TMDs from PB

REF2018

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Plan for today

Motivation:
We want to develop an approach in which transverse momentum kinematics will be treated without any mismatch between matrix element (ME) and PS

Introduction and motivation given by Hannes yesterday

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Plan for today:

- brief reminder of the Parton Branching (PB) method
- comparison of PB with another existing approaches

DGLAP and Sudakov form factor

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution equation

DGLAP evolution equation

$$\frac{d\,\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^1 dz\; P_{ab}(\mu^2, z)\; \widetilde{f}_b(x/z, \mu^2)$$

$xf(x,\mu^2) = \widetilde{f}(x,\mu^2)$

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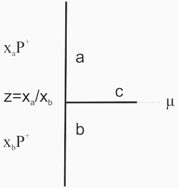
DGLAP evolution equation



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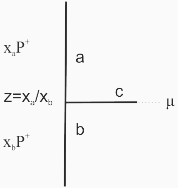

$$P_{ab} = D_{ab}\delta(1-z) + K_{ab}\frac{1}{(1-z)_+} + R_{ab}\ ,$$

$\int_0^1 f(x)g(x)_+ \mathrm{d}x = \int_0^1 (f(x)-f(1))g(x)\mathrm{d}x$

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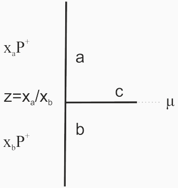

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problems for numerical solution:

1. $\delta(1-z)$ $\rightarrow$ *momentum sum rule* $\sum_c \int_0^1 \mathrm{d}z z P_{ca}(\mu^2, z) = 0$
2. integrals separately divergent: $\int_0^1 \rightarrow \int_0^{z_M}$, $z_M \approx 1$:

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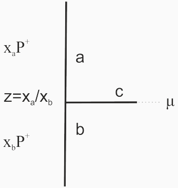

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    - resolvable $z < z_M$ and non-resolvable $z > z_M$ branchings

Introduce the *Sudakov form factor*: $\Delta_a(\mu^2) = \exp\left(-\int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \sum_b \int_0^{z_M} dz z P^R_{ba}(\mu'^2,z)\right)$

Advantages:

- Δ_a: probability of an evolution without any resolvable branching

Iterative solution

After integration:

$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2) + \int_{\ln \mu_0^2}^{\ln \mu^2} d\ln\mu_1^2 \frac{\Delta_a(\mu^2)}{\Delta_a(\mu_1^2)} \sum_b \int_x^{z_M} dz_1 P_{ab}^R\left(\mu_1^2, z_1\right) \widetilde{f}_b\left(\frac{x}{z_1}, \mu_0^2\right) \Delta_b(\mu_1^2) + \ldots$$

a is a gluon at the scale μ^2 and x. Where does it come from?

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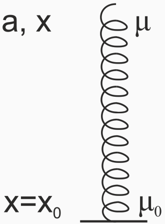

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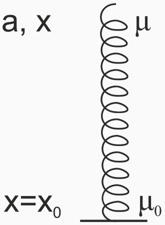

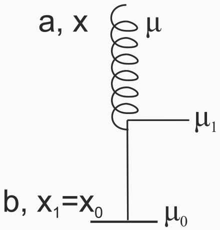

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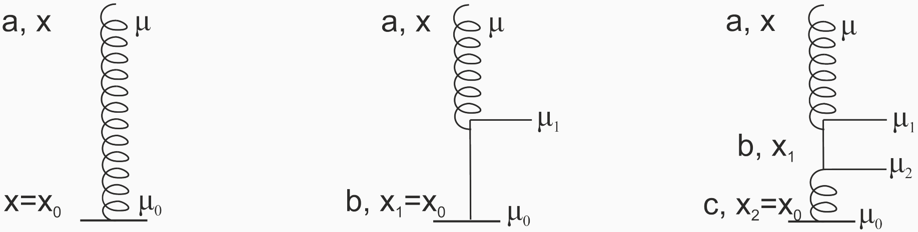

Validation of the method with QCDNUM

QCDNUM - semi-analytical solution of DGLAP Comput. Phys. Commun., 2011, 182, 490-532

input at μ_0^2: QCDNUM $\rightarrow$ evolve with PB up μ^2 $\rightarrow$ compare

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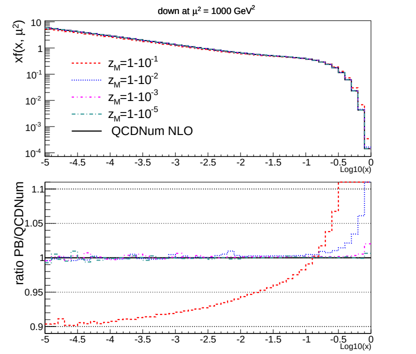
How are the collinear distributions affected by the z_M parameter?



Very good agreement with QCDNUM ✓

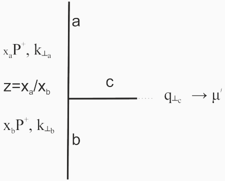
Parton Branching method to obtain TMDs

Interpretation of the evolution scale: virtuality and $p_\perp$- ordering

Momentum conservation

$$k_b = k_a + q_c$$

Assumptions: $k_a^+ = z k_b^+,\ q_c^+ = (1-z)k_b^+,\ k_b^2 = 0,\ q_c^2 = 0$
$k_{\perp,b} = 0 \to k_{\perp,a} = -q_{\perp,c}$



$$k_a^2(1-z) = -q_{\perp,c}^2$$

associate: $\mu'^2 = -k_a^2$

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$\to$ *virtuality ordering* condition, partons in the cascade are ordered in virtuality

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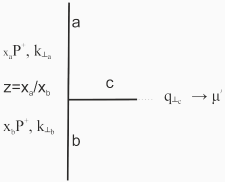

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Limit of $z \rightarrow 0$:

$$\mu'^2 = q_{\perp,c}^2$$

$\rightarrow$ *$p_\perp$-ordering* condition, partons in the cascade are ordered in $p_\perp$

Interpretation of the evolution scale: angular ordering

colour coherence phenomena:
angular ordering of the soft gluons emissions

$$\Theta_{i+1} > \Theta_i$$

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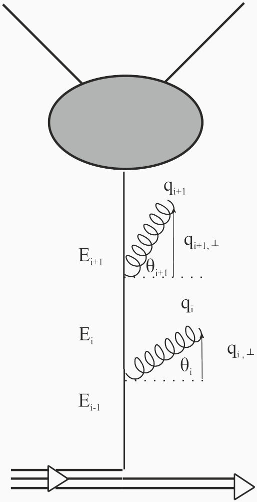
$$|q_{\perp,i}| = (1 - z_i)|\overrightarrow{k}_{i-1}| \sin\Theta_i$$

Associate:

$$q^2_{\perp,i} = (1 - z_i)^2 \mu'^2$$

→ *angular ordering* condition



TMD from DGLAP

replace $q_{\perp,c}$ with q_0 $\rightarrow$ conditions for the z_M value:

- $p_\perp$- ordering: $\mu'^2 1 = q^2_{\perp,c} \rightarrow z_M = \text{fixed}$
- virtuality ordering: $\mu'^2(1-z) = q^2_{\perp,c} \rightarrow z_M = 1 - \left(\frac{q_0}{\mu'}\right)^2$
- angular ordering: $\mu'^2(1-z)^2 = q^2_{\perp,c} \rightarrow z_M = 1 - \left(\frac{q_0}{\mu'}\right)$

renormalization scale in α_s should be chosen to be $q^2_\perp$, rather than the evolution scale μ'^2

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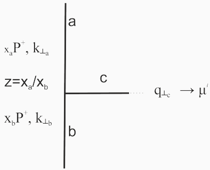

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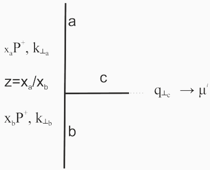

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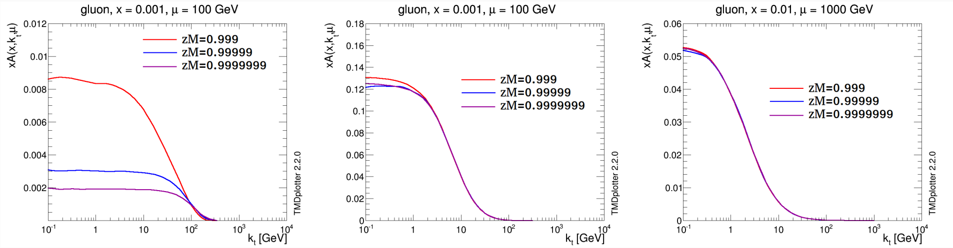
$\vec{k}_{\perp,a} = \vec{k}_{\perp,b} - \vec{q}_{\perp,c}$

- $\vec{k}_{\perp,a}$ contains the whole history of the evolution
- PB method: effect of every individual part of the ordering definition can be studied separately
- collinear PDFs not affected by the ordering if $z_M \approx 1$ and $\alpha_s(\mu'^2)$

Results

Effect of ordering choice and z_M on TMDs



$p_\perp$- ordering	virtuality ordering	angular ordering
$q_\perp^2 = 1\mu'^2$	$q_\perp^2 = (1-z)\mu'^2$	$q_\perp^2 = (1-z)^2\mu'^2$

Everywhere: fixed z_M, $\alpha_s\left(1\mu'^2\right)$

$p_\perp$- ordering: not stable TMD

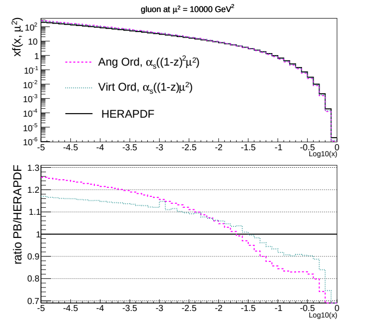
$p_\perp$- ordering valid only for $z \to 0$, otherwise violates the energy-momentum conservation

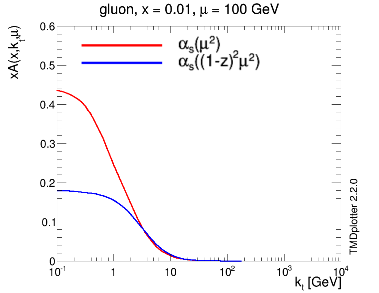
virtuality-ordering: difference between z_M only in the small $k_\perp$ region at higher scales. large improvement compared to $p_\perp$-ordering

angular-ordering: no visible difference between dirrefent z_M $\to$ stable TMD

Renormalization scale

virtuality ordering $q_\perp^2 = (1-z)\mu'^2$	angular ordering $q_\perp^2 = (1-z)^2\mu'^2$
$z_M = 1 - \left(\frac{q_0}{\mu'}\right)^2$	$z_M = 1 - \left(\frac{q_0}{\mu'}\right)$
$\alpha_s\left(q_\perp^2\right)$	



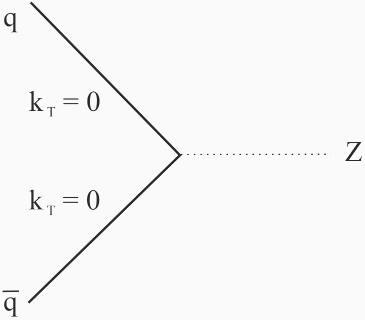


angular ordering, the same conclusions for virtuality ordering.

virtuality ordering, the same conclusions for angular ordering

Collinear and TMD PDFs affected significantly by the change of renormalization scale
$\rightarrow$ by full virtuality or angular ordering extra large logarithms resummed.

Prediction for Z boson $p_\perp$ spectrum using TMDs



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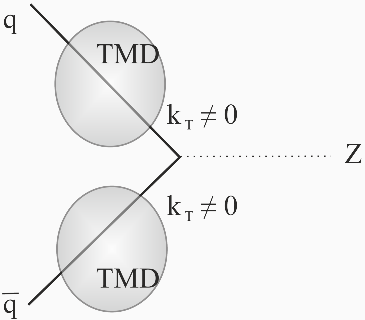

Procedure:

- DY LO matrix element: $q\bar{q} \rightarrow Z$
- Generate $k_\perp$ of $q\bar{q}$ according to TMDs (m_{DY} fixed, x_1, x_2 change)
- compare with the 8 TeV ATLAS measurement

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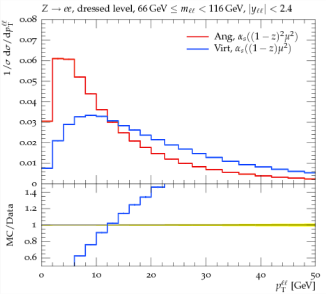

Results after the fit (experimental and model uncertainty)

- difference between angular and virtuality ordering visible
- angular ordering: the shape of Z boson $p_\perp$ spectrum reproduced

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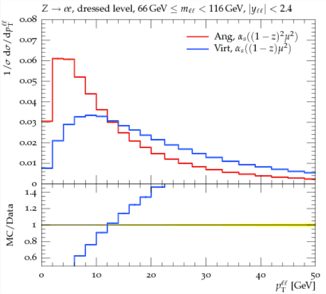

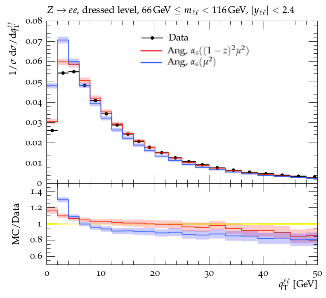

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- with $\alpha_s\left((1-z)^2\mu'^2\right)$ agreement within the data much better than for $\alpha_s(\mu'^2)$
- All the $p_\perp$ dependence directly from the PB method
- prediction for the whole spectrum from one method
- no tuning/adjustment of free parameters
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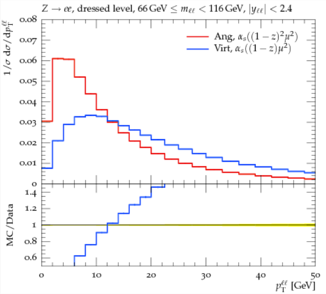
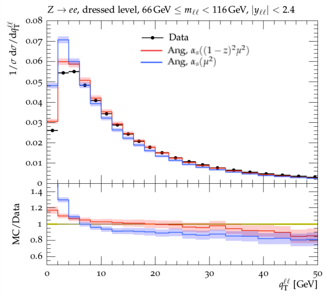
Prediction for Z boson $p_\perp$ spectrum using TMDs





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For more applications and fit procedure see the talks of:
Armando Bermudez Martinez
Daniela Dominguez Damiani
Jindrich Lidrych

PB and other approaches

PB and Marchesini, Webber

PB with angular ordering is very successful

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PB for angular ordering:

$$
\begin{aligned}
&\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2) \\
+ \quad &\int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \sum_b \int_x^{1-\frac{q_0}{\mu'}} dz \alpha_s \left((1-z)^2\mu'^2\right) P_{ab}^R\left(\mu'^2, z\right) \widetilde{f}_b\left(\frac{x}{z},\mu'^2\right) \qquad (1)
\end{aligned}
$$

where

$$q_{\perp,i}^2 = (1-z_i)^2\mu'^2$$

Eq. (1) is identical to the Marchesini and Webber (MarWeb1988) prescription
Nuclear Physics B310 (1988) 461-526

PB and KMR/MRW

Reminder: Kimber, Martin, Ryskin (KMR) (and Martin, Ryskin, Watt (MRW)): method to obtain TMDs (unintegrated PDFs) from the integrated PDFs and the Sudakov form factors Phys. Rev. D63 (2001) 114027

$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2) + \int_{\mu_0^2}^{\mu^2} \frac{dq_\perp^2}{q_\perp^2} \underbrace{\left(\Delta_a(\mu^2, q_\perp^2) \sum_b \int_x^{1-C(q_\perp)} dz P_{ab}^R\left(q_\perp^2, z\right) \widetilde{f}_b\left(\frac{x}{z}, q_\perp^2\right)\right)}_{\widetilde{f}(x,\mu^2,q_\perp^2)} \tag{2}$$

at this last step of the evolution the unintegrated distribution becomes dependent on two scales: $q_\perp$ and μ

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This would be almost equivalent to PB formula for $p_\perp$-ordering where $q_t = \mu$
Why almost? In PB:

- for $p_\perp$-ordering: C is a fixed number e.g. 10^{-3}
- $k_\perp$ does not come only from the last step! $\rightarrow$ I will come back to this later

PB and KMR/MRW

In KMR:

- "Strong ordering": $C(q_\perp) = \frac{q_\perp}{\mu}$ and $q_\perp < \mu(1-x)$
- "Angular ordering" $C(q_\perp) = \frac{\mu}{q_\perp + \mu}$ and $q_\perp < \mu\frac{1-x}{x}$

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PB for angular ordering written in terms of integral over $q_\perp$ (identical to MarWeb1988):

$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2) + \int_{q_0^2}^{(1-x)^2\mu^2} \frac{dq_\perp^2}{q_\perp^2}\Delta_a\left(\mu^2, \frac{q_\perp^2}{(1-z)^2}\right)$$

$$\times \quad \sum_b \int_x^{1-\frac{q_\perp}{\mu}} dz \alpha_s\left(q_\perp^2\right) P_{ab}^R\left(\frac{q_\perp^2}{(1-z)^2}, z\right) \widetilde{f}_b\left(\frac{x}{z}, \frac{q_\perp^2}{(1-z)^2}\right)$$

KMR for "strong ordering" :

at first sight KMR for "angular ordering" doesn't look similar to Webber, Marchessini or PB with angular ordering. How does the distributions look like?

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at first sight KMR for "angular ordering" doesn't look similar to MarWeb1988 or PB with angular ordering. How do the distributions look like?

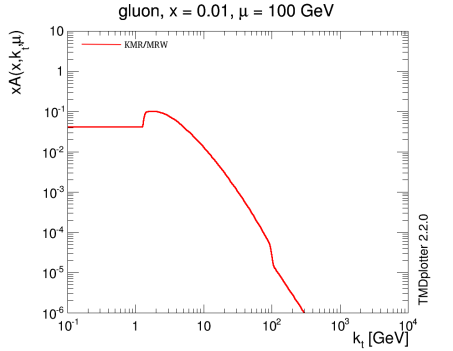
PB and KMR/MRW: distributions

PB: intrinsic $k_\perp$ is a Gauss distribution with width=0.5 GeV

KMR/MRW parametrization for $k_\perp < k_0 = 1\mathrm{GeV}$:

$$\frac{\widetilde{f}_a(x, k_\perp, \mu^2)}{k_\perp^2} = \frac{1}{\mu_0^2}\widetilde{f}_a(x, k_\perp, \mu_0^2)\Delta_a(\mu^2, \mu_0^2) = \mathrm{const}$$

TMD sets obtained according to KMR/MRW formalism with angular ordering included in TMDlib in TMD set called MRW-ct10nlo Eur.Phys.J.C78(2018)no.2,137



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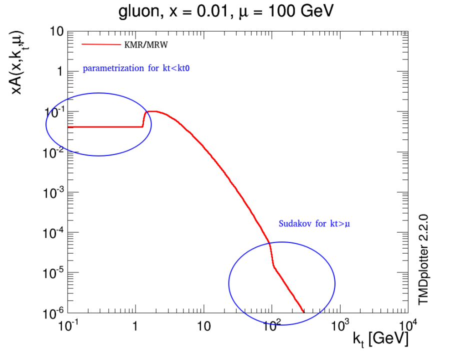

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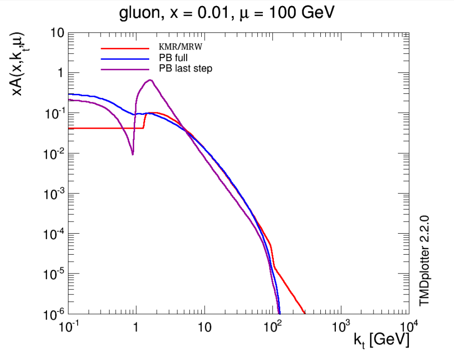


exercise:

PB last Step: try to obtain KMR from PB: take PB with angular ordering but take $k_\perp$ only from the last emission

do $\vec{k}_{\perp,a} = -\vec{q}_{\perp,c}$ instead $\vec{k}_{\perp,a} = \vec{k}_{\perp,b} - \vec{q}_{\perp,c}$ (PB full)

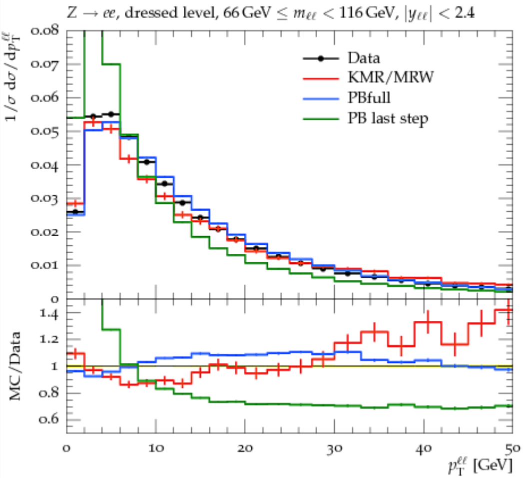
$k_t < 1\mathrm{GeV}$:

- KMR/MRW: initial parametrization
- PB last Step: matching of intrinsic $k_\perp$ and evolution clearly visible
- PB full: matching of intrinsic $k_\perp$ and evolution smeared during evolution

For $k_t \in (\approx 10\mathrm{GeV}, \approx \mu)$:

PB full and KMR/MRW very similar!

Z boson $p_\perp$ spectrum



- PB with angular ordering and full evolution works very well
- KMR/MRW works well for small and middle-range $k_\perp$ but for higher $k_\perp$ it overestimates the data
- PB with last step evolution not sufficent

PB and CSS

WORK IN PROGRESS

Reminder: Collins, Soper and Sterman TMD factorization formula for the DY cross section:

Nuclear Physics B250 (1985) 199-224

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}y\mathrm{d}Q_T^2} \sim \frac{4\pi^2\alpha^2}{9Q^2 s}\frac{1}{(2\pi)^2}\int \mathrm{d}^2 b \exp(iQ_T\cdot b)\sum_j e_j^2 \cdot \sum_a \int_{x_A}^1 \frac{\mathrm{d}\xi_A}{\xi_A} f_{a/A}(\xi_A, 1/b)$$
$$\sum_b \int_{x_B}^1 \frac{\mathrm{d}\xi_B}{\xi_B} f_{b/B}(\xi_B, 1/b) \exp\left(-\int_{1/b^2}^{Q^2}\frac{\mathrm{d}\overline{\mu}^2}{\overline{\mu}^2}\left[\ln\left(\frac{Q^2}{\overline{\mu}^2}\right)A(g(\overline{\mu})) + B(g(\overline{\mu}))\right]\right) \tag{6}$$
$$\cdot C_{ja}\left(\frac{x_A}{\xi_A}, g(1/b)\right) C_{jb}\left(\frac{x_B}{\xi_B}, g(1/b)\right) + \frac{4\pi^2\alpha^2}{9Q^2 s} Y(Q_T, Q, x_A, x_B)$$

where $A = \sum_i \left(\frac{\alpha_s(\mu)}{\pi}\right)^i A^i$, the same for B and C.

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where $A = \sum_i \left(\frac{\alpha_s(\mu)}{\pi}\right)^i A^i$, the same for B and C.

- one scale evolution up to a scale $1/b$
- in the last step of the evolution the dependence on the second scale enters

a bit like KRM with "strong ordering"

PB and CSS

WORK IN PROGRESS

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PB and CSS

WORK IN PROGRESS

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PB: Sudakov form factor with P_{ba}^R but possible also with P_a^V (momentum sum rule) .
For angular ordering:

$$\Delta_a(\mu^2) = \exp\left(-\int_{q_0^2}^{\mu^2}\frac{dq_\perp^2}{q_\perp^2}\left(\int_0^{1-\frac{q_\perp}{\mu}} dz\left(k_a\frac{1}{1-z}\right) - d\right)\right) .$$

notice: $2\int_0^{1-\frac{q_\perp}{\mu}} dz\left(\frac{1}{1-z}\right) = \ln\left(\frac{\mu}{q_\perp}\right)^2$

PB with angular ordering: in Sudakov the same coefficients as $\underbrace{\frac{1}{2}A^1}_{\mathrm{LL}}, \underbrace{\frac{1}{2}A^2 \text{ and } \frac{1}{2}B^1}_{\mathrm{NLL}}$ in CSS

in PB we have also $\underbrace{\frac{1}{2}B^2}_{\mathrm{NNLL}} + T$ where $T \sim \left(\frac{\alpha_s}{\pi}\right)^2 f(n_f)$

Summary and Conclusions

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- DGLAP evolution equation solved with Parton Branching method
- collinear PDFs and TMDs obtained
- different ordering definitions studied
- application of the TMDs to the Z boson $p_{\perp}$, a very good description of the 8 TeV data with angular ordering
- studies on comparison with Marchesini and Webber, KMR and CSS ongoing
- results in: Phys.Lett. B772 (2017) 446-451, JHEP 1801 (2018) 070, arXiv:1804.11152

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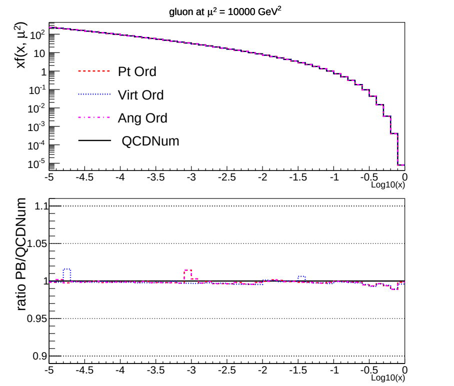
Thank you!

Backup

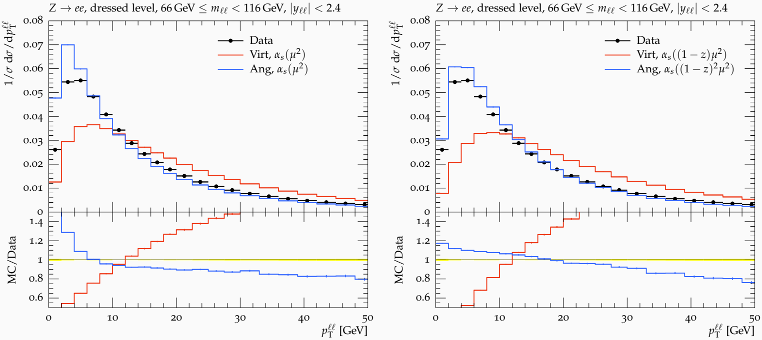
Cross check: Effect of ordering choice on collinear PDFs



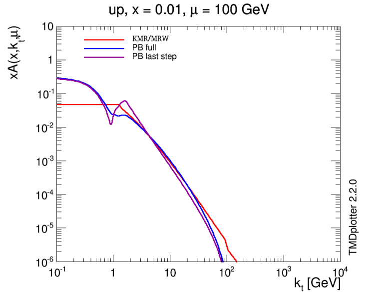
$z_M = 1 - 10^5$, $\alpha_s\left(1\mu'^2\right)$

As expected: collinear PDFs not affected by the ordering (if $z_M \approx 1$) ✓

Application to Z boson $p_\perp$ spectrum

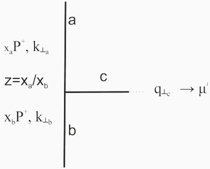


MWR-ct10nlo and PB for quarks



replace $q_{\perp,c}$ with q_0 $\rightarrow$ conditions for the z_M value:

- $p_\perp$- ordering: $\mu'^2 1 = q_{\perp,c}^2 \quad \rightarrow \quad z_M = \text{fixed}$
- virtuality ordering: $\mu'^2 (1-z) = q_{\perp,c}^2 \quad \rightarrow \quad z_M = 1 - \left(\frac{q_0}{\mu'}\right)^2$
- angular ordering: $\mu'^2 \underbrace{(1-z)^2}_{a(z)^2} = q_{\perp,c}^2 \quad \rightarrow \quad z_M = 1 - \left(\frac{q_0}{\mu'}\right)$



renormalization scale in α_s should be chosen to be $q_\perp^2$, rather than the evolution scale μ'^2

$\vec{k}_{\perp,a} = \vec{k}_{\perp,b} - \vec{q}_{\perp,c}$

$$\widetilde{A}_a\left(x, k_\perp, \mu^2\right) = \Delta_a\left(\mu^2\right) \widetilde{A}_a\left(x, k_\perp, \mu_0^2\right) +$$
$$\sum_b \int_{\mu_0^2}^{\mu^2} \frac{\mathrm{d}^2 \mu'_\perp}{\pi \mu'^2} \frac{\Delta_a\left(\mu^2\right)}{\Delta_a\left(\mu'^2\right)} \int_x^{z_M \approx 1} \mathrm{d}z P_{ab}^R\left(z, \mu'^2, \alpha_s(1\mu'^2)\right) \widetilde{A}_b\left(\frac{x}{z}, k_\perp + a(z)\mu_\perp, \mu'^2\right) \Big| \int \mathrm{d}k_\perp^2$$

PB method: effect of every individual part of the ordering definition can be studied separately

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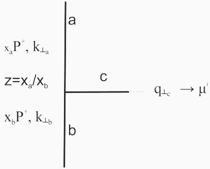

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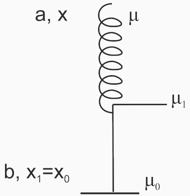
PB method: effect of every individual part of the ordering definition can be studied separately

collinear PDFs not affected by the ordering if $z_M \approx 1$ and $\alpha_s(\mu^2)$

Parton Branching (PB) method and Monte Carlo (MC) techniques

$$\int_{\ln \mu_0^2}^{\ln \mu^2} d \ln \mu_1^2 \frac{\Delta_a(\mu^2)}{\Delta_a(\mu_1^2)} \sum_b \int_x^{z_M} dz_1 P_{ab}^R \left(\mu_1^2, z_1\right) \widetilde{f}_b \left(\frac{x}{z_1}, \mu_0^2\right) \Delta_b(\mu_1^2)$$



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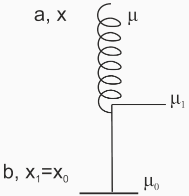

$R_1, R_2 \in [0, 1]$ - uniformly distributed random numbers

probabilistic interpretation:

- generate μ_1^2: Δ_b:
  if $\mu_1^2 > \mu^2$ stop, otherwise splitting

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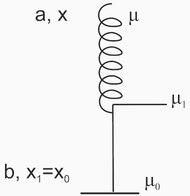

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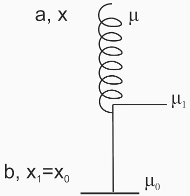

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- generate the next scale μ_2^2:
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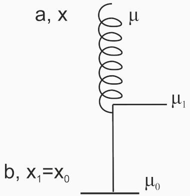
Parton Branching (PB) method and Monte Carlo (MC) techniques

$$\int_{\ln \mu_0^2}^{\ln \mu^2} d \ln \mu_1^2 \frac{\Delta_a(\mu^2)}{\Delta_a(\mu_1^2)} \sum_b \int_x^{z_M} dz_1 P_{ab}^R \left(\mu_1^2, z_1\right) \widetilde{f}_b \left(\frac{x}{z_1}, \mu_0^2\right) \Delta_b(\mu_1^2)$$



$R_1, R_2 \in [0, 1]$ - uniformly distributed random numbers

probabilistic interpretation:

- generate μ_1^2: Δ_b:
  if $\mu_1^2 > \mu^2$ stop, otherwise splitting
- LO: decide $b \to b$ or $b \to a$ and generate z_1: P_{ab}^R
- generate the next scale μ_2^2:
  if $\mu_2^2 > \mu^2$ stop, otherwise next splitting

Important:
P_{ab} can be negative but $\int \mathrm{d}z \; P_{ab} > 0$ - we can use the same method also at higher orders

Fit to HERA σ_r data

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- kernel $K_a(x'', \mu^2)$ from evolution ($x_0 = 1 - 10^{-6}$)
- K_a folded with the starting distribution A_0

$$\widetilde{f}_a(x, \mu^2) = x \int \mathrm{d}x' \int \mathrm{d}x'' A_0(x') K_a(x'', \mu^2) \delta(x' x'' - x)$$

- $\widetilde{f}_a(x, \mu^2)$ convoluted with ME and fitted to data
- procedure repeated with different A_0 until the minimal χ^2 is found

Fit to HERA σ_r data

- data: HERA H1 and ZEUS combined DIS measurement
  [Eur.Phys.J. C75 (2015) no.12, 580]
- range: $3.5 < Q^2 < 50000\ \mathrm{GeV}^2$, $4 \cdot 10^{-5} < x < 0.65$
- systematic uncertainty: in the χ^2 definition in `xFitter`
- experimental uncertainties: Hessian method in `xFitter`
- model uncertainties: variation of m_c, m_b and μ_0
- initial parametrization in a form of HERAPDF2.0
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Fit performed twice for angular ordering:

- $\alpha_s\left(\mu'^2\right)$ (Set1): $\chi^2/(\mathrm{d.o.f}) \approx 1.2$
- $\alpha_s\left((1-z)^2\mu'^2\right)$ (Set2): $\chi^2/(\mathrm{d.o.f}) \approx 1.2$

Set1: reproduces HERAPDF2.0 ✓

Set2: very different from HERAPDF2.0 ✓