Excited Hadrons and Quark-Hadron Duality

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Research with **Pere Masjuan** and **Wojciech Broniowski**

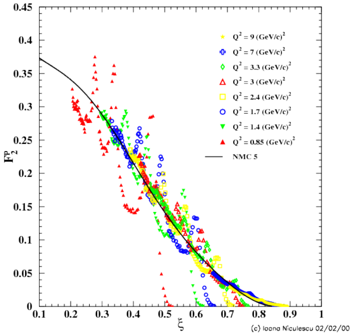
DUALITY

What is duality ?

- QED: Electrons+Nuclei vs Atoms or Molecules

$$\Psi = |pe^- pe^- \ldots\rangle \sim |HH \ldots\rangle \sim |H_2 H_2 \ldots\rangle$$

- QCD Quarks+Gluons vs Hadrons



- Is the PDG complete ? Are there missing states ?

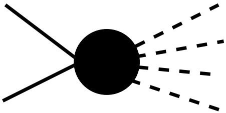
Completeness and Inclusive Processes

- $e^+e^- \to X$ (hadrons)

$$T = e^2 \bar{v}(p_+)\gamma_\mu u(p_-)(1/q^2)\langle n|J_\mu(0)|0\rangle$$

- $q = p_- + p_+$, $X = \pi^+\pi^-, \bar{p}p, \bar{K}K, \ldots$

$$\sigma(e^+e^- \to X) = \sum_n |T|^2 \delta(p_n - q) \equiv w_{\mu\nu}(q) L^{\mu\nu}(q)$$



- Hadronic tensor

$$W_{\mu\nu}(q) = \sum_n (2\pi)^4 \delta(q - p_n)\langle 0|J_\mu(0)|n\rangle\langle n|J_\nu(0)|0\rangle \qquad \sum_n |n\rangle\langle n| = 1$$

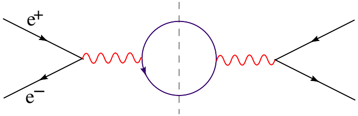
- Optical theorem

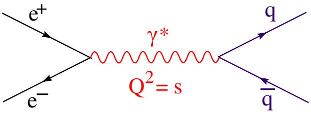
$$W_{\mu\nu}(q) = 2\mathrm{Disc}\Pi_{\mu\nu}(q) = 2\mathrm{Im}\Pi_{\mu\nu}(q)$$

- Vacuum polarization

$$\Pi_{\mu\nu}(q) = i\int d^4x e^{iq\cdot x}\langle 0|T\{J_\mu(x)J_\nu(0)\}|0\rangle$$

Completeness and Inclusive Processes: High energy





- $e^+e^- \to X = \bar{q}q$

$$T(e^+e^- \to \bar{q}q) = e^2 \bar{v}(p_+)\gamma_\mu u(p_-)(1/q^2)\bar{v}_q(k_+)Q_q\gamma^\mu u_q(k_-)$$

- $q = p_- + p_+$

$$\sigma(e^+e^- \to X) = \int \frac{d^3k_+}{(2\pi)^3 E_+}\frac{d^3k_-}{(2\pi)^3 E_-}|T|^2\delta(k_+ + k_- - q) + \dots$$

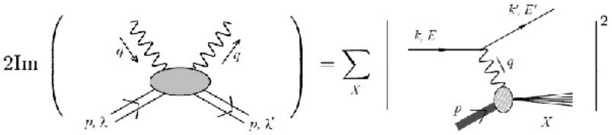
- Duality

$$1 = \sum_X |X\rangle\langle X| = \underbrace{|\bar{q}q\rangle\langle\bar{q}q| + |\bar{q}qg\rangle\langle\bar{q}qg| + \dots}_{\text{QCD}} = |\pi^+\pi^-\rangle\langle\pi^+\pi^-| + |\bar{p}p\rangle\langle\bar{p}p| + |\bar{K}K\rangle\langle\bar{K}K| + \dots$$

- Ratio

$$\frac{\sigma(e^+e^- \to X)}{\sigma(e^+e^- \to e^+e^-)} = \frac{\sum_{q=u,d,\dots}\sigma(e^+e^- \to \bar{q}q)}{\sigma(e^+e^- \to e^+e^-)} = \sum_q Q_q^2 \to \sum_q Q_q^2\theta(s - 4m_q^2)$$

Completeness and Inclusive Processes: DIS



- Hadronic tensor

$$W_{\mu\nu}(P,q) = \sum_n (2\pi)^4 \delta(P+q-p_n) \langle P|J_\mu(0)|n\rangle\langle n|J_\nu(0)|P\rangle$$

- Completeness relation

$$\sum_n |n\rangle\langle n| = 1$$

- Optical theorem

$$W_{\mu\nu}(P,q) = 2\mathrm{Disc}T_{\mu\nu}(P,q) = 2\mathrm{Im}T_{\mu\nu}(P,q)$$

$$T_{\mu\nu}(P,q) = i\int d^4x e^{iq\cdot x}\langle P|T\left\{J_\mu(x)J_\nu(0)\right\}|P\rangle$$

Questions to consider

Quark-Hadron Duality: Physical observables can be computed in quark-gluon or hadronic language: Measurements can only be made in terms of hadrons

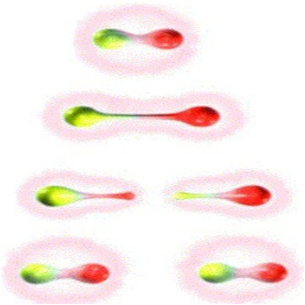
- What states can we use to saturate sum rules ?
- How do we reconcile the picture of intermediate $q\bar{q}$ free states with confinement ?
- Does quark hadron duality impose constraints on results ?

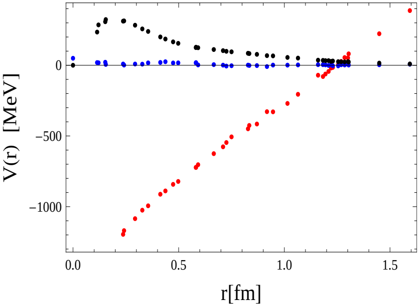
- Two point functions (Spectrum)
- Three point functions (Form factors, Decays)
- Four point functions (Hadron-Hadron scattering, Structure functions, GPD's)

RESONANCES AND WIDTHS

Quark potential and string breaking

Transition $Q\bar{Q} \to B\bar{B}$





- Energy of two heavy quarks

$$E(r) = m_{\bar{Q}} + m_Q + V(r)$$

- Meson masses

$$M_{\bar{q}Q} = \Delta_{\bar{q}Q} + m_Q \qquad M_{q\bar{Q}} = \Delta_{q\bar{Q}} + m_Q$$

- Uncoupled Born-Oppenheimer (diabatic crossings)

$$V_{\bar{Q}Q}(r) = \sigma r\,, \qquad V_{\bar{Q}q\bar{q}Q}(r) = \Delta_{\bar{q}Q} + \Delta_{\bar{Q}q} \equiv 2\Delta\,.$$

- Orthogonal eigenstates

The radial Regge spectrum for light quarks

- Two scalar quarks in the CM frame

$$M = 2\sqrt{p^2 + m^2} + \sigma r \to 2m + \frac{p^2}{m} + \sigma r + \dots \qquad (1)$$

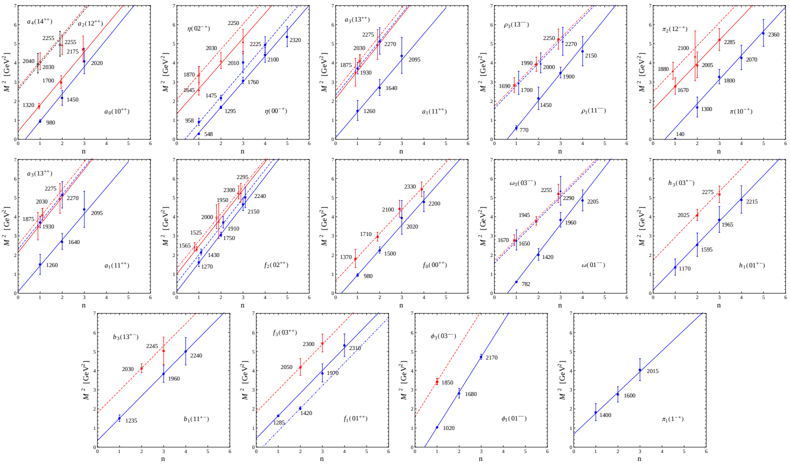
- Bohr-Sommerfeld quantization $L = 0$ and $m = 0$

$$2\int_0^{M/\sigma} p_r dr = 2\pi(n + \alpha) \qquad (2)$$

- Radial Regge spectrum for large n Anisovich, Anisovich, '02

$$M_n^2 = \mu^2 n + M_0^2 \qquad \mu^2 = 1.25(15)\text{GeV}^2 \qquad (3)$$

Empirical Regge Spectrum and Widths

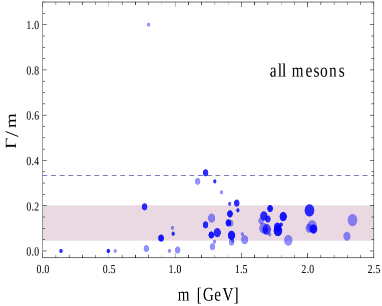


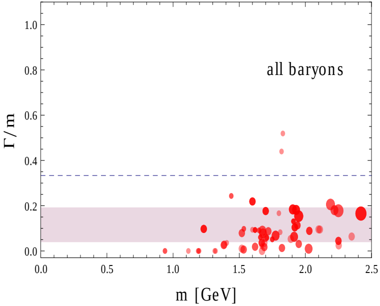
- The **half-width rule**: $\Delta M_R = \Gamma_R/2$ or $\Delta M_R^2 = M_R \Gamma_R$

$$M^2 = 1.38(4)n + 1.12(4)J - 1.25(4)\,.$$

The hadron resonance spectrum

- Most states listed by the PDG are resonances (unstable with strong interactions)
- Many are classified according to the quark-model
- Meson and baryon spectrum resemble large N_c QCD properties





$$\frac{\Gamma}{m} \equiv \sum_{J,\alpha} (2J+1) \frac{\Gamma_{J,\alpha}}{m_{J,\alpha}} = 0.12(8) \qquad = \mathcal{O}({N_c}^{-1})$$

Large N_c scaling rules

$N_c \to \infty$ with αN_c fixed (t'Hoot, Witten)

- Hadronic spectrum (baryons and mesons are stable)

$$m_{\pi,\rho,\omega,\sigma} \sim N_c^0 \quad \Gamma_{\sigma,\rho,\omega} \sim 1/N_c \quad m_{N,\Delta} \sim N_c \quad \Gamma_\Delta \sim 1/N_c$$

- Couplings

$$g_{MMM} \sim 1/\sqrt{N_c} \quad g_{MMMM} \sim 1/N_c \quad g_{BMM} \sim \sqrt{N_c}$$

- Scattering

$$T(\pi\pi \to \pi\pi) \sim 1/N_c \, , \, T(\pi N \to \pi N) \sim N_c^0 \, , \, T(NN \to NN) \sim N_c$$

LARGE MOMENTUM

Operator Product Expansion

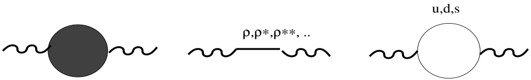
- Vector-Vector and Axial-Axial correlators

$$\begin{aligned}
\Pi_V^{\mu a,\nu b}(q) &= i\int d^4x e^{-iq\cdot x}\langle 0|T\left\{J_V^{\mu a}(x)J_V^{\nu b}(0)\right\}|0\rangle \\
&= \Pi_V^T(q^2)\,\left(q^\mu q^\nu - g^{\mu\nu}q^2\right)\delta^{ab}, \\
\Pi_A^{\mu a,\nu b}(q) &= i\int d^4x e^{-iq\cdot x}\langle 0|T\left\{J_A^{\mu a}(x)J_A^{\nu b}(0)\right\}|0\rangle \\
&= \Pi_A^T(q^2)\,\left(q^\mu q^\nu - g^{\mu\nu}q^2\right)\delta^{ab} + \Pi_A^L(q^2)\,q^\mu q^\nu \delta^{ab},
\end{aligned}$$

with $J_{V,A}^{\mu a} = \bar\psi i\gamma^\mu\{1,\gamma_5\}\frac{\tau^a}{2}\psi$ QCD currents. At large Q

$$\begin{aligned}
\Pi_{V+A}^T(Q^2) &= \frac{1}{4\pi^2}\left\{-\frac{N_c}{3}\log\frac{Q^2}{\mu^2} + \frac{\pi}{3}\frac{\langle\alpha_S G^2\rangle}{Q^4} + \frac{256\pi^3}{81}\frac{\alpha_S\langle\bar q q\rangle^2}{Q^6}\right\} + \dots\,, \\
\Pi_{V-A}^T(Q^2) &= -\frac{32\pi}{9}\frac{\alpha_S\langle\bar q q\rangle^2}{Q^6} + \dots\,,
\end{aligned} \tag{4}$$

Narrow Meson resonances



- Saturating with narrow resonances

$$\Pi_V^T(Q^2) = \sum_{n=0}^{\infty} \frac{F_{V,n}^2}{M_{V,n}^2 + Q^2} + c.t.,$$

$$\Pi_A^T(Q^2) = \frac{f_\pi^2}{Q^2} + \sum_{n=0}^{\infty} \frac{F_{A,n}^2}{M_{A,n}^2 + Q^2} + c.t.,$$

- Spectral function

$$\text{Im}\Pi_V^T(s) \rightarrow \sum_{n=0}^{\infty} F_{V,n}^2 \delta(M_{V,n}^2 - s) \sim \lim_{n\to\infty} \frac{F_{V,n}^2}{dn/dM_{V,n}^2}$$

$$\text{Im}\Pi_V^T(s) \rightarrow \frac{N_c}{24\pi^2} \text{Im} \log(-s) = \frac{N_c}{24\pi}$$

Regge phase-space

- For large meson masses, the level density is given by

$$\begin{aligned}\rho(M^2) &= \sum_n \delta(M^2 - M_n^2) \to \int dn \delta(M^2 - M_n^2) \\ &= \frac{1}{dM_n^2/dn}\Big|_{M_n^2=M^2} = \frac{dn}{dM^2} = \frac{1}{2\pi\sigma}\end{aligned} \tag{5}$$

- So, using the WKB approximation we get

$$\frac{dn}{dM^2} = \frac{1}{\pi}\int_0^a \frac{dr}{p_r} = \frac{1}{2\pi\sigma} \tag{6}$$

- Finite quark mass corrections

$$\frac{dn}{dM^2} = \frac{1}{2\pi\sigma}\sqrt{1 - \frac{4m^2}{M^2}}\theta(M^2 - 4m^2) \tag{7}$$

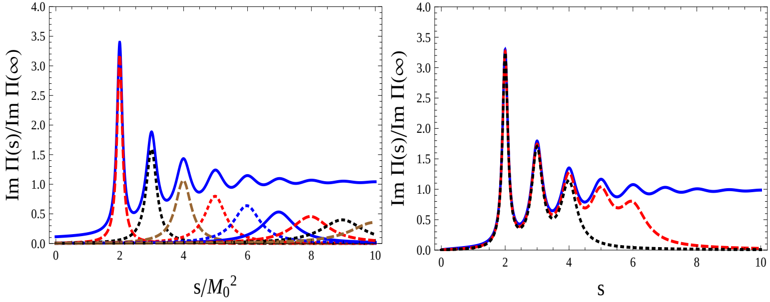
- Two body phase space factor = Absortive part of two point correlators for FREE PARTICLES.

Finite width

Polarization operator

$$\frac{1}{\pi}\mathrm{Im}\Pi(s) = \frac{1}{\pi}\sum_{n=0}^{N}\frac{f_n^2\Gamma_n\sqrt{s}}{(s-M_n^2)^2+\Gamma_n^2 s},$$

$$\mathrm{f}_n^2 = 1/(24\pi^2 N_c), \quad M_n^2 = n+1, \quad M_n\Gamma_n = \gamma n, \quad n = 0,1,2,\ldots,N.$$



FORM FACTORS

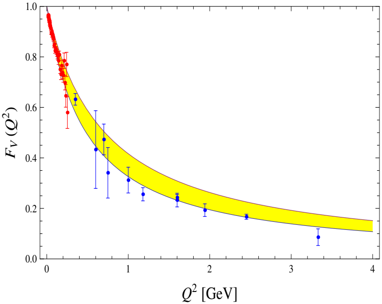
Pion Form Factors

- Large N_c (generalized meson dominance)

$$F_{em}(Q^2) = \sum_V \frac{c_V M_V^2}{M_V^2 + Q^2} \rightarrow \underbrace{\frac{c}{Q^2}}_{\text{pQCD}}$$

- Minimal Ansatz

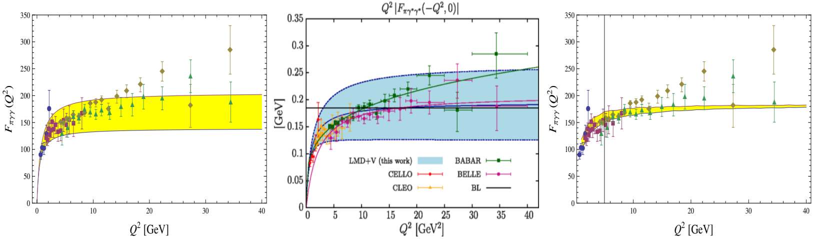
$$F_{em}(Q^2) = \frac{M_V^2}{M_V^2 + Q^2} \quad , \qquad M_V = m_V \pm \frac{\Gamma_V}{2}$$



Pion Transition Form Factor

$$F_{\pi\gamma\gamma^*}(Q^2) = \frac{1}{4\pi^2 f_\pi} \sum_V \frac{c_V M_V^2}{M_V^2 + Q^2} \to \qquad \sum_V c_V = 1 \tag{8}$$

- 1 state $V = \rho$ (no pQCD constraint) Lattice: Gerardin et al. PRD94 (2016) 074507
- 2 states $V = \rho, \rho'$ (with pQCD constraint)



Nucleon Form Factors

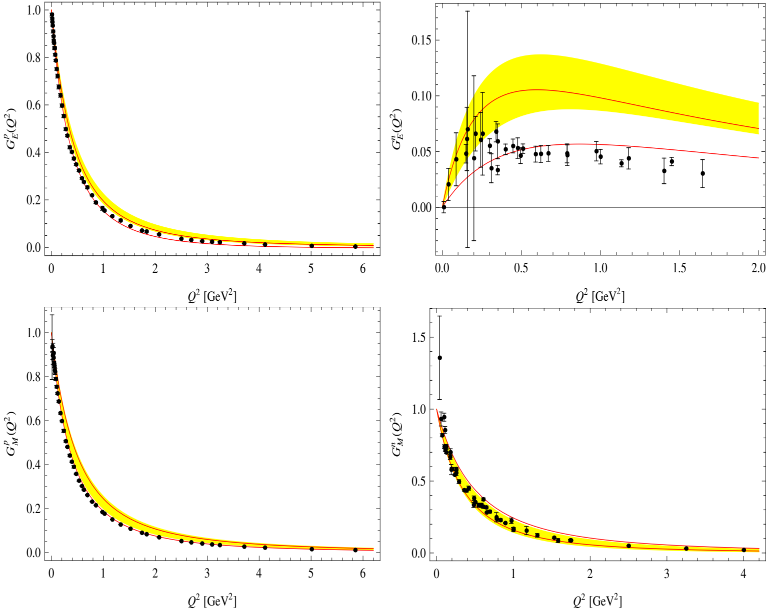
$$\langle N(p')|J_V^{\mu a}(0)|N(p)\rangle = \bar{u}(p')\frac{\tau^a}{2}\left[\gamma^\mu F_1^{I=1}(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M_N}F_2^{I=1}(q^2)\right]u(p)$$

$$t^{i+3}F_i(t) \to 0$$

$$F_1^{I=0}(t) = \sum_V \frac{g_{\omega NN} f_{\omega\gamma}}{m_\omega^2 - t} \qquad F_2^{I=0}(t) = \sum_V \frac{f_{\omega NN} f_{\omega\gamma}}{m_\omega - t}$$

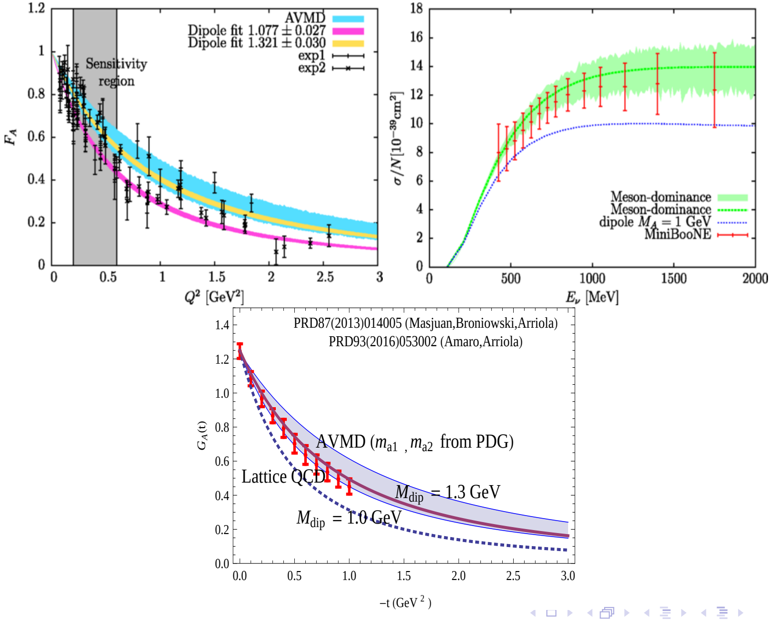
$$F_1^{I=1}(t) = \sum_V \frac{g_{\rho NN} f_{\rho\gamma}}{m_\rho - t} \qquad F_2^{I=1}(t) = \sum_V \frac{f_{\rho NN} f_{\rho\gamma}}{m_\rho - t}$$

SU(3) $\to g_{\omega NN} = 3 g_{\rho NN} \sim 9$ Violations of 30%



$G_E^p(Q^2)$
Q^2 [GeV^2]
$G_E^n(Q^2)$
Q^2 [GeV^2]
$G_M^p(Q^2)$
Q^2 [GeV^2]
$G_M^n(Q^2)$
Q^2 [GeV^2]

Nucleon Axial Form Factor



Single resonance saturation

T. Ledwig, J. Nieves, A. Pich, E. Ruiz Arriola and J. Ruiz de Elvira, Phys. Rev. D **90**, no. 11, 114020 (2014)

- Pion Transition form factor

$$F_{\pi\gamma\gamma^*}(q^2) = \underbrace{\frac{1}{4\pi^2 f_\pi}}_{\text{ChiralAnomaly}} \times \underbrace{\frac{m_V^2}{m_V^2 - q^2}}_{\text{VMD}} \to \underbrace{-\frac{6f_\pi}{N_c q^2}}_{\text{BL}}$$

$$m_V^2 = \frac{24\pi^2 f_\pi^2}{N_c}, \qquad f_\pi = 88(\text{MeV}) \to \qquad m_V = 781(\text{MeV})$$

- Analysis involving OTHER form factors

$$m_S = m_V = \frac{m_P}{\sqrt{2}} = \frac{m_A}{\sqrt{2}} = \pi\sqrt{\frac{24}{N_c}} f_\pi \,.$$

- Widths

$$\frac{\Gamma_V}{m_V} = \frac{\pi}{4N_c} = 0.26 \qquad (\text{exp} : 0.20)$$
$$\frac{\Gamma_S}{m_S} = \frac{9\pi}{8N_c} = 1.2 \qquad (\text{exp} : 1.)$$

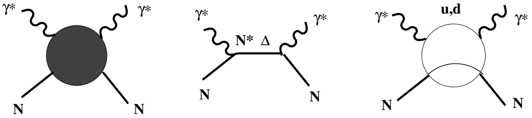
- The Low Energy Constants of Chiral Perturbation Theory

$$2L_1^{\text{SRA}} = L_2^{\text{SRA}} = -\frac{1}{2}L_3^{\text{SRA}} = \frac{1}{2}L_5^{\text{SRA}} = \frac{4}{3}L_8^{\text{SRA}} = \frac{f^2}{8m_V^2} = \frac{N_c}{192\pi^2}$$

- EVERYTHING IS DETERMINED FROM f_π !!

DIS

Deep inelastic scattering



- Forward Compton scattering

$$W^{ab}_{\mu\nu}(p,q;s) = \frac{1}{4\pi}\int d^4x e^{iq\cdot\xi}\left\langle p,s\middle|[J^a_\mu(\xi),J^{b\dagger}_\nu(0)]\middle|p,s\right\rangle \quad (9)$$

$$= \left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)M_N W_1(\nu,Q^2)+\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)\frac{1}{M_N}W_2(\nu,Q^2)$$

$$= \sum_R \langle N|J_\mu|R\rangle\langle R|J_\nu|N\rangle\delta((p+q)^2-M_R^2) \quad (10)$$

- Bjorken limit:

$$Q^2\to\infty \qquad \text{with} \qquad x=Q^2/2p\cdot q \quad \text{fixed}\,.$$

- Scaling and Spin 1/2 (parton model)

$$M_N W_1(x,Q^2)\ \to, F_1(x) \quad \text{and} \quad \frac{p\cdot q}{M_N}W_2(x,Q^2)\ \to F_2(x) \qquad F_2(x)=2xF_1(x)$$

Narrow resonance approximation

- Invariant mass

$$W^2 = (p+q)^2 = M_N^2 + Q^2 \left(\frac{1}{x} - 1 \right)$$

- Bjorken limit

$$W_2 = \sum_R \left[G_{N\to R}(q^2/M_R^2) \right]^2 \delta(W^2 - M_R^2) \to \int d\mu^2 \sum_i \left[G_{N\to i}(-Q^2/\mu^2) \right]^2 \frac{dn}{d\mu^2} \quad (11)$$

$$F_2(x) = \sum_i \left[G_{N\to i}(x/(1-x)) \right]^2 \frac{dn}{2\mu^2} \quad (12)$$

- Scaling limit

$$M_R^2 = \mu^2 n + M^2 \qquad G_{N\to R}(q) \equiv F(q^2/M_R^2)$$

- Drell-Yang relation

$$G_{N\to R}(-Q^2) \to \frac{1}{Q^n} \qquad F_2(x) \to (1-x)^{n-1}$$

The radial Regge spectrum for quark-diquark states

- Quark-diquark in the CM frame

$$M = \sqrt{p^2 + m_D^2} + p + \sigma r \tag{13}$$

- Bohr-Sommerfeld quantization $L = 0$

$$2 \int_0^{M/\sigma} p_r dr = 2\pi(n + \alpha) \to \frac{dn}{M^2} = \frac{1}{2\pi\sigma}(1 - \frac{m_D^2}{M^2}) \tag{14}$$

- Radial Regge spectrum for large n

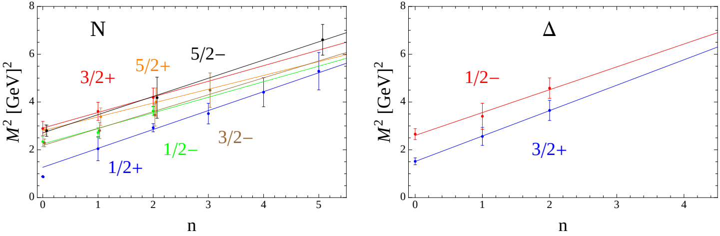
$$M_n^2 = \mu^2 n + M_0^2 \tag{15}$$

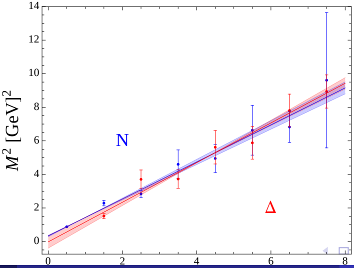
- Quark-Hadron duality in DIS requires Quark-Diquark dynamics !
- Problem of missing resonances →Quark model with qqq states predicts more states than observed

Regge Fits

Radial Regge trajectory parameterized as:

$$M_{n,J}^2 = a + \mu^2 n + \beta J \tag{16}$$

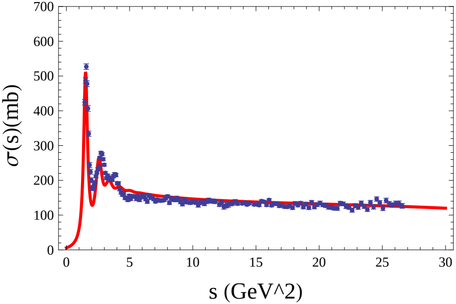




Photoproduction

- We take

$$M_0 = 0.98 \qquad M_1 = 1.3 \qquad M_2 = 1.6 \qquad M_n^2 = an + M_0^2$$



- For electroproduction many more details are needed.
- High spin formalism
- Callan-Gross relation $2xF_1(x) = F_2(x)$ spin 1/2 quarks (e.g. pion structure function)

Electroproduction

- Structure function

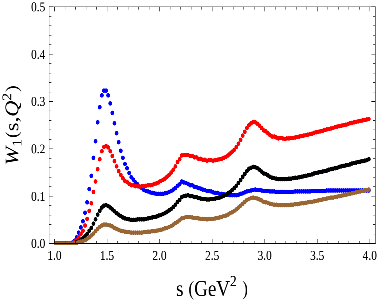
$$W_1(s,Q^2) = \frac{1}{\pi}\sum_{n=0}^{N} \frac{f_n^2\,[G_n(Q)]^2\,\Gamma_n\sqrt{s}}{(s-M_n^2)^2+\Gamma_n^2 s}\,,$$

$$\mathrm{f}_n^2 = 1/(24\pi^2 N_c)\,, \quad M_n^2 = n+1\,, \quad M_n\Gamma_n = \gamma n\,, \quad n = 0,1,2,\ldots,N\,.$$

- Form Factors

$$G_n(Q^2) = \frac{1}{(1+rQ^2/M_n^2)^2} \qquad W_1(s,Q^2) \underbrace{\rightarrow}_{\mathrm{Bj}} F_1(x,Q^2)$$

- Phenomenological analysis by Bosted and Christy (Phys.Rev. C81 (2010) 055213)



Conclusions

- Quark-Hadron Duality requires infinitely many resonance states at high energies
- Large N_C inspired approach including widths provides a working scheme
- Minimal number of resonances leads to parameter reduction at low energies
- Radial Regge formulas for hadrons (mesons and baryons)

$$M_n^2 \sim an + bJ + c$$

- Effective two-body dynamics (Quark-diquark for baryons)
- Detailed description requires higher spin fields

Can we describe quark properties of hadrons WITHOUT explicit ?

References

1. **"Excited Hadrons and Quark-Hadron Duality,"**
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