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TMD factorization: status and implications

Recent results in collaboration with
A. Vladimirov (Regensburg)
and ongoing related work with
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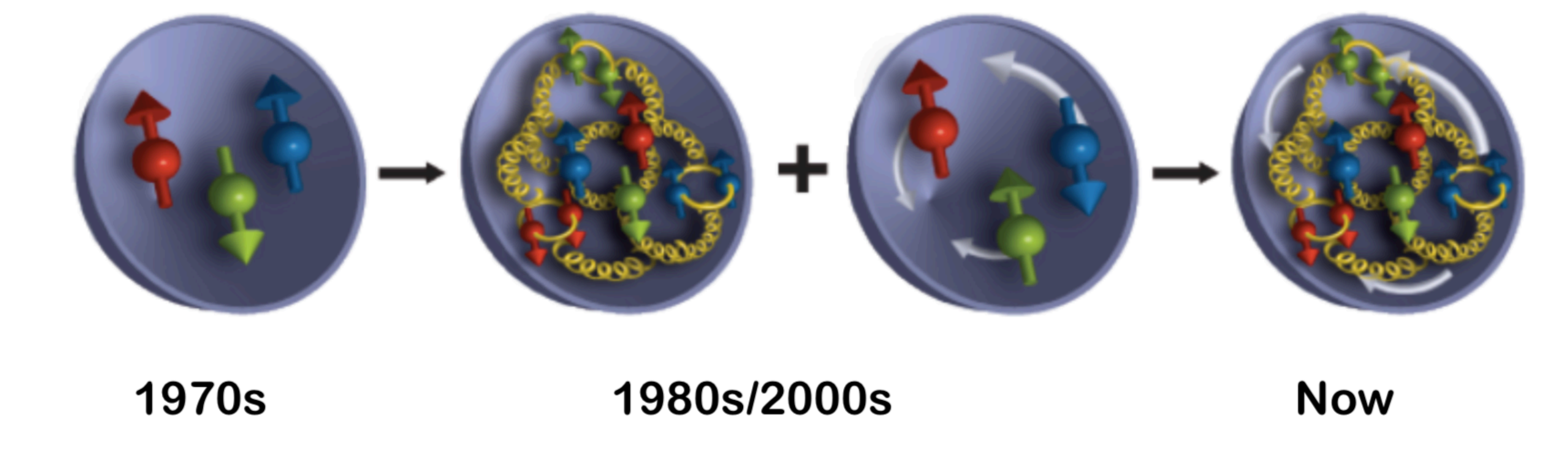
W. Waalewijn (NIKHEF),
L. Zoppi (NIKHEF)

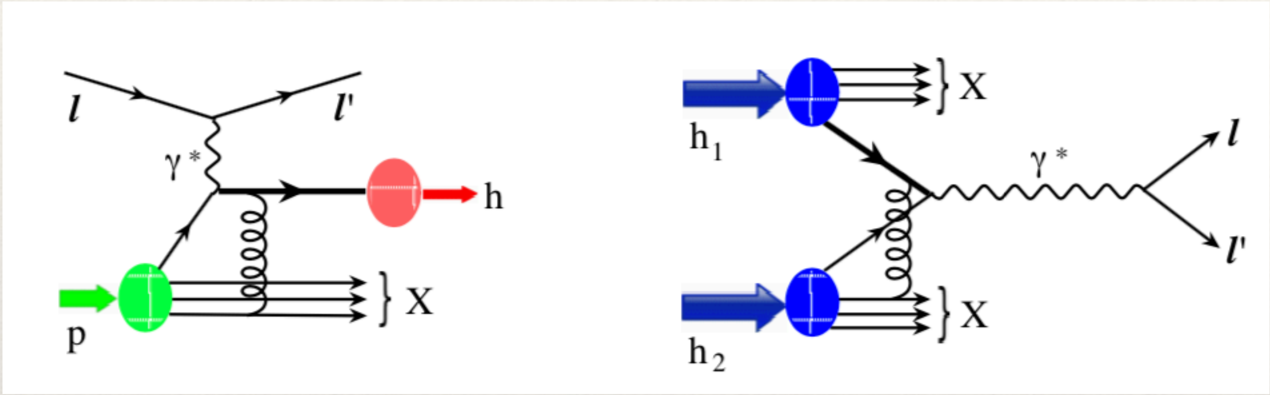
Outline



- Overview of hadron structure concepts
- Factorization for TMDs
- 2-D Evolution
- Phenomenology

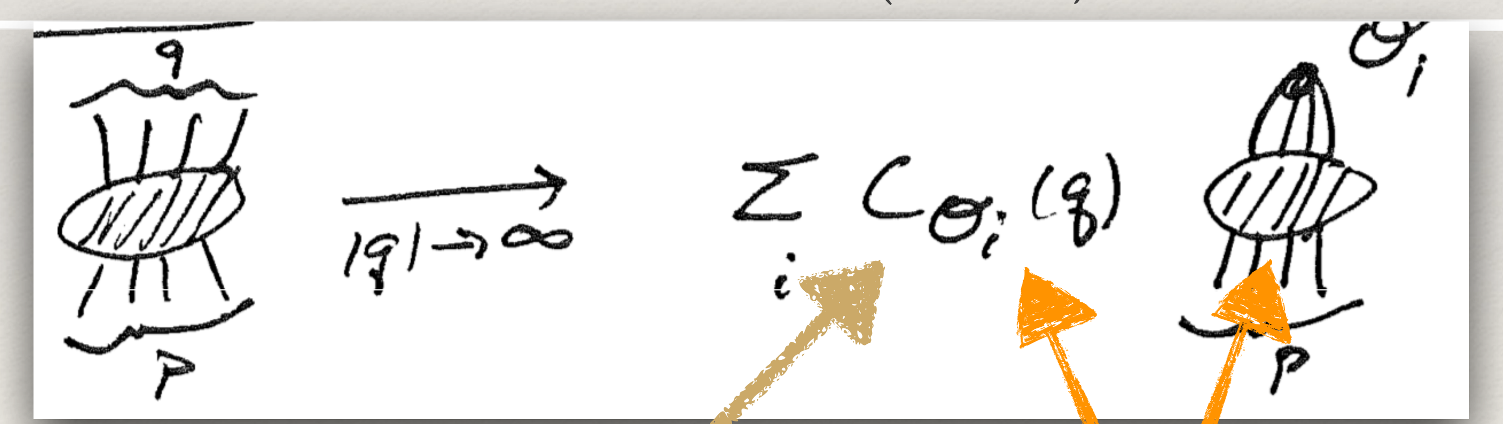
Drell-Yan and Deep Inelastic Scattering classical knowhow

Defining pure quark states is not an easy task in QCD:
quarks/gluons can only be treated in an asymptotic regime!



Altarelli, Parisi, Dokshitzer, Gribov, Lipatov,Collins, Soper, Sterman, Catani, Martinelli, Nason,.. '70-'90

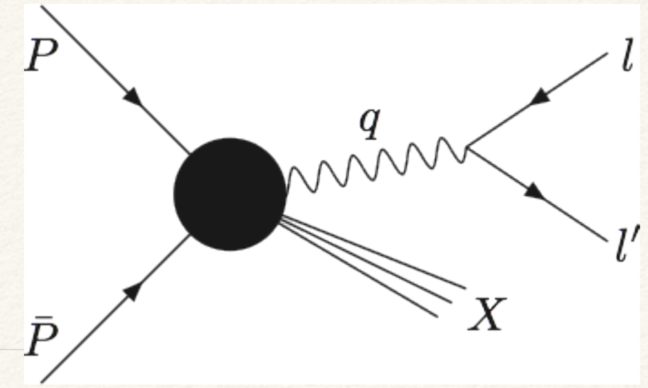
Basic Principle: OPERATOR PRODUCT EXPANSION (Wilson, Zimmermann…),



Drawing from:
J. Preskill lectures

Wilson Coefficients

Factorization Scale Dependence: μ



Drell-Yan and Deep Inelastic Scattering classical knowhow

Factorization!

$$\frac{d\sigma}{dQ^2} = \sum_{i,j=q,\bar{q},g} \int_0^1 dx_1\, dx_2\, \mathcal{H}_{ij}(x_1, x_2, Q^2, \mu^2)\, f_{i/P}(x_1, \mu^2)\, f_{j/\bar{P}}(x_2, \mu^2)$$

PDF: Parton Distribution Functions

PDF describe the probability of finding a parton "i" in the hadron "P":

We can define the probability of one parton in one hadron **independently** of the probability of the other parton.

The quark states are totally disentangled!

PDF: Parton Distribution Functions

All non-perturbative QCD information is encoded in PDFs

Is this true for all processes that we know?

No,....(But all MonteCarlo assume yes! Precision?)

Do exist similar processes with different functions which encode a non-perturbative information different from PDFs?

Trying to extend our knowledge naively

Suppose to measure more differential cross sections, can we write (Collins, Soper, Sterman '80)

$$\frac{d\sigma}{dQ^2 dq_T dy} = \sum_q \sigma_q^\gamma H(Q^2, \mu^2) \int \frac{d^2\mathbf{b}}{4\pi} e^{-i\mathbf{q_T}\cdot\mathbf{b}} \Phi_{q/A}(x_A, \mathbf{b}, \mu) \Phi_{q/B}(x_B, \mathbf{b}, \mu)?$$

Unfortunately we cannot be so naive

$$\Phi_{q\leftarrow q}(x, \mathbf{b}, \mu) = \delta(1-x) + a_s 2C_F \mathbf{B}^\varepsilon \Gamma(-\varepsilon) \left(p_{qq}(x) - \varepsilon(1-x) - 2\delta(1-x)\lambda_\delta\right) + \ldots$$

$p_{qq} =$ splitting function

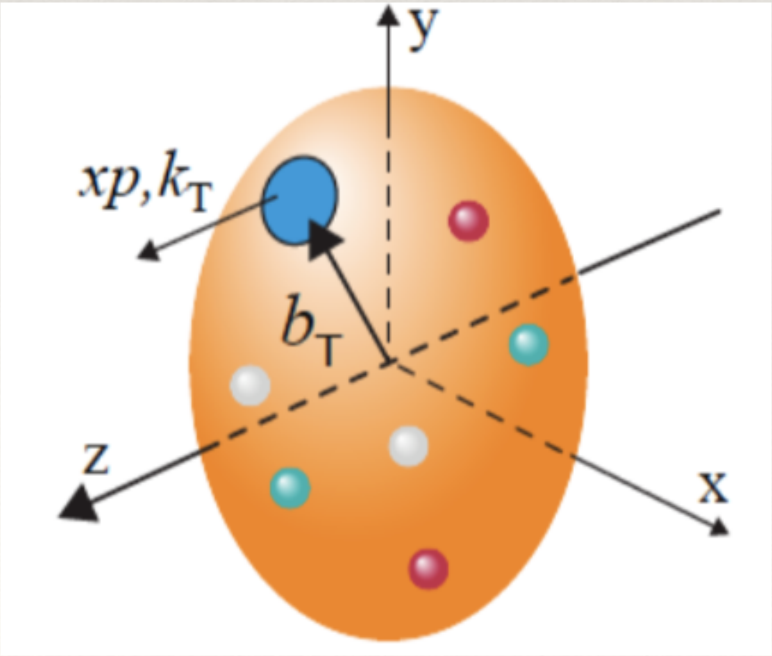
$\mathbf{B} = \mathbf{b}^2/4$

$\lambda_\delta = \ln \delta^+/p^+$

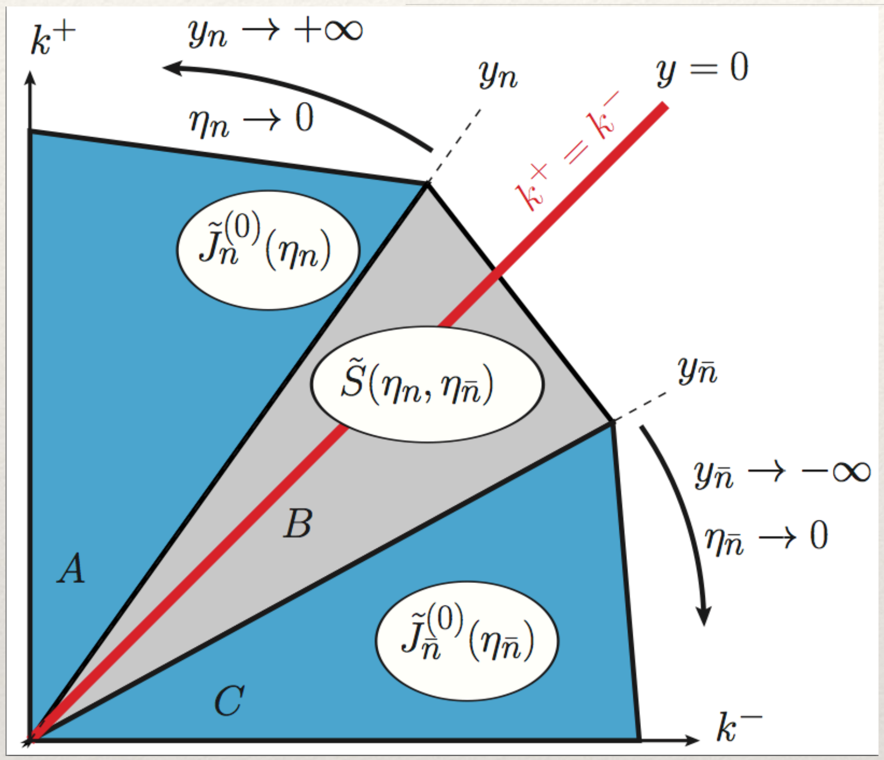
One does not achieve a full separation of UV and IR singularities!

*In standard QCD calculations this new **rapidity divergent** part cancels in the whole cross section computation: it seems that at small transverse momentum the initial states are **entangled** …*

Factorization theorem for TMDs



Factorization theorem basics: siDIS case



$$d\sigma \sim \int d^4x e^{iqx} \sum_X \langle h_1 | J^\mu(x) | X, h_2 \rangle \langle X, h_2 | J^\nu(0) | h_1 \rangle$$

$$d\sigma \sim \int d^2b_T e^{-iq_T b_T} H(Q^2) \Phi_{h_1}(z_1, b_T) S(b_T) \Delta_{h_2}(z_2, b_T) + Y$$

TMDPDF — Soft Factor — TMDFF

- PDF, Soft Factor, FF have "rapidity divergences"
- Soft Factor mixes PDF and FF through rapidity divergent pieces ..
- We have the **splitting of rapidity singularities in the Soft Factor**:

$$S(b_T) = \sqrt{S(b_T, \zeta)}\sqrt{S(b_T, \zeta^{-1})} \quad \longrightarrow \quad d\sigma \sim H(Q) \int d^2b_T e^{-iq_T b_T} F(x; b_T) D(z; b_T)$$

Each TMD is now free of rapidity singularity: nice renormalizable non-perturbative object

TMD factorization in nutshell

.. for DY and heavy boson production we have (Collins 2011, Echevarria, Idilbi, Scimemi (EIS) 2012)

$$\frac{d\sigma}{dQ^2 dq_T dy} = \sum_q \sigma_q^\gamma H(Q^2,\mu^2) \int \frac{d^2\mathbf{b}}{4\pi} e^{-i\mathbf{q_T}\cdot\mathbf{b}} \Phi_{q/A}(x_A,\mathbf{b},\zeta_A,\mu)\Phi_{q/B}(x_B,\mathbf{b},\zeta_B,\mu)$$

$$\sqrt{\zeta_A\zeta_B} = Q^2$$

…and similar formulas are valid for SIDIS (EIC) and hadron production in e+e- colliders

The pathological behavior is associated to a particular kind of divergences: rapidity divergences

We have new non-perturbative effects which cannot be included in PDFs.

TMD factorization in nutshell

The renormalization of the rapidity divergences is responsible for the a new resummation scale: **2-D evolution**

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$$\frac{d\sigma}{dQ^2 dq_T dy} = \sum_q \sigma_q^\gamma H(Q^2,\mu^2) \int \frac{d^2\mathbf{b}}{4\pi} e^{-i\mathbf{q_T}\cdot\mathbf{b}} \Phi_{q/A}(x_A,\mathbf{b},\zeta_A,\mu)\Phi_{q/B}(x_B,\mathbf{b},\zeta_B,\mu)$$

$$\sqrt{\zeta_A\zeta_B} = Q^2$$

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We have new non-perturbative effects which cannot be included in PDFs.

The case of unpolarized TMDs:
the perturbative calculable part of unpolarized TMDs is known at NNLO!
How can we use this information?
Which scale prescription allows an optimal extraction of TMD's?
What is the range of validity of the TMD factorization theorem?
Do LHC data have an impact on TMD extraction?

Evolution kernel for TMDs

PDF

$$\frac{d}{d\ln\zeta_F}\ln\tilde{F}^{[\Gamma]}_{f/N}(x,\mathbf{b}_\perp,S;\zeta_F,\mu^2) = -D(b_T;\mu^2)\,,$$

Fragmentation

$$\frac{d}{d\ln\zeta_D}\ln\tilde{D}^{[\Gamma]}_{h/f}(z,\mathbf{b}_\perp,S_h;\zeta_D,\mu^2) = -D(b_T;\mu^2)\,.$$

$$\tilde{F}^{[\Gamma]}_{f/N}(x,\mathbf{b}_\perp,S;\zeta_{F,f},\mu_f^2) = \tilde{F}^{[\Gamma]}_{f/N}(x,\mathbf{b}_\perp,S;\zeta_{F,i},\mu_i^2)\,\tilde{R}\left(b_T;\zeta_{F,i},\mu_i^2,\zeta_{F,f},\mu_f^2\right)$$

$$\tilde{D}^{[\Gamma]}_{h/f}(z,\mathbf{b}_\perp,S_h;\zeta_{D,f},\mu_f^2) = \tilde{D}^{[\Gamma]}_{h/f}(z,\mathbf{b}_\perp,S_h;\zeta_{D,i},\mu_i^2)\,\tilde{R}\left(b_T;\zeta_{D,i},\mu_i^2,\zeta_{D,f},\mu_f^2\right)$$

$$\tilde{R}(b;\zeta_i,\mu_i^2,\zeta_f,\mu_f^2) = \exp\left\{\int_{\mu_i}^{\mu_f}\frac{d\bar{\mu}}{\bar{\mu}}\,\gamma\left(\alpha_s(\bar{\mu}),\ln\frac{\zeta_f}{\bar{\mu}^2}\right)\right\}\left(\frac{\zeta_f}{\zeta_i}\right)^{-D(b_T;\mu_i)}$$

The evolution kernel is the same for pdf and fragmentation and it is also polarization independent

Mulders-Tangerman '96,
Boer Mulders ' 98
Mulders, 2001 (gluons)
Boer, Mulders, Collins
Mulders, Buffing, Mukherjee 2013
....

Polarization effects with a universal evolution

Quark Polarization

Nucleon Polarization

QUARKS	*unpolarized*	*chiral*	*transverse*
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$h_{1T}, h_{1T}^{\perp}$

$$
\begin{aligned}
\Phi_{q\leftarrow h}^{[\gamma^+]}(x,\boldsymbol{b}) &= f_1(x,\boldsymbol{b}) + i\epsilon_T^{\mu\nu} b_\mu s_{T\nu} M f_{1T}^{\perp}(x,\boldsymbol{b}),\\
\Phi_{q\leftarrow h}^{[\gamma^+\gamma_5]}(x,\boldsymbol{b}) &= \lambda g_{1L}(x,\boldsymbol{b}) + i b_\mu s_T^\mu M g_{1T}(x,\boldsymbol{b}),\\
\Phi_{q\leftarrow h}^{[i\sigma^{\alpha+}\gamma_5]}(x,\boldsymbol{b}) &= s_T^\alpha h_1(x,\boldsymbol{b}) - i\lambda b^\alpha M h_{1L}^{\perp}(x,\boldsymbol{b})\\
&\quad + i\epsilon_T^{\alpha\mu} b_\mu M h_1^{\perp}(x,\boldsymbol{b}) + \frac{M^2\boldsymbol{b}^2}{2}\left(\frac{g_T^{\alpha\mu}}{2} + \frac{b^\alpha b^\mu}{\boldsymbol{b}^2}\right) s_{T\mu} h_{1T}^{\perp}(x,\boldsymbol{b}).
\end{aligned}
$$

Gluon Polarization

Nucleon Polarization

GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_{1T}^g, h_{1T}^{\perp g}$

Time-reversal flip

Table from 1804.08148

Name	Function	Leading matching function	Twist of leading matching	Maximum known order of coef.function
unpolarized	$f_1(x,\boldsymbol{b})$	f_1	tw-2	NNLO (a_s^2)
Sivers	$f_{1T}^{\perp}(x,\boldsymbol{b})$	T	tw-3	LO (a_s^0)
helicity	$g_{1L}(x,\boldsymbol{b})$	g_1	tw-2	NLO (a_s^1)
worm-gear T	$g_{1T}(x,\boldsymbol{b})$	g_1, T, ΔT	tw-2/3	LO (a_s^0)
transversity	$h_1(x,\boldsymbol{b})$	h_1	tw-2	NNLO(a_s^2)
Boer-Mulders	$h_1^{\perp}(x,\boldsymbol{b})$	δT_ϵ	tw-3	LO (a_s^0)
worm-gear L	$h_{1L}^{\perp}(x,\boldsymbol{b})$	h_1, δT_g	tw-2/3	LO (a_s^0)
pretzelosity**	$h_{1T}^{\perp}(x,\boldsymbol{b})$	–	tw-4	–

D. Gutierrez-Reyes et al. 1805.07243

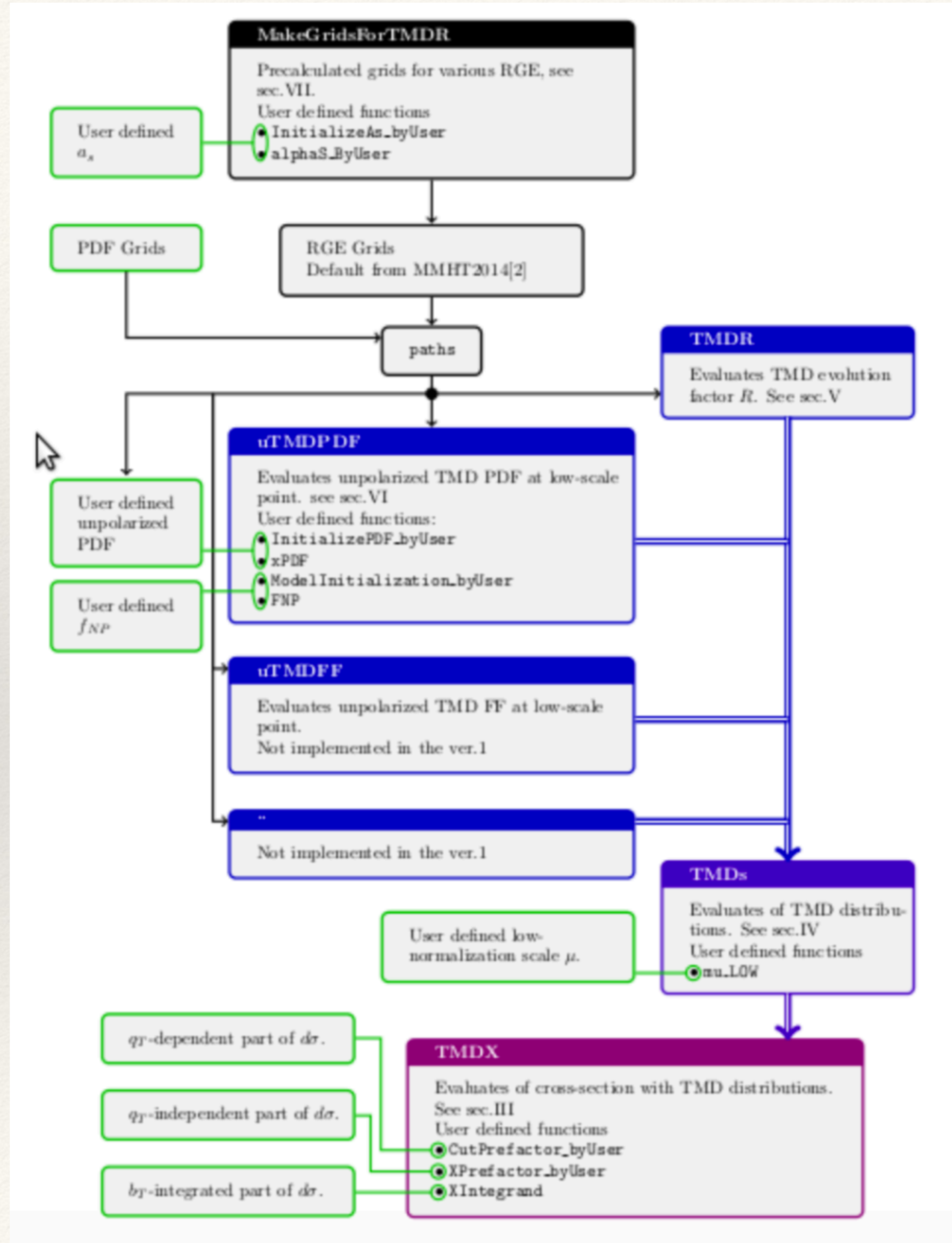
arTeMiDe

- FORTRAN 90 code
- Module structure
- Convolutions, evolution (LO,NLO,NNLO)
- Fourier to q_T-space, integrations over phase space
- Scale-variation (ζ-prescription)
- User defined PDFs, scales, f_{NP}
- Efficient code ($\sim 10^9$ TMDs $\sim$ 6. min at NNLO)

Currently ver ~~1~~ 1.3

Available at: https://teorica.fis.ucm.es/artemide

Future plans: add modules for fragmentations, and polarized TMDs



Cross section and TMD structure

Lepton tensor cuts

E.w. charges

Hard Coefficient

$$\frac{d\sigma}{dQ^2 dy d(q_T^2)} = \frac{4\pi}{3N_c}\frac{\mathcal{P}}{sQ^2}\sum_{GG'} z_{ll'}^{GG'}(q)\sum_{ff'} z_{ff'}^{GG'}|C_V(q,\mu)|^2 \int \frac{d^2\vec{b}}{4\pi} e^{i(\vec{b}\vec{q})}$$

$$\times F_{f\leftarrow h_1}(x_1,\vec{b};\mu,\zeta)F_{f'\leftarrow h_2}(x_2,\vec{b};\mu,\zeta) + Y,$$

Y-term
1/Q^2 corrections

Evolution factor

Low energy TMD

$$F_{f\leftarrow h}(x,\mathbf{b};\mu_f,\zeta_f) = R^f[\mathbf{b};(\mu_f,\zeta_f)\leftarrow(\mu_i,\zeta_i)]F_{f\leftarrow h}(x,\mathbf{b};\mu_i,\zeta_i)$$

$$R^f[\mathbf{b};(\mu_f,\zeta_f)\leftarrow(\mu_i,\zeta_i)] = \exp\left[\int_P \left(\gamma_F^f(\mu,\zeta)\frac{d\mu}{\mu} - \mathcal{D}^f(\mu,\mathbf{b})\frac{d\zeta}{\zeta}\right)\right]$$

$$F_{f\leftarrow h}(x,\vec{b};\mu,\zeta) = \sum_q \int_x^1 \frac{dz}{z} C_{f\leftarrow q}(z,\mathbf{L}_\mu;\mu,\zeta) f_{q\leftarrow h}\left(\frac{x}{z},\mu\right) f_{NP}(z,\mathbf{b})$$

Matching (Wilson) coefficient

PDF

Non-perturbative input

Cross section and TMD structure

Lepton tensor cuts

E.w. factors

Hard Coefficient

$$\frac{d\sigma}{dQ^2dyd(q_T^2)} = \frac{4\pi}{3N_c}\frac{\mathcal{P}}{sQ^2}\sum_{GG'} z_{ll'}^{GG'}(q)\sum_{ff'} z_{ff'}^{GG'}\,|C_V(q,\mu)|^2\int\frac{d^2\vec{b}}{4\pi}e^{i(\vec{b}\vec{q})}$$

$$\times F_{f\leftarrow h_1}(x_1,\vec{b};\mu,\zeta)F_{f'\leftarrow h_2}(x_2,\vec{b};\mu,\zeta) + Y,$$

Only small qT data

Evolution factor

Low energy TMD

$$F_{f\leftarrow h}(x,\mathbf{b};\mu_f,\zeta_f) = R^f[\mathbf{b};(\mu_f,\zeta_f)\leftarrow(\mu_i,\zeta_i)]F_{f\leftarrow h}(x,\mathbf{b};\mu_i,\zeta_i)$$

$$R^f[\mathbf{b};(\mu_f,\zeta_f)\leftarrow(\mu_i,\zeta_i)] = \exp\left[\int_P\left(\gamma_F^f(\mu,\zeta)\frac{d\mu}{\mu} - \mathcal{D}^f(\mu,\mathbf{b})\frac{d\zeta}{\zeta}\right)\right]$$

$$F_{f\leftarrow h}(x,\vec{b};\mu,\zeta) = \sum_q\int_x^1\frac{dz}{z}C_{f\leftarrow q}(z,\mathbf{L}_\mu;\mu,\zeta)f_{q\leftarrow h}\left(\frac{x}{z},\mu\right)f_{NP}(z,\mathbf{b})$$

ζ-prescription

Matching (Wilson) coefficient

PDF

Gaussian? Exponential?

2-D TMD evolution

Coupled evolution of TMD ...

TMD (standard) anomalous dimension

$$\mu^2 \frac{d}{d\mu^2} F_{f\leftarrow h}(x, b; \mu, \zeta) = \frac{\gamma_F(\mu, \zeta)}{2} F_{f\leftarrow h}(x, b; \mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} F_{f\leftarrow h}(x, b; \mu, \zeta) = -\mathcal{D}(\mu, b) F_{f\leftarrow h}(x, b; \mu, \zeta)$$

TMD rapidity anomalous dimension

Collinear overlap

$$\zeta \frac{d}{d\zeta} \gamma_F(\mu, \zeta) = -\Gamma(\mu)$$

$$\mu \frac{d}{d\mu} \mathcal{D}(\mu, b) = \Gamma(\mu)$$

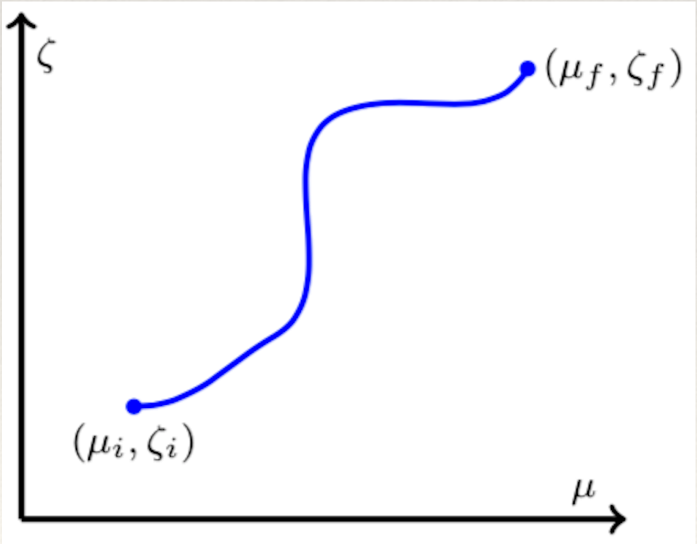
Ambiguity in the TMD evolution

Coupled evolution of TMD and truncation of the perturbative series

Integrability Condition…

$$\zeta \frac{d}{d\zeta} \gamma_F(\mu, \zeta) = -\mu \frac{d}{d\mu} \mathcal{D}(\mu, b)$$

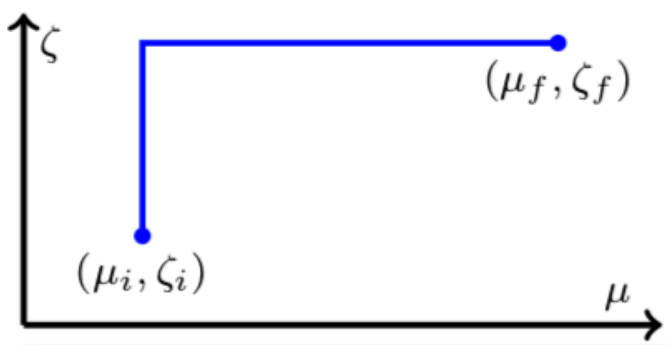
…ensures the path independence of the evolution factor…



$$R[b; (\mu_f, \zeta_f) \to (\mu_i, \zeta_i)] = \exp\left[\int_P \left(\gamma_F(\mu, \zeta)\frac{d\mu}{\mu} - \mathcal{D}(\mu, b)\frac{d\zeta}{\zeta}\right)\right]$$

Ambiguity in the TMD evolution

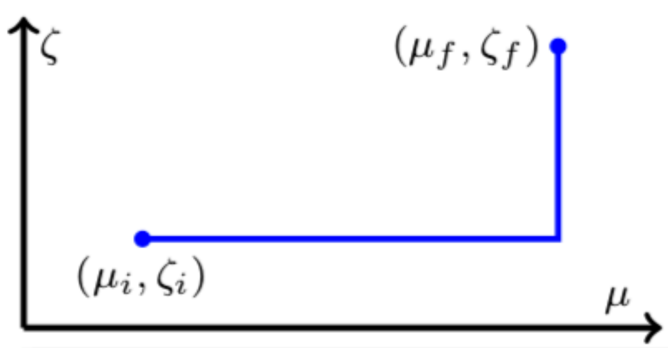
Coupled evolution of TMD and truncation of the perturbative series



Solution 1

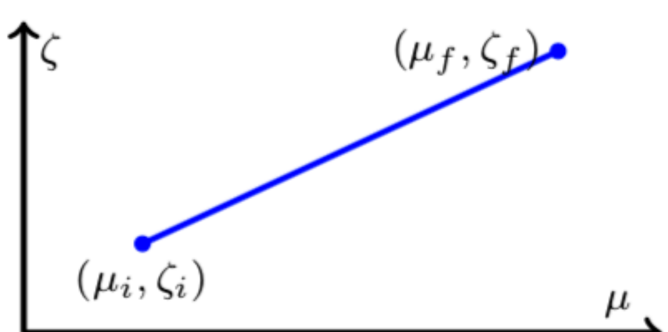
$$\ln R = \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F(\mu, \zeta_f) - \mathcal{D}(\mu_i, b) \ln\left(\frac{\zeta_f}{\zeta_i}\right)$$

given in [Collins' textbook]



Solution 2

$$\ln R = \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F(\mu, \zeta_i) - \mathcal{D}(\mu_f, b) \ln\left(\frac{\zeta_f}{\zeta_i}\right)$$



Solution 3

$$\ln R = \int_0^1 \Big(\gamma_F(\mu(t), \zeta(t)) \frac{\mu_f - \mu_i}{(\mu_f - \mu_i)t + \mu_i} - \mathcal{D}(\mu(t), b) \frac{\zeta_f - \zeta_i}{(\zeta_f - \zeta_i)t + \zeta_i}\Big) dt$$

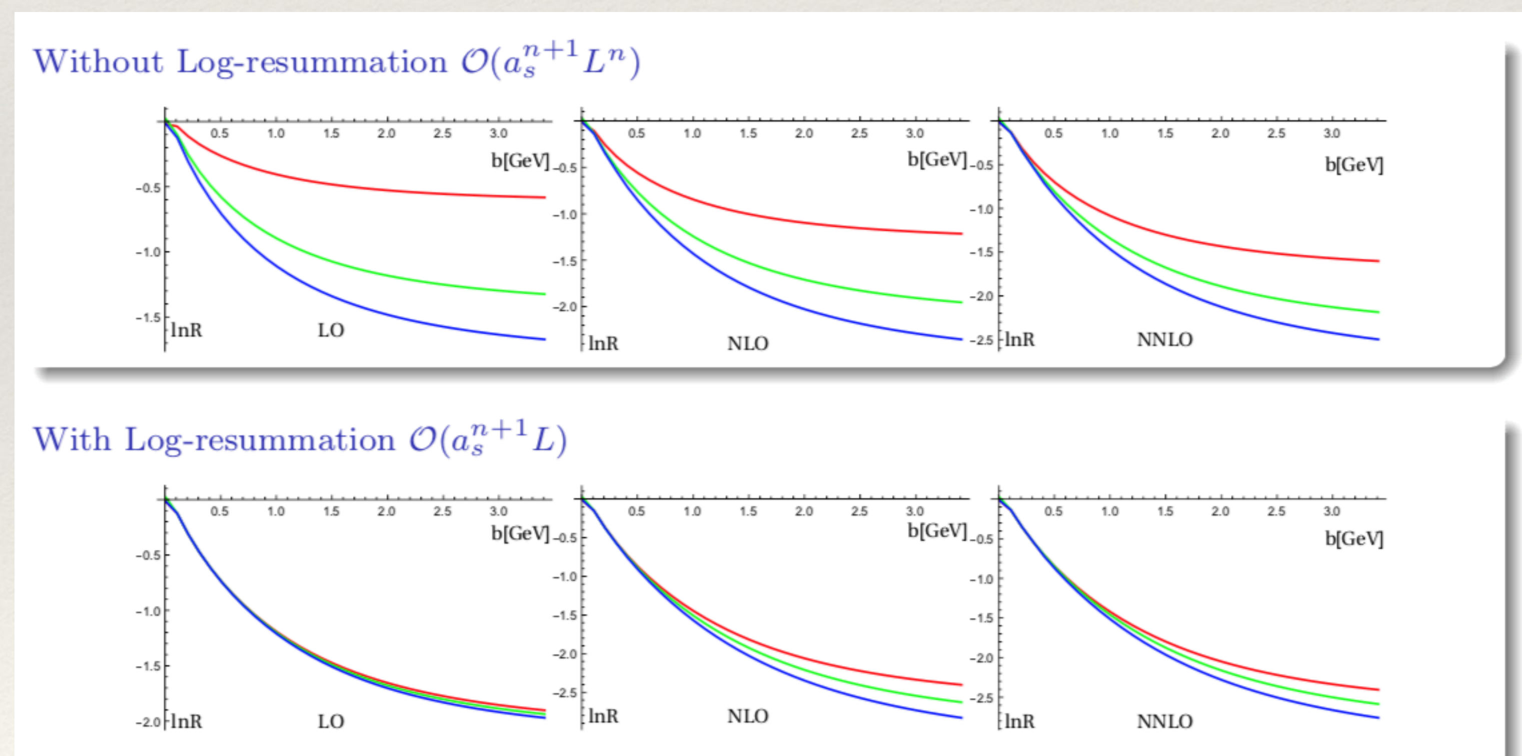
Ambiguity in the TMD evolution

Coupled evolution of TMD and truncation of the perturbative series

In practice due to the truncation of the perturbative series:
Transitivity and reversibility of evolution is lost

$$\zeta \frac{d}{d\zeta}\gamma_F(\mu,\zeta) \neq -\mu \frac{d}{d\mu}\mathcal{D}(\mu,b)$$

For Q=Mz the solution path dependence enormous



2D Evolution field: Notation and ideal case

The evolution scales are treated equally

$$\vec{\nu} = \left(\ln \frac{\mu^2}{1\ \mathrm{GeV}^2}, \ln \frac{\zeta}{1\ \mathrm{GeV}^2}\right)$$

Differentiation

$$\vec{\nabla} = \frac{d}{d\vec{\nu}} = (\mu^2 \frac{d}{d\mu^2}, \zeta \frac{d}{d\zeta}), \qquad \mathbf{curl} = (-\zeta \frac{d}{d\zeta}, \mu^2 \frac{d}{d\mu^2})$$

Evolution field

$$\mathbf{E}(\vec{\nu}, b) = (\frac{\gamma_F(\vec{\nu})}{2}, -\mathcal{D}(\vec{\nu}, b))$$

TMD Evolution

$$\vec{\nabla} F(x, b; \vec{\nu}) = \mathbf{E}(\vec{\nu}, b) F(x, b; \vec{\nu})$$

Integrability Condition and Scalar Potential

$$\vec{\nabla} \times \mathbf{E} = 0 \Rightarrow \mathbf{E}(\vec{\nu}, b) = \vec{\nabla} U(\vec{\nu}, b)$$

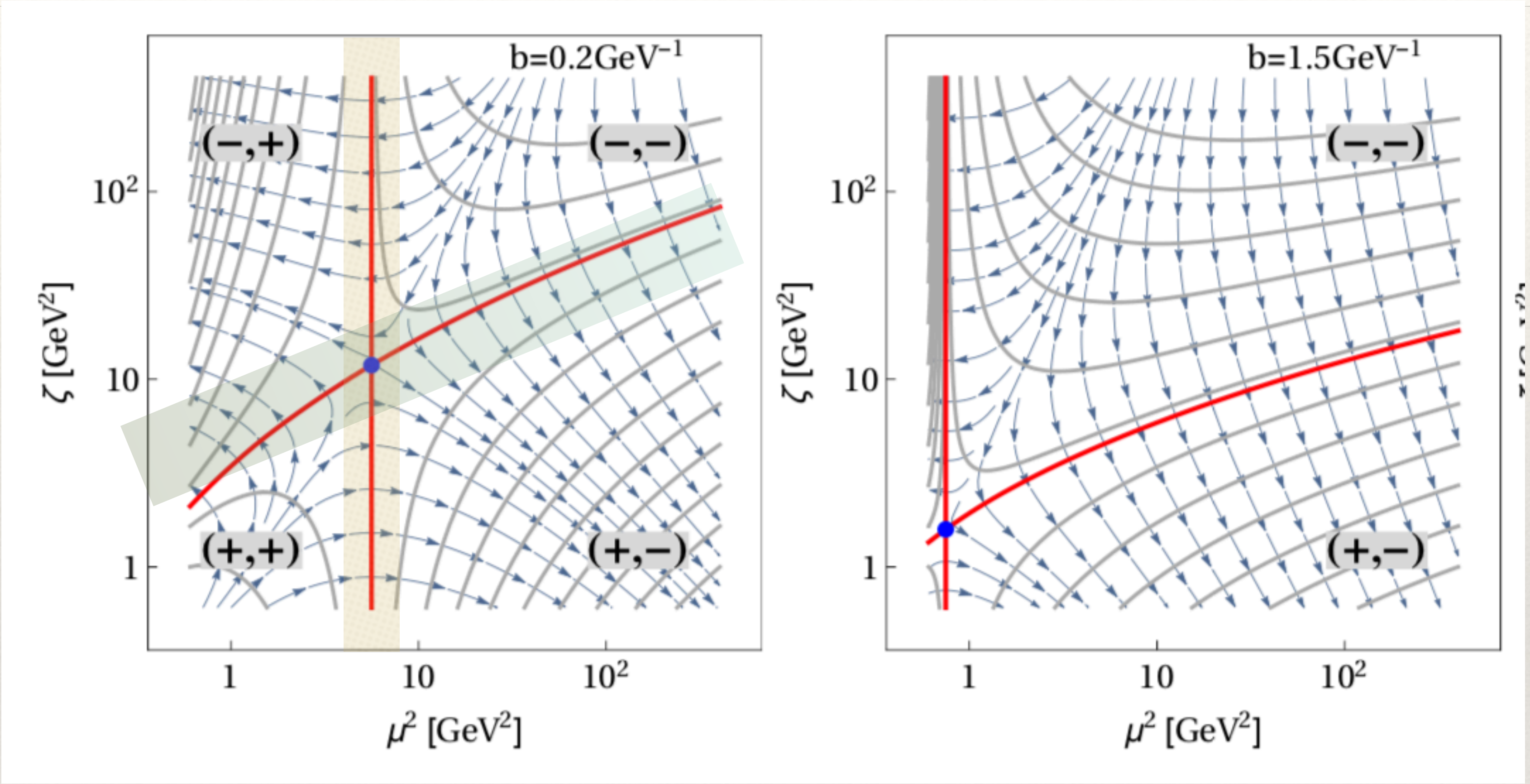
Evolution kernel

$$\ln R[b; \vec{\nu}_f \to \vec{\nu}_i] = U(\vec{\nu}_f, b) - U(\vec{\nu}_i, b)$$

with

$$U(\vec{\nu}, b) = \int^{\nu_1} \frac{\Gamma(s) s - \gamma_V(s)}{2} ds - \mathcal{D}(\vec{\nu}, b)\nu_2 + \mathrm{const}(b)$$

2D Evolution field: Notation and ideal case



● =saddle point

Singularities: Landau pole (on the left, not shown) and saddle point $\mathbf{E}(\vec{\nu}_{\text{saddle}}, b) = \vec{0}$

Equipotential/null-evolution curves: $\vec{\omega}(t, \vec{\nu}_B, b) = (t, \omega(t, \vec{\nu}_B, b)) \quad \rightarrow \frac{d\vec{\omega}}{dt} \cdot \vec{\nabla} U(\vec{\omega}, b) = 0$

Special null-evolution curves: $\mu = \mu_{\text{saddle}}$ and $\vec{\nu}_B = \vec{\nu}_{\text{saddle}}$

Truncation of the perturbative series

The truncation introduces a path difference

$$\delta\Gamma(\mu,b)=\Gamma(\mu)-\mu\frac{d\mathcal{D}(\mu,b)}{d\mu},$$

$$\delta\Gamma^{(N)}=2\sum_{n=1}^{N}\sum_{k=0}^{n}n\bar{\beta}_{n-1}(a_s)a_s^{n-1}d^{(n,k)}\mathbf{L}_\mu^k$$

$$\text{with } \bar{\beta}_n(a_s)=\beta(a_s)-\sum_{k=0}^{n-1}\beta_k a_s^{k+2}$$

$$\mathbf{L}_\mu=\ln\left(\frac{X^2b^2}{4e^{-2\gamma_E}}\right)$$

$\delta\Gamma^{(N)}\sim\mathcal{O}(a_s^{N+1}\mathbf{L}_\mu^N)$ with perturbative D

$\delta\Gamma^{(N)}\sim\mathcal{O}(a_s^{N+1}\mathbf{L}_\mu)$ with resummed D

$$\ln\frac{R[b;\{\mu_1,\zeta_1\}\xrightarrow{P_1}\{\mu_2,\zeta_2\}]}{R[b;\{\mu_1,\zeta_1\}\xrightarrow{P_2}\{\mu_2,\zeta_2\}]}=\frac{1}{2}\int_{\Omega(P_1\cup P_2)}d^2\nu\,\delta\Gamma(\vec{\nu},b)=\int_{\mu_2}^{\mu_1}\frac{d\mu}{\mu}\delta\Gamma(\mu,b)\ln\left(\frac{\zeta_1(\mu)}{\zeta_2(\mu)}\right)$$

The path dependence is enhanced by the difference in rapidity scale

At large value of impact parameter the breaking of integrability condition becomes crucial

Recovering path independence

Helmholtz decomposition of evolution fields

$$\mathbf{E}(\vec{\nu}, b) = \tilde{\mathbf{E}}(\vec{\nu}, b) + \boldsymbol{\Theta}(\vec{\nu}, b)$$

Ideally one could repair the truncation using decomposition of the evolution field

Basic properties of evolution fields

$$\text{curl}\tilde{\mathbf{E}} = 0, \qquad \vec{\nabla} \cdot \vec{\Theta} = 0, \qquad \tilde{\mathbf{E}} \cdot \boldsymbol{\Theta} = 0.$$

Scalar potentials

$$\tilde{\mathbf{E}}(\vec{\nu}, b) = \vec{\nabla}\tilde{U}(\vec{\nu}, b) \qquad \boldsymbol{\Theta}(\vec{\nu}, b) = \mathbf{curl}\, V(\vec{\nu}, b)$$

$$\text{curl}\mathbf{E} = \text{curl}\boldsymbol{\Theta} = \frac{\delta\Gamma(\vec{\nu}, b)}{2} \neq 0$$

THE INTEGRABILITY CONDITION IS RE-ESTABLISHED DEFINING THE EVOLUTION KERNEL AS

$$\ln R[b; \vec{\nu}_f \to \vec{\nu}_i] = \tilde{U}(\vec{\nu}_f, b) - \tilde{U}(\vec{\nu}_i, b)$$

$$\nabla^2 \tilde{U}(\vec{\nu}, b) = \frac{1}{2} \frac{d\gamma_F(\vec{\nu})}{d\nu_1}$$

However in order to fix completely the evolution potential one needs boundary condition for the evolution field:
at the moment no theoretically solid non-perturbative input is known

Recovering path independence

We modify anomalous dimensions such that integrability restored

$$\mu \frac{d\mathcal{D}(\mu, b)}{d\mu} = -\zeta \frac{d\gamma_F(\mu, \zeta)}{d\zeta}$$

It can be done from both sides of the equation.

Improved $\mathcal{D}$

Facilitate

$$\mu \frac{d\mathcal{D}}{d\mu} = \Gamma.$$

by

$$\mathcal{D}(\mu, b) = \int_{\mu_0}^{\mu} \frac{d\mu}{\mu} \Gamma(\mu) + \mathcal{D}(\mu_0, b)$$

- In the spirit of [Collins' text book].
- Already used in many studies
- However, it is not the best way

Improved γ

We set

$$\zeta \frac{d\gamma_F}{d\zeta} \equiv -\mu \frac{d\mathcal{D}}{d\mu} = \delta\Gamma - \Gamma$$

Or

$$\gamma_F(\mu, \zeta) \to \gamma_M(\mu, \zeta, b)$$

$$\gamma_M = (\Gamma - \delta\Gamma) \ln\left(\frac{\mu^2}{\zeta}\right) - \gamma_V$$

- Completely self consistent
- Very natural

Universität Regensburg

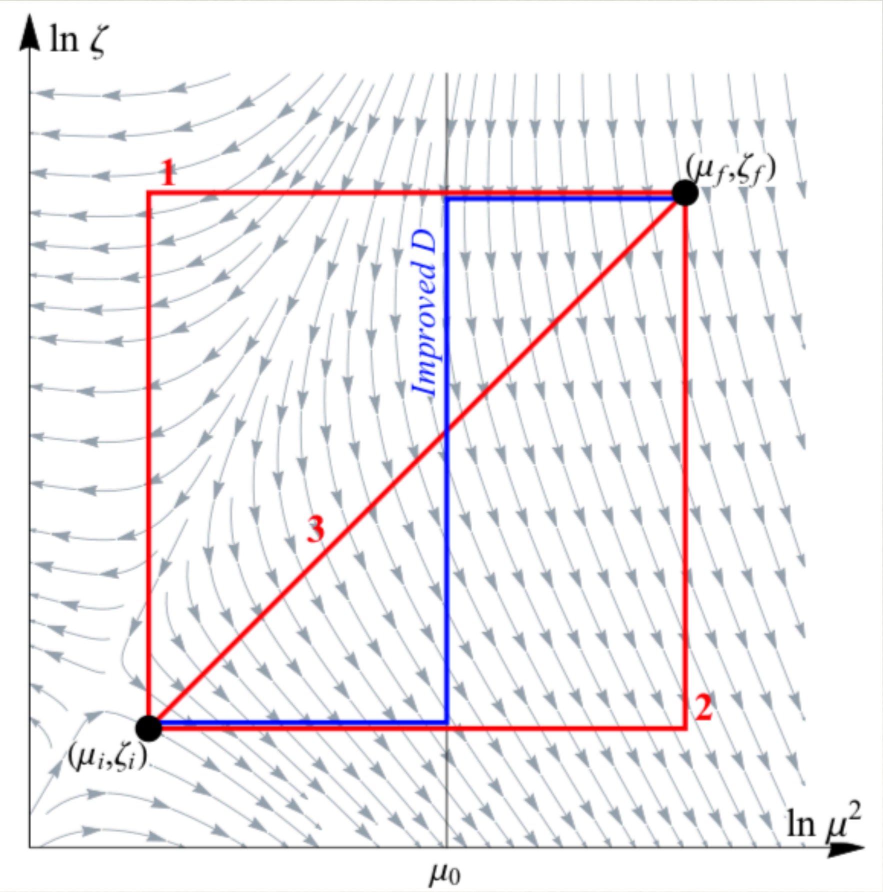
Improved D scenario

$$\mathcal{D}(\mu,b)=\int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\Gamma(\mu')+\mathcal{D}(\mu_0,b) \longrightarrow \tilde{U}(\vec{\nu},b;\mu_0)=\int_{\ln\mu_0^2}^{\nu_1}\frac{\Gamma(s)(s-\nu_2)-\gamma_V(s)}{2}ds-\mathcal{D}(\mu_0,b)\nu_2+const(b)$$

The truncation effects should be minimized by the choice of μ_0

$$\ln R[b;(\mu_f,\zeta_f)\to(\mu_i,\zeta_i);\mu_0]=\int_{\mu_i}^{\mu_f}\frac{d\mu}{\mu}\left(\Gamma(\mu)\ln\left(\frac{\mu^2}{\zeta_f}\right)-\gamma_V(\mu)\right)-\int_{\mu_0}^{\mu_i}\frac{d\mu}{\mu}\Gamma(\mu)\ln\left(\frac{\zeta_f}{\zeta_i}\right)-\mathcal{D}(\mu_0,b)\ln\left(\frac{\zeta_f}{\zeta_i}\right).$$

This is a mixture of solution 1 and 2.
The solution dependence is parameterized by μ_0



The minimization occurs only when one finds a μ_0 such that

$$\delta\Gamma(\mu_0,b)=0$$

In order to compare fits one should agree on a conventional μ_0 scale

Improved γ scenario

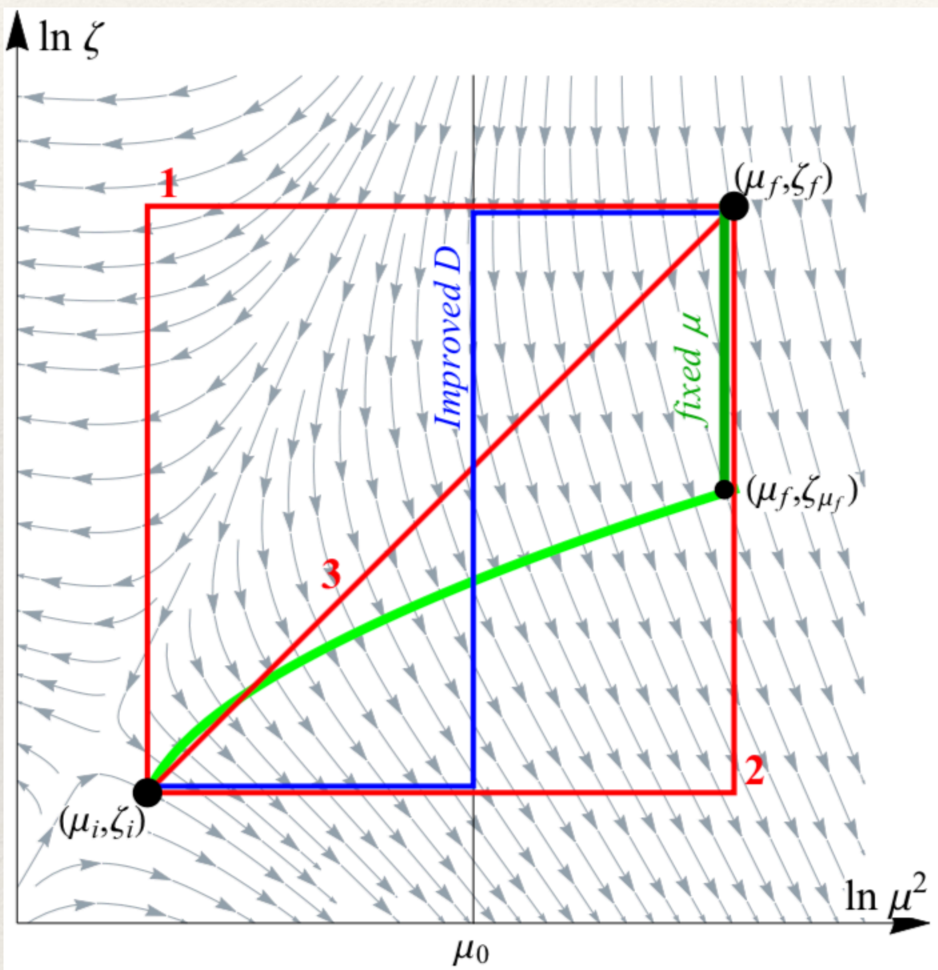
$$\gamma_M(\mu,\zeta,b) = (\Gamma(\mu) - \delta\Gamma(\mu,b))\mathbf{l}_\zeta - \gamma_V(\mu) \longrightarrow \tilde{U}(\vec{\nu},b) = \int^{\nu_1} \frac{(\Gamma(s) - \delta\Gamma(s,b))s - \gamma_V(s)}{2} ds - \mathcal{D}(\vec{\nu},b)\nu_2 + const(b)$$

$$\ln R[b;(\mu_f,\zeta_f) \to (\mu_i,\zeta_i)] = -\int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} (2\mathcal{D}(\mu,b) + \gamma_V(\mu)) + \mathcal{D}(\mu_f,b) \ln\left(\frac{\mu_f^2}{\zeta_f}\right) - \mathcal{D}(\mu_i,b) \ln\left(\frac{\mu_i^2}{\zeta_i}\right)$$

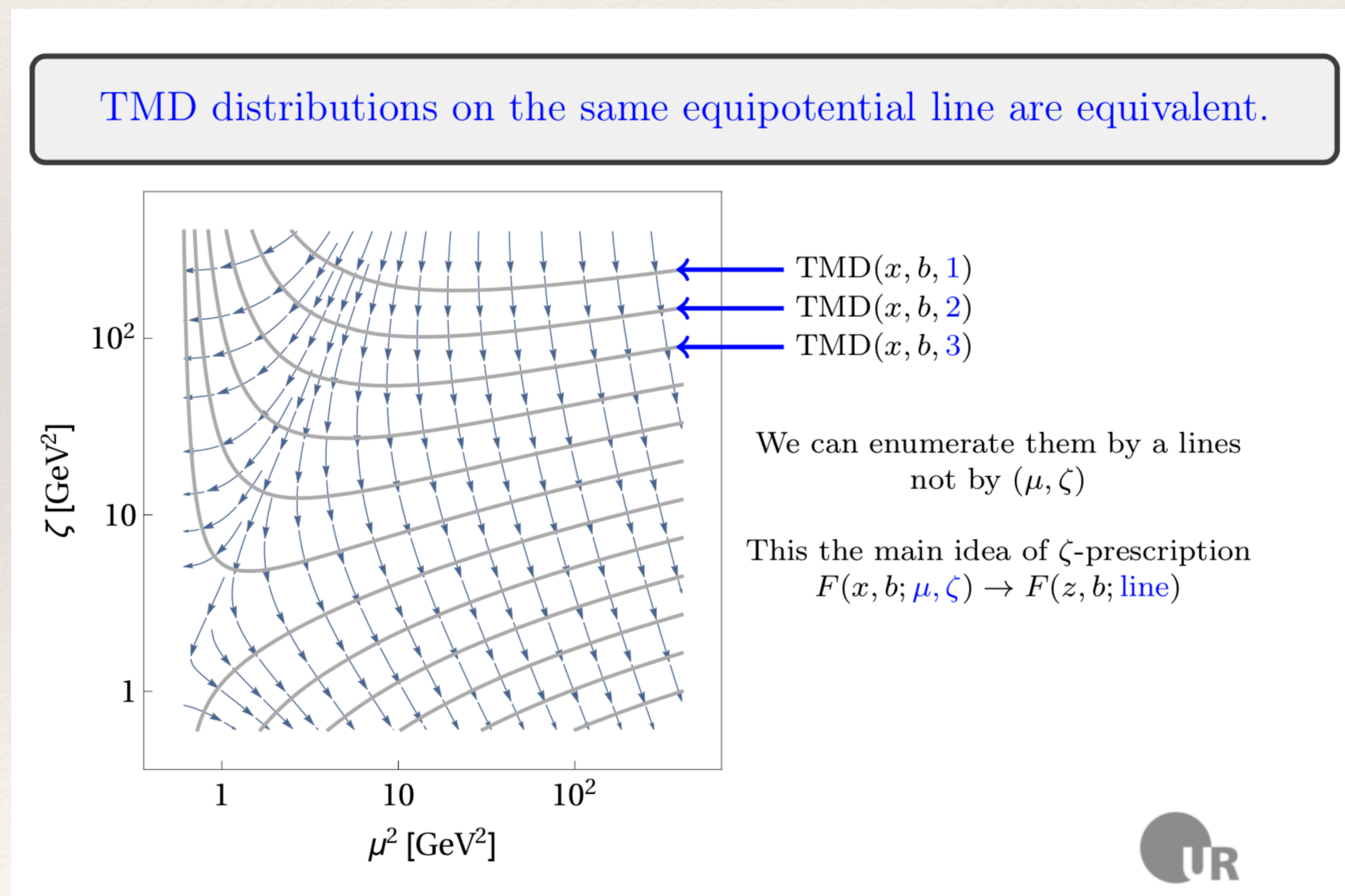
Clear advantages:

- No more the intermediate scale μ_0
- Path independence
- Simplicity
- We achieve a clear separation of evolution and non-perturbative part of the TMD

Equivalent TMDs: no evolution on equipotential lines



The 2-D evolution just connects TMDs on different equipotential lines



We can provide evolution first on an *equipotential line* and then on a vertical line.

Left for Technical discussion

TMD on equipotential lines

The TMDs one equipotential lines are not evolved so one can define a TMD by a single parameter line

$$F(x,b;\vec{\nu}_B) = F(x,b;\vec{\nu}\,'_B), \qquad \vec{\nu}\,'_B \in \vec{\omega}(\vec{\nu}_B, b).$$

ONE CAN HAVE AN EVOLUTION ONLY WHEN MOVING BETWEEN DIFFERENT LINES

$$F(x,b;\vec{\nu}_B) = R[b;\vec{\nu}_B \to \vec{\nu}\,'_B]F(x,b;\vec{\nu}\,'_B)$$

Outcome: the modeling of the non-perturbative part of the TMD does not depend anymore on the relation between renormalization scale and impact parameter.

Question: Is there a preferred line?

Left for Technical discussion

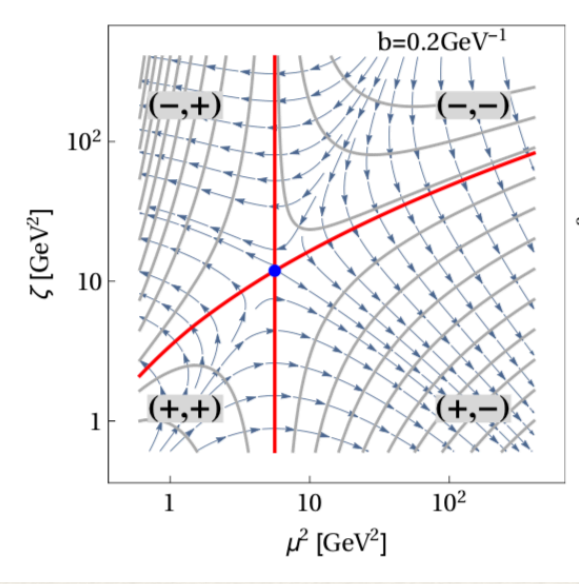
The optimal TMD distribution

There is a consistency constraint in the TMD matching to PDFs

$$\vec{\nu} = \left(\ln \frac{\mu^2}{1\ \mathrm{GeV}^2}, \ln \frac{\zeta}{1\ \mathrm{GeV}^2}\right)$$

$$F_{f\to k}(x,b;\vec{\nu}_B) = \sum_n \sum_{f'} C^{(n)}_{f\to f'}(x,b,\vec{\nu}_B,\mu_{\mathrm{OPE}}) \otimes f^{(n)}_{f'\to h}(x,\mu_{\mathrm{OPE}})$$

The values of μ_{OPE} are restricted to the values of μ taken along the null-evolution curve



$$\begin{aligned}
&\text{if } \nu_{B,1} < \ln \mu^2_{\mathrm{saddle}} \Rightarrow && \mu_{\mathrm{OPE}} < \mu_{\mathrm{saddle}},\\
&\text{if } \nu_{B,1} > \ln \mu^2_{\mathrm{saddle}} \Rightarrow && \mu_{\mathrm{OPE}} > \mu_{\mathrm{saddle}},\\
&\text{if } \vec{\nu}_B = (\ln \mu^2_{\mathrm{saddle}}, \ln \zeta_{\mathrm{saddle}}) \Rightarrow && \mu_{\mathrm{OPE}} \text{ unrestricted}
\end{aligned}$$

Left for Technical discussion

ζ-prescription

We have just to evolve between different equipotential/null-evolution line

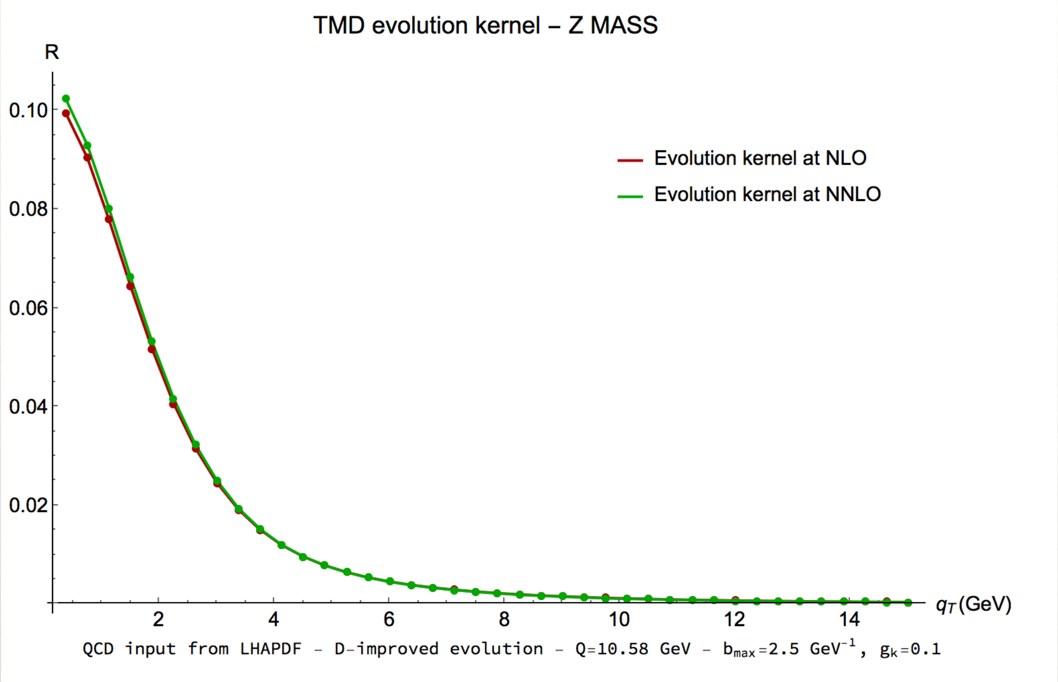
$$F(x,b;\mu_f,\zeta_f) = R[b;(\mu_f,\zeta_f) \to (\mu_f,\zeta_{\mu_f}(\vec{\nu}_B,b))]F(x,b;\vec{\nu}_B)$$

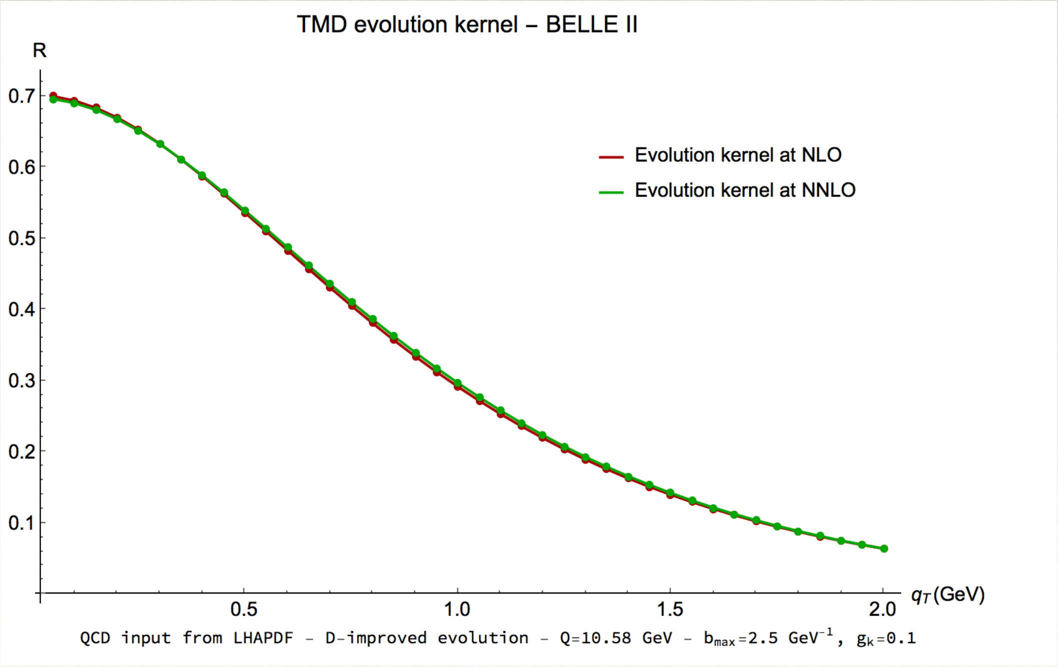
This is realized choosing $\zeta_\mu(b)$ such that

$$\frac{\gamma_F(\mu,\zeta_\mu(b))}{2\mathcal{D}(\mu,b)} = \frac{\mu^2}{\zeta_\mu(b)}$$

$$\mu^2 \frac{dF(x,\mathbf{b};\mu,\zeta_\mu)}{d\mu^2} = 0.$$

Perturbative stability of R with ζ prescription





Drawings from D. Gutierrez-Reyes. See talk of L. Zoppi

Perturbative orders…

name	$\mathcal{D}$	γ_V	H	$C_{f\leftarrow f'}$	a_s(run)	PDF (evolution)
LO	a_s^1	a_s^1	a_s^0	a_s^0	lo	lo
NLO	a_s^2	a_s^2	a_s^1	a_s^1	nlo	nlo
NNLO	a_s^3	a_s^3	a_s^2	a_s^2	nnlo	nnlo

…Theoretical uncertainties in QCD analysis…

Matching Scales

In the implementation we must choose matching prescriptions such that the perturbative series is as convergent as possible, undesired power corrections are not introduced

Hard Scale

Low Scale

$$\frac{d\sigma}{dQ^2 dy d(q_T^2)} = \frac{4\pi}{3N_c}\frac{\mathcal{P}}{sQ^2}\sum_{GG'} z_{ll'}^{GG'}(q)\sum_{ff'} z_{ff'}^{GG'}\int\frac{d^2\vec{b}}{4\pi}e^{i(\vec{b}\vec{q})}|C_V(Q,c_2Q)|^2\Big\{R^f[\vec{b};(c_2Q,Q^2)\to(c_3\mu_i,\zeta_{c_3\mu_i});c_1\mu_i]\Big\}$$

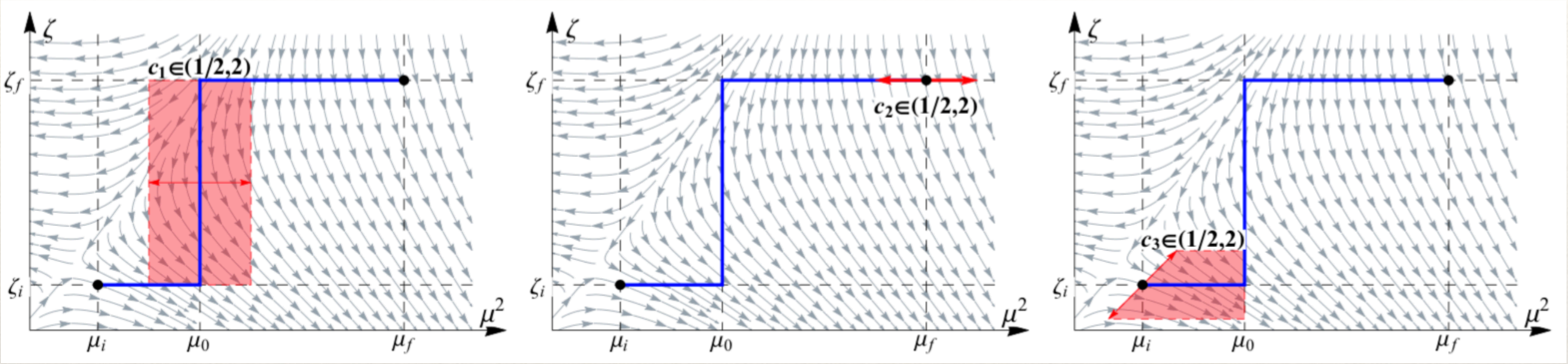
$$\times F_{f\leftarrow h_1}(x,\vec{b};c_4\mu_{\mathrm{OPE}},\zeta_{c_4\mu_{\mathrm{OPE}}})F_{f'\leftarrow h_2}(x,\vec{b};c_4\mu_{\mathrm{OPE}},\zeta_{c_4\mu_{\mathrm{OPE}}})$$

Rapidity Evolution

Small b Scale

Parameters and quality of the fits depend strongly on the choices made for the implementation

Details of scale variations



- ~~c_1 measure only solution dependence~~
- c_2 measure mismatch between H and R + solution dependence
- ~~c_3 measure mismatch between F and R + solution dependence~~
- c_4 measure mismatch between C and f

Eliminated by gamma-scenario

Included in c2 by optimal TMD definition

A new error analysis: LHC

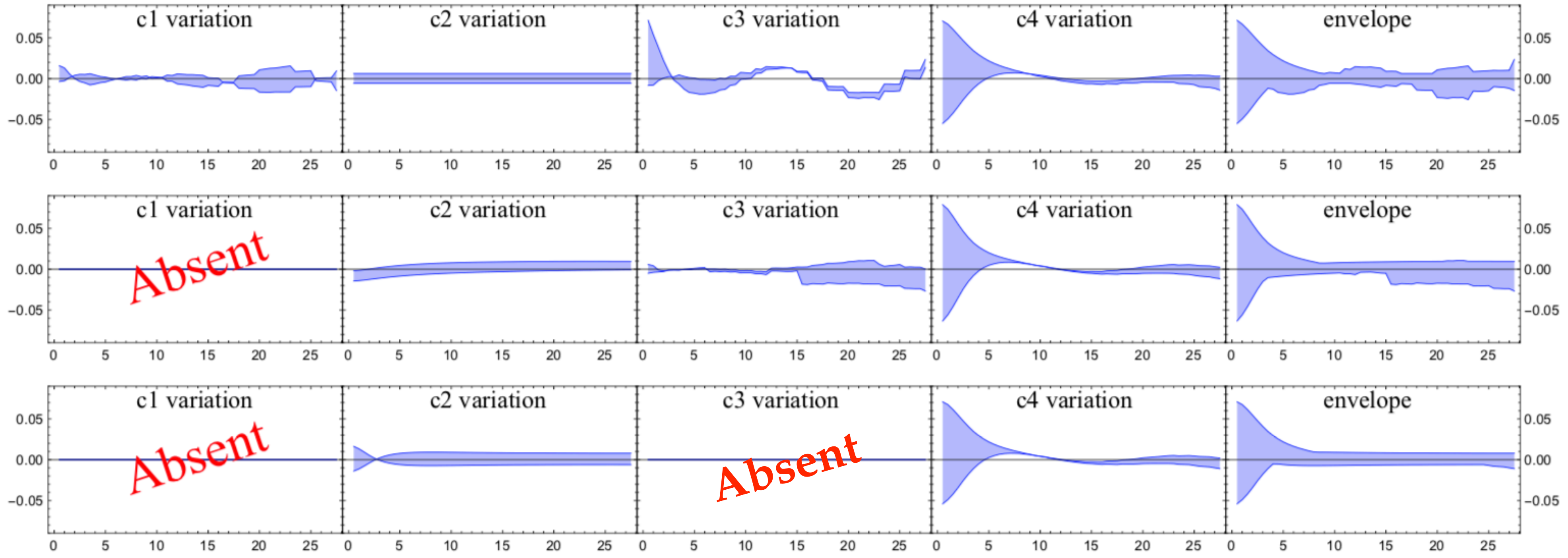


FIG. 10: Comparison of error bands obtained by the scale-variations for cross-sections given by (6.2) (top), (6.5) (middle), (6.12) (bottom). Here, the kinematics bin-integration, etc., is for the Z-boson production measure at ATLAS at 8 TeV [29].

A new error analysis: CDF

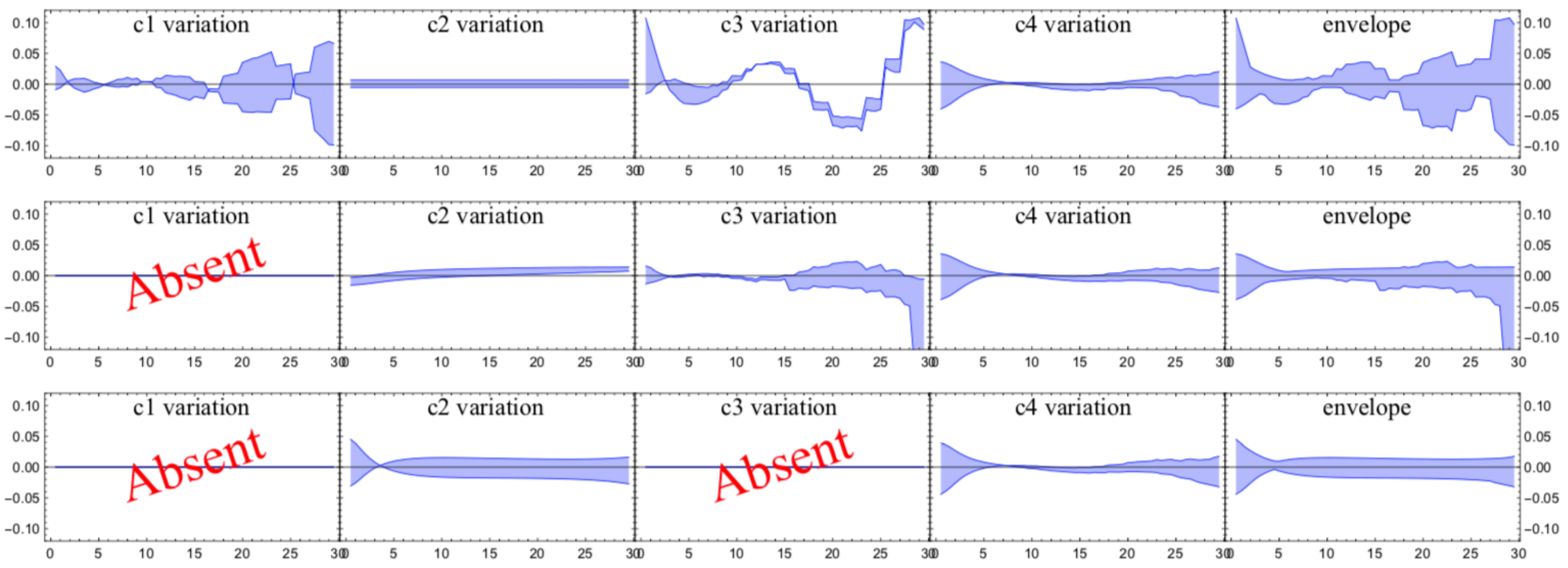


FIG. 9: Comparison of error bands obtained by the scale-variations for cross-sections given by (6.2) (top), (6.5) (middle), (6.12) (bottom). Here, the kinematics bin-integration, etc., is for the Z-boson production at CDF at run 2 [36].

A new error analysis: E288

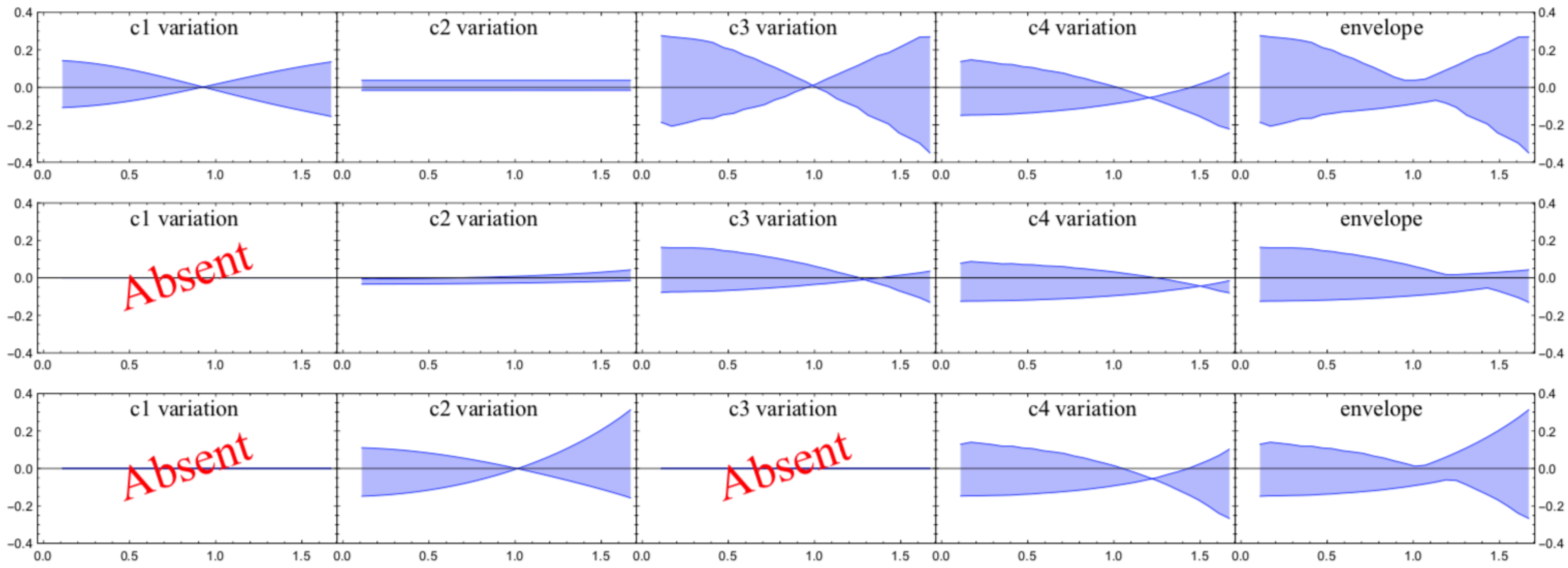


FIG. 11: Comparison of error bands obtained by the scale-variations for cross-sections given by (6.2) (top), (6.5) (middle), (6.12) (bottom). Here, the kinematics bin-integration, etc., is for Drell-Yan process measured at E288 experiment at $E_{\text{beam}} = 200\text{GeV}$ and $Q = 6 - 7\text{GeV}$ [38].

Modeling non-perturbative inputs for TMD extraction

The TMD modeling is not very constrained

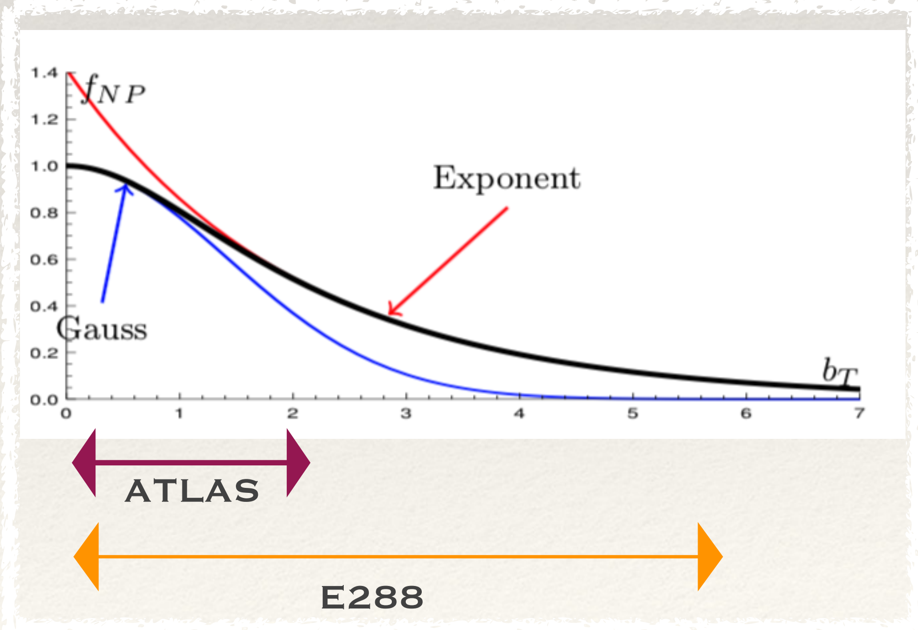
$$F_{q\leftarrow h}(x,\boldsymbol{b}) = \int_x^1 \frac{dz}{z} \sum_f C_{q\leftarrow f}\left(z,\boldsymbol{b};\mu,\zeta_\mu\right) f_{f\leftarrow h}\left(\frac{x}{z},\mu\right) f_{NP}\left(z,\boldsymbol{b}\right)$$

Perturbative Wilson coefficient matching

PDF

Non-perturbative non-asymptotic part

Asymptotic limit of TMD for large transverse momentum



+ a nonperturbative contribution to evolution factor

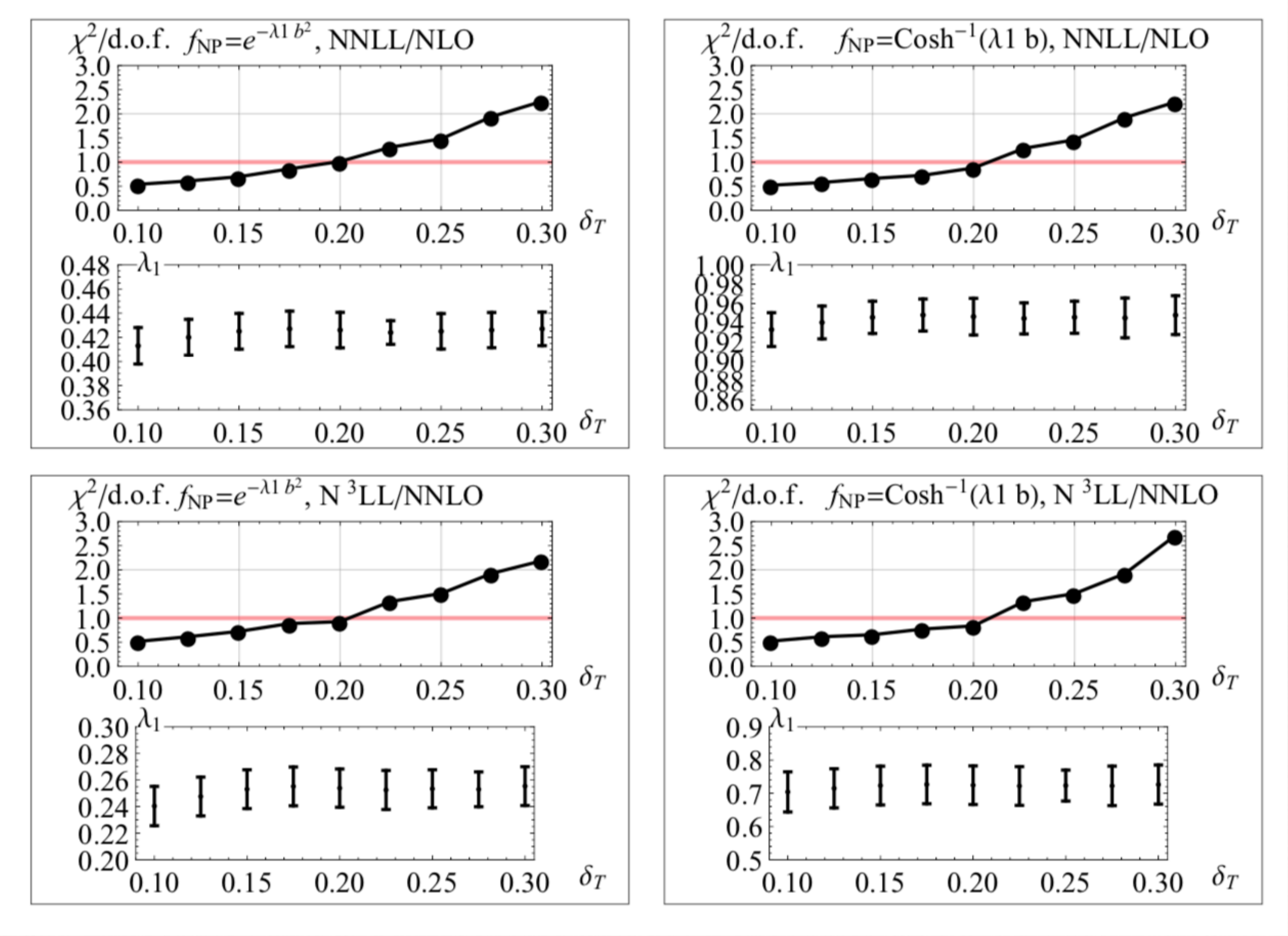
$$D^{NP} \sim g_K b b^*$$

Data and limit of TMD analysis

The limits of the TMD analysis are defined by the limit of factorization and are independent of the non-perturbative parametrization of TMDs or perturbative order

$$\delta_t = q_t/M$$

For high energy data we find $\delta_t \lesssim 0.2$



ATLAS experiment has an extraordinary precision: is this criterium sufficient?

Table from arXiv:1706:01473

Statistics

χ^2 minimization requires the analysis of experimental correlations

$$\text{Eperimental result (i)} \simeq m_i \pm \sigma_{i,\text{stat}} \pm \sigma_{i,\text{unc}} \pm \sum_{k=1}^{N} \sigma_{i,\text{corr}}^{(k)}$$

$$m_i = \text{central value of measurement i}$$

$$\sigma_{i,stat} = \text{uncorrelated statistical error}$$

$$\sigma_{i,unc} = \text{uncorrelated systematic error}$$

$$\sigma_{i,corr} = \text{correlated error}$$

All this information is provided by experiments and should be used to make the correlation matrix

Statistics

χ^2 minimization requires the analysis of experimental correlations

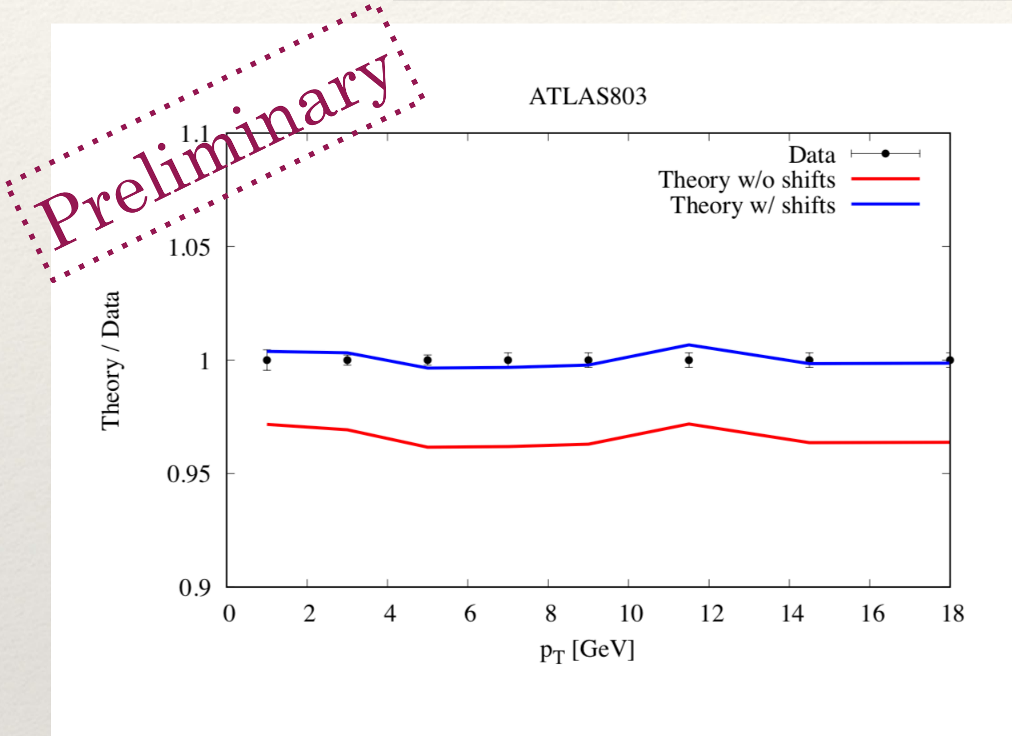
$$V_{ij} = (\sigma^2_{i,stat} + \sigma^2_{i,unc})\delta_{ij} + \sum_{i=1}^{N} \sigma^{(k)}_{i,corr}\sigma^{(k)}_{j,corr}$$

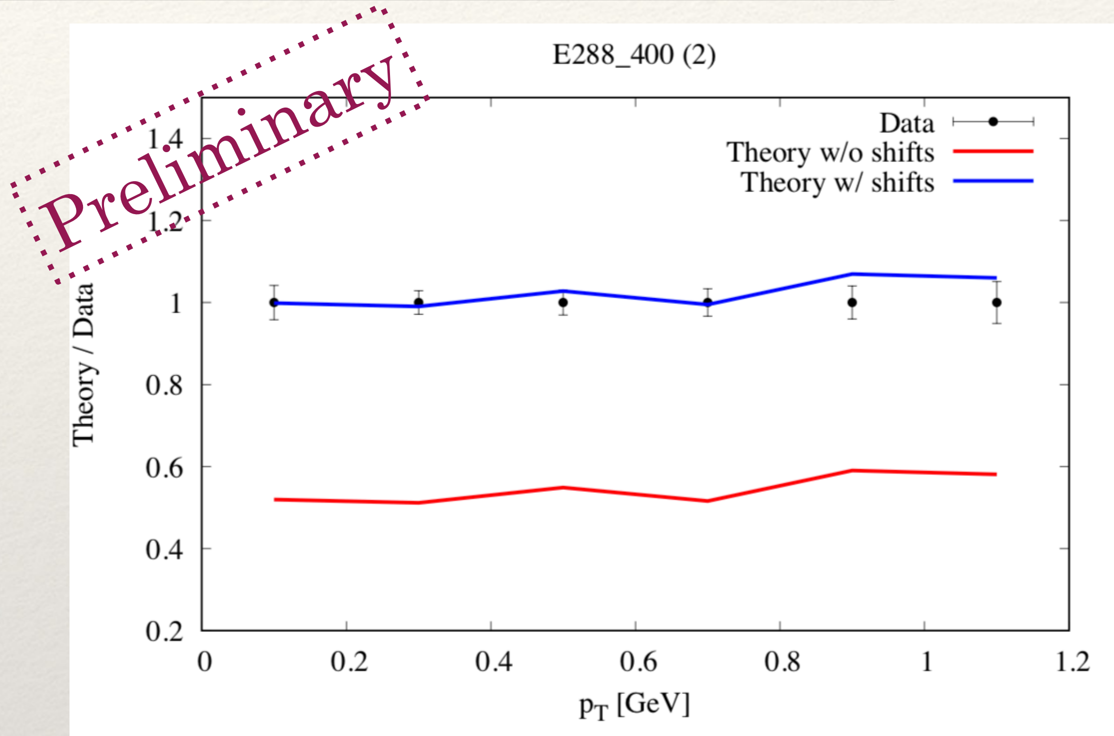
$$\chi^2 = \sum_{i,j=1}^{N} (m_i - t_i)V^{-1}_{ij}(m_j - t_j)$$

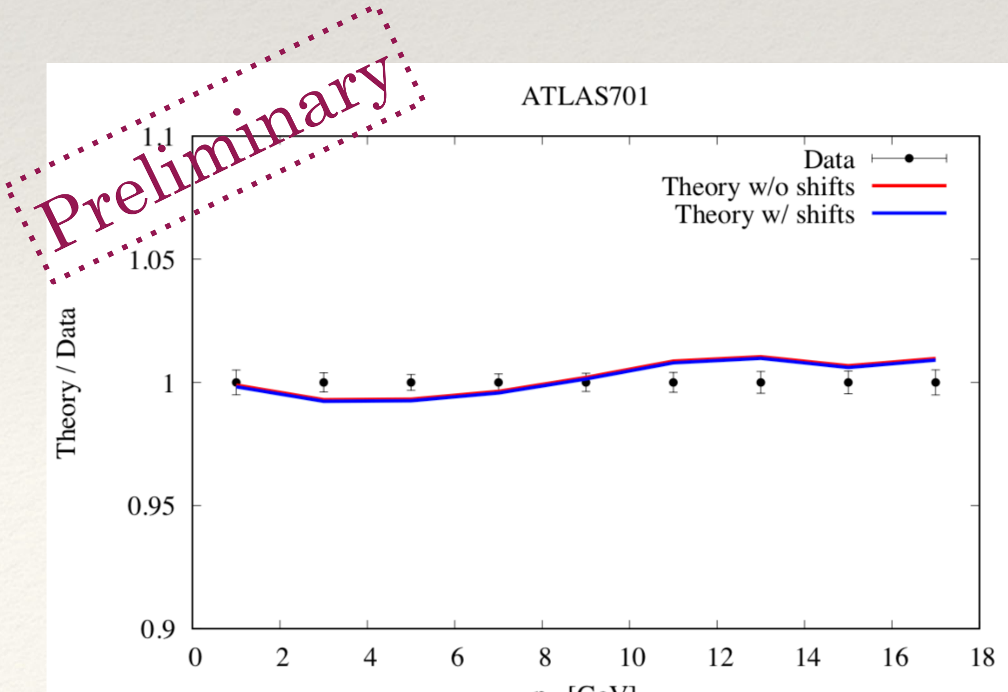
The effects of correlation in systematics can buy visualized calculating the systematic **SHIFTS**

We reformulate this in terms of
nuisance parameters,
uncorrelated uncertainties,
shifted theoretical predictions

Comparison shifted/unshifted results







The amount of shifts depends on sets..
but it is generally significative.

Resume fo main uncertainties

1. The factorization establishes some limits for its validity.
2. Some data sets (ATLAS) can be more sensitive to Y-terms and un-factorizable contributions due to their precision
3. Scale variations
4. PDFs: different sets and replicas can provide different results
5. Model building: still in a guess and try process ...

QED corrections .
See talk of Miguel

Jets!!
See talk of Lorenzo

Conclusions

- A NNLO analysis is necessary for fitting data and extracting unpolarized TMDs.
- LHC provides very precise data that should be included in fits (especially data off the Z-boson peak). ATLAS and CMS could do better at 13 TeV!!
- The data analysis should combine high energy and low energy data, because they are sensitive to different non-perturbative regions, both compatible with TMD factorization
- Scale choices and prescription should be critically analyzed (2D-evolution and zeta-prescription, optimal TMDs provide a better description of errors and separation of perturbative/non-perturbative effects)
- All this is/will be included in arTeMiDe (already new 1.3 release)

More to be done

Improve the statistical analysis, especially for LHC data

COMPASS data for DY and SIDIS

Extend the code to polarized processes and Jets

prepare for the advent of EIC

Back up

.. Up back

Some recent literature on TMDs in perturbation theory and some phenomenology

Perturbative Calculations

- Evolution to N3LO Y. Li, H.X. Zhu, arXiv:1604.01404 A. Vladimirov, arXiv:1610.05791
- Soft function at NNLO M.G. Echevarría, I.S., A. Vladimirov, arXiv:1511.05590.
- NNLO coefficients for TMDPDFs M.G. Echevarría, I.S., A. Vladimirov, arXiv1604.07869, T. Lübbert, J. Oredsson, M. Stahlhofen, arXiv:1602.01829, T. Gehrmann, T. Lübbert, Li Lin Yang arXiv: 1403.6451
- NNLO coefficients for TMD Fragmentation Functions M.G. Echevarría, I.S., A. Vladimirov, arXiv:1509.06392, arXiv:1604.07869

Phenomenology

- Global Fits (SIDIS+DY) A. Bacchetta et al. arxiv:1703.10157,
- DY and Z-boson fits (ResBos, D'Alesio et al. arXiv:1410.4522 up to NNLL)
- Implementation of standard CSS (DYres/DyqT, Cute)
- LHC data
- TMD extraction using higher order corrections (ARTEMIDE) arXiv:1706.01473 and 1803.11089

ζ-prescription

IT IS POSSIBLE TO MAKE A COMPLETE ANALYSIS OF UNPOLARIZED TMD IN DRELL-YAN AND SIDIS USING **NNLO** RESULTS

Also new studies with JETS! D. Gutierrez-Reyes, I.S., W. Waalewijn, L. Zoppi 1807.07573, PRL,...

The study of polarized TMDs at the same precision is just started (see Daniel Gutierrez work):

D. Gutierrez-Reyes, I.S., A. Vladimirov, arXiv:1702.06558 and 1805.07243