Estimating nucleon Transverse Momentum Dependent structure functions on the lattice

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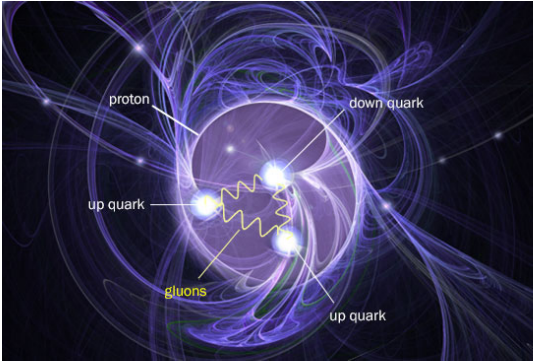
Hadrons' internal structure

Standard Model of elementary particles: electrons, muons, quarks, gluons, photons, $W^{\pm}$, Z, Higgs, ...



Credit: *Brookhaven National Lab website*

Experiment: HERA, LHC, Electron-Ion Collider study hadron structure functions and try to discover the origin of mass,
Theory: Quantum Chromodynamics (QCD) is the theory describing the interations of quarks and gluons.

Transverse momentum distribution

Straight-link distributions

- B. Musch [LHPC Collaboration], *Transverse momentum distribution of quarks from the lattice using extended gauge links*, PoS Lattice 2007, arXiv:0710.4423
- P. Hägler, B. Musch, J. Negele, A. Schäfer, *Intrinsic quark transverse momentum in the nucleon from lattice QCD*, EPL 88 (2009) 61001, arXiv:0908.1283
- B. Musch, P. Hägler, J. Negele, A. Schäfer, *Exploring quark transverse momentum distributions with lattice QCD*, Phys. Rev. D 83 (2011) 094507, arXiv:1011.1213

Staple-link distributions

- B. Musch, P. Hägler, M. Engelhardt, J. Negele, A. Schäfer, *Sivers and Boer-Mulders observables from lattice QCD*, Phys. Rev. D 85 (2012) 094510, arXiv:1111.4249
- M. Engelhardt, P. Hägler, B. Musch, J. Negele, A. Schäfer, *Lattice QCD study of the Boer-Mulders effect in a pion*, Phys. Rev. D 93 (2016) 054501, arXiv:1506.07826
- B. Yoon et al., *Nucleon Transverse Momentum-dependent Parton Distributions in Lattice QCD: Renormalization Patters and Discretization Effects*, Phys. Rev. D 96 (2017) 094508, arXiv:1706.03406

Transverse momentum distribution

x-dependence:

- X. Ji, P. Sun, X. Xiong, F. Yuan, *Soft factor subtraction and transverse momentum dependent parton distributions on the lattice*, Phys. Rev. D91, 074009 (2015), arXiv:1405.7640
- X. Ji, L. C. Jin, F. Yaun, J. H. Zhang, Y. Zhao, *Transverse Momentum Dependent Quasi-Parton Distributions*, arXiv: 1801.05930
- M. Ebert, I. Stewart, Y. Zhao, *Determining the Nonperturbative Collins-Soper Kernel From Lattice QCD*, arXiv:1811.00026

The method is still being developed, no numerical results yet.

Transverse momentum distribution

Functions

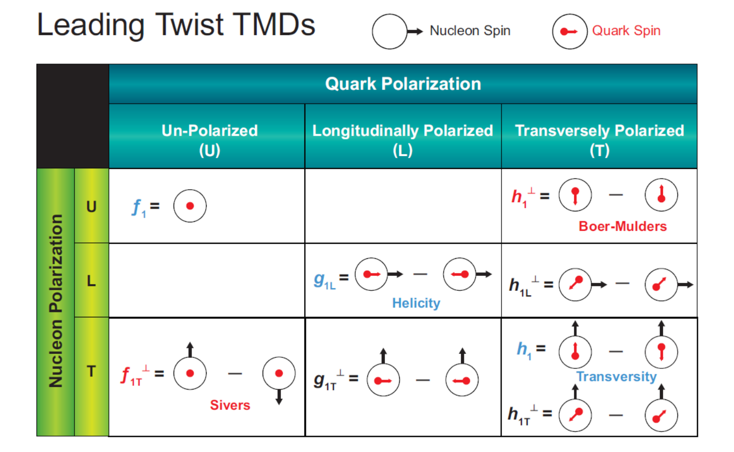


Figure from EIC White Book

Transverse momentum distribution

Lattice formulation

$$\tilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,...) = \frac{1}{2}\langle P,S|\bar{q}(0)\Gamma\mathcal{U}[0,...,b]q(b)|P,S\rangle$$

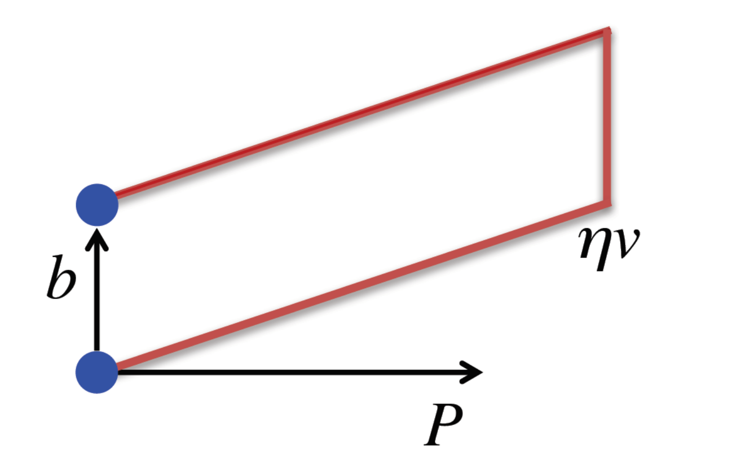
$$\tilde{\Phi}^{[\Gamma]}_{\text{subtr.}}(b,P,S,...) = \tilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,...)\ \mathcal{S}\ Z_{\text{TMD}}\ Z_2$$

where

- $\mathcal{S}$ regulates the soft and collinear divergences
- Z_2 is the quark field renormalization factor
- Z_{TMD} contains dependence on the specific tensor structure of the TMD operator

$$\Phi^{[\Gamma]}(x,k_T,P,S,....) =$$
$$= \int \frac{d^2 b_T}{(2\pi)^2} \int \frac{d(b\cdot P)}{(2\pi)P^+} \exp(ix(b\cdot P) - ib_T\cdot k_T)\tilde{\Phi}^{[\Gamma]}_{\text{subtr.}}(b,P,S,...)\Big|_{b^+=0}$$

Lattice staple operator



The four vector v points in a spatial direction. From B. Yoon et al., Phys. Rev. D 96 (2017) 094508

Transverse momentum distribution

Lattice staple operator

$$P = (0,0,0),(1,0,0),(2,0,0),(3,0,0),(4,0,0),(5,0,0),$$

$\vec{v} =$

- 1 1 1 1 1 ...
- 3 3 3 3 3 ...
- 1 2 1 2 1 ...
- 2 1 2 1 2 ...
- 1 3 1 3 1 ...
- 3 1 3 1 3 ...
- 1 -2 1 -2 ...
- -2 1 -2 1 ...
- 1 -3 1 -3 ...
- -3 1 -3 1 ...

$\vec{b} =$3, 3 3, 3 3 3, ..., -3, ..., 2, ..., -2, ..., 2 3, ..., 3 2, ..., -2 -3, ...,
Approx. 30000 contractions for each momentum and sink-source separation!

Transverse momentum distribution

Three-point correlation function

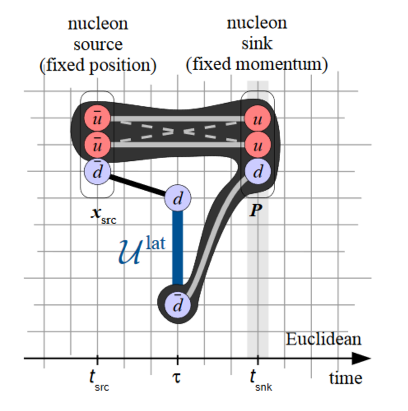


Figure from B. Musch PhD thesis

Disconnected diagrams are neglected

Bare matrix elements

$$C_{3\mathrm{pt}}[\hat{O}](t_i, t, t_f, P) = \sum_{x_i, x_f} e^{-i(x_f - x_i)\cdot P} \langle \phi(t_f, x_f) \hat{O}_t \phi^\dagger(t_i, x_i) \rangle$$

$$C_{2\mathrm{pt}}(t_i, t_f, P) = \sum_{x_i, x_f} e^{-i(x_f - x_i)\cdot P} \langle \phi(t_f, x_f) \phi^\dagger(t_i, x_i) \rangle$$

The bare matrix element is obtained from plateaux in t for $t_i \ll t \ll t_f$,

$$\tilde{\Phi}^{[\Gamma]}_{\mathrm{unsubtr.}} = E(P) \frac{C_{3\mathrm{pt}}[\hat{O}](t_i, t, t_f, P)}{C_{2\mathrm{pt}}(t_i, t_f, P)}$$

Transverse momentum distribution

Lattice amplitudes

$$\frac{1}{2}\tilde{\Phi}^{[1]}_{\text{unsubtr.}} = m_N \tilde{A}_1 - i\frac{m_N^2}{v\cdot P}\varepsilon^{\mu\nu\rho\sigma}P_\mu b_\nu v_\rho S_\sigma \tilde{B}_5$$
$$\frac{1}{2}\tilde{\Phi}^{[\gamma^5]}_{\text{unsubtr.}} = -m_N^2(b\cdot S)\tilde{A}_5 + \frac{im_N^2}{v\cdot P}(v\cdot S)\tilde{B}_6$$
$$\frac{1}{2}\tilde{\Phi}^{[\gamma^\mu]}_{\text{unsubtr.}} = P^\mu \tilde{A}_2 - im_N^2 b^\mu \tilde{A}_3 - im_N \varepsilon^{\mu\nu\alpha\beta}P_\nu b_\alpha S_\beta \tilde{A}_{12} + \dots$$

where Λ is the helicity, S_j the transverse spin and m_N the nucleon mass.

Transverse momentum distribution

Double moments

x-integrated TMDs in Fourier space correspond directly to amplitudes $\tilde{A}_{iB}$ evaluated at $b \cdot P = 0$

$$
\begin{aligned}
\tilde{f}_1^{[1](0)}(b_T^2, \hat{\zeta}, \eta v \cdot P) &= 2\tilde{A}_{2B}/\mathcal{Z}^f, \\
\tilde{g}_1^{[1](0)}(b_T^2, \hat{\zeta}, \eta v \cdot P) &= -2\tilde{A}_{6B}/\mathcal{Z}^g, \\
\tilde{g}_{1T}^{[1](1)}(b_T^2, \hat{\zeta}, \eta v \cdot P) &= -2\tilde{A}_{7B}/\mathcal{Z}^g, \\
\tilde{h}_1^{[1](0)}(b_T^2, \hat{\zeta}, \eta v \cdot P) &= -2\left(\tilde{A}_{9B} - (m_N^2 b^2/2)\tilde{A}_{11B}\right)/\mathcal{Z}^h, \\
\tilde{h}_{1L}^{\perp[1](1)}(b_T^2, \hat{\zeta}, \eta v \cdot P) &= -2\tilde{A}_{10B}/\mathcal{Z}^h, \\
\tilde{h}_{1T}^{\perp[1](0)}(b_T^2, \hat{\zeta}, \eta v \cdot P) &= 4\tilde{A}_{11B}/\mathcal{Z}^h, \\
\tilde{f}_{1T}^{\perp[1](1)}(b_T^2, \hat{\zeta}, \eta v \cdot P) &= -2\tilde{A}_{12B}/\mathcal{Z}^f, \\
\tilde{h}_1^{\perp[1](1)}(b_T^2, \hat{\zeta}, \eta v \cdot P) &= 2\tilde{A}_{4B}/\mathcal{Z}^h,
\end{aligned}
$$

$$
\tilde{f}^{[m](n)}(b_T^2, \ldots) = n!\left(-\frac{2}{m_N^2}\partial_{b_T^2}\right)^n \int_{-1}^{1} dx x^{m-1} \int d^2 k_T e^{i b_T \cdot k_T} f(x, k_T^2)
$$

Transverse momentum distribution

Lattice formulation

Sivers shift $\langle \vec{k}_y \rangle$

$$\langle \vec{k}_y \rangle_{TU}(b_T^2, \ldots) = m_N \frac{\tilde{f}_{1T}^{\perp [1](1)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)}$$

"TU" $\rightarrow$ distribution of Unpolarized quarks in a Transversely polarized proton.

Boer-Mulders shift $\langle \vec{k}_y \rangle$

$$\langle \vec{k}_y \rangle_{UT}(b_T^2, \ldots) = m_N \frac{\tilde{h}_1^{\perp [1](1)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)}$$

"UT" $\rightarrow$ distribution of Transversely polarized quarks in a Unpolarized proton.

Assuming multiplicative renormalization the ratios depend only on b_T^2, $\hat{\zeta}$, $\eta v \cdot P$. All other renormalization and soft factor related dependencies cancel out in the ratio.

Transverse momentum distribution

Lattice staple operator

The four vector v enters only in the Collins-Soper parameter

$$\hat{\zeta} = \zeta/(2m_N) = \frac{v \cdot P}{\sqrt{|v^2|}\sqrt{P^2}}$$

The light-cone limit can be approached even with a purely spatial choice of v, as used in lattice calculations, if the spatial momentum P is chosen large.

Transverse momentum distribution

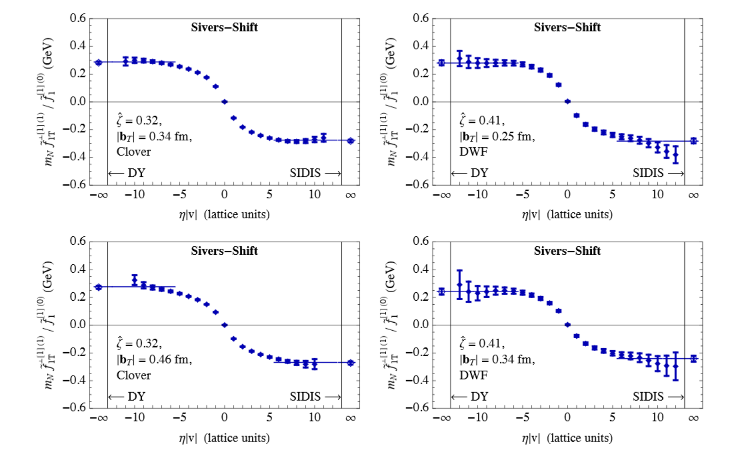
Lattice details: Phys. Rev. D 85 (2012) 094510

am_u	am_s	L^3T/a^4	a fm	m_π MeV	m_N GeV	# conf
0.01	0.05	$28^3 \times 64$	0.1196(10)	369(4)	1.197(12)	273
0.01	0.05	$20^3 \times 64$	0.1196(10)	369(4)	1.197(12)	658
0.02	0.05	$28^3 \times 64$	0.1185(10)	518(6)	1.348(15)	486

Lattice details: Phys. Rev. D 96 (2017) 094508

action	L^3T/a^4	a fm	m_π MeV	m_N GeV	# conf
clover	$32^3 \times 96$	0.11403(77)	317(4)	-	967
DWF	$32^3 \times 64$	0.0840(14)	297(5)	-	533

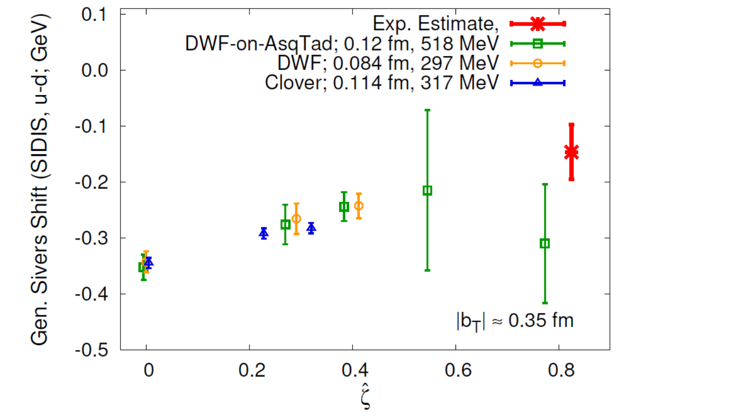
Lattice results for the Sivers shift



From B. Yoon et al., Phys. Rev. D 96 (2017) 094508

Transverse momentum distribution

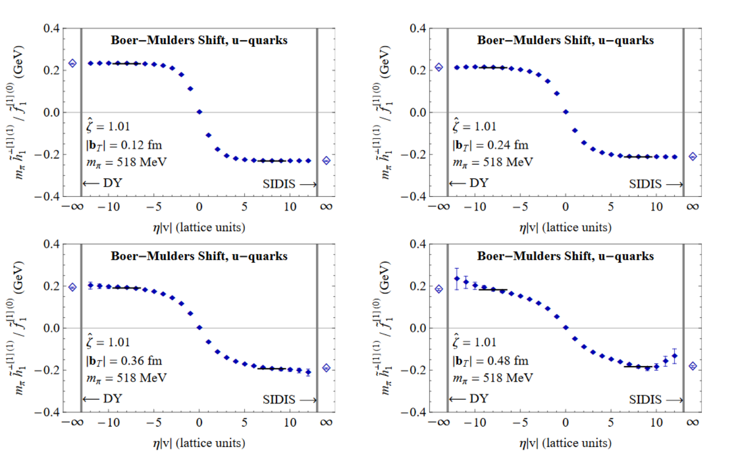
Lattice results for the Sivers shift



From B. Yoon et al., Phys. Rev. D 96 (2017) 094508

Transverse momentum distribution

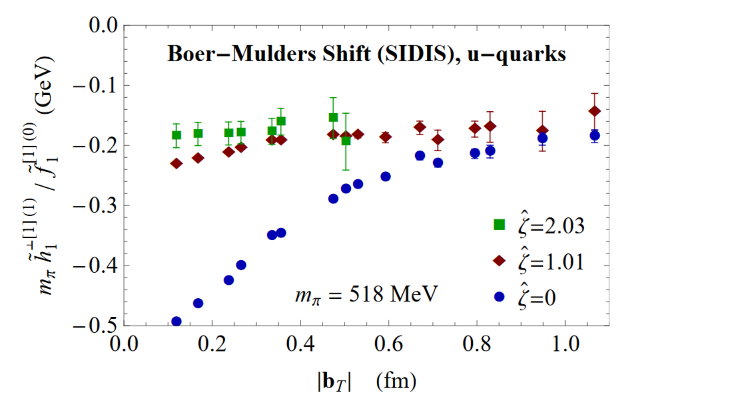
Lattice results for the Boer-Mulders shift



$\hat{\zeta} = 1.01$. From M. Engelhardt et al, Phys. Rev. D93, 054501 (2016)

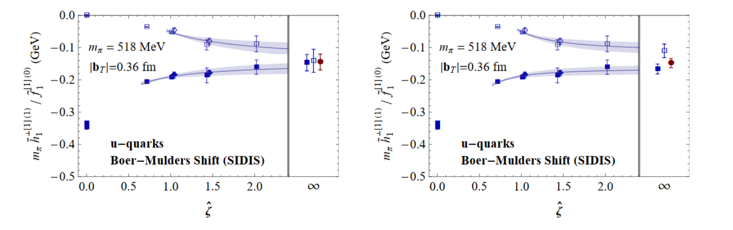
Transverse momentum distribution

Lattice results for the Boer-Mulders shift



From M. Engelhardt et al, Phys. Rev. D93, 054501 (2016)

Lattice results for the Boer-Mulders shift



From M. Engelhardt et al, Phys. Rev. D93, 054501 (2016)
Note that for the nucleon one reaches $|\hat{\zeta}| \approx 0.4$.

Transverse momentum distribution

Next steps: study the continuum limit

$$\tilde{f}_1^{[1](0)}(b_T^2, \hat{\zeta}, \eta v \cdot P) = 2\tilde{A}_{2B}/\mathcal{Z}^f,$$
$$\tilde{h}_1^{\perp[1](1)}(b_T^2, \hat{\zeta}, \eta v \cdot P) = 2\tilde{A}_{4B}/\mathcal{Z}^h,$$

and

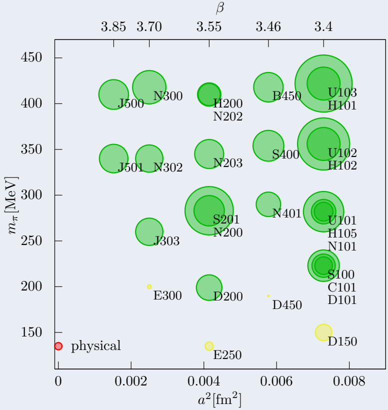
$$\mathcal{Z} = \mathcal{S} Z_2 Z_{\text{TMD}}$$

The renormalization factors do not cancel explicitly in the Boer-Mulders shift. It was postulated that for large b Z_{TMD} becomes independent of the Γ structure.

- Is that true?
- Are the anomalous dimensions equal to their continuum predictions? [M. G. Echevarria, I. Scimemi, A. Vladimirov, JHEP 1609 (2016) 004]

Transverse momentum distribution

Next steps: study the continuum limit



Landscape of gauge field ensembles generated by the CLS consortium.

Conclusions

TMDs on the lattice - exciting project

- pioneering work showed that double moments are accessible on the lattice
- many further questions open $\Rightarrow$ continuum limit
- we are pushing the calculations further
- problems to be solved in the quasi-TMD approach

Thank you for your attention!