Off-shell initial state effects and gauge invariance of amplitude in Drell-Yan process.

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Outline.

1. Introduction
2. Parton Reggeization Approach (PRA)
3. Structure functions for DY process in PRA
4. Spectra and angular coefficients for DY process in LO PRA
5. Helicity structure functions and Gauge Invariance
6. Conclusions

Introduction, I

- The Drell-Yan(DY) process of production of lepton pairs with large invariant mass in hadronic collisions is one of the most important test of perturbative QCD, as well as the unique source of information about partonic structure of hadrons.
- We discuss DY process in the framework of the parton Reggeization approach (PRA), which includes off-shell initial state effects in a gauge-invariant way.
- Structure functions or angular coefficients, which parametrize the angular distribution of leptons in the rest frame of the lepton pair are often under consideration. Behavior of these observables in the region of relatively small $q_T \leq Q$ will be the main subject of the present paper.

Introduction, II

- Un-physical divergence at $q_T \to 0$ is regulated through the resummation of higher-order corrections in α_s enhanced by $\log^2(q_T/Q)$ and $\log(q_T/Q)$ through Collins-Soper-Sterman formalism (CSS), which later has been reformulated in a form of Transverse Momentum Dependent (TMD) factorization theorem.
- In TMD-factorization, the hard-scattering coefficient (HSC) doesn't depend explicitly on the transverse momenta of colliding partons. Instead, it is calculated with on-shell initial-state partons and corresponding partonic tensor automatically satisfies the QED Ward identity

Introduction, III

- However, it is possible to develop a complementary approach to TMD factorization, starting not from the collinear limit but from Multi-Regge limit for QCD scattering amplitudes
- In PRA, the HSC, although being gauge-invariant, nevertheless explicitly depends on the transverse momenta and virtualities of initial-state partons. Below we demonstrate, that this dependence is important for the calculation of the **helicity structure functions** in DY process at $q_T << Q$

LO PRA framework

LO PRA framework

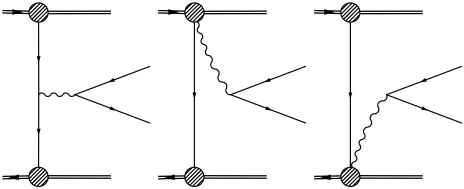
For details, see
[A. V. Karpishkov, M. A. Nefedov, V. A. Saleev, $B\bar{B}$ angular correlations at the LHC in parton Reggeization approach merged with higher-order matrix elements. Phys.Rev. **D96** 096019 (2017)] .

- **The aim** of **PRA** is to improve the description of **multi-scale** correlational observables ($|\mathbf{q}_T| \sim Q$) in comparison with the *fixed-order CPM* calculations.
- **The wider task** is to understand the role of transverse momentum in Initial-State Radiation (ISR) and put it under theoretical control at the level of quantum field theory at all transverse momenta, $0 \leq |\mathbf{q}_T| \leq Q$.
- To provide predictions with controllable accuracy and understand our formalism better we can go to NLO in **PRA**.

1. M. Nefedov, "On the one-loop calculations of multiscale quantities in Lipatov's EFT ", Talk at this Conference.
2. M. Nefedov and V. Saleev"From LO to NLO in the parton Reggeization approach", EPJ Web Conf. 191 (2018) 04007.
3. M. Nefedov and V. Saleev,"DIS structure functions in the NLO approximation of the Parton Reggeization Approach," EPJ Web Conf. **158** (2017) 03011.
4. M. Nefedov and V. Saleev,"On the one-loop calculations with Reggeized quarks," Mod. Phys. Lett. A **32** (2017) no.40, 1750207.

LO PRA framework

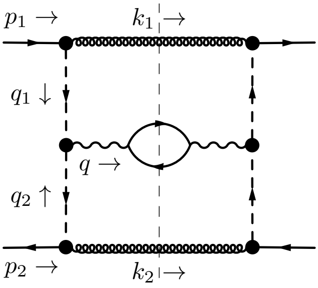
In order for **Gauge Invariance to be satisfied**, in addition to the annihilation diagram, one must also take into account the *direct interaction of the photon with the proton or its fragments*:



- Such a consideration in the general case does not lead to a simple factorization formula.
- However, in the **multi-Regge limit**, factorization is possible, it is **MRK** factorization.

Derivation of the LO factorization formula

[A. V. Karpishkov, M. A. Nefedov, V. A. Saleev, Phys.Rev. **D96** 096019 (2017)]



Auxiliary hard CPM subprocess:

$$q(p_1) + \bar{q}(p_2) \to g(k_1) + l\bar{l}(P_{\mathcal{A}}) + g(k_2),$$

Modified MRK approximation: $z_{1,2}$ and $\mathbf{q}_{T1,2}^2$ – arbitrary ($q_1^+ = z_1 p_1^+, \quad q_2^- = z_2 p_2^-$):

$$\overline{|\mathcal{M}|^2}_{\text{mMRK}} \simeq \frac{4g_s^4}{q_1^2 q_2^2} P_{qq}(z_1) P_{qq}(z_2) \frac{\overline{|\mathcal{A}_{PRA}|^2}}{z_1 z_2},$$

where $q_{1,2}^2 = -\mathbf{q}_{T1,2}^2/(1 - z_{1,2})$, has correct **collinear** and **Multi-Regge** limits!

Conjecture: mMRK approximation is reasonable zero-approximation for exact $\overline{|\mathcal{M}|^2}$ *away from collinear limit.*

Factorization formula

Substituting the $\overline{|\mathcal{M}|^2}_{\rm mMRK}$ to the factorization formula of CPM and changing the variables we get:

$$d\sigma = \int\limits_0^1 \frac{dx_1}{x_1} \int \frac{d^2\mathbf{q}_{T1}}{\pi} \tilde{\Phi}_q(x_1, t_1, \mu^2) \int\limits_0^1 \frac{dx_2}{x_2} \int \frac{d^2\mathbf{q}_{T2}}{\pi} \tilde{\Phi}_{\bar{q}}(x_2, t_2, \mu^2) \cdot d\hat{\sigma}_{\rm PRA},$$

where $x_1 = q_1^+/P_1+$, $x_2 = q_2^-/P_2^-$, $\tilde{\Phi}(x, t, \mu^2)$ – "tree-level" **unintegrated PDFs**, the partonic cross-section in PRA is:

$$d\hat{\sigma}_{\rm PRA} = \frac{\overline{|\mathcal{A}_{PRA}|^2}}{2x_1x_2S} \cdot (2\pi)^4 \delta\left(\frac{1}{2}\left(q_1^+ n_- + q_2^- n_+\right) + q_{T1} + q_{T2} - P_{\mathcal{A}}\right) d\Phi_{\mathcal{A}}.$$

Note the usual **flux-factor** $2x_1x_2S$ for **off-shell** initial-state partons.

LO unintegrated PDF

The "tree-level" unPDF:

$$\tilde{\Phi}_q(x,t,\mu^2) = \frac{1}{t}\frac{\alpha_s}{2\pi}\int_x^1 dz \frac{x}{z}\left[P_{qq}(z) f_q\left(\frac{x}{z},\mu^2\right) + P_{qg}(z) f_g\left(\frac{x}{z},\mu^2\right)\right].$$

contains collinear divergence at $t \to 0$ and IR divergence at $z \to 1$.
In the "dressed" unPDF collinear divergence is regulated by **Sudakov formfactor** $T(t,\mu^2)$:

$$\Phi_q(x,t,\mu^2) = \frac{T_q(t,\mu^2)}{t} \quad \times \quad \frac{\alpha_s(t)}{2\pi}\int_x^1 dz\, \theta_z^{\text{cut}} \frac{x}{z}\left[P_{qq}(z) f_q\left(\frac{x}{z},\mu^2\right) + \right.$$

$$\left. + \quad P_{qg}(z) f_g\left(\frac{x}{z},\mu^2\right)\right]$$

where: $\theta_z^{\text{cut}} = \theta\left((1-\Delta_{KMR}(t,\mu^2)) - z\right)$, and the Kimber-Martin-Ryskin(KMR) **cut condition** [KMR, 2001]:

$$\Delta_{KMR}(t,\mu^2) = \frac{\sqrt{t}}{\sqrt{\mu^2}+\sqrt{t}},$$

follows from the **rapidity ordering** between the last emission and the hard subprocess

LO unintegrated PDF

When $\Delta_{KMR}(t,\mu^2) << 1$, (DY: $q_T << Q$)

$$\Phi_q(x,t,\mu^2) \simeq \boxed{\frac{\partial}{\partial t}\left[T_q(t,\mu^2)\cdot xf_q(x,t)\right]} \leftarrow \text{derivative form of unPDF} \quad (1)$$

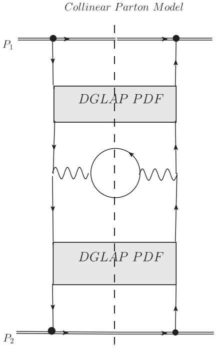
$\Rightarrow$ **LO normalization condition:** (since $T(0,\mu^2)=0, \quad T(\mu^2,\mu^2)=1$)

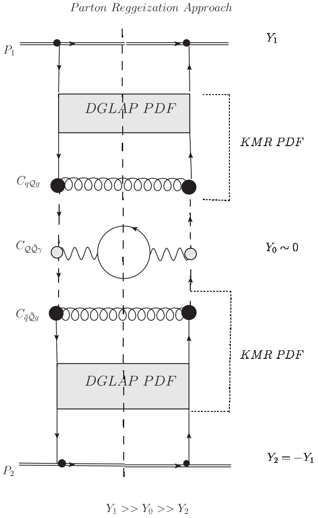
$$xf_q(x,\mu^2) = \int_0^{\mu} \Phi_q(x,t,\mu^2)dt \quad (2)$$

KMR unPDF:

- can be related to MRK, the same as Lipatov's ET.
- contains $\ln(Q^2/\Lambda^2)$, $\ln^{1,2}(Q^2/q_T^2)$.
- High-energy resummation ($\ln(\frac{1}{x})$) are absent.
- at the $t < t_0$ we put $\Phi(x,t,\mu^2) = f(x,\mu^2)(a + b * t)$ and matching point is $t_0 = 1$ GeV.

General Scheme of PRA





Gauge-invariant off-shell amplitudes

$\overline{|\mathcal{A}_{\mathrm{PRA}}|^2}$ is obtained from Lipatov's **gauge-invariant effective theory for MRK processes in QCD** [Lipatov 1995]
Feynman rules for Reggeized gluons:[Antonov, Lipatov, Kuraev, Cherednikov 2005].
Feynman rules for Reggeized quarks : [Lipatov, Vyazovsky, 2001].
Effective vertex for $Q + \bar{Q} \to \gamma^*$ – [Fadin, Sherman, 1976]:

$$\Gamma_\mu(q_1, q_2) = \gamma_\mu - \hat{q}_1 \frac{n_\mu^-}{q_2^-} - \hat{q}_2 \frac{n_\mu^+}{q_1^+}. \tag{3}$$

The QED Ward identity $(q_1 + q_2)^\mu \Gamma_\mu(q_1, q_2) = 0$ is satisfied by this vertex for any q_1 and q_2.
The first use for description of DY process:
M. A. Nefedov, N. N. Nikolaev and V. A. Saleev, "Drell-Yan lepton pair production at high energies in the Parton Reggeization Approach," Phys. Rev. D **87** (2013) no.1, 014022

Structure functions for DY process in PRA

Structure functions for DY process in PRA

$$\frac{d\sigma}{dQ^2 dq_T^2 dy d\Omega} = \frac{\alpha^2}{64\pi^3 S Q^4} L_{\mu\nu} W^{\mu\nu},$$

where y is the rapidity of virtual photon (or l^+l^- lepton pair), $d\Omega = d\phi d\cos\theta$ is the spatial angle of producing positive lepton in the rest frame of virtual photon.

The convolution of hadronic and leptonic tensors reads as a sum of contributions of the so-called helicity structure functions $F_{UU}^{1,2,\cos\phi,\cos 2\phi}$:

$$\begin{aligned} \frac{d\sigma}{dx_A dx_B d^2\mathbf{q}_T d\Omega} &= \frac{\alpha^2}{4Q^2}\Big[F_{UU}^{(1)} \cdot (1+\cos^2\theta) + F_{UU}^{(2)} \cdot (1-\cos^2\theta) + \\ &+ F_{UU}^{(\cos\phi)} \sin(2\theta)\cos\phi + F_{UU}^{(\cos 2\phi)} \cdot \sin^2\theta \cos(2\phi)\Big], \end{aligned}$$

were $x_{A,B} = Qe^{\pm y}/\sqrt{S}$, angles θ and ϕ are defined in the Collins-Soper frame.

With the help of factorization formula, SFs can be represented as:

$$F_{UU}^{(1,\dots)} = \frac{S}{6\pi^2 Q_T^4} \int dt_1 \int d\phi_1 \sum_q \Phi_q^p(x_1, t_1, \mu^2) \Phi_{\bar{q}}^p(x_2, t_2, \mu^2) \cdot e_q^2 f^{(1,\dots)}, \quad (4)$$

where $t_2 = (\mathbf{q}_T - \mathbf{q}_{T1})^2$, $Q_T^2 = Q^2 + q_T^2$ and e_q is the quark electric charge in units of electron charge.
The partonic tensor:

$$w_{\mu\nu}^{\mathrm{PRA}} = \frac{1}{4} \mathrm{tr} \left[\left(\frac{q_2^-}{2} \hat{n}^+ \right) \Gamma_\mu(q_1, q_2) \left(\frac{q_1^+}{2} \hat{n}^- \right) \Gamma_\nu(q_1, q_2) \right],$$

Projecting the partonic tensor on transverse, longitudinal, single spin-flip and double spin-flip helicity states of the virtual photon, one obtains the following expressions for partonic SFs in PRA:

$$f_{\mathrm{PRA}}^{(1)} = Q^2 + \frac{q_T^2}{2}, \; f_{\mathrm{PRA}}^{(2)} = (\mathbf{q}_{T1} - \mathbf{q}_{T2})^2,$$

$$f_{\mathrm{PRA}}^{(\cos\phi)} = \sqrt{\frac{Q^2}{q_T^2}} (\mathbf{q}_{T1}^2 - \mathbf{q}_{T2}^2), \; f_{\mathrm{PRA}}^{(\cos 2\phi)} = \frac{q_T^2}{2}.$$

Structure functions for DY process in PRA

$$\frac{dN}{d\Omega} = \frac{4}{\lambda + 3}\left(1 + \lambda \cos^2\theta + \frac{\nu}{2}\sin^2\theta \cos 2\phi\right),$$

with the normalization

$$\int \left(\frac{dN}{d\Omega}\right) d\Omega = \frac{16\pi}{3}.$$

The angular coefficients are presented via helicity structure functions:

$$\lambda = \frac{F_{UU}^{(1)} - F_{UU}^{(2)}}{F_{UU}^{(1)} + F_{UU}^{(2)}} \qquad \nu = \frac{F_{UU}^{\cos(2\phi)}}{F_{UU}^{(1)} + F_{UU}^{(2)}}.$$

Structure functions for DY process in PRA

For the calculation of cross-sections, the π^2-resummation K-factor is included:

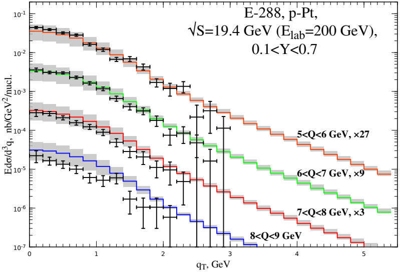
$$K = \exp\left(C_F \frac{\alpha_s(\mu_K^2)}{2\pi}\pi^2\right),$$

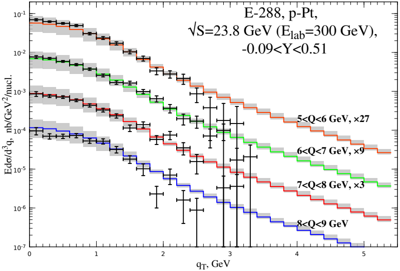
with $\mu_K^2 = Q^{2/3}Q_T^{4/3}$, and $\mu_F = Q_T$. The theoretical uncertainty was obtained by independent variation of this scales. Typical values of K-factor are 1.3 – 1.8.

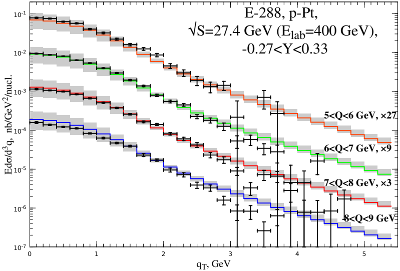
To justify the use of PRA at relatively low $\sqrt{S} = 24$ GeV, which is expected to be achieved during the operation of NICA collider in the pp-mode, we compare our numerical results for the differential cross-section $Ed\sigma/d^3\mathbf{q}$ as a function of q_T, Y and Q with experimental data of E-288 Collaboration, obtained in the collisions of the proton beam with platinum fixed target at the same energies.

Spectra and angular coefficients for DY process in PRA

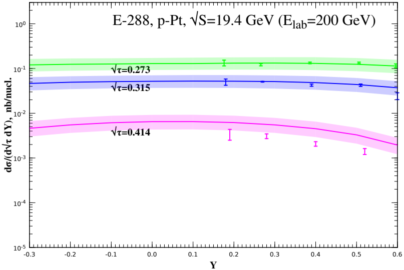
Spectra and angular coefficients for DY process in PRA

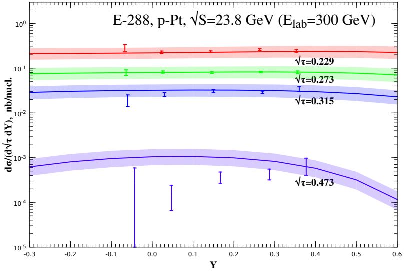


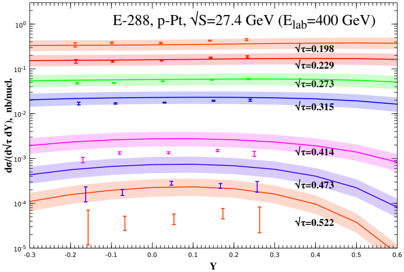




Spectra and angular coefficients for DY process in PRA



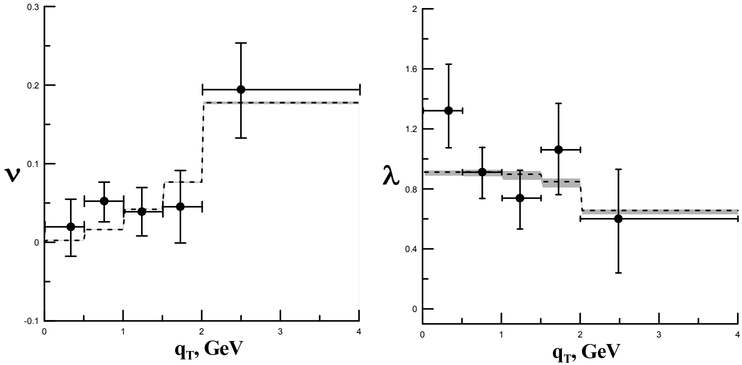




$$\tau = \frac{Q^2}{S} = x_A \times x_B$$

EMC-effect is not included in calculations

Spectra and angular coefficients for DY process in PRA



Angular coefficients ν and λ as function of q_T. The histogram corresponds to LO calculation in PRA. The data are from NuSea Collaboration ($E_{lab} = 800$ GeV, $\sqrt{S} = 39$ GeV, $4.5 < Q < 15$ GeV, $0 < x_F < 0.8$).

Helicity structure functions and Gauge Invariance

Quasi on-shell schemes

- From the point of view of standard TMD factorization (CSS) terms which restore the Ward identity for $t_{1,2} \neq 0$ can be viewed as corrections sub-leading in powers of q_T/Q. Therefore it is not obvious that these terms have significant numerical effect on the SFs at moderate $q_T < Q$ and especially for $q_T \ll Q$.
- The simplest way to restore gauge-invariance, retaining the transverse momentum of initial-state partons, is to artificially put their virtuality to zero on the level of hard-scattering coefficient.
- Below we will compare the results of PRA with two versions of QOS-scheme.

Quasi on-shell scheme - I

In the Ref. [J.C. Collins, Foundations of perturbative QCD, (Sec. 14.5.2)] the hard-scattering coefficient does not depend explicitly on $\mathbf{q}_{T1}$ and $\mathbf{q}_{T2}$, and four-momenta of initial-state partons, which has been used for the calculation of the partonic tensor, has been chosen as follows:

$$(\tilde{q}_1^{(\mathrm{QOS}-1)})^\mu = \frac{1}{4\kappa}\left(q^+(\kappa+1)n_-^\mu + q^-(\kappa-1)n_+^\mu\right) + \frac{q_T^\mu}{2},$$
$$(\tilde{q}_2^{(\mathrm{QOS}-1)})^\mu = \frac{1}{4\kappa}\left(q^+(\kappa-1)n_-^\mu + q^-(\kappa+1)n_+^\mu\right) + \frac{q_T^\mu}{2}, \qquad (5)$$

where $\kappa = \sqrt{Q_T^2/Q^2}$ and $q^\pm = Q_T e^{\pm Y}$, so that $\tilde{q}_1 + \tilde{q}_2 = q$ while $\tilde{q}_{1,2}^2 = 0$. In the QOS-1-approximation, the partonic tensor reads:

$$w_{\mu\nu}^{\mathrm{QOS}} = \frac{1}{4}\mathrm{tr}\left[\hat{\tilde{q}}_2\gamma_\mu\hat{\tilde{q}}_1\gamma_\nu\right],$$

and the only nonzero partonic SF is $f^{(1)}_{\mathrm{QOS}-1} = Q^2$ while $f^{(2)}_{\mathrm{QOS}-1} = f^{(\cos\phi)}_{\mathrm{QOS}-1} = f^{(\cos 2\phi)}_{\mathrm{QOS}-1} = 0$ like in CPM.

Quasi on-shell scheme - 2

One can try to introduce the $\mathbf{q}_{T1,2}$-dependence into the QOS-scheme. To this end, one adds the “small” light-cone components q_1^- and q_2^+ to put vectors $\tilde{q}_{1,2}$ on-shell:

$$(\tilde{q}_1^{(\mathrm{QOS}-2)})^\mu = \frac{1}{2}\left(q_1^+ n_-^\mu + \frac{\mathbf{q}_{T1}^2}{q_1^+} n_+^\mu\right) + q_{T1}^\mu,$$

$$(\tilde{q}_2^{(\mathrm{QOS}-2)})^\mu = \frac{1}{2}\left(\frac{\mathbf{q}_{T2}^2}{q_2^-} n_-^\mu + q_2^- n_+^\mu\right) + q_{T2}^\mu,$$

where “large” light-cone components are determined from the condition $\tilde{q}_1 + \tilde{q}_2 = q$.

Quasi on-shell scheme - 2

$$q_1^+ = (Q_T^2 + t_1 - t_2 + \sqrt{D})/(2q^-)$$

and

$$q_2^- = (Q_T^2 - t_1 + t_2 + \sqrt{D})/(2q^+)$$

where

$$D = (Q_T^2 - t_1 - t_2)^2 - 4t_1 t_2.$$

Quasi on-shell scheme - 2

Partonic SFs in the new QOS-2 scheme are equal to:

$$f^{(1)}_{\rm QOS-2} = Q^2 - \frac{(\mathbf{q}_{T1}-\mathbf{q}_{T2})^2}{2} + \frac{(\mathbf{q}^2_{T1}-\mathbf{q}^2_{T2})^2}{2Q_T^2},\ f^{(2)}_{\rm QOS-2} = (\mathbf{q}_{T1}-\mathbf{q}_{T2})^2 - \frac{(\mathbf{q}^2_{T1}-\mathbf{q}^2_{T2})^2}{Q_T^2},$$

$$f^{(\cos\phi)}_{\rm QOS-2} = \sqrt{\frac{Q^2 D}{q_T^2}}\,\frac{\mathbf{q}^2_{T1}-\mathbf{q}^2_{T2}}{Q_T^2},$$

$$f^{(\cos 2\phi)}_{\rm QOS-2} = -\frac{(\mathbf{q}_{T1}-\mathbf{q}_{T2})^2}{2} + \frac{Q^2+Q_T^2}{2Q_T^2}\,\frac{(\mathbf{q}^2_{T1}-\mathbf{q}^2_{T2})^2}{q_T^2}.$$

In this version of QOS-2-scheme, the helicity SFs $f^{(1)}_{\rm QOS-2}$, $f^{(2)}_{\rm QOS-2}$ and $f^{(\cos\phi)}_{\rm QOS-2}$ are equal to PRA results at leading power in $|\mathbf{q}_{T1,2}|/Q$, however the SF $f^{(\cos 2\phi)}_{\rm QOS-2}$ is completely different from the PRA result. At small $\mathbf{q}_T = \mathbf{q}_{T1} + \mathbf{q}_{T2}$ the first term dominates and this coefficient is negative.

Numerical results for SFs

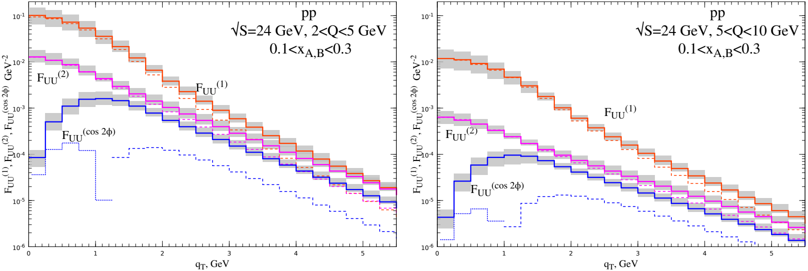


Рис.: Predictions for unpolarized Drell-Yan SFs $F_{UU}^{(1)}$, $F_{UU}^{(2)}$ and $F_{UU}^{(\cos 2\phi)}$ in pp-collisions at $\sqrt{S} = 24$ GeV. Solid lines with uncertainty bands – PRA predictions. Dashed lines – predictions in the QOS-2 scheme for the default scale-choice. Short-dashed line – plot of the $(-F_{UU}^{(\cos 2\phi)})$ in the QOS-2 scheme, since this SF in QOS scheme is negative at low q_T.

TMD versus PRA at $q_T << Q$

Predictions of TMD-factorization, based solely on the $q\bar{q}$-annihilation picture

$$F_{UU}^{(1)} \sim \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2} \cdot f_1^q(x_1, q_{T1}) f_1^{\bar{q}}(x_2, q_{T2}) + O\left(\frac{q_T^2}{Q^2}\right),$$

$$F_{UU}^{(2)} \sim O\left(\frac{q_T^2}{Q^2}\right),$$

$$F_{UU}^{(\cos 2\phi)} \sim \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2} \cdot h_1^{\perp q}(x_1, q_{T1}) h_1^{\perp \bar{q}}(x_2, q_{T2}) \times$$
$$\times \frac{2(\mathbf{q}_T\mathbf{q}_{T1})(\mathbf{q}_T\mathbf{q}_{T2}) - \mathbf{q}_T^2(\mathbf{q}_{T1}\mathbf{q}_{T2})}{\mathbf{q}_T^2\Lambda^2} + O\left(\frac{q_T^2}{Q^2}\right),$$

where $f_1^q(x, q_T)$ and $h_1^{\perp q}(x, q_T)$ are *unpolarized and Boer-Mulders* leading twist TMD PDFS.

Predictions of PRA:

$$F_{UU}^{(1)} \sim \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2} \cdot f_1^q(x_1, q_{T1}) f_1^{\bar{q}}(x_2, q_{T2}) + O\left(\frac{q_T^2}{Q^2}\right),$$

$$F_{UU}^{(2)} \sim \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2} \cdot f_1^q(x_1, q_{T1}) f_1^{\bar{q}}(x_2, q_{T2}) \sim O\left(\frac{q_T^2}{Q^2}\right),$$

$$F_{UU}^{(\cos 2\phi)} \sim \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2} \cdot f_1^q(x_1, q_{T1}) f_1^{\bar{q}}(x_2, q_{T2}) \sim F_{UU}^{(2)} \sim O\left(\frac{q_T^2}{Q^2}\right),$$

TMD versus PRA at $q_T << Q$

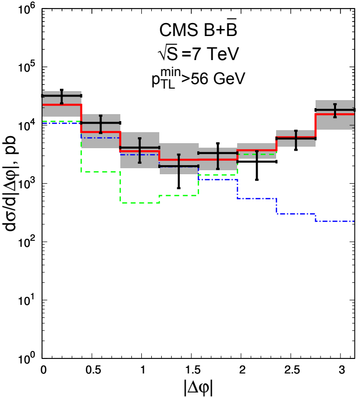
- The **Gauge Invariance** formalism leads to reduction of number of independent TMD PDFS or form-factors.
- Numerical value of $F_{UU}^{\cos 2\phi}$ even at $q_T^2 \ll Q^2$ strongly depends on the details of the procedure of restoration of gauge-invariance of the hard-scattering coefficient.
- On the other hand, in the TMD-factorization, based on the only one diagram (a), the hadronic tensor does not satisfy Ward identity for $q_T \neq 0$.
- This raises serious doubts about the Boer-Mulders function as well-defined physical quantity in TMD-factorization.

General Conclusions

- PRA can be applied for study of hard processes at wide energy range, from NICA to LHC
- PRA takes into account effects of transverse momenta and off-shell effects in the initial state **in a Gauge-Invariant way.**
- PRA can be applied at all values of q_T/Q in DY process. One smoothly interpolates between hard processes at low p_T and large p_T.
- In the PRA, instead of some other approaches, we can do NLO calculations (real and loop's corrections).

Thank you for your attention!

NLO contribution in $\triangle\phi_{BB}$ and $\triangle\phi_{jj}$ spectra



Azimuthal angle difference spectra $p\bar{p} \to B\bar{B}X$: LO ($RR \to b\bar{b}$, NLO ($RR \to b\bar{b}g$) and their sum