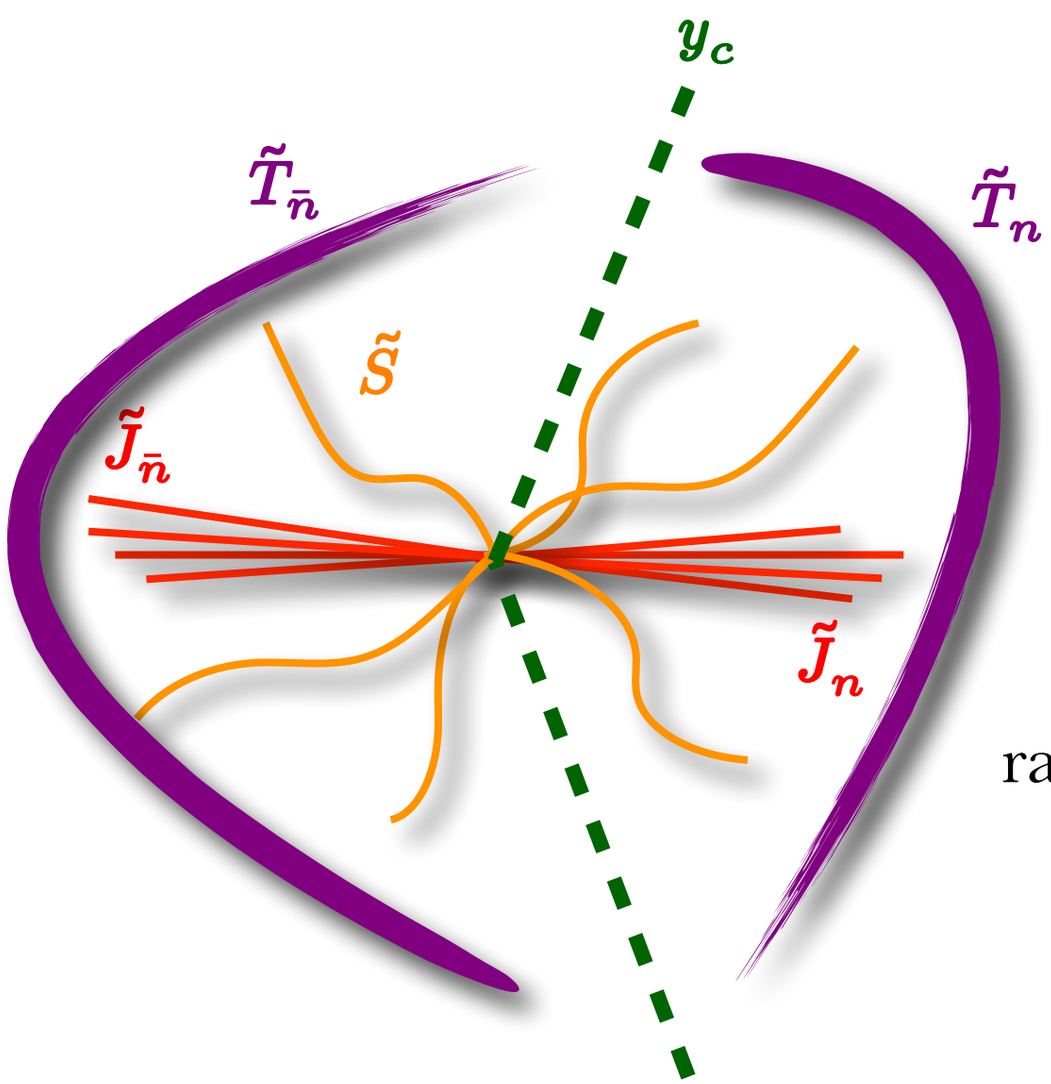
$$k_n \sim Q(1, \lambda^2, \lambda)$$
$$k_{\bar{n}} \sim Q(\lambda^2, 1, \lambda)$$
$$k_s \sim Q(\lambda, \lambda, \lambda)$$

$$y = \frac{1}{2}\ln\left|\frac{k^+}{k^-}\right|$$

Different rapidities (mixed under boosts)

$$k_n^2 \sim k_{\bar{n}}^2 \sim k_s^2 \sim Q^2\lambda^2$$

Same invariant mass!



Cancel spurious rapidity divergences

$$\zeta_A = (p^+)^2 e^{-2y_c}$$

$$\tilde{T}_n(x_A, \vec{k}_{n\perp}, S_A; \zeta_A, \mu) = \tilde{J}_n \sqrt{\tilde{S}}$$

$$\tilde{T}_{\bar{n}}(x_B, \vec{k}_{\bar{n}\perp}, S_B; \zeta_B, \mu) = \tilde{J}_{\bar{n}} \sqrt{\tilde{S}}$$

$$\zeta_B = (\bar{p}^-)^2 e^{+2y_c}$$

[MGE, Idilbi, Scimemi 1111.4996 (JHEP), 1211.1947 (PLB), 1402.0869 (PRD)]
[MGE, Kasemets, Mulders, Pisano 1502.05354 (JHEP)]
[Collins' book (2011)]

Evolution of TMDs in QCD

TMDs depend on **two scales**.

Evolution universal for all (un)polarized TMDPDFs and TMDFFs

- **Renormalization scale**:

$$\frac{d}{d\ln\mu}\ln\tilde{T}^{[pol]}_{j\leftrightarrow A}(x,b_\perp,S_A;\zeta_A,\mu)=\gamma_j\left(\alpha_s(\mu),\ln\frac{\zeta_A}{\mu^2}\right)$$

Known at 3-loops.
Numerical at 4-loops

$$\gamma_j\left(\alpha_s(\mu),\ln\frac{\zeta_A}{\mu^2}\right)=-\Gamma^j_{cusp}(\alpha_s(\mu))\ln\frac{\zeta_A}{\mu^2}-\gamma^j_{nc}(\alpha_s(\mu))$$

[Moch, Vermaseren, Vogt hep-ph/0507039 (JHEP), hep-ph/0403192 (NPB)]
[Moch, Ruijl, Ueda, Vermaseren, Vogt 1707.08315 (JHEP)]

- **Rapidity scale**:

$$\frac{d}{d\ln\zeta_A}\ln\tilde{T}^{[pol]}_{j\leftrightarrow A}(x,b_\perp,S_A;\zeta_A,\mu)=-D_j(b_T;\mu)$$

Known at 3-loops (almost 4-loops)

$$\frac{dD_j}{d\ln\mu}=\Gamma^j_{cusp}(\alpha_s(\mu))$$

Cusp does *not* completely determine D_j

Indirect 2-loops: *[Becher, Neubert 1007.4005 (EPJC)]*
Direct 2-loops: *[MGE, Scimemi, Vladimirov 1511.05590 (PRD)]*

Direct 3-loops:
[Li, Zhu 1604.01404 (PRL)]
[Vladimirov 1610.05791 (PRL)]

Refactorization of TMDs in QCD

- TMDs contain perturbative information when transverse momentum is large:

$$\tilde{T}_{i\leftrightarrow A}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \tilde{C}^T_{i\leftrightarrow j}(x, b_T; \zeta, \mu) \otimes t_{j\leftrightarrow A}(x; \mu) + O(b_T \Lambda_{QCD})$$

- For each TMD we have a different OPE. For example:

$$\tilde{f}_1^{q/A}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}^f_{q/j}(\bar{x}, b_T; \zeta, \mu)\, f_{j/A}(x/\bar{x}; \mu)$$

$$\tilde{h}_1^{\perp g/A\,(2)}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}^h_{g/j}(\bar{x}, b_T; \zeta, \mu)\, f_{j/A}(x/\bar{x}; \mu)$$

$$\tilde{g}_{1L}^{g/A}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}^g_{g/j}(\bar{x}, b_T; \zeta, \mu)\, g_{j/A}(x/\bar{x}; \mu)$$

$$\tilde{f}_{1T}^{\perp g/A\,(1)}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}_1}{\bar{x}_1} \frac{d\bar{x}_2}{\bar{x}_2} \tilde{C}^{sivers}_{g/j}(\bar{x}_1, \bar{x}_2, b_T; \zeta, \mu)\, T_{F\,j/A}(x_1/\bar{x}_1, x_2/\bar{x}_2; \mu)$$

Unpolarized quark/gluon TMD distribution and fragmentation functions at NNLO in
[MGE, Scimemi, Vladimirov 1604.07869 (JHEP)]

Transversely polarized TMDs at NNLO in
[Gutiérrez-Reyes, Scimemi, Vladimirov 1805.07243 (JHEP)]

(more to come…)

TMDs in full glory

$$\tilde{T}_{i\leftrightarrow A}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \tilde{C}^T_{i\leftrightarrow j}(x,\hat{b}_T;\mu_b^2,\mu_b)\otimes t_{j\leftrightarrow A}(x;\mu_b)$$
$$\times \exp\left[\int_{\mu_b}^{\mu}\frac{d\hat{\mu}}{\hat{\mu}}\gamma_j\left(\alpha_s(\hat{\mu}),\ln\frac{\zeta}{\hat{\mu}^2}\right)\right]\left(\frac{\zeta}{\mu_b^2}\right)^{-D_j(\hat{b}_T;\mu_b)}$$
$$\times \tilde{T}^{NP}_{i\leftrightarrow A}(x,b_T;\zeta)$$

- **General goal: extract from data all the non-perturbative inputs**
- How: **only parametrize what cannot be calculated**
- Nonperturbative part of **D_j is universal** (for all (un)polarized TMDs)
- At large and low b_T we need **cutoffs** (q_T<Lambda and q_T>Q regions)
- There are subtleties with the **evolution path**: *[Scimemi, Vladimirov 1803.11089 (JHEP)]*
- The **determination of nonperturbative pieces is not easy** (Fourier transform mixes regions, overlap of perturbative and non-perturbative)
- **Several schemes/conventions (thus difficult to be included in TMDlib)**

The higher the theoretical precision, the better!

3. QED corrections to TMD evolution

Definition of quark TMDs in QCDxQED

- We can now consider Drell-Yan where colliding quarks exchange/emit also photons
- Factorization follows the same steps as in pure QCD…

$$J_{i/P}(x,k_{nT}) = \frac{1}{2}\int \frac{dy^- d^2y_\perp}{(2\pi)^3} e^{-i(\frac{1}{2}y^- x P^+ - y_\perp\cdot k_{n\perp})}$$
$$\times \frac{1}{2}\sum_S \langle PS|\left[\bar\xi_n W_n^T \widehat{W}_{i,n}^T\right](0^+,y^-,y_\perp)\frac{\bar{\not n}}{2}\left[\widehat{W}_{i,n}^{T\dagger} W_n^{T\dagger}\xi_n\right](0)|PS\rangle$$

$$S_i(k_{sT}) = \int \frac{d^2y_\perp}{(2\pi)^2} e^{iy_\perp\cdot k_{s\perp}} \frac{Tr_c}{N_c}\langle 0|\left[S_n^{T\dagger} S_{\bar n}^T \widehat{S}_{i,n}^{T\dagger}\widehat{S}_{i,\bar n}^T\right](0^+,0^-,y_\perp)\left[S_{\bar n}^{T\dagger} S_n^T \widehat{S}_{i,\bar n}^{T\dagger}\widehat{S}_{i,n}^T\right](0)|0\rangle$$

$$W_n(x) = \bar P \exp\left[ig_s\int_{-\infty}^0 ds\,\bar n\cdot A_n(x+s\bar n)\right]$$

$$\widehat{W}_{i,n}(x) = \exp\left[ieQ_i\int_{-\infty}^0 ds\,\bar n\cdot B_n(x+s\bar n)\right]$$

photon

- **New photon Wilson lines** introduce **rap. divs.**, which cancel as in QCD
- **All (un)polarized TMDPDFs and TMDFFs defined similarly**

QCDxQED evolution of quark TMDs

- Evolution equations in QCDxQED are:

$$\frac{d}{d\ln\mu}\ln\tilde{F}_i(x,b_T;\zeta,\mu) \equiv \gamma_{F_i}\left(\alpha_s(\mu),\alpha(\mu),\ln\frac{\zeta}{\mu^2}\right) = -\gamma_i(\alpha_s(\mu),\alpha(\mu)) - \Gamma_i(\alpha_s(\mu),\alpha(\mu))\ln\frac{\zeta}{\mu^2}$$

$$\frac{d}{d\ln\zeta}\ln\tilde{F}_i(x,b_T;\zeta,\mu) = -D_i(L_\perp;\alpha_s(\mu),\alpha(\mu))$$

$$L_\perp = \ln(\mu^2 b_T^2 e^{2\gamma_E}/4)$$

$$\frac{d}{d\ln\mu}D_i(L_\perp;\alpha_s(\mu),\alpha(\mu)) = \Gamma_i(\alpha_s(\mu),\alpha(\mu))$$

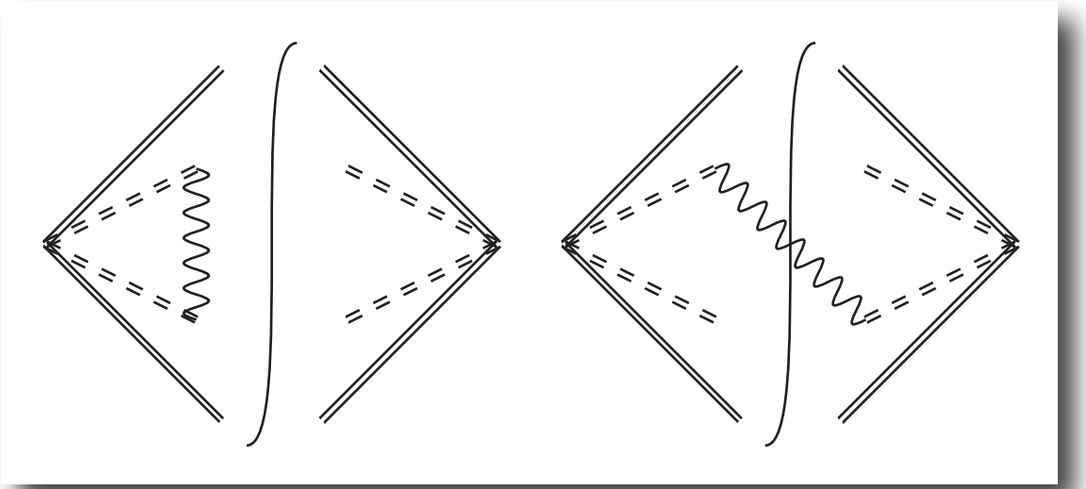
- QCD & QED scales are not distinguished for simplicity
- QED corrections break flavor universality of pure QCD evolution
- No relation between α_s and α holds for all scales (μ_b integrated over). Either we impose a relation, or consider each contribution independently

Will derive now QED corrections to:
D and anomalous dimension

$$A(\alpha_s,\alpha) = \sum_{n,m} A^{(n,m)}\left(\frac{\alpha_s}{4\pi}\right)^n\left(\frac{\alpha}{4\pi}\right)^m$$

Pure QED corrections to the soft function at LO

$$S_i(k_{sT}) = \int \frac{d^2 y_\perp}{(2\pi)^2} e^{i y_\perp \cdot k_{s\perp}} \frac{Tr_c}{N_c} \langle 0| \left[S_n^{T\dagger} S_{\bar{n}}^T \widehat{S}_{i,n}^{T\dagger} \widehat{S}_{i,\bar{n}}^T \right] (0^+, 0^-, y_\perp) \left[S_{\bar{n}}^{T\dagger} S_n^T \widehat{S}_{i,\bar{n}}^{T\dagger} \widehat{S}_{i,n}^T \right] (0) |0\rangle$$



Double lines:
gluon Wilson lines
Double dashed:
photon Wilson lines

$$\tilde{S}(b_T; \mu; \delta^+, \delta^-) = 1 + \frac{\alpha Q_i^2}{2\pi} \left[-\frac{2}{\varepsilon_{UV}^2} + \frac{2}{\varepsilon_{UV}} \ln \frac{\delta^+ \delta^-}{\mu^2} + L_T^2 + 2 L_T \ln \frac{\delta^+ \delta^-}{\mu^2} + \frac{\pi^2}{6} \right]$$

$$D_i^{(0,1)}(L_\perp) = \frac{\Gamma_i^{(0,1)}}{2} L_\perp \qquad \Gamma_i^{(0,1)} = 4 Q_i^2$$

$$\Gamma^{(1,0)} = 4 C_F$$

Pure QED corrections to the soft function at NLO

$$D_i^{(0,2)}(L_\perp) = \frac{\Gamma_i^{(0,1)}}{4\hat{\beta}^{(0,1)}}\left(\hat{\beta}^{(0,1)}L_\perp\right)^2 + \left(\frac{\Gamma_i^{(0,2)}}{2\hat{\beta}^{(0,1)}}\right)\left(\hat{\beta}^{(0,1)}L_\perp\right) + D_i^{(0,2)}(0)$$

$$D_i^{(0,2)}(0) = -\left(\frac{112}{27}\right)Q_i^2\left[N_c\sum_j^{n_f}Q_j^2 + n_l Q_l^2\right],$$

$$\Gamma_i^{(0,2)}/\Gamma_i^{(0,1)} = -\frac{20}{9}\left[N_c\sum_j^{n_f}Q_j^2 + n_l Q_l^2\right]$$

Results in pure QCD

$$D^{(2,0)}(0) = C_F C_A\left(\frac{404}{27} - 14\zeta_3\right) - \left(\frac{112}{27}\right)C_F T_F n_f$$

$$\Gamma^{(2,0)}/\Gamma^{(1,0)} = \left(\frac{67}{9} - \frac{\pi^2}{3}\right)C_A - \frac{20}{9}T_F n_f$$

QCD and QED beta functions

$$\frac{d\ln\alpha_s}{d\ln\mu^2} \equiv \beta(\alpha_s(\mu),\alpha(\mu)) = -\sum_{n,m} \beta^{(n,m)} \left(\frac{\alpha_s}{4\pi}\right)^n \left(\frac{\alpha}{4\pi}\right)^m$$

$$\frac{d\ln\alpha}{d\ln\mu^2} \equiv \hat{\beta}(\alpha_s(\mu),\alpha(\mu)) = -\sum_{n,m} \hat{\beta}^{(n,m)} \left(\frac{\alpha_s}{4\pi}\right)^n \left(\frac{\alpha}{4\pi}\right)^m$$

Mixed QCDxQED and pure QED coefficients

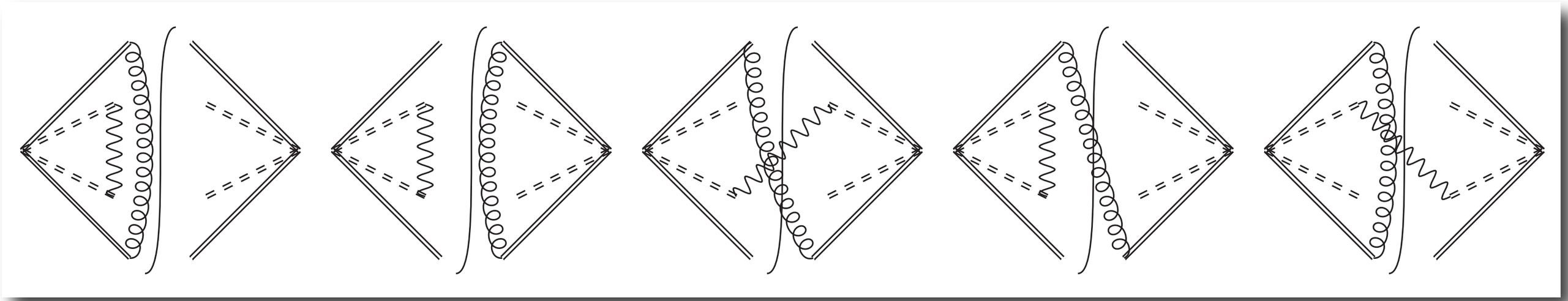
$$\beta^{(1,1)} = -2\sum_j^{n_f} Q_j^2$$

$$\hat{\beta}^{(0,1)} = -\frac{4}{3}[N_c \sum_j^{n_f} Q_j^2 + n_l Q_l^2]$$

$$\hat{\beta}^{(0,2)} = -4[N_c \sum_j^{n_f} Q_j^4 + n_l Q_l^4]$$

$$\hat{\beta}^{(1,1)} = -4C_F N_c \sum_j^{n_f} Q_j^2$$

Mixed QCDxQED corrections to the soft function (1/2)



Sample diagrams

- **Soft function** can be partially ***factorized***:

$$S_i(k_{sT}) = \int \frac{d^2 y_\perp}{(2\pi)^2} e^{i y_\perp \cdot k_{s\perp}} \tilde{S}^{QCD}(y_T)\, \tilde{S}_i^{QED}(y_T) + O(\alpha_s^n \alpha^m)\Big|_{n\cdot m>1}$$

$$\tilde{S}^{QCD}(y_T) = \frac{Tr_c}{N_c} \langle 0| \left[S_n^{T\dagger} S_{\bar{n}}^T\right](0^+, 0^-, y_\perp) \left[S_{\bar{n}}^{T\dagger} S_n^T\right](0)\, |0\rangle$$

$$\tilde{S}_i^{QED}(y_T) = \langle 0| \left[\widehat{S}_{i,n}^{T\dagger} \widehat{S}_{i,\bar{n}}^T\right](0^+, 0^-, y_\perp) \left[\widehat{S}_{i,\bar{n}}^{T\dagger} \widehat{S}_{i,n}^T\right](0)\, |0\rangle$$

Mixed QCDxQED corrections to the soft function (2/2)

- **Cancellation** of QCD & QED **rapidity divergences** only possible if:

$$\ln\tilde{S}_i(b_T) = A_i(L_\perp;\alpha_s,\alpha) + 2D_i(L_\perp;\alpha_s,\alpha)\ln\frac{\delta^+\delta^-}{\mu^2}$$

- And we already know that for each soft function we can write:

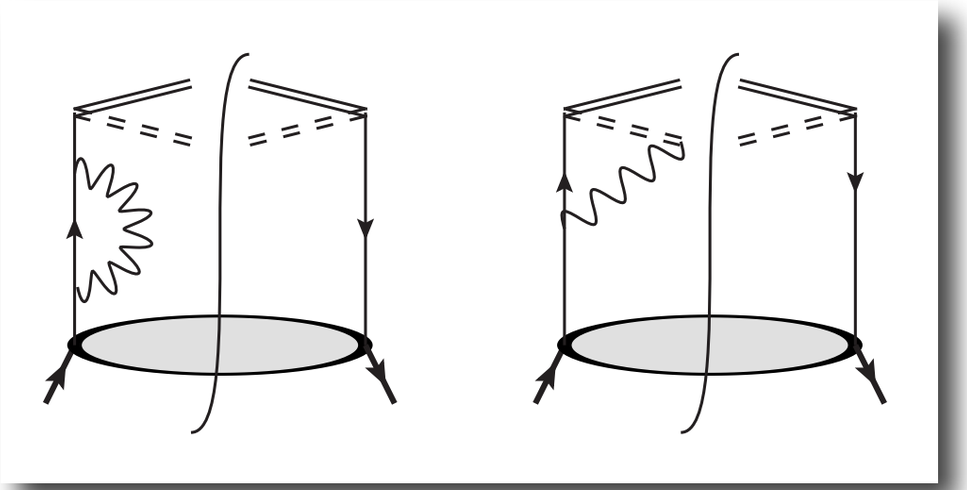
$$\ln\tilde{S}^{QCD}(b_T) = A^{QCD}(L_\perp;\alpha_s,\alpha) + 2D^{QCD}(L_\perp;\alpha_s,\alpha)\ln\frac{\delta^+\delta^-}{\mu^2}$$

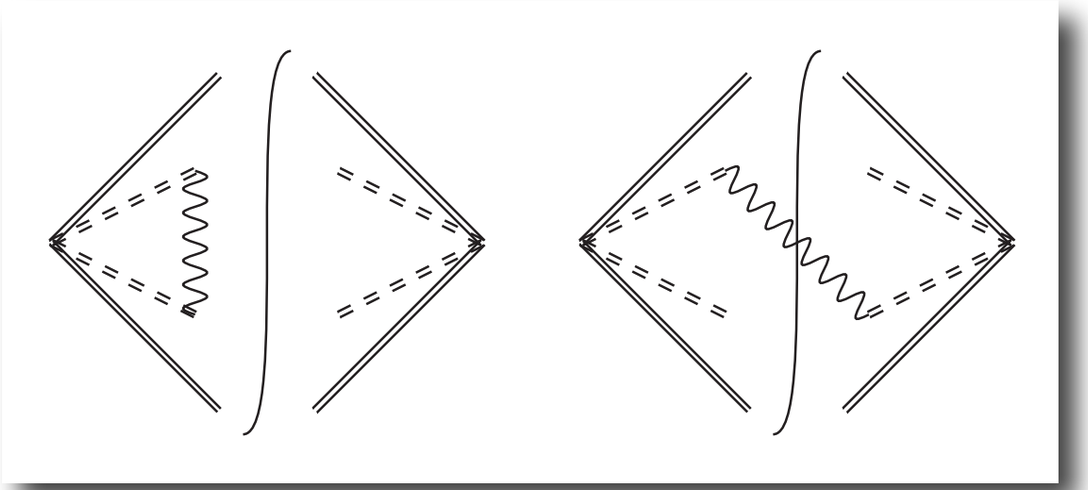
$$\ln\tilde{S}_i^{QED}(b_T) = A_i^{QED}(L_\perp;\alpha_s,\alpha) + 2D_i^{QED}(L_\perp;\alpha_s,\alpha)\ln\frac{\delta^+\delta^-}{\mu^2}$$

Then we have:

$$D_i^{(1,1)}(L_\perp) = 0$$

Pure QED corrections to the anomalous dimension at LO





$$\gamma^{(1,0)} = -6C_F$$

Calculation is analogous to the one in pure QCD

$$\gamma_i^{(0,1)} = -6Q_i^2$$
$$\Gamma_i^{(0,1)} = 4Q_i^2$$

$$\Gamma^{(1,0)} = 4C_F$$

Pure QED corrections to the anomalous dimension at NLO

$$\gamma^{(2,0)} = C_F^2\left(-3+4\pi^2-48\zeta_3\right) + C_F C_A\left(-\frac{961}{27}-\frac{11\pi^2}{3}+52\zeta_3\right)$$
$$+ C_F T_F n_f\left(\frac{260}{27}+\frac{4\pi^2}{3}\right)$$

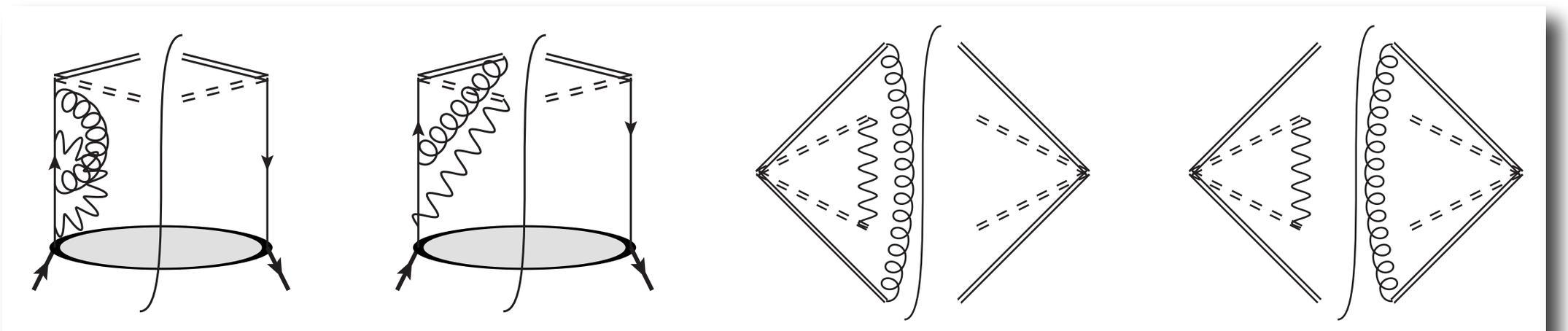
$$\gamma_i^{(0,2)} = Q_i^4\left(-3+4\pi^2-48\zeta_3\right) + Q_i^2\left[N_c\sum_j^{n_f} Q_j^2 + n_l Q_l^2\right]\left(\frac{260}{27}+\frac{4\pi^2}{3}\right)$$

$$\Gamma_i^{(0,2)}/\Gamma_i^{(0,1)} = -\frac{20}{9}\left[N_c\sum_j^{n_f} Q_j^2 + n_l Q_l^2\right]$$

$$\Gamma^{(2,0)}/\Gamma^{(1,0)} = \left(\frac{67}{9}-\frac{\pi^2}{3}\right)C_A - \frac{20}{9}T_F n_f$$

Mixed QCDxQED corrections to the anomalous dimension

- **Mixed QCD-QED** virtual corrections for **non-cusp**:



$$\gamma^{(2,0)} = C_F^2\left(-3+4\pi^2-48\zeta_3\right) + C_F C_A\left(-\frac{961}{27}-\frac{11\pi^2}{3}+52\zeta_3\right)$$
$$+ C_F T_F n_f\left(\frac{260}{27}+\frac{4\pi^2}{3}\right)$$

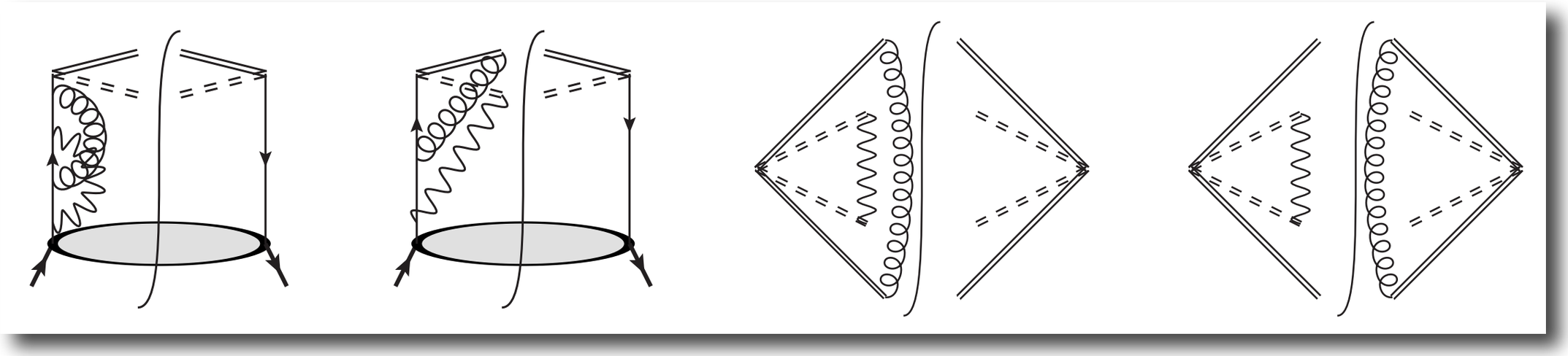
no fermion loops possible at this order

factor 2 needed because of 2 ways of replacing internal g-g with g-γ and γ-g

$$\gamma_i^{(1,1)} = 2C_F Q_i^2\left(-3+4\pi^2-48\zeta_3\right)$$

Mixed QCDxQED corrections to the anomalous dimension

- **Mixed QCD-QED** virtual corrections for **cusp**:



$$\Gamma^{(2,0)}/\Gamma^{(1,0)} = \left(\frac{67}{9} - \frac{\pi^2}{3}\right) C_A - \frac{20}{9} T_F n_f$$

no fermion loops possible at this order

$$\Gamma_i^{(1,1)} = 0$$

Consistent with

[Kilgore, Sturm 1107.4798 (PRD)]
[Kilgore 1308.1055 (EPJC)]

QCDxQED evolution of quark TMDs: summary

$\mathcal{O}(\alpha)$

$$D_i^{(0,1)}(L_\perp) = \frac{\Gamma_i^{(0,1)}}{2} L_\perp$$

$$\gamma_i^{(0,1)} = -6Q_i^2$$

$$\Gamma_i^{(0,1)} = 4Q_i^2$$

$\mathcal{O}(\alpha_s\alpha)$

$$D_i^{(1,1)}(L_\perp) = 0$$

$$\gamma_i^{(1,1)} = 2C_F Q_i^2 \left(-3 + 4\pi^2 - 48\zeta_3\right)$$

$$\Gamma_i^{(1,1)} = 0$$

$\mathcal{O}(\alpha^2)$

$$D_i^{(0,2)}(L_\perp) = \frac{\Gamma_i^{(0,1)}}{4\hat{\beta}^{(0,1)}} \left(\hat{\beta}^{(0,1)} L_\perp\right)^2 + \left(\frac{\Gamma_i^{(0,2)}}{2\hat{\beta}^{(0,1)}}\right) \left(\hat{\beta}^{(0,1)} L_\perp\right) + D_i^{(0,2)}(0)$$

$$D_i^{(0,2)}(0) = -\left(\frac{112}{27}\right) Q_i^2 [N_c \sum_j^{n_f} Q_j^2 + n_l Q_l^2],$$

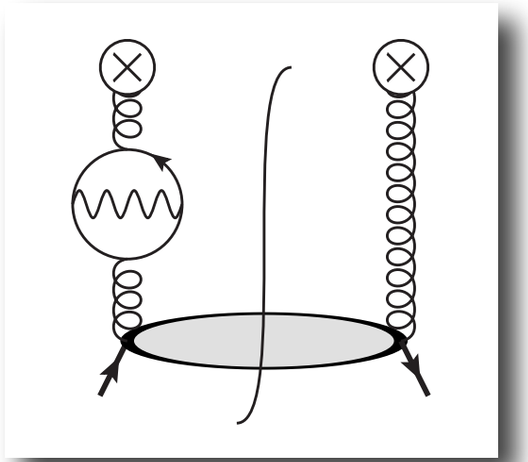
$$\Gamma_i^{(0,2)}/\Gamma_i^{(0,1)} = -\frac{20}{9} [N_c \sum_j^{n_f} Q_j^2 + n_l Q_l^2]$$

$$\gamma_i^{(0,2)} = Q_i^4 \left(-3 + 4\pi^2 - 48\zeta_3\right) + Q_i^2 [N_c \sum_j^{n_f} Q_j^2 + n_l Q_l^2] \left(\frac{260}{27} + \frac{4\pi^2}{3}\right)$$

$$\Gamma_i^{(0,2)}/\Gamma_i^{(0,1)} = -\frac{20}{9} [N_c \sum_j^{n_f} Q_j^2 + n_l Q_l^2]$$

Definition & Evolution of gluon TMDs in QCDxQED

- Gluon TMDs are **defined as in pure QCD**: **no photon Wilson lines**!
- Glue-Glue bi-local operator is already QED gauge invariant
- QED effects appear only as higher-order corrections
- **No pure QED corrections** for anomalous dimension nor D term
- **Mixed QCD-QED corrections for anomalous dimension**:



No contribution of soft function at this order

$$\gamma_g^{(2,0)} = 2C_A^2\left(-\frac{692}{27}+\frac{11\pi^2}{18}+2\zeta_3\right)+2C_AT_Fn_f\left(\frac{256}{27}-\frac{2\pi^2}{9}\right)+8C_FT_Fn_f$$

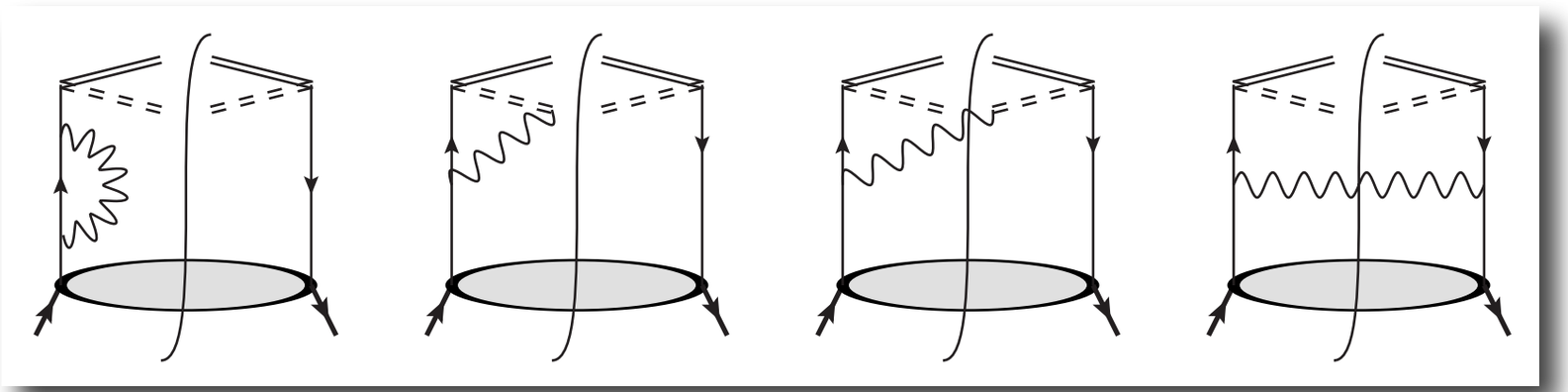
$$\gamma_g^{(1,1)} = 8T_F\sum_j^{n_f} Q_j^2$$

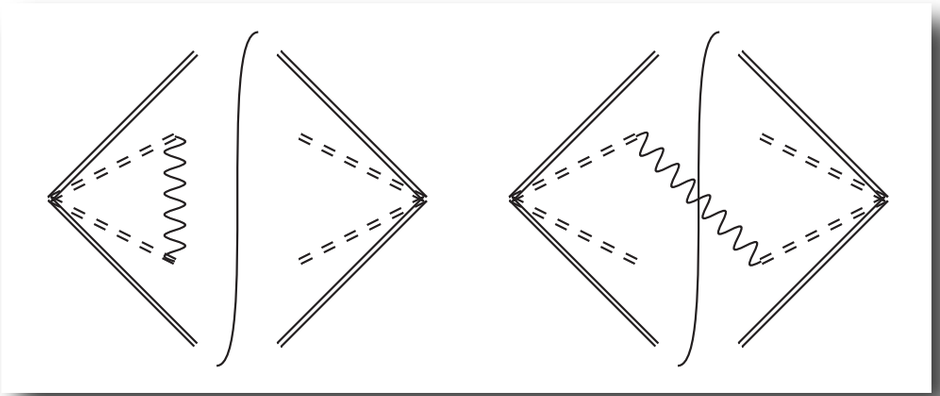
(No factor N_c here)

- **No mixed corrections for cusp nor D** (soft function zero <u>at this order</u>)

QED corrections to f_1 at large p_T

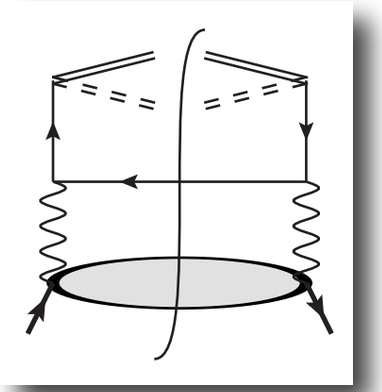
$$\tilde{f}_{i/A}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g,\gamma} \tilde{C}_{i/j}(x, b_T; \zeta, \mu) \otimes f_{j/A}(x; \mu) + O\Big(b_T \Lambda_{QCD}\Big)$$





QED rapidity divergences cancel in the properly defined TMDs

$$\tilde{C}^{(0,1)}_{i/i}(x, b_T; \mu) = Q_i^2 \left[\delta(1-x)\left(-L_T^2 + 3L_T + 2L_T \ln\frac{\mu^2}{Q^2} - \frac{\pi^2}{6}\right) - 2L_T P_{i\leftarrow i} + 2(1-x)\right]$$



$$\tilde{C}^{(0,1)}_{i/\gamma}(x, b_T; \mu) = N_c Q_i^2 \Big[-2L_T(x^2 + (1-x)^2) + 4x(1-x)\Big]$$

(Factor N_c due to color multiplicity)

Conclusions & Outlook

- **Theoretical precision is needed to extract nonperturbative Physics**
- **New: QED corrections to TMD evolution** obtained at $O(\alpha_s\alpha)$ and up to $O(\alpha^2)$ **(universal but flavor-dependent)**
- **QED corrections to f_1** at large p_T obtained at $O(\alpha)$. Similarly for other TMDs.
- **"TMD community" is catching up with the "PDF community"!**
- **ToDo: phenomenology** (study numerical impact of higher-orders and QED)
- **Need more experimental data**! (JLab, RHIC, EIC, Fixed-target@LHC,…)

Thank you!