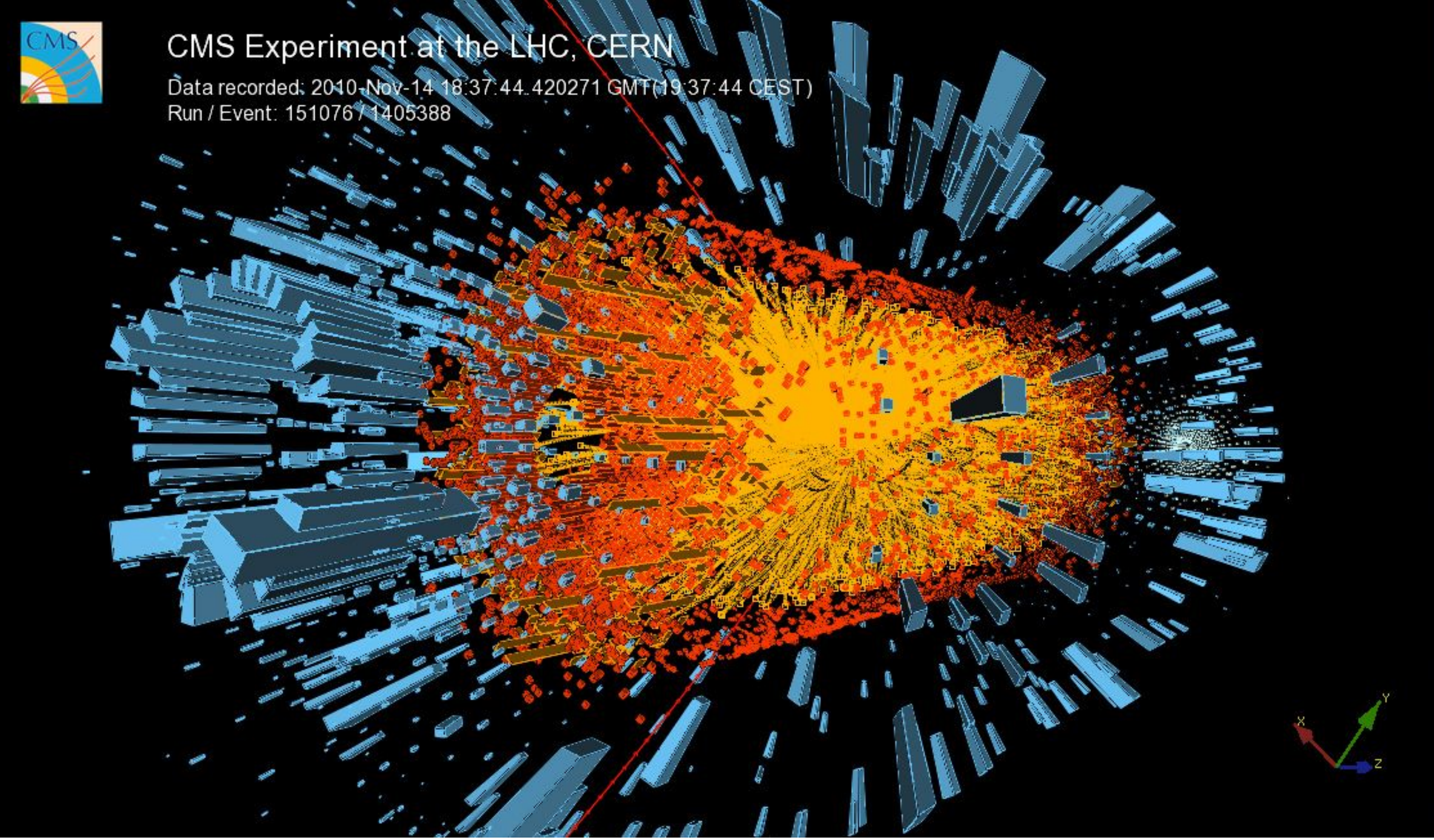
Sudakov suppression of jets in heavy-ion collisions

Konrad Tywoniuk (UiB)

Work done in collaboration with Y. Mehtar-Tani (BNL)

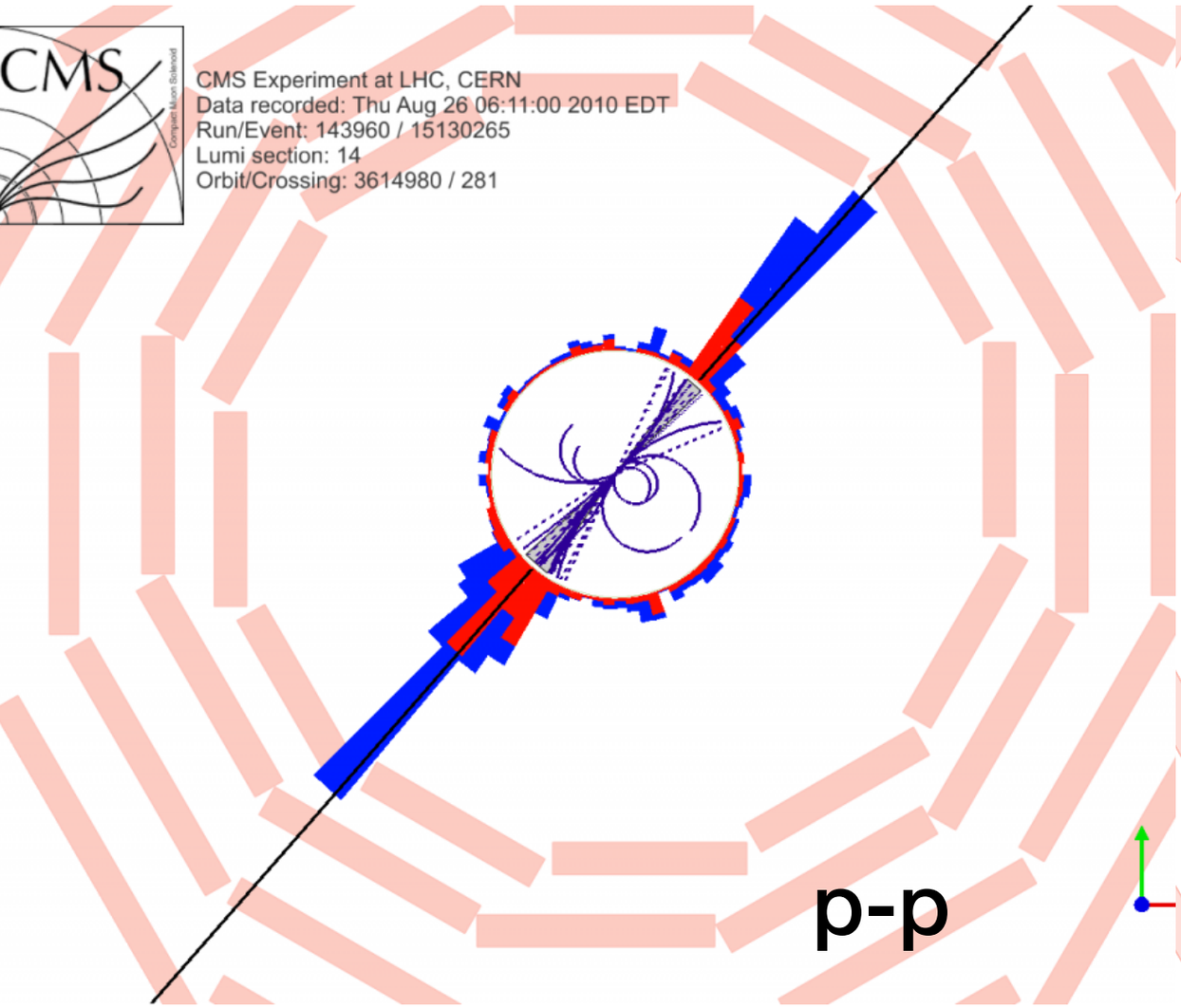
Workshop on Resummation, Evolution and Factorization (REF 2018)
19-23 November 2018, Kraków

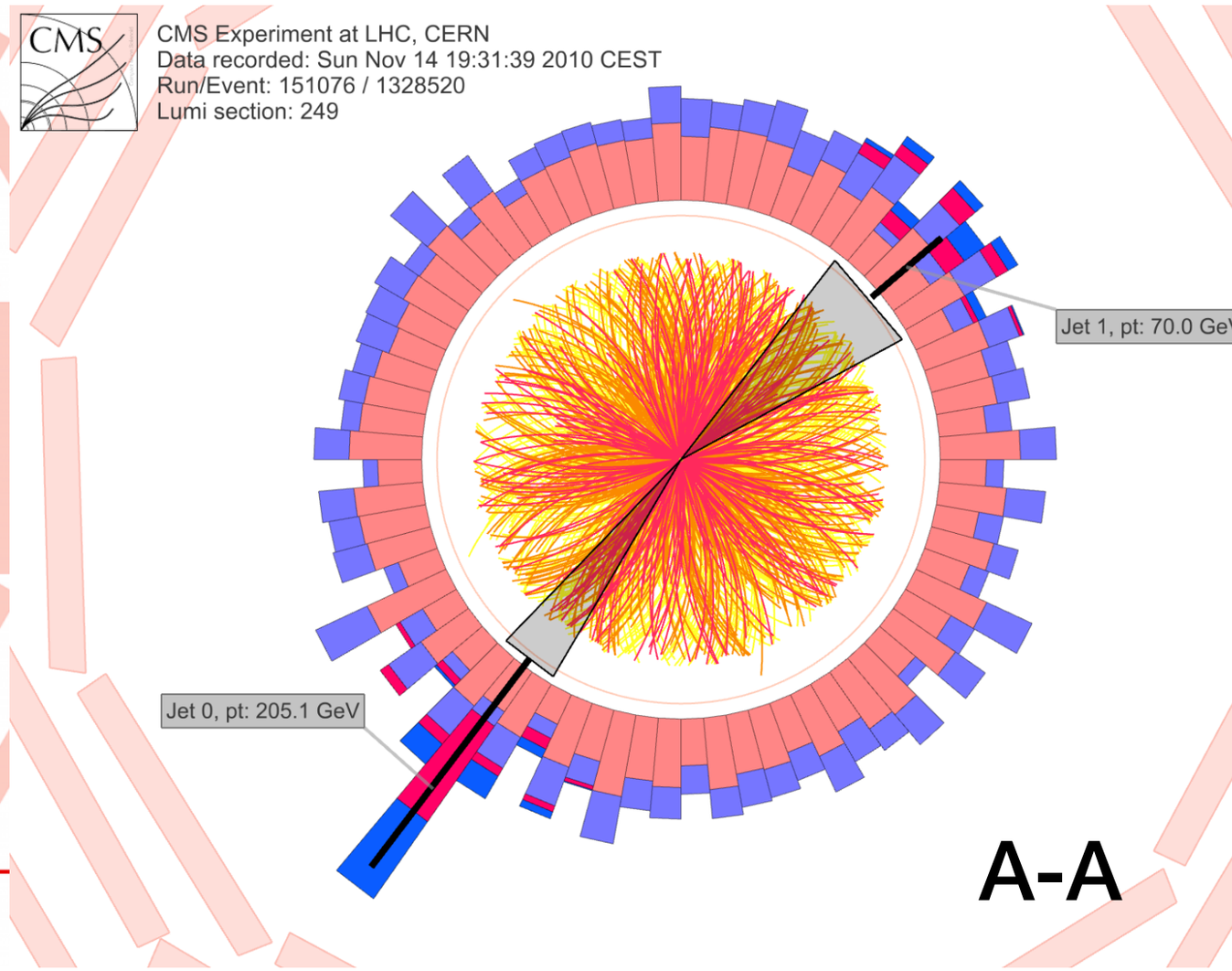
UNIVERSITY OF BERGEN



Heavy-ion collisions

Extreme nuclear environment





Jet Quenching

Energy imbalance & acoplanarity

JET QUENCHING

JET QUENCHING

- copious emergence of jet structures in heavy-ion collisions
  → with reduced energy & substructure modifications

JET QUENCHING

- copious emergence of jet structures in heavy-ion collisions → with reduced energy & substructure modifications
- models of jet evolution in medium typically adopt a multi-stage framework (factorization):

ME/shower (in) → medium modifications → shower (out)

JET QUENCHING

- copious emergence of jet structures in heavy-ion collisions → with reduced energy & substructure modifications
- models of jet evolution in medium typically adopt a multi-stage framework (factorization):

  ME/shower (in) → *medium modifications* → *shower (out)*
- hence, modeling of jets does not only depend on the details of stage 2, but also how we separate the stages

JET QUENCHING

- copious emergence of jet structures in heavy-ion collisions → with reduced energy & substructure modifications
- models of jet evolution in medium typically adopt a multi-stage framework (factorization):

  ME/shower (in) → *medium modifications* → *shower (out)*
- hence, modeling of jets does not only depend on the details of stage 2, but also how we separate the stages
- theoretical guidance at high-p_T
  - what drives quenching & substructure modifications?

Two showers

Two showers

- Medium cascade
  - at high-p_T, radiative processes dominate (medium-induced emissions)
  - energy is transported from hard particles via multiple branchings to soft particles (thermalization)
  - time-ordering

Two showers

- Medium cascade
  - at high-p_T, radiative processes dominate (medium-induced emissions)
  - energy is transported from hard particles via multiple branchings to soft particles (thermalization)
  - time-ordering
- Vacuum-like jet shower
  - color coherence angular-ordered cascade (collimation)

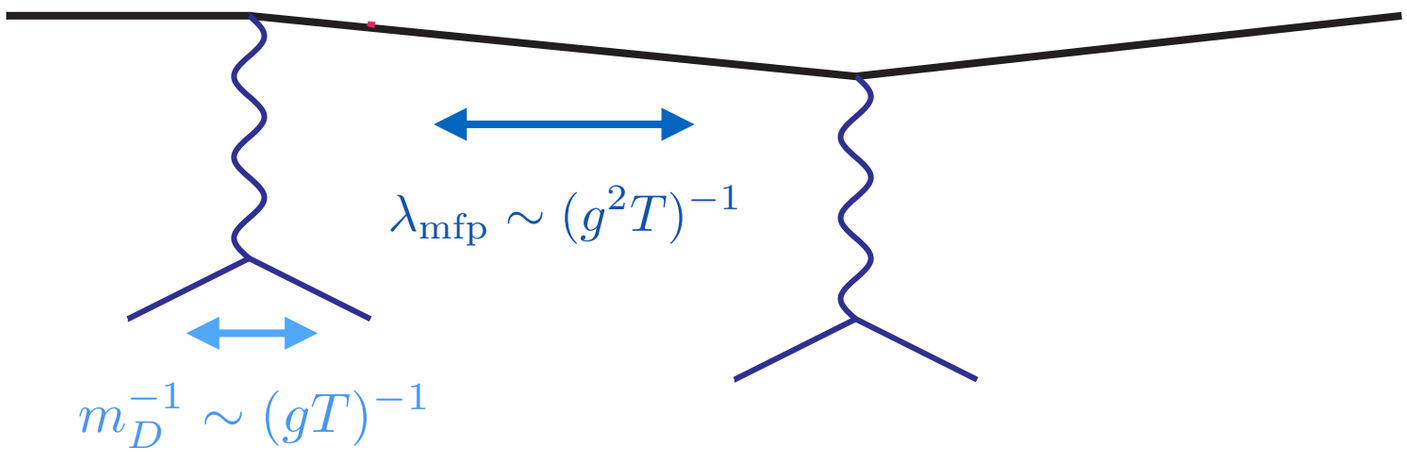
Two showers

- Medium cascade
  - at high-p_T, radiative processes dominate (medium-induced emissions)
  - energy is transported from hard particles via multiple branchings to soft particles (thermalization)
  - time-ordering
- Vacuum-like jet shower
  - color coherence angular-ordered cascade (collimation)
- Is there a way to combine the two?

OUTLINE

- radiative processes in the medium
- phase space analysis
- first application: single-inclusive jet spectrum
- generating functional approach

MEDIUM INTERACTIONS @ HIGH-ENERGY



- high-temperature: separation of scales
- elastic interactions with transverse momentum exchange
  - dressed propagators: 2D non-relativistic QM
- light-cone perturbation theory in background

two-point correlator

$$\langle A^a_{\rm med}(t,\boldsymbol{x}) A^b_{\rm med}(t',\boldsymbol{y}) = \delta^{ab} n(t) \delta(t-t') \gamma(\boldsymbol{x}-\boldsymbol{y})$$

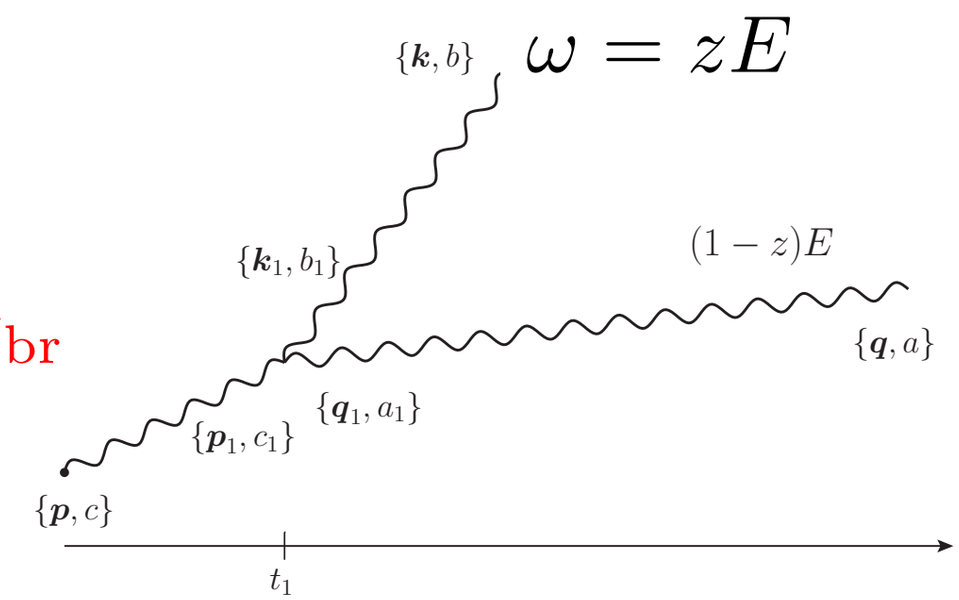
BDMPS-Z SPECTRUM

momentum broadening $\langle k_\perp^2 \rangle \sim \hat{q} t$

modified kinematics

lack of collinear singularity!

$$t_{\rm f} = \frac{\omega}{k_\perp^2} \sim \sqrt{\frac{\omega}{\hat{q}}} \equiv t_{\rm br}$$



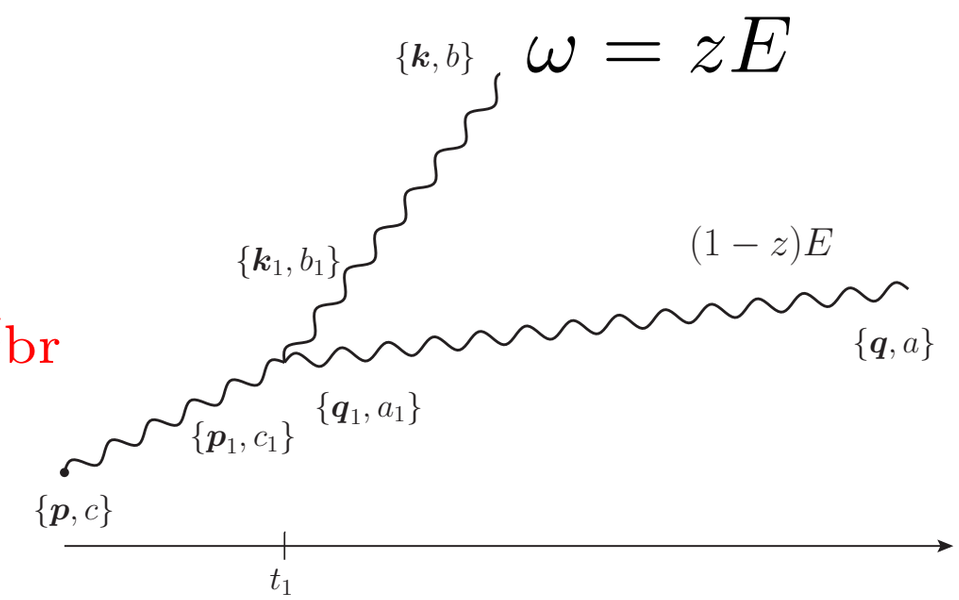
BDMPS-Z SPECTRUM

momentum broadening $\langle k_\perp^2 \rangle \sim \hat{q} t$

modified kinematics

lack of collinear singularity!

$$t_{\rm f} = \frac{\omega}{k_\perp^2} \sim \sqrt{\frac{\omega}{\hat{q}}} \equiv t_{\rm br}$$



Energy spectrum

$$\frac{{\rm d}I}{{\rm d}z} = \frac{\alpha_s}{\pi} P(z) \ln \left| \cos \frac{1+i}{2} \sqrt{\frac{\hat{q}_{\rm eff}}{z(1-z)p_{\rm T}}} L \right|$$

$z \ll 1$

$$\bar{\alpha} \sqrt{\frac{\hat{q} L^2}{\omega^3}} \qquad \omega \ll \hat{q} L^2$$

$$\frac{\bar{\alpha}}{6\omega} \left(\frac{\hat{q} L^2}{2\omega} \right)^2 \qquad \omega \gg \hat{q} L^2$$

Landau-Pomeranchuk-Migdal (LPM) interference effect

Two regimes

Characteristic angle of radiation $\theta_{\rm br}(\omega) \sim k_{\rm br}/\omega \sim (\hat{q}/\omega^3)^{1/4}$

Multiplicity of gluons $N(\omega) = \int_\omega^\infty {\rm d}\omega' \, \frac{{\rm d}I}{{\rm d}\omega} = 2\bar{\alpha}\sqrt{\frac{\hat{q}L^2}{\omega}}$

Two regimes

Characteristic angle of radiation $\theta_{\rm br}(\omega) \sim k_{\rm br}/\omega \sim (\hat{q}/\omega^3)^{1/4}$

Multiplicity of gluons $$N(\omega) = \int_\omega^\infty {\rm d}\omega' \, \frac{{\rm d}I}{{\rm d}\omega} = 2\bar{\alpha}\sqrt{\frac{\hat{q}L^2}{\omega}}$$

rare, small-angle emissions $\omega_c = \hat{q}L^2$

$$\theta_{\rm br}(\omega_c) \sim \sqrt{\frac{1}{\hat{q}L^3}} \equiv \theta_c$$

copious, large-angle emissions $\omega_s = \bar{\alpha}^2 \hat{q}L^2$

$$\theta_{\rm br}(\omega_s) \sim \frac{1}{\bar{\alpha}^{3/2}}\theta_c$$

Two regimes

Characteristic angle of radiation $\theta_{\rm br}(\omega) \sim k_{\rm br}/\omega \sim (\hat{q}/\omega^3)^{1/4}$

Multiplicity of gluons $$N(\omega) = \int_\omega^\infty {\rm d}\omega' \, \frac{{\rm d}I}{{\rm d}\omega} = 2\bar{\alpha}\sqrt{\frac{\hat{q}L^2}{\omega}}$$

rare, small-angle emissions $\omega_c = \hat{q}L^2$

$$\theta_{\rm br}(\omega_c) \sim \sqrt{\frac{1}{\hat{q}L^3}} \equiv \theta_c$$

copious, large-angle emissions $\omega_s = \bar{\alpha}^2 \hat{q}L^2$

$$\theta_{\rm br}(\omega_s) \sim \frac{1}{\bar{\alpha}^{3/2}}\theta_c$$

Average energy of one gluon: $$\Delta E = \int_0^\infty {\rm d}\omega \, \omega \frac{{\rm d}I}{{\rm d}\omega} = 2\bar{\alpha}\hat{q}L^2$$

significant for a large medium (dominated by ω_c)

MULTIPLE SOFT EMISSIONS

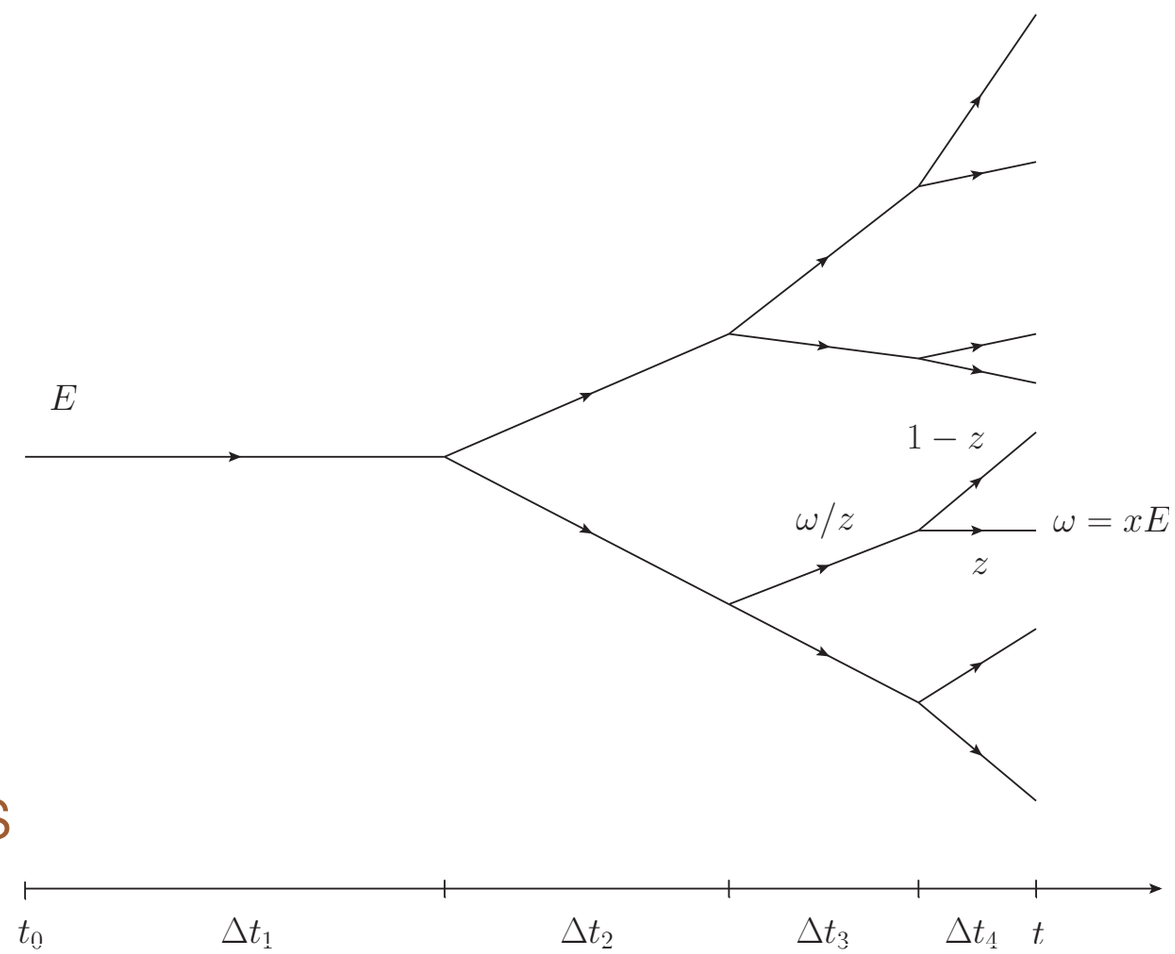
Blaizot, Iancu, Mehtar-Tani 1301.6102
Blaizot, Dominguez, Iancu, Mehtar-Tani 1311.5823

$$\omega_{\rm BH} \ll \omega \ll \bar{\alpha}^2 \omega_c \qquad t_{\rm split} = \frac{t_{\rm br}}{\bar{\alpha}} \ll L$$

short splitting time → many splittings inside the medium!

$$\mathcal{P}_2 \sim \omega \frac{{\rm d}I}{{\rm d}\omega} \simeq \frac{\bar{\alpha}}{t_{\rm br}} L$$

$$\mathcal{P}_3^{(\rm indep)} \sim \left(\frac{\bar{\alpha}}{t_{\rm br}} L \right)^2$$



any overlap (interference) reduces this probability by a small factor $t_{\rm br}/L$

TURBULENT CASCADE

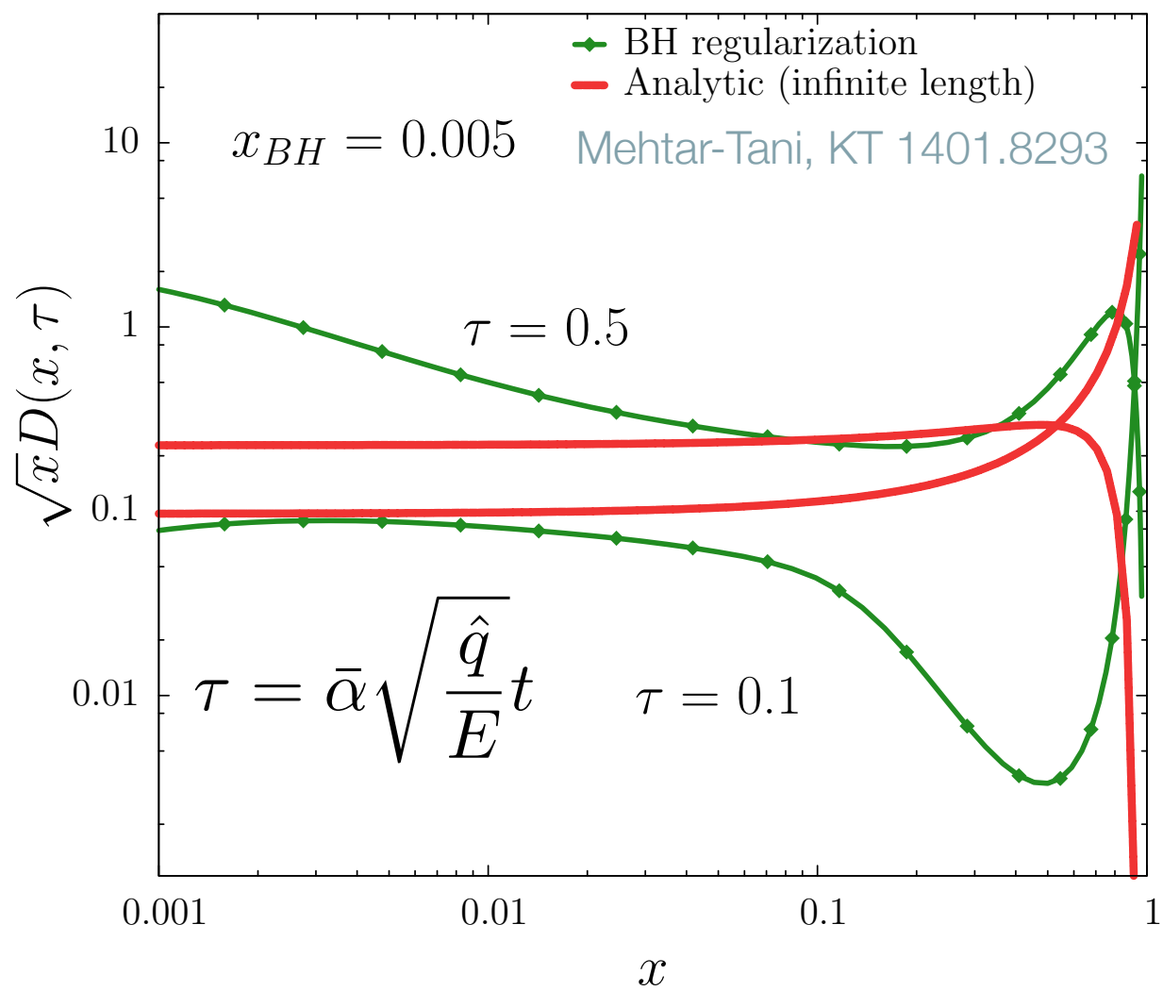
Blaizot, Iancu, Mehtar-Tani 1301.6102
Blaizot, Dominguez, Iancu, Mehtar-Tani 1311.5823

$$\frac{\partial D(x,\tau)}{\partial \tau} = \int \mathrm{d}z\, \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z},\tau\right) - \frac{z}{\sqrt{x}} D(x,\tau) \right]$$

$$\mathcal{K}(z) = \frac{\mathrm{d}I}{\mathrm{d}z\,\mathrm{d}\tau}$$

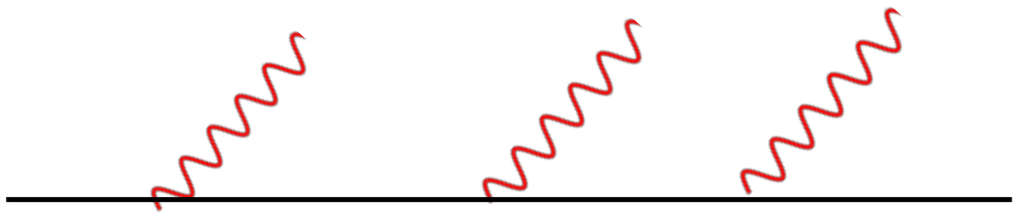
- energy flow to the IR (thermal scale)
- quasi-democratic branching
- generalization to transverse momentum dependence
- overlapping formation times: radiative corrections to q̂

Mehtar-Tani, Schlichting 1807.06181
Kutak, Płaczek, Straka 1811.06390



QUENCHING WEIGHTS

Baier, Dokshitzer, Mueller, Schiff JHEP 0109 (2001) 033
Salgado, Wiedemann PRD 68 (2003) 014008

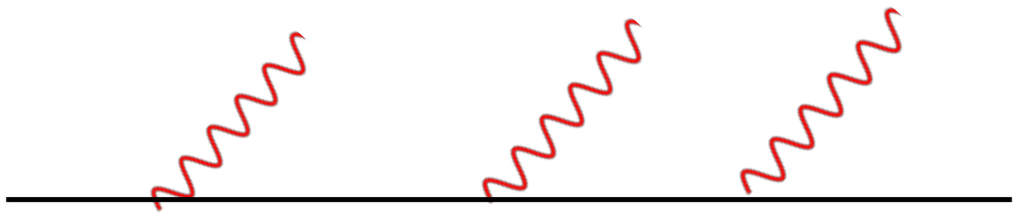


compute energy radiated off leading particle (z≪1)

$$\mathcal{P}(\epsilon, L) = \delta(\epsilon) + \int_0^L \mathrm{d}t \int \mathrm{d}\omega \left[\frac{\mathrm{d}I}{\mathrm{d}\omega \mathrm{d}t} - \delta(\omega) \int \mathrm{d}\omega' \, \frac{\mathrm{d}I}{\mathrm{d}\omega' \mathrm{d}t} \right] \mathcal{P}(\epsilon - \omega, t)$$

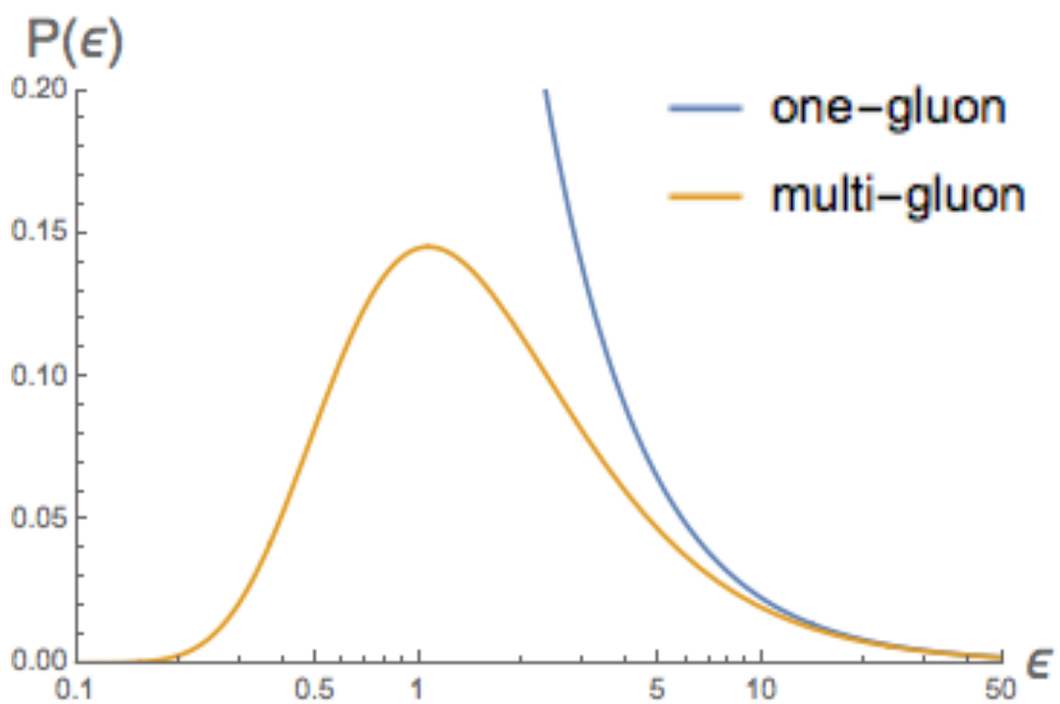
QUENCHING WEIGHTS

Baier, Dokshitzer, Mueller, Schiff JHEP 0109 (2001) 033
Salgado, Wiedemann PRD 68 (2003) 014008



compute energy radiated off leading particle (z≪1)

$$\mathcal{P}(\epsilon, L) = \delta(\epsilon) + \int_0^L \mathrm{d}t \int \mathrm{d}\omega \left[\frac{\mathrm{d}I}{\mathrm{d}\omega \mathrm{d}t} - \delta(\omega) \int \mathrm{d}\omega' \frac{\mathrm{d}I}{\mathrm{d}\omega' \mathrm{d}t} \right] \mathcal{P}(\epsilon - \omega, t)$$



Using the soft BDMPS spectrum:

$$\mathcal{P}(\epsilon) = \sqrt{\frac{\omega_s}{\epsilon^3}} \mathrm{e}^{-\frac{\pi \omega_s}{\epsilon}}$$

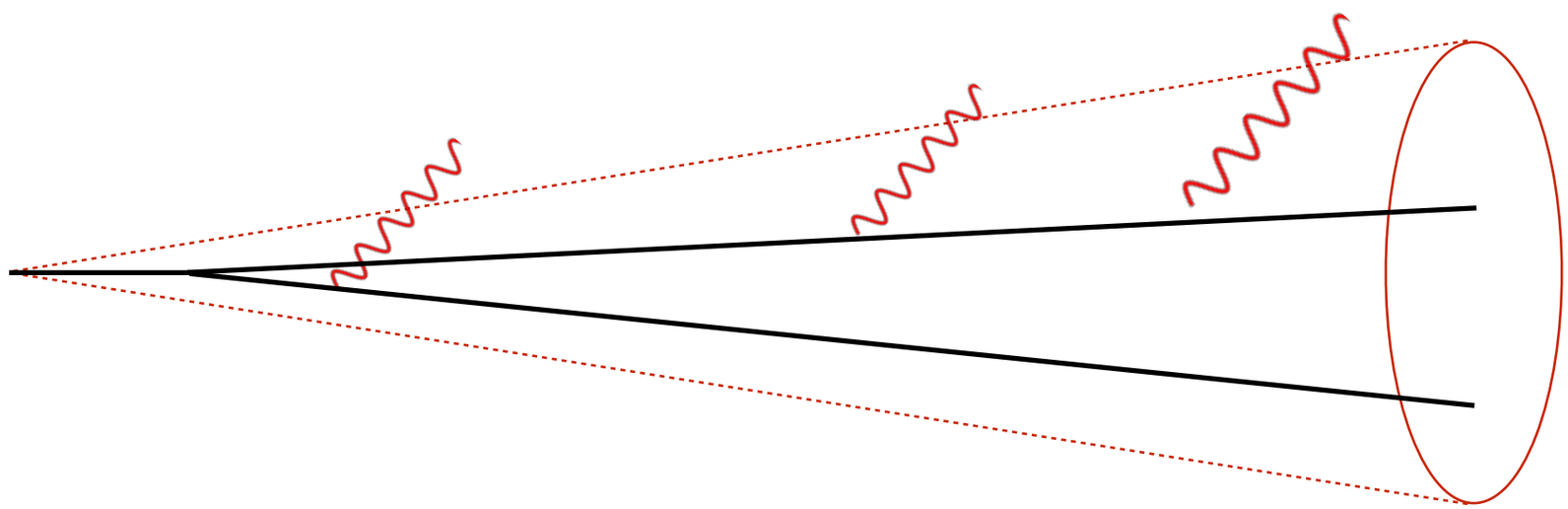
$$\omega_s \sim \bar{\alpha}^2 \hat{q} L^2$$

e-loss dominated by max of distribution!

Two-prong quenching weights

Y. Mehtar-Tani, KT arXiv:1706.06047 [hep-ph]

So far, we have only dealt with a single hard parton propagating through the plasma (shower initiator). What about two?



$$\mathcal{P}_{qg} = \mathcal{P}_q \otimes \mathcal{P}_{\rm sing}$$

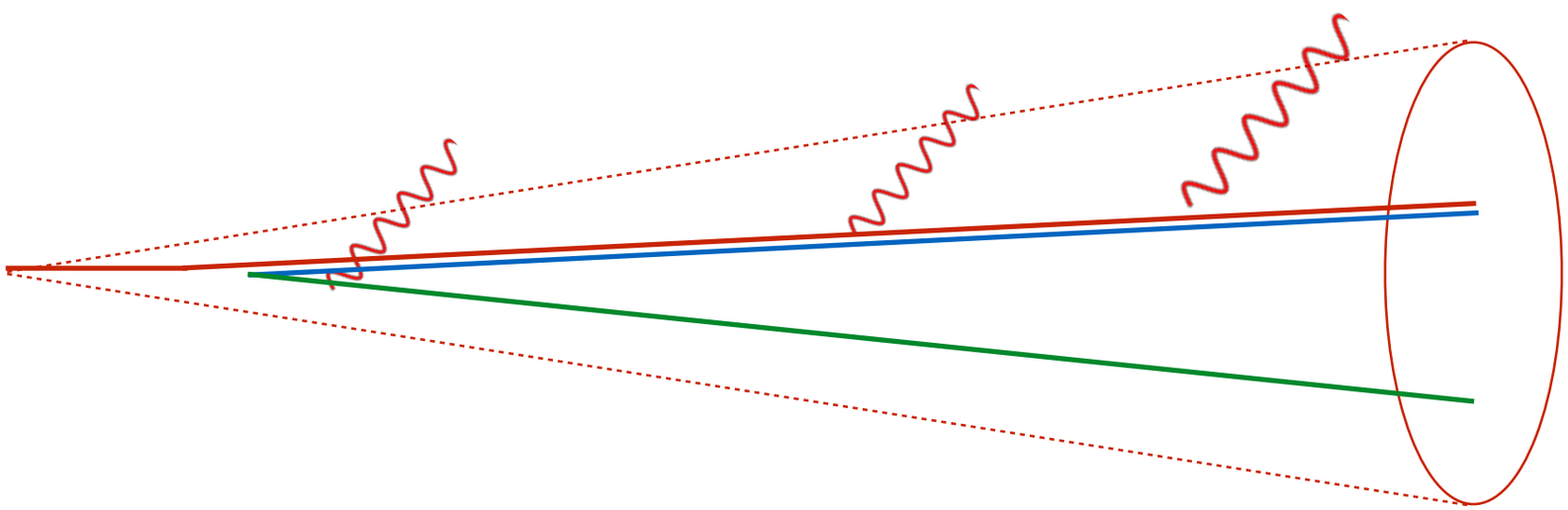
Interferences are important at $t<t_d$!

Generalizes to all higher-order splittings at large-N_c.

Two-prong quenching weights

Y. Mehtar-Tani, KT arXiv:1706.06047 [hep-ph]

So far, we have only dealt with a single hard parton propagating through the plasma (shower initiator). What about two?



$$\mathcal{P}_{qg} = \mathcal{P}_q \otimes \mathcal{P}_{\rm sing}$$

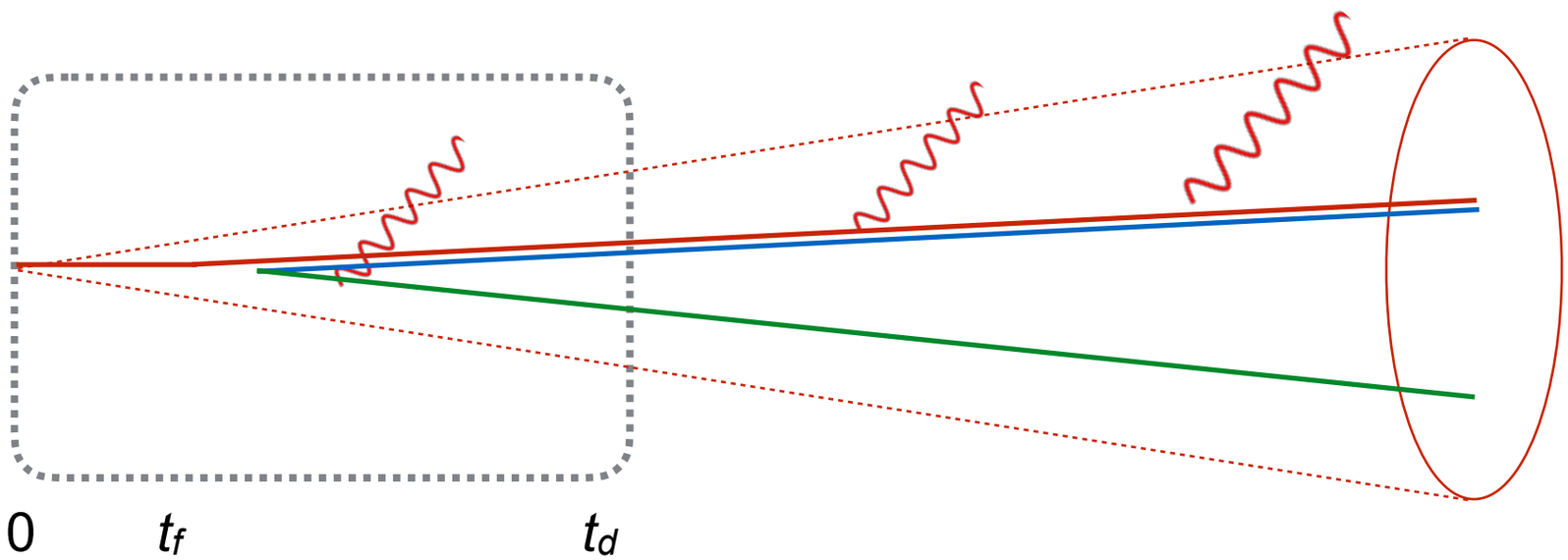
Interferences are important at $t<t_d$!

Generalizes to all higher-order splittings at large-N_c.

Two-prong quenching weights

Y. Mehtar-Tani, KT arXiv:1706.06047 [hep-ph]

So far, we have only dealt with a single hard parton propagating through the plasma (shower initiator). What about two?



$$t_{\rm d} \sim (\hat{q}\theta_{12})^{-1/3}$$

$$t_{\rm d} = L$$

$$\theta_c \sim (\hat{q}L^3)^{-1/2}$$

$$\mathcal{P}_{qg} = \mathcal{P}_q \otimes \mathcal{P}_{\rm sing}$$

Interferences are important at $t<t_d$!

Generalizes to all higher-order splittings at large-N_c.

Quenching weights evolution equations

Regularized rates in Laplace space: $\gamma(\nu, t) = \int_0^\infty \mathrm{d}\omega \left(\mathrm{e}^{-\nu\omega} - 1\right) \frac{\mathrm{d}I}{\mathrm{d}\omega \mathrm{d}t}$

Rate equation for one charge

$$\frac{\partial}{\partial t}\mathcal{P}(\nu, t) = \gamma_{\mathrm{dir}}(\nu, t)\mathcal{P}(\nu, t)$$

Rate equation for two charges

$$\frac{\partial}{\partial t}\mathcal{P}_{\mathrm{sing}}(\nu, t) = \gamma_{\mathrm{dir}}(\nu, t)\mathcal{P}_{\mathrm{sing}}(\nu, t) + \gamma_{\mathrm{int}}(\nu, t)\mathcal{S}_2(t)$$

Quenching weights evolution equations

Regularized rates in Laplace space: $\gamma(\nu, t) = \int_0^\infty \mathrm{d}\omega \left(\mathrm{e}^{-\nu\omega} - 1\right) \frac{\mathrm{d}I}{\mathrm{d}\omega \mathrm{d}t}$

Rate equation for one charge

$$\frac{\partial}{\partial t}\mathcal{P}(\nu, t) = \gamma_{\mathrm{dir}}(\nu, t)\mathcal{P}(\nu, t)$$

Rate equation for two charges

$$\frac{\partial}{\partial t}\mathcal{P}_{\mathrm{sing}}(\nu, t) = \gamma_{\mathrm{dir}}(\nu, t)\mathcal{P}_{\mathrm{sing}}(\nu, t) + \gamma_{\mathrm{int}}(\nu, t)\mathcal{S}_2(t)$$

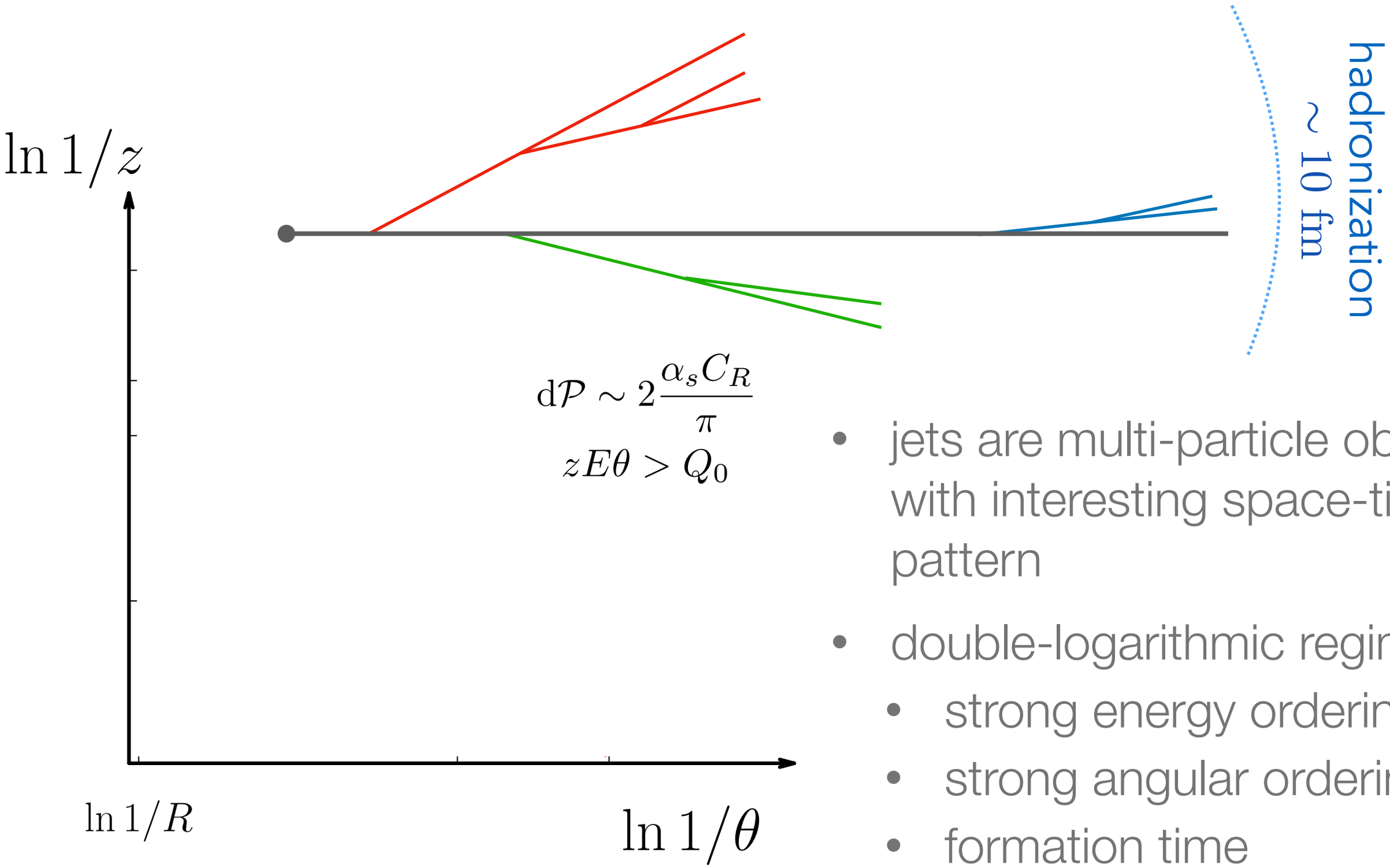
$$\mathcal{S}_2(t) = \exp\left[-\frac{1}{4}\int_0^t \mathrm{d}s\, \hat{q}(\boldsymbol{x}_{12}, t)\boldsymbol{x}_{12}^2(s)\right]$$

"decoherence parameter"

color randomization of a $q\bar{q}$ pair

JET SPACETIME EVOLUTION

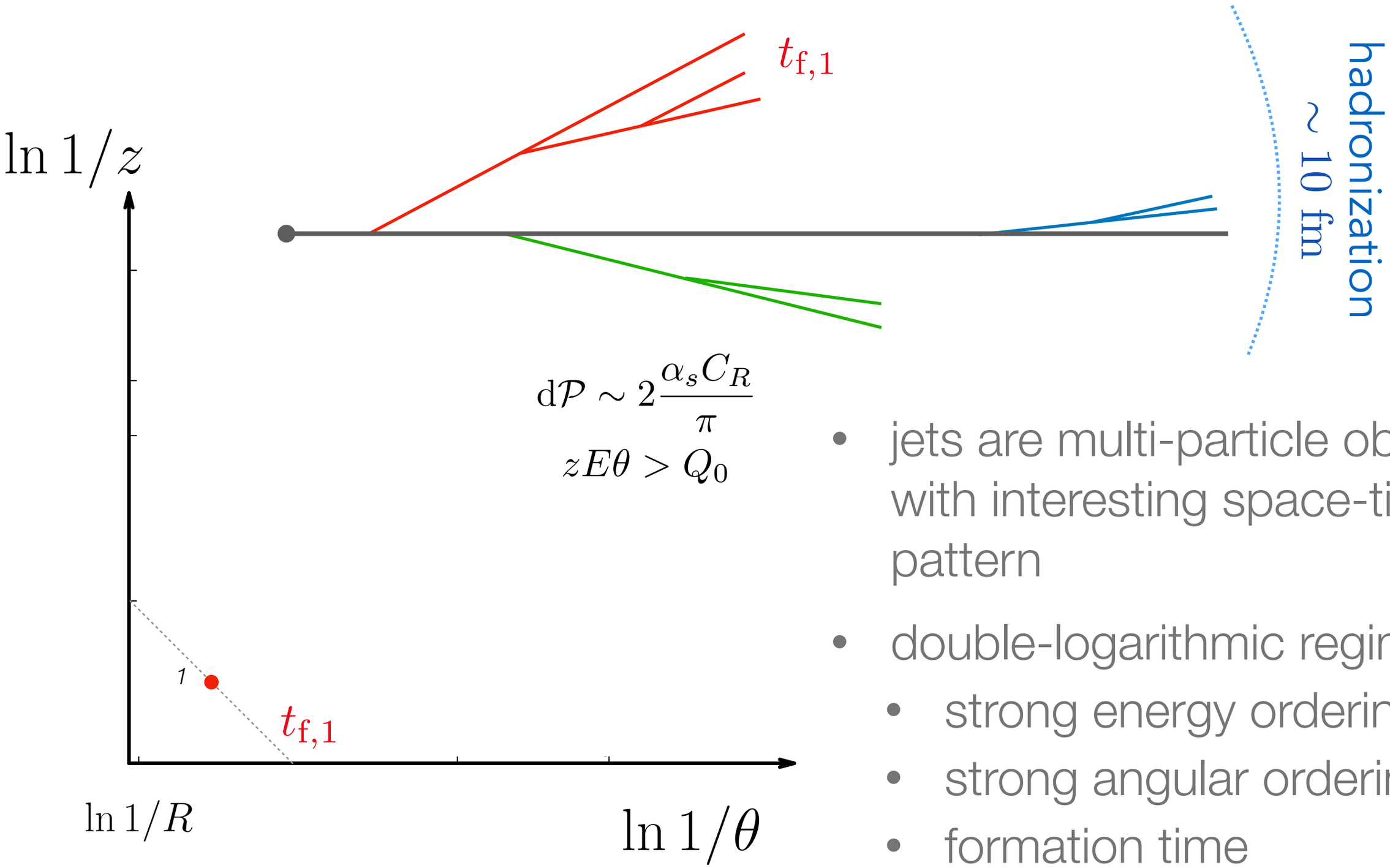
Bassetto, Ciafaloni, Marchesini (1983);
Dokshitzer, Troyan, Khoze, Mueller (1991)



- jets are multi-particle objects with interesting space-time pattern
- double-logarithmic regime
  - strong energy ordering
  - strong angular ordering
  - formation time

JET SPACETIME EVOLUTION

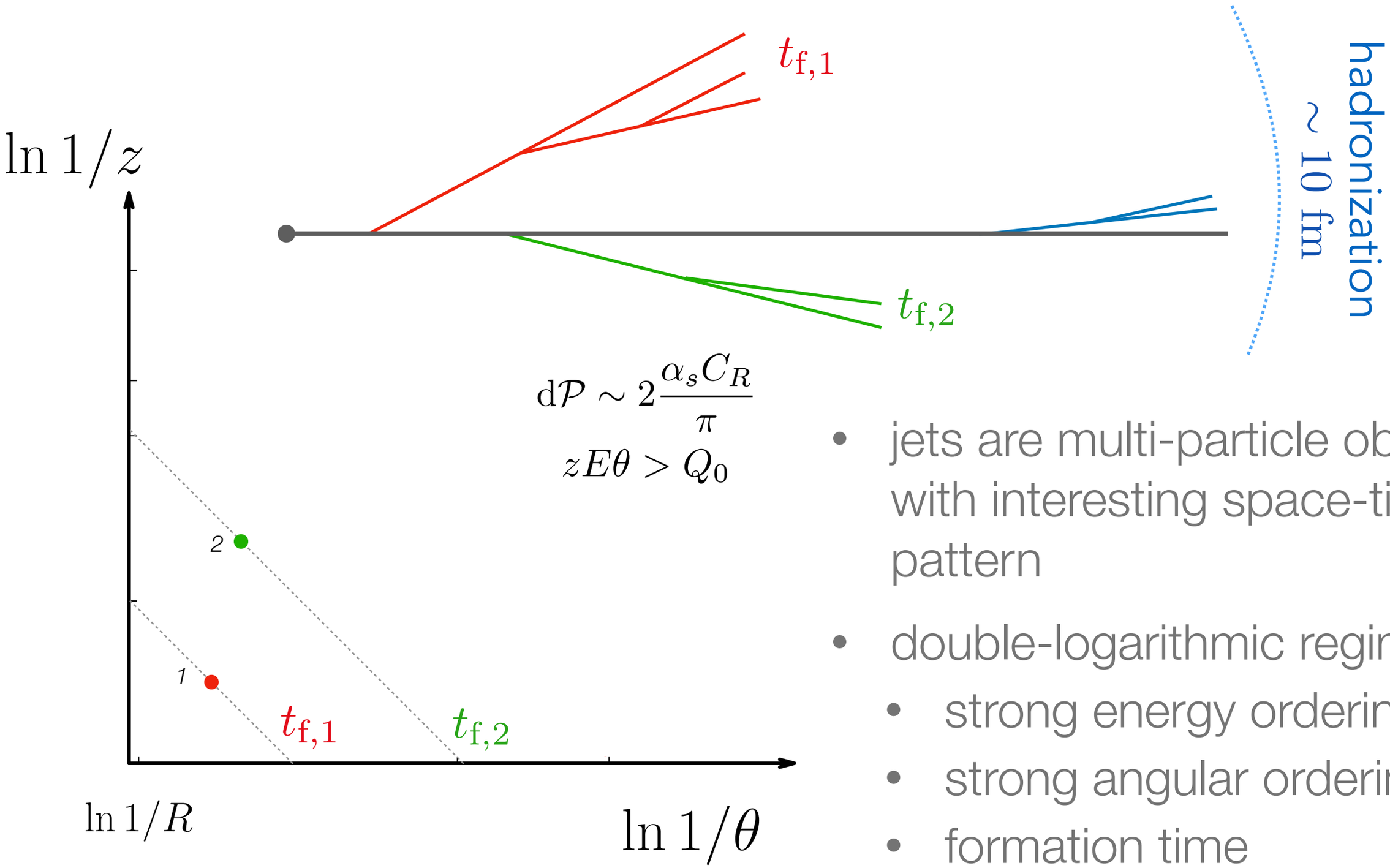
Bassetto, Ciafaloni, Marchesini (1983);
Dokshitzer, Troyan, Khoze, Mueller (1991)



- jets are multi-particle objects with interesting space-time pattern
- double-logarithmic regime
  - strong energy ordering
  - strong angular ordering
  - formation time

JET SPACETIME EVOLUTION

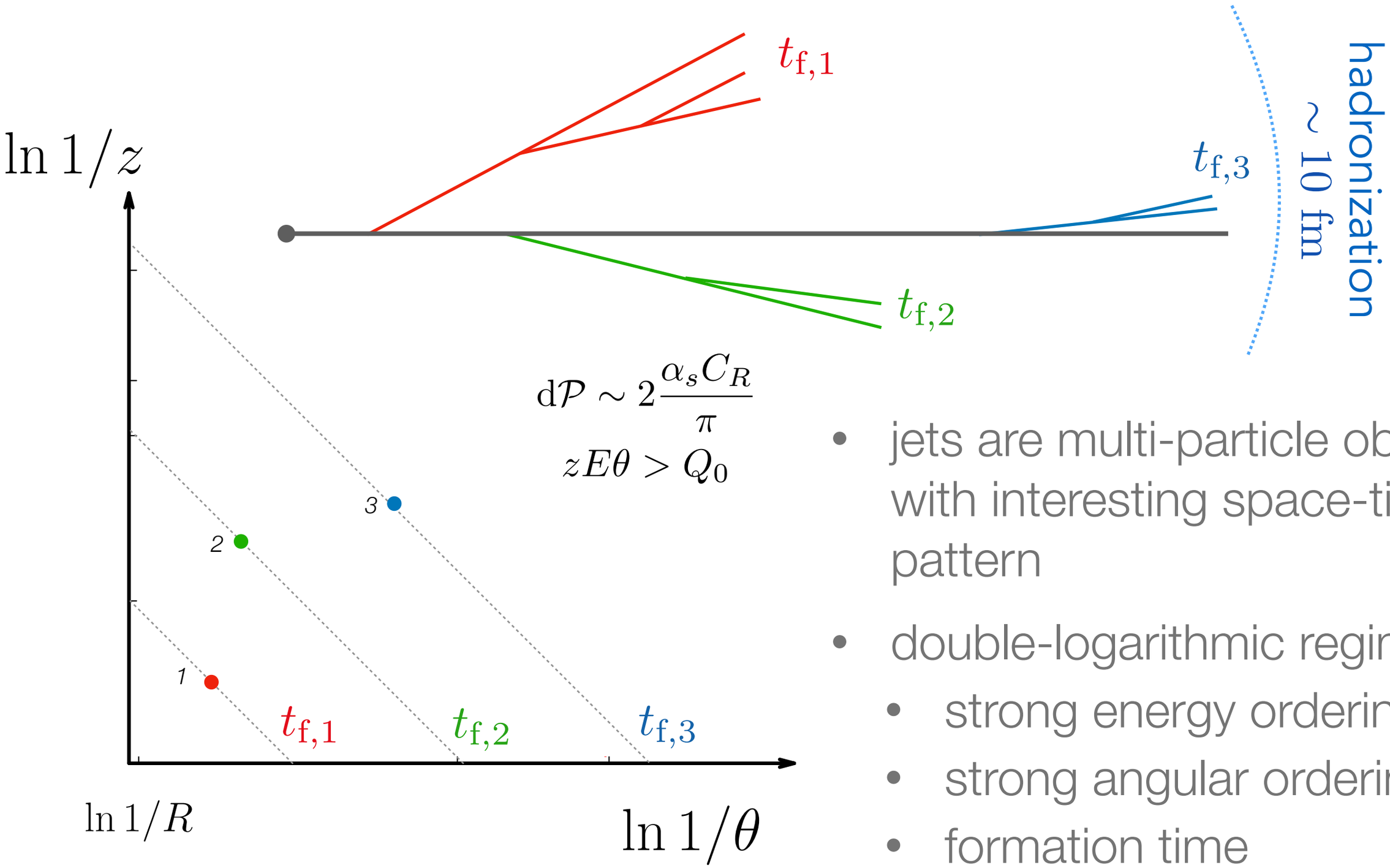
Bassetto, Ciafaloni, Marchesini (1983);
Dokshitzer, Troyan, Khoze, Mueller (1991)



- jets are multi-particle objects with interesting space-time pattern
- double-logarithmic regime
  - strong energy ordering
  - strong angular ordering
  - formation time

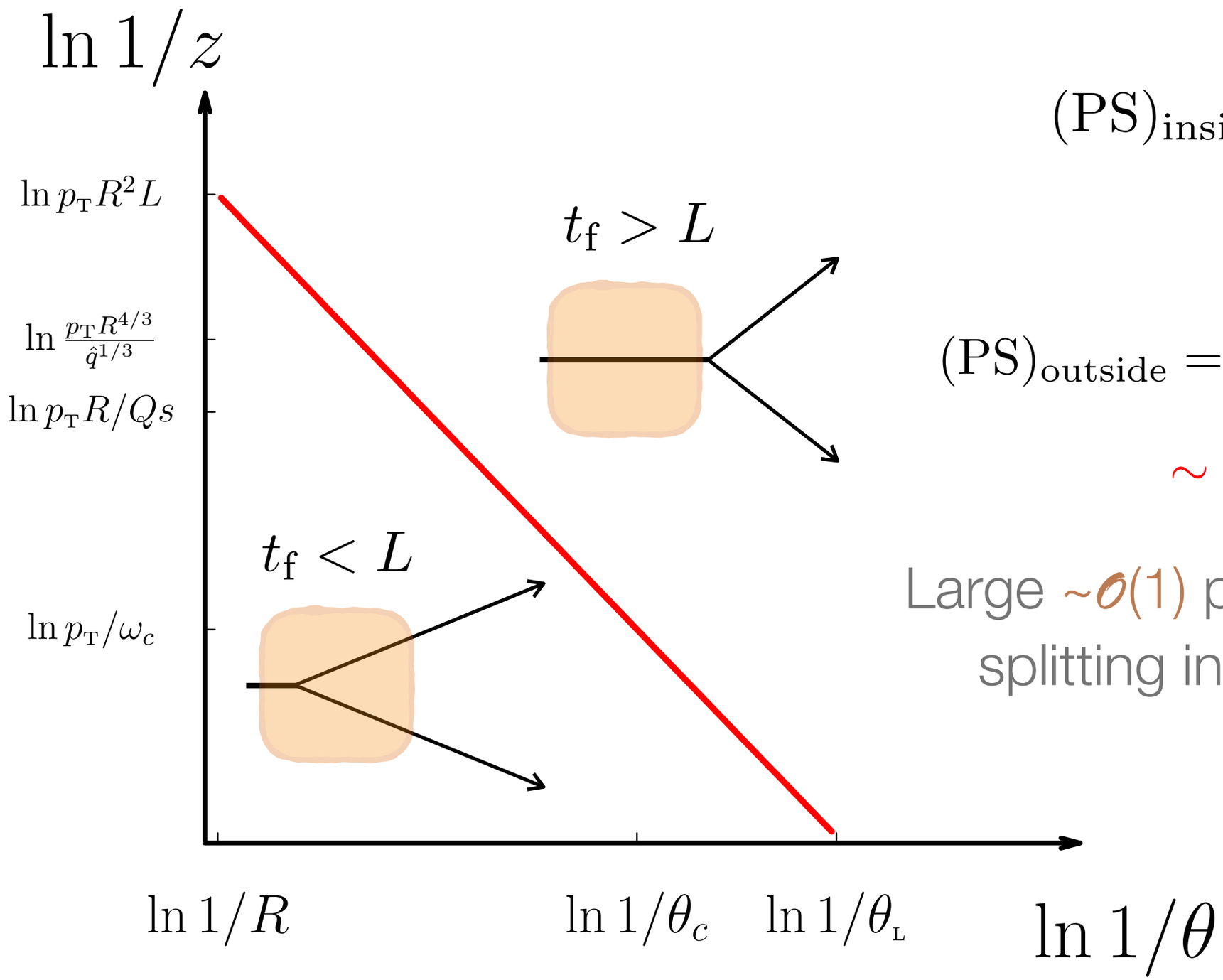
JET SPACETIME EVOLUTION

Bassetto, Ciafaloni, Marchesini (1983);
Dokshitzer, Troyan, Khoze, Mueller (1991)



- jets are multi-particle objects with interesting space-time pattern
- double-logarithmic regime
  - strong energy ordering
  - strong angular ordering
  - formation time

REDRAWING PHASE SPACE



$$(\mathrm{PS})_{\mathrm{inside}} = \frac{\bar{\alpha}}{4} \ln^2 \left(p_{\mathrm{T}} R^2 L \right)$$

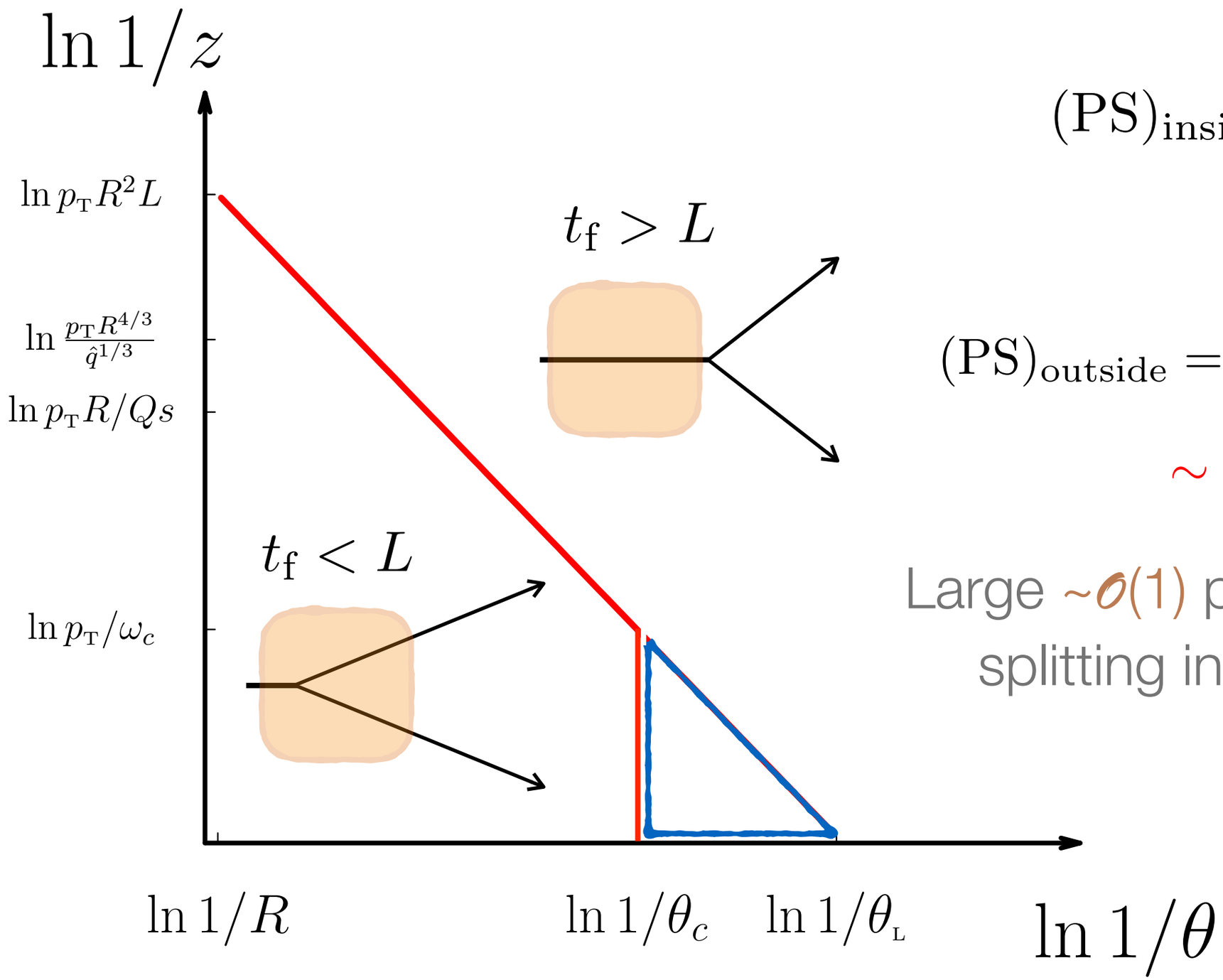
$$\sim 3.2$$

$$(\mathrm{PS})_{\mathrm{outside}} = \frac{\bar{\alpha}}{2} \ln^2 \left(\frac{p_{\mathrm{T}} R}{Q_0} \right) - (\mathrm{PS})_{\mathrm{inside}}$$

$$\sim 2.3$$

Large ~$\mathcal{O}(1)$ probability of multiple splitting inside the medium.

REDRAWING PHASE SPACE



$$(\mathrm{PS})_{\mathrm{inside}} = \frac{\bar{\alpha}}{4} \ln^2 \left(p_{\mathrm{T}} R^2 L \right)$$

$$\sim 3.2$$

$$(\mathrm{PS})_{\mathrm{outside}} = \frac{\bar{\alpha}}{2} \ln^2 \left(\frac{p_{\mathrm{T}} R}{Q_0} \right) - (\mathrm{PS})_{\mathrm{inside}}$$

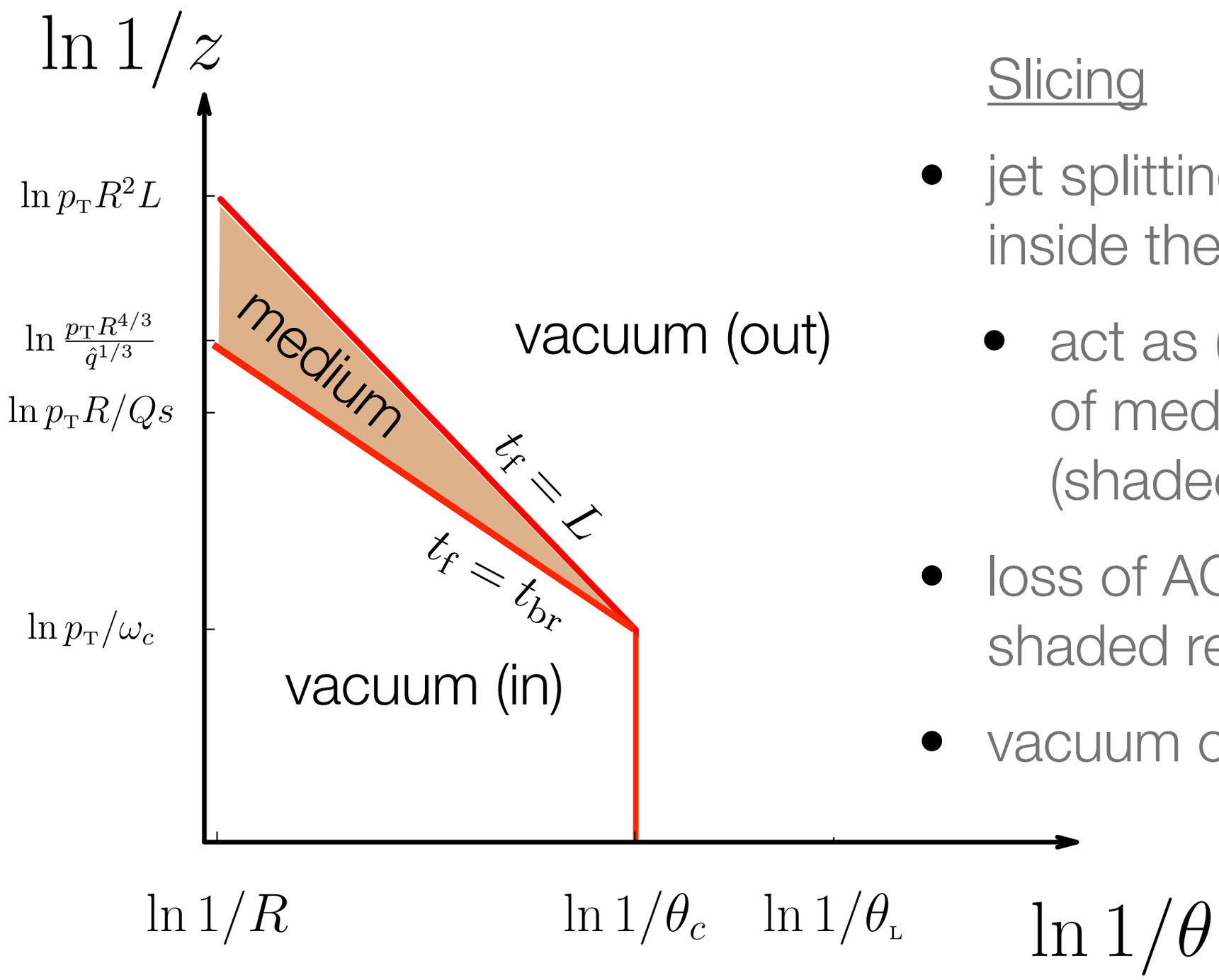
$$\sim 2.3$$

Large ~$\mathcal{O}(1)$ probability of multiple splitting inside the medium.

Blue region corresponds to unresolved splittings.

REDRAWING PHASE SPACE

see also Caucal, Iancu, Mueller, Soyez 1801.09703



Slicing

- jet splittings in the "hard" regime inside the medium
  - act as (independent) sources of medium-induced radiation (shaded region)
- loss of AO when crossing shaded region
- vacuum cascade at late times

CONSEQUENCES

- energy-loss/quenching depends on the amount (phase space) of radiation inside the medium
- mismatch of real and virtual contributions!

Remainder: A warm-up exercise to show the appearance of medium-induced logs.

JET SPECTRUM IN HEAVY-ION COLLISIONS

The spectrum of outgoing particles is affected by energy loss.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p} = \int_0^\infty \mathrm{d}\epsilon\, \mathcal{P}(\epsilon) \frac{\mathrm{d}\sigma_0(p_\mathrm{T}+\epsilon)}{\mathrm{d}p'}$$

$$\approx \frac{\mathrm{d}\sigma_0}{\mathrm{d}p} \underbrace{\int_0^\infty \mathrm{d}\epsilon\, \mathcal{P}(\epsilon) \mathrm{e}^{-n\epsilon/p_\mathrm{T}}}_{\mathcal{Q}(p_\mathrm{T})}$$

For $\epsilon/p_\mathrm{T} \ll 1$ and large n: $\frac{p_\mathrm{T}^n}{(p_\mathrm{T}+\epsilon)^n} \simeq \mathrm{e}^{-n\epsilon/p_\mathrm{T}} \left[1 + \mathcal{O}\left(\frac{n\epsilon^2}{2p_\mathrm{T}^2}\right)\right]$

$$R_\mathrm{jet} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}p}\right) \Big/ \left(\frac{\mathrm{d}\sigma_0}{\mathrm{d}p}\right)$$

nuclear modification factor

RADIATIVE CORRECTIONS

Y. Mehtar-Tani, KT arXiv:1707.07361 [hep-ph]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p} = \frac{\mathrm{d}\sigma_{\mathrm{Born}}}{\mathrm{d}p}\left[1 + \alpha_s\left((\mathrm{PS})_{\mathrm{real}} - (\mathrm{PS})_{\mathrm{virt}}\right) + \mathcal{O}(\alpha_s^2)\right]$$

- higher-order corrections not enhanced by phase space when balance between real & virtual emissions
  - for sufficiently inclusive observables
  - is this the case in the medium?

RADIATIVE CORRECTIONS

Y. Mehtar-Tani, KT arXiv:1707.07361 [hep-ph]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p} = \frac{\mathrm{d}\sigma_{\mathrm{Born}}}{\mathrm{d}p}\left[1 + \alpha_s\left((\mathrm{PS})_{\mathrm{real}} - (\mathrm{PS})_{\mathrm{virt}}\right) + \mathcal{O}(\alpha_s^2)\right]$$

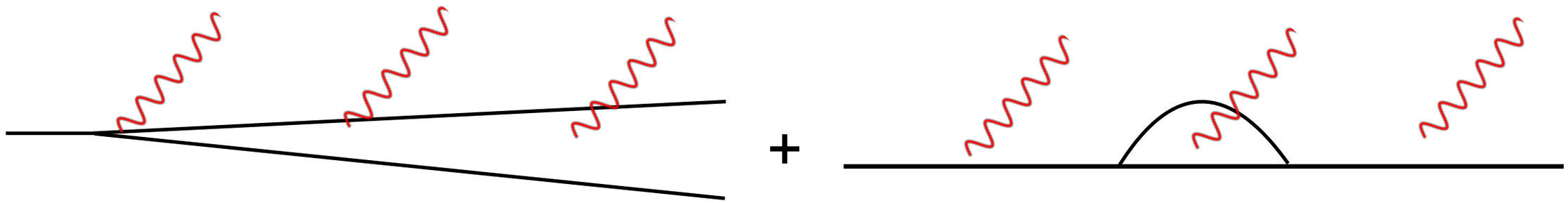
- higher-order corrections not enhanced by phase space when balance between real & virtual emissions
  - for sufficiently inclusive observables
  - is this the case in the medium?

$$R_{\mathrm{jet}} = \mathcal{Q}^{(0)}(p_{\mathrm{T}}) + \mathcal{Q}^{(1)}(p_{\mathrm{T}}) + \mathcal{O}(\alpha_s^2)$$

- expanding quenching factor corresponds to accounting for the quenching of higher-order emissions (substructure fluctuations)

FIRST CORRECTION TO QUENCHING

Y. Mehtar-Tani, KT arXiv:1707.07361 [hep-ph]



sensitive to quark+gluon quenching sensitive to quark quenching

$$\mathcal{Q}^{(1)}(p_{\rm T}) = \int_0^1 {\rm d}z\, P_{gq}(z) \int_0^R \frac{{\rm d}\theta}{\theta}\, \frac{\alpha_s(k_\perp)}{\pi} \left[\mathcal{Q}_{gq}(p_{\rm T}) - \mathcal{Q}_q(p_{\rm T})\right]$$

$$\mathcal{Q}_{gq}(p_{\rm T}) = \mathcal{Q}_q(p_{\rm T})\mathcal{Q}_{\rm sing}(p_{\rm T})$$

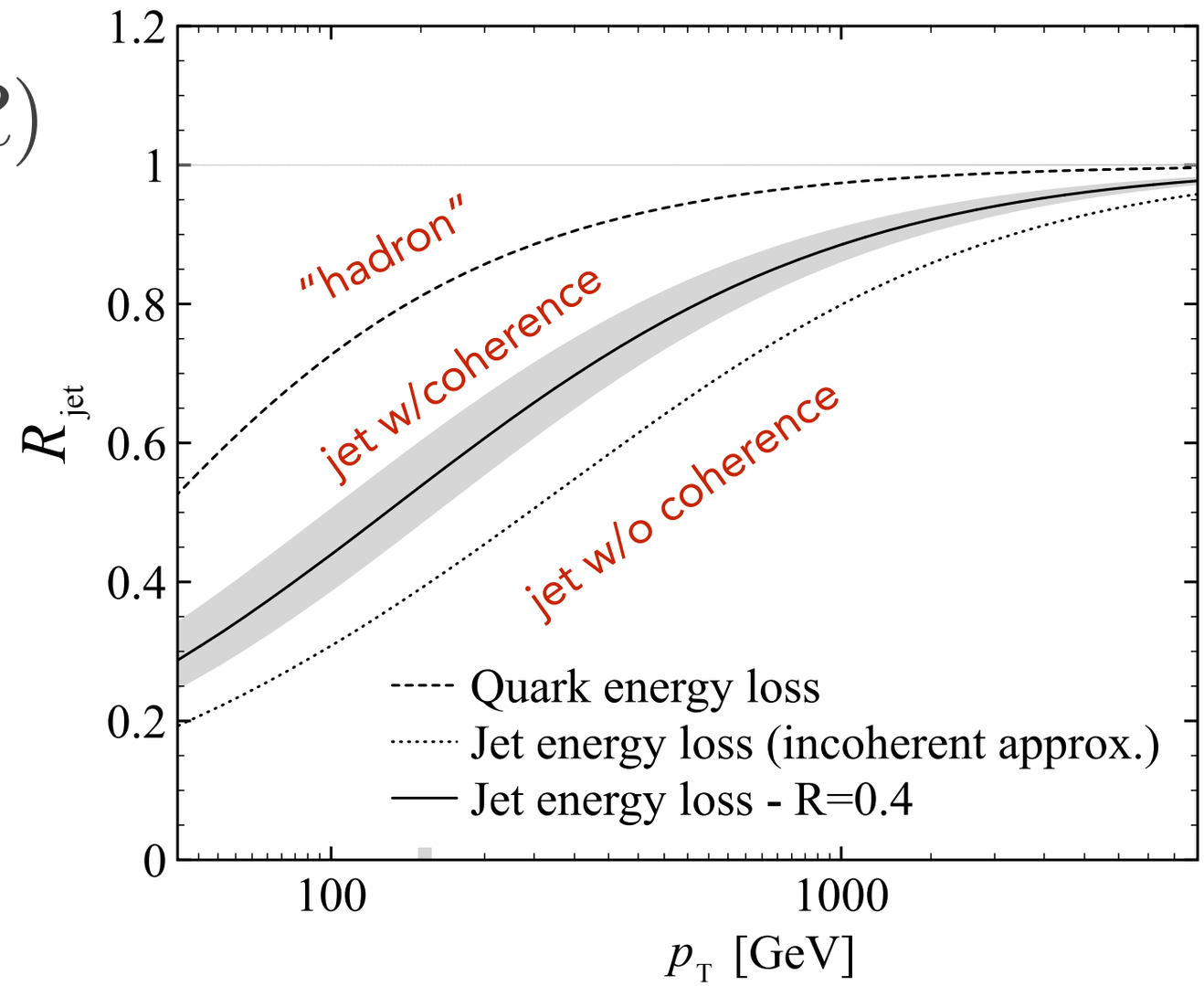
- real & virtual are differently affected by energy loss effects!
- the mismatch is largest at short formation times $t_{\rm f} \ll t_{\rm br}$
  - enhances a subset of higher-order corrections

"SUDAKOV" SUPPRESSION FACTOR

Y. Mehtar-Tani, KT arXiv:1707.07361 [hep-ph]

$$R_{\rm jet} = \mathcal{Q}_q(p_{\rm T}) \times \mathcal{C}(p_{\rm T}, R)$$

- resummation of medium logs via non-linear equation
- strong-quenching limit: dominance of virtual fluctuations (Sudakov)



$$\mathcal{C}(1, p_{\rm T}, R) = 1 + \int_0^1 {\rm d}z \int_{\theta_c}^R \frac{{\rm d}\theta}{\theta} \frac{\alpha_s(k_\perp)}{\pi} P_{gq}(z) \Theta(t_{\rm f} < t_{\rm d}) \times \left[\mathcal{C}(z, p_{\rm T}, \theta)\mathcal{C}(1-z, p_{\rm T}, \theta)\mathcal{Q}_q^2(p_{\rm T}) - \mathcal{C}(1, p_{\rm T}, \theta)\right]$$

GENERATING FUNCTIONAL APPROACH: A SKETCH

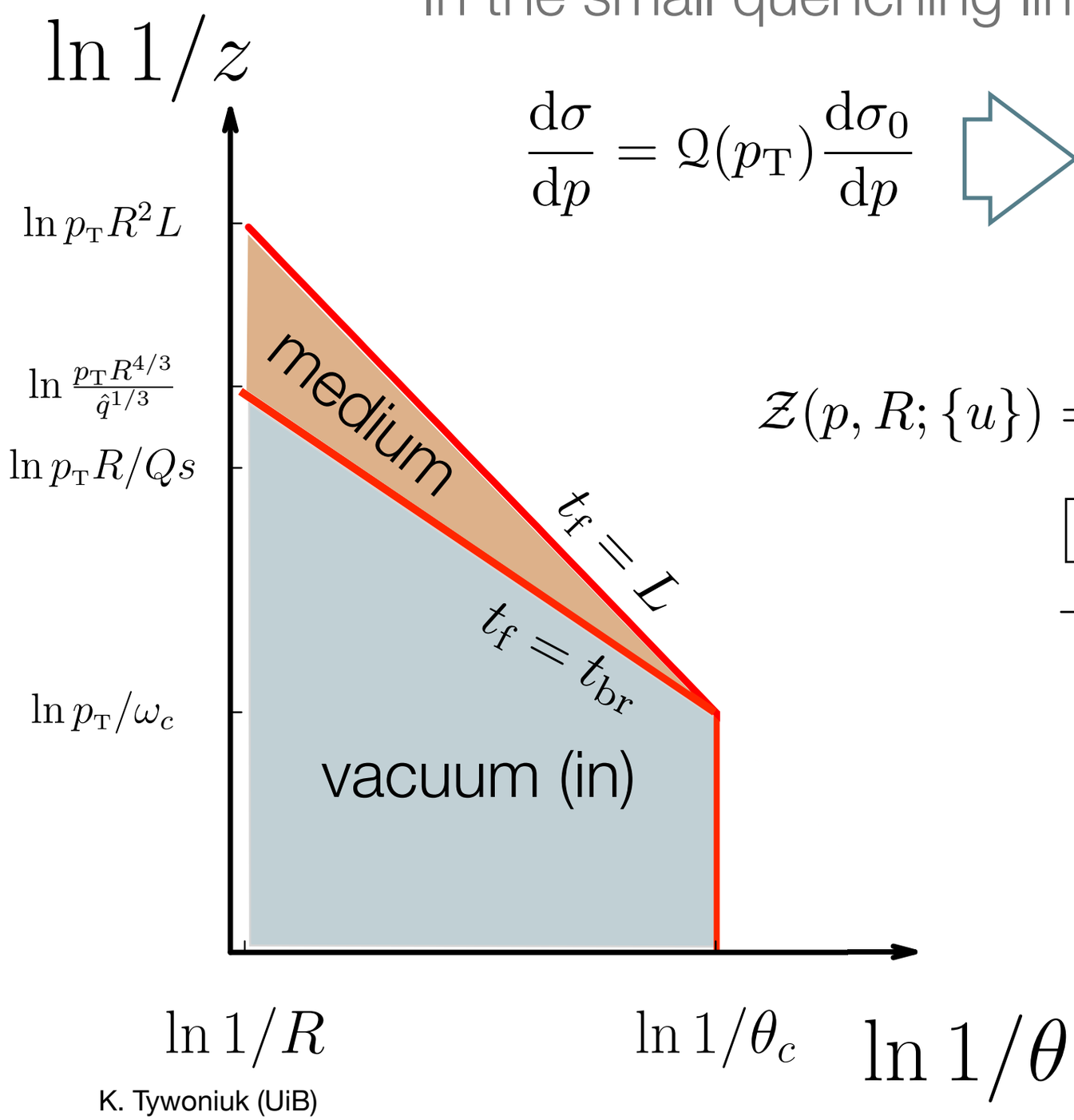
In the small quenching limit, we can generalize:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p} = \mathcal{Q}(p_{\mathrm{T}})\frac{\mathrm{d}\sigma_0}{\mathrm{d}p} \quad \Rightarrow \quad \frac{\mathrm{d}\sigma^{(\mathrm{excl})}}{\mathrm{d}k_1 \ldots \mathrm{d}k_N} = \mathcal{Q}^N(p_{\mathrm{T}})\frac{\mathrm{d}\sigma_0^{(\mathrm{excl})}}{\mathrm{d}k_1 \ldots \mathrm{d}k_N}$$

GENERATING FUNCTIONAL APPROACH: A SKETCH

In the small quenching limit, we can generalize:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p} = \mathcal{Q}(p_\mathrm{T})\frac{\mathrm{d}\sigma_0}{\mathrm{d}p} \quad \Rightarrow \quad \frac{\mathrm{d}\sigma^{(\mathrm{excl})}}{\mathrm{d}k_1\ldots\mathrm{d}k_N} = \mathcal{Q}^N(p_\mathrm{T})\frac{\mathrm{d}\sigma_0^{(\mathrm{excl})}}{\mathrm{d}k_1\ldots\mathrm{d}k_N}$$

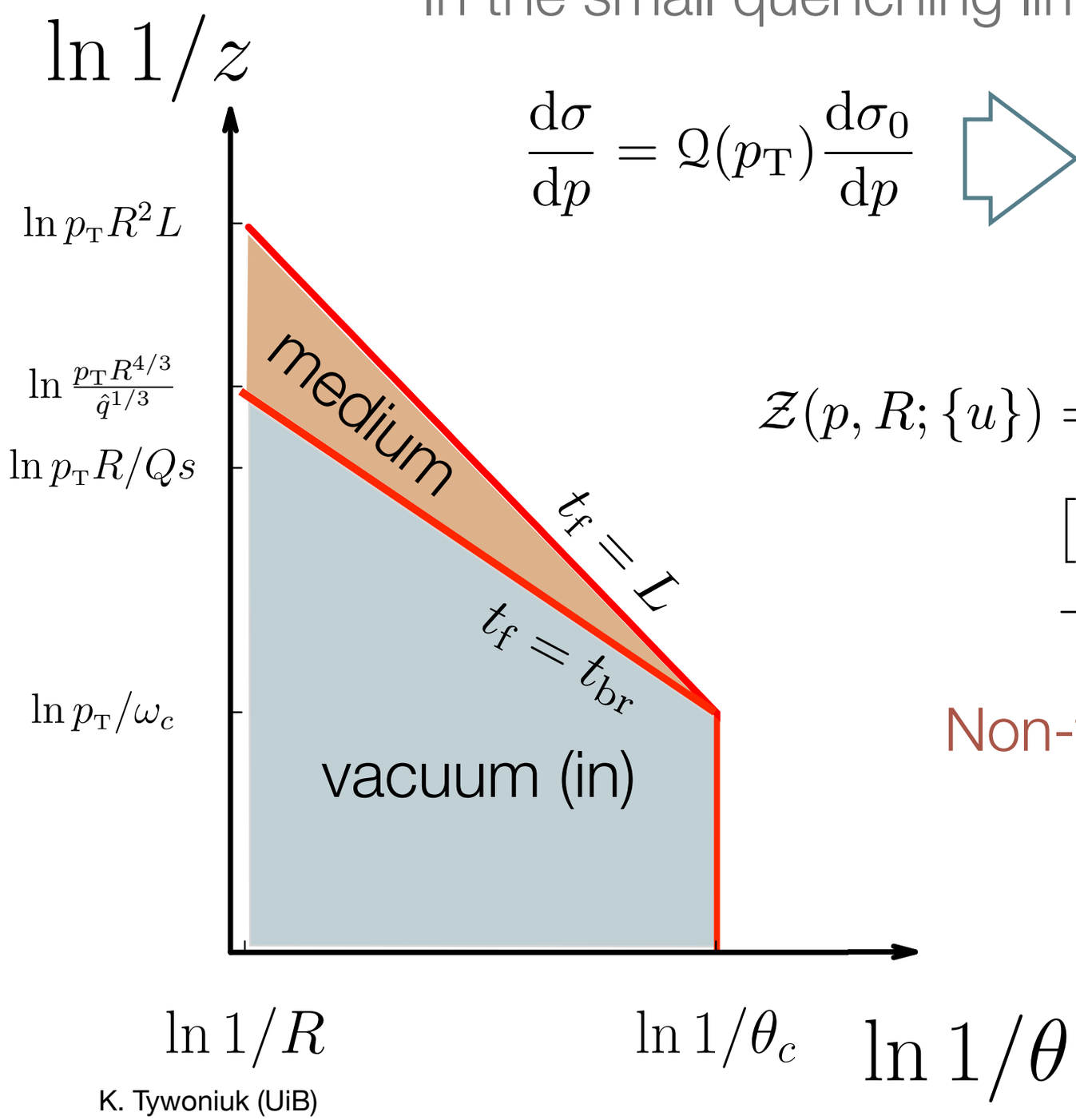


$$\mathcal{Z}(p,R;\{u\}) = u(p) + \int^R \mathrm{d}\Pi\,\Theta(t_\mathrm{f} < t_\mathrm{br} < L)$$
$$\left[\mathcal{Z}(zp,\theta)\mathcal{Z}\big((1-z)p,\theta\big)\,\mathcal{Q}^2(p_\mathrm{T}) - \mathcal{Z}(p,\theta)\right]$$
$$+\ldots$$

GENERATING FUNCTIONAL APPROACH: A SKETCH

In the small quenching limit, we can generalize:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p} = \mathcal{Q}(p_\mathrm{T})\frac{\mathrm{d}\sigma_0}{\mathrm{d}p} \quad \Rightarrow \quad \frac{\mathrm{d}\sigma^{(\mathrm{excl})}}{\mathrm{d}k_1\ldots\mathrm{d}k_N} = \mathcal{Q}^N(p_\mathrm{T})\frac{\mathrm{d}\sigma_0^{(\mathrm{excl})}}{\mathrm{d}k_1\ldots\mathrm{d}k_N}$$



$$\mathcal{Z}(p,R;\{u\}) = u(p) + \int^R \mathrm{d}\Pi\,\Theta(t_\mathrm{f} < t_\mathrm{br} < L)$$
$$\left[\mathcal{Z}(zp,\theta)\mathcal{Z}\big((1-z)p,\theta\big)\,\mathcal{Q}^2(p_\mathrm{T}) - \mathcal{Z}(p,\theta)\right]$$
$$+\ldots$$

Non-trivial normalization due to e-loss:

$$\mathcal{Z}_\mathrm{vac}(p,R;u=1) = 1$$

$$\mathcal{Z}(p,R;u=1) = \mathcal{C}(p,R)$$

CONCLUSIONS

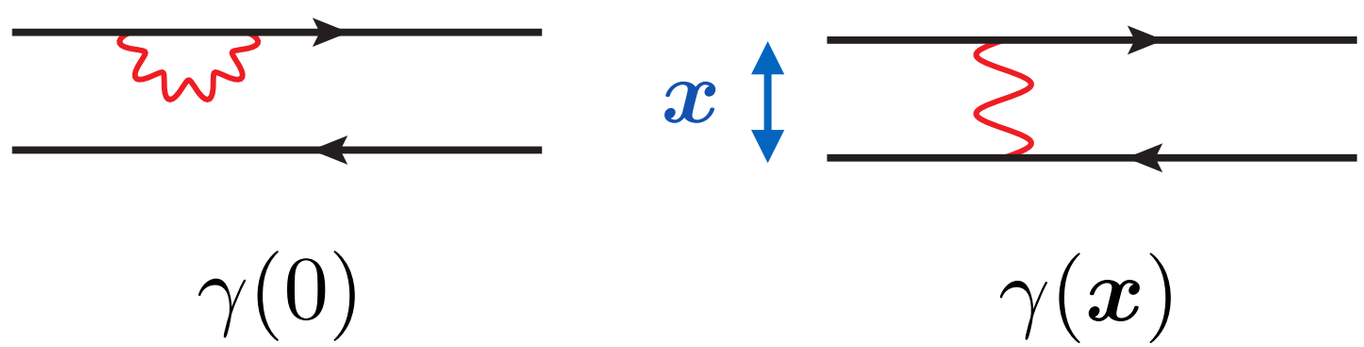
- medium cascade: efficient transport of energy to large angles
- hard, vacuum splittings in the medium act as sources
- mismatch of real & virtual: logarithmic sensitivity to jet scales (Sudakov-like resummation)
- merging the cascades: generating functional with phase space considerations

Thank you!

MEDIUM TRANSPORT COEFFICIENT

$$\gamma(\boldsymbol{x}) = g^2 \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2} \frac{\mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{x}}}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$$

Sensitive to the transverse extension of the “dipole”.



“Harmonic oscillator” approximation

$$\sigma(\boldsymbol{x}) = 2g^2[\gamma(0) - \gamma(\boldsymbol{x})] \simeq \frac{1}{2N_c}\hat{q}\boldsymbol{x}^2$$