5D γ conversion model

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http://llr.in2p3.fr/∼dbernard/polar/harpo-t-p.html

Talk Layout

- The context: HARPO: high-performance γ-ray astronomy and linear polarimetry with conversion to e^+e^- pairs with a gas TPC

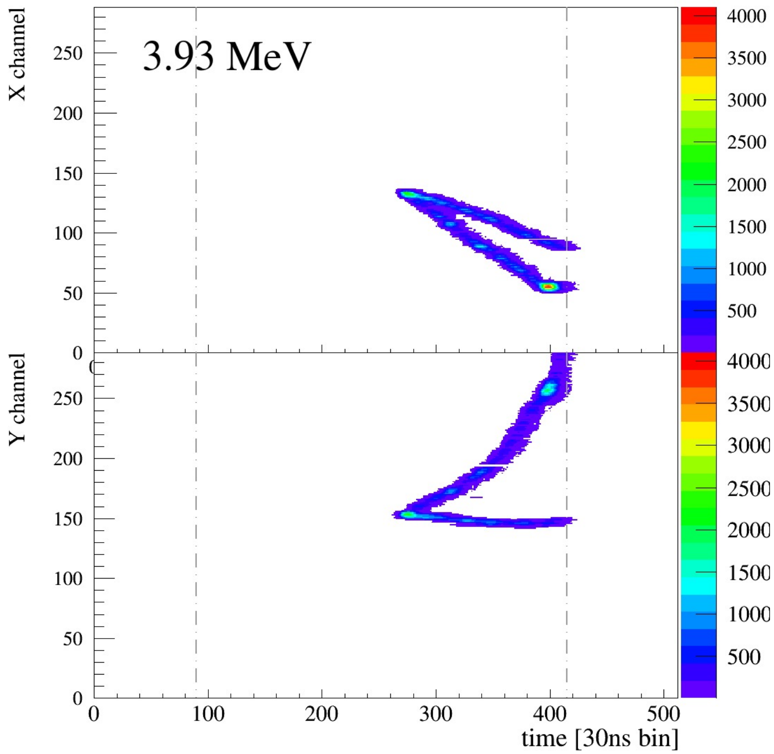
  telescope performance studies — NIM A 701 (2013) 225
  polarimeter performance studies — NIM A 729 (2013) 765
  cosmic-rays TPC tracker characterization — NIM A 718 (2013) 395
  polarized γ-ray beam data-taking campaign — PoS (ICRC2015) 1016
  high dilution factor polarimetry on beam demonstrated — Astroparticle Physics 97 (2018) 10
  Recent Summary — Il Nuovo Cimento 40 C (2017) 117

- Event generator:
  - Past achievements:
    - a VEGAS-based generator (FORTRAN)
      Exact, 5D, polarized, Event generator — NIM A 729 (2013) 765
      Event generator comparison — Astropart. Phys. 88 (2017) 60
      G4SUW, Guildford, April 2017
    - a VEGAS-Free generator (FORTRAN) — PoS(IFS2017)127
  - Present activities: Fortran $\rightarrow$ C++

Context : Experimental project HARPO

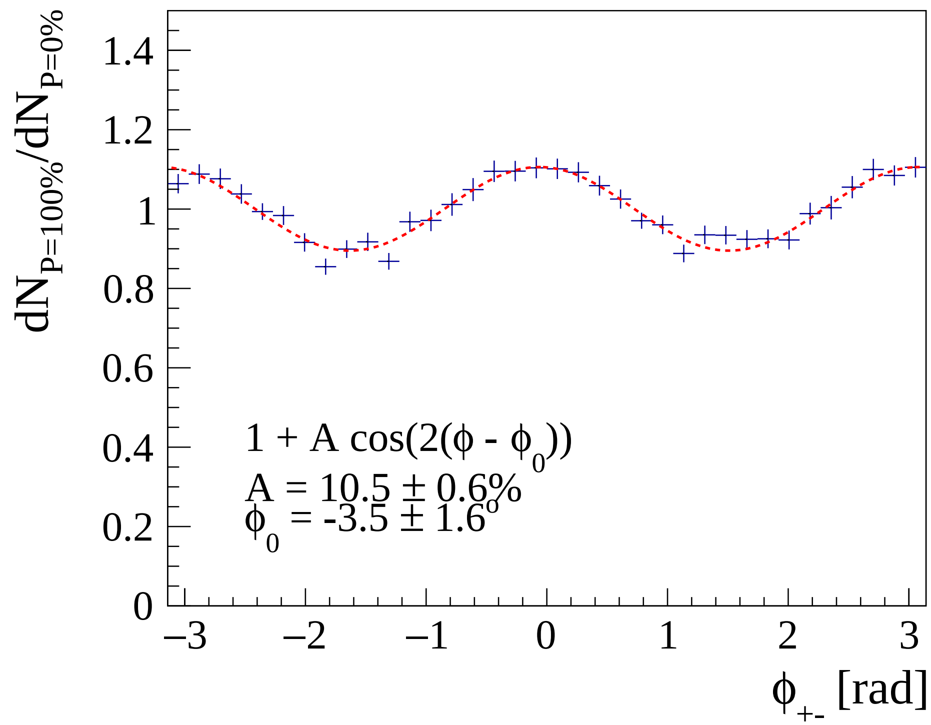
- Gas (argon-isobutane) Time projection chamber (TPC)
- Validated in (polarized) gamma-ray beam

(x, t) and (y, t) signal maps
$4\,\mathrm{MeV}\ \gamma$ in 2.1 bar Ar-iC4H10 95-5%



SPIE (2016) 99052R

azimuthal angle distribution
$11.8\,\mathrm{MeV}\ \gamma$ in 2.1 bar Ar-iC4H10 95-5%



Astroparticle Physics 97 (2018) 10

γ-conversion pre-existing situation

- Geant4 and EGS5 physics models designed for EM showers
- No recoil generation
  - pair wrongly generated in the conversion plane
- e^+ and e^- polar angles generated independantly
  - no energy-momentum conservation
- 5D final state generated from product of 1D pdfs
- Often afflicted by high-energy and/or low-angle approximations
- Validation of polarized models unpublished, if any.

An exact, 5D, Bethe-Heitler, Polarized Generator

- 5D final state: $\varphi_+, \theta_+, \varphi_-, \theta_-, x_+ \equiv E_+/E_\gamma$
- BASES / SPRING version of the VEGAS method, S. Kawabata, Comp. Phys. Comm. 88, 309 (1995).
- Bethe-Heitler "BH" differential cross section (2 dominant diagrams only)
- Exact:
  - no low energy approximation
  - no small angle approximation
  - 5D differential Xsection sampled, no product of 1D differential Xsections
  - strict energy-momentum conservation
- Polarized:
  - linearly polarized photons
  - unpolarized photons
  - partially polarized photons
  - no circular-polarisation dependence at 1rst order Born approximation (BH)
- Target: nucleus or electron (triplet conversion)
- Atomic electron field screening: form factor $F(q^2)$, coherent (nuclear), incoherent (triplet)
- (2012 - 2013): extensive validations for high-energy photons, $E = 4 - 400, \mathrm{MeV}$

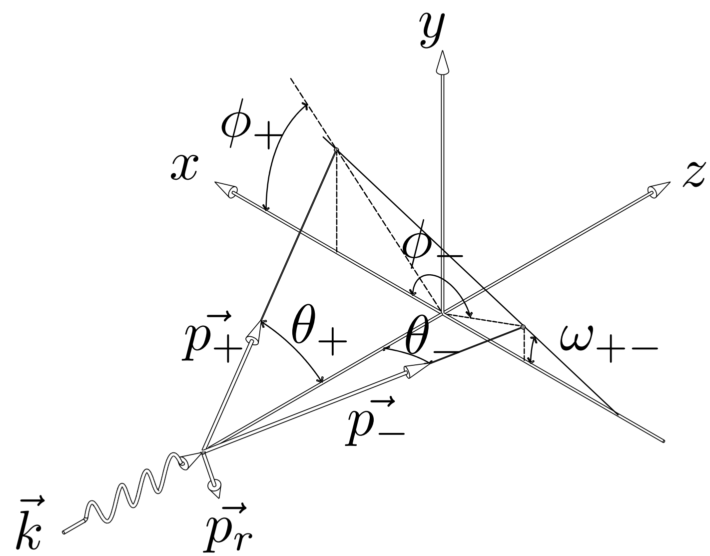
NIM A 729 (2013) 765

Target, Variables

- Target :

ion	"nuclear conversion"	$\gamma Z \rightarrow e^+e^-Z$
electron	"triplet conversion"	$\gamma e^- \rightarrow e^+e^-e^-$

- Variables: azimuthal (ϕ_+, ϕ_-) and polar (θ_+, θ_-) angles of e^+ and e^-, and $x_+ \equiv E_+/E$



Bethe-Heitler differential cross section: 1

- Linearly polarized gamma rays: $(c = 1, \quad \hbar = 1, \quad E = \hbar\omega = \omega)$

$$
\begin{aligned}
\mathrm{d}\sigma \quad = \quad & \frac{-2\alpha Z^2 r_0^2 m^2}{(2\pi)^2\omega^3} \mathrm{d}E_+ \mathrm{d}\Omega_+ \mathrm{d}\Omega_- \frac{|p_-||p_+|}{|\vec{q}|^4} \left[\left(2E_+ \frac{p_- \sin\theta_- \cos(\psi + \phi)}{E_- - p_- \cos\theta_-} + 2E_- \frac{p_+ \sin\theta_+ \cos\psi}{E_+ - p_+ \cos\theta_+} \right)^2 \right. \\
& -q^2 \left(\frac{p_- \sin\theta_- \cos(\psi + \phi)}{E_- - p_- \cos\theta_-} - \frac{p_+ \sin\theta_+ \cos\psi}{E_+ - p_+ \cos\theta_+} \right)^2 \\
& \left. -\omega^2 \frac{(p_+ \sin\theta_+)^2 + (p_- \sin\theta_-)^2 + 2p_+ p_- \sin\theta_+ \sin\theta_- \cos\phi}{(E_- - p_- \cos\theta_-)(E_+ - p_+ \cos\theta_+)} \right]
\end{aligned}
$$

with: $|\vec{q}|^2 = |\vec{p_+} + \vec{p_-} - \vec{k}|^2$, $\psi \equiv \varphi^+$ and $\psi + \phi \equiv \varphi^-$

M. May, Phys. Rev. 84, 265 (1951).

Correction by a factor of 2: Jauch and Rohrlich, *The theory of photons and electrons* (Springer Verlag, 1976).

- Non polarized gamma rays:

$$
\begin{aligned}
& \frac{-\alpha Z^2 r_0^2 m^2}{(2\pi)^2\omega^3} \mathrm{d}E_+ \mathrm{d}\Omega_+ \mathrm{d}\Omega_- \frac{|p_-||p_+|}{|\vec{q}|^4} \left[\left(\frac{p_+ \sin\theta_+}{E_+ - p_+ \cos\theta_+} \right)^2 (4E_-^2 - q^2) + \left(\frac{p_- \sin\theta_-}{E_- - p_- \cos\theta_-} \right)^2 (4E_+^2 - q^2) + \right. \\
& \left. \frac{2p_+ p_- \sin\theta_+ \sin\theta_- \cos\phi}{(E_- - p_- \cos\theta_-)(E_+ - p_+ \cos\theta_+)} (4E_+ E_- + q^2 - 2\omega^2) - 2\omega^2 \frac{(p_+ \sin\theta_+)^2 + (p_- \sin\theta_-)^2}{(E_+ - p_+ \cos\theta_+)(E_- - p_- \cos\theta_-)} \right]
\end{aligned}
$$

"The quantum theory of radiation", W. Heitler, 1954.

Bethe-Heitler differential cross section: 2

- That is: $\mathrm{d}\sigma = \Phi(X_u + P \times X_p)\mathrm{d}E_+\mathrm{d}\Omega_+\mathrm{d}\Omega_-$ with $\Phi = \frac{-\alpha Z^2 r_0^2 m^2}{(2\pi)^2\omega^3}\frac{|p_-||p_+|}{|\vec{q}|^4}$
- P the **linear** polarization fraction of the incident photon, And:

$$X_u = \left[\left(\frac{p_+ \sin\theta_+}{E_+ - p_+\cos\theta_+}\right)^2(4E_-^2 - q^2) + \left(\frac{p_- \sin\theta_-}{E_- - p_-\cos\theta_-}\right)^2(4E_+^2 - q^2) + \right.$$
$$\left. \frac{2p_+p_-\sin\theta_+\sin\theta_-\cos\phi}{(E_- - p_-\cos\theta_-)(E_+ - p_+\cos\theta_+)}(4E_+E_- + q^2 - 2\omega^2) - 2\omega^2\frac{(p_+\sin\theta_+)^2 + (p_-\sin\theta_-)^2}{(E_+ - p_+\cos\theta_+)(E_- - p_-\cos\theta_-)}\right]$$

$$X_p = \cos 2(\phi+\psi)(4E_+^2 - q^2)\left(\frac{p_-\sin\theta_-}{E_- - p_-\cos\theta_-}\right)^2 + \cos 2\psi(4E_-^2 - q^2)\left(\frac{p_+\sin\theta_+}{E_+ - p_+\cos\theta_+}\right)^2$$
$$+2\cos(\phi+2\psi)(4E_+E_- + q^2)\frac{p_-\sin\theta_- p_+\sin\theta_+}{(E_- - p_-\cos\theta_-)(E_+ - p_+\cos\theta_+)}$$

- Partial screening effect of (other) atomic electrons parametrized by form factor (fn of q)
- Differential cross section does not depend on **circular** polarization fraction at this first order of Born approximation

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Gears

- Variables that are actually generated:

variable	related to	
$\lambda(1)$	recoil polar angle	in the CMS (center of mass system)
$\lambda(2)$	recoil azimutal angle	
$\lambda(3)$	pair Invariant Mass(MeV)	
$\lambda(4)$	positron polar angle	in the pair CMS frame
$\lambda(5)$	positron azimutal angle	in the pair CMS frame

- Output:
    - e^+, e^- and recoil 4-vectors
    - A series of additional (redundant) information .. pair opening angle, azimutal angle ..

- Photon energy validity range:

real*8	threshold to 100 GeV
real*16	threshold to 1 PeV,

above which the differential cross section becomes negative in some corners of the variable space.

First batch of validations (2012)

- Comparison of the total mass attenuation coefficients (argon; with and without screening; triplet and nuclear) with the data from NIST

  J. H. Hubbell et al., J. Phys. Chem. Ref. Data 9, 1023 (1980).

- Comparison triplet / nuclear q-distribution ratio with analytical calculations

  K. J. Mork, Phys. Rev. 160, 1065 - 1071 (1967).

- Triplet: $\sigma(q > q_0)$, cross section for recoil electron momentum larger than q_0, compared with high-energy asymptotic expression

  M. L. Iparraguirre and G. O. Depaola, Eur. Phys. J. C 71, 1778 (2011).

- Most probable opening compared with $\hat{\theta}_{+-} \approx \frac{1.6\,\mathrm{MeV}}{E}$

  H. Olsen, Phys. Rev. 131, 406 - 415 (1963)

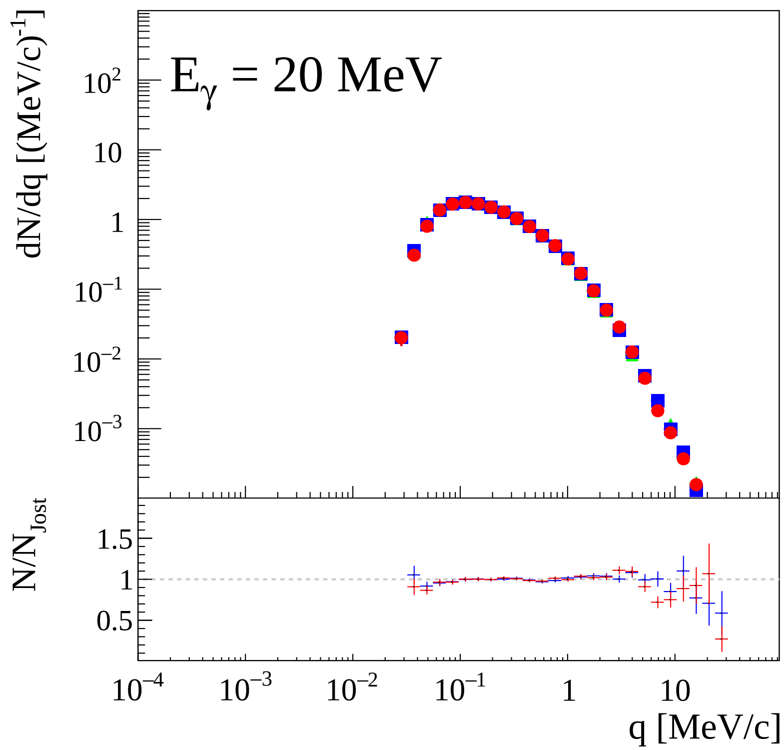
D. Bernard, NIM A 729 (2013) 765

See slides 11-14 of my presentation at G4SUW

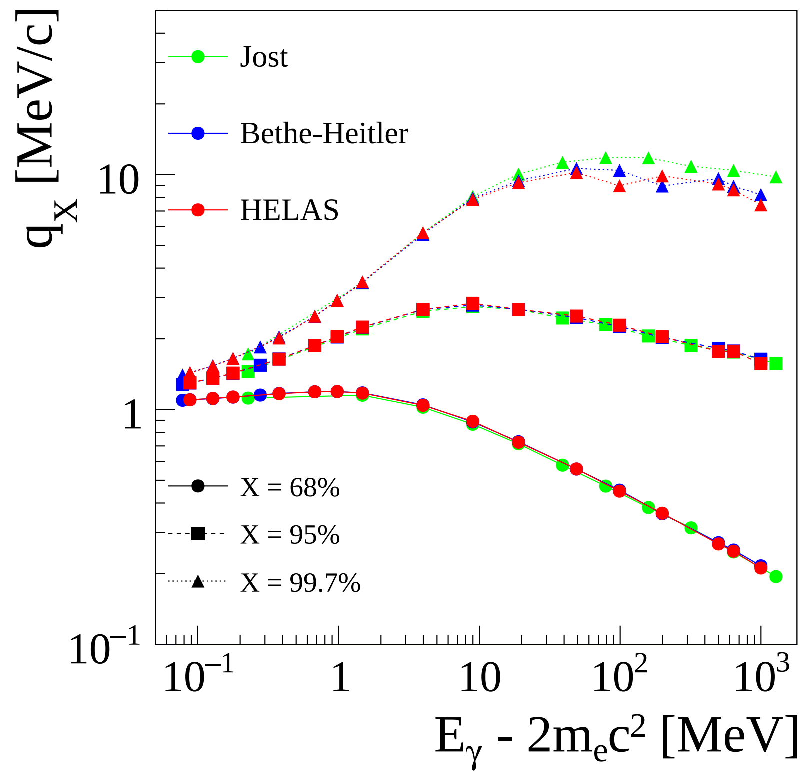
2nd batch of validations (2016); extract

- Extend to 1.1 MeV - 1 GeV

recoil momentum q



recoil momentum q containment



R. Jost, "Distribution of Recoil Nucleus in Pair Production by Photons," Phys. Rev. 80, 189 (1950).

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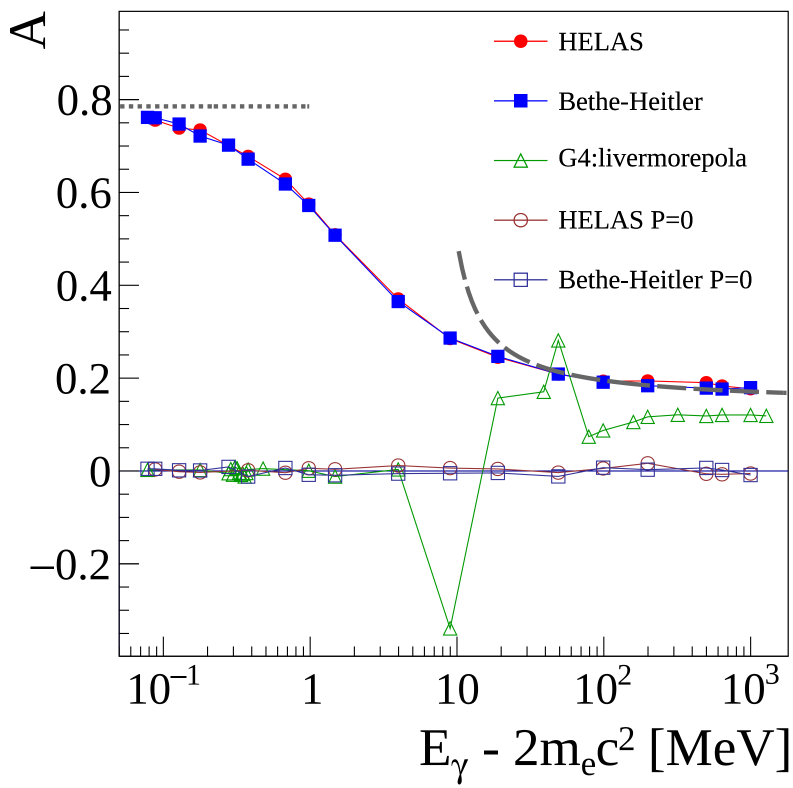
γ *Conversion Event Generators: A Comparison*

Name	Model	Generator
HELAS	HELAS Feynman amplitudes	BASES/SPRING
Bethe-Heitler	Bethe-Heitler	BASES/SPRING
G4:emstandard	G4BetheHeitler	Geant4 10.02.01
G4:livermorepola	G4LivermorePolarizedGammaConversion	Geant4 10.02.01
G4:penelope	G4PenelopeGammaConversion	Geant4 10.02.01
EGS0	egs5, IPRDST$= 0$	egs5 1.0.6
EGS2	egs5, IPRDST$= 2$	egs5 1.0.6

- See slides 17-19 of my presentation at G4UW Wollongong

P. Gros, D. Bernard, Astropart. Phys. 88 (2017) 60

Polarisation Asymmetry



- Low energy

$$A = \frac{\pi}{4}.$$

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- High energy

$$A \approx \frac{\frac{4}{9}\ln 2E - \frac{20}{28}}{\frac{28}{9}\ln 2E - \frac{218}{27}}$$

V. F. Boldyshev et al., Yad. Fiz. 14 (1971) 1027, (Sov.J.Nucl.Phys. 14 (1972) 576).

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VEGAS: Multi-dimensional integration and event generation with package BASES/SPRING

- Method: The VEGAS algorithm, due to G. P. Lepage (J. Comput. Phys. 27 (1978) 192)
- Instantiation: BASES/SPRING (S. Kawabata, Comput. Phys. Commun. 88 (1995) 309.)
- Integration:
  - n-D hyper-volume first segmented in identical cells (n-dimensional bins);
  - pdf and its variance evaluated in each cell, by shooting a number of points;
  - hyper-volume segmentation then tuned iteratively, so as to minimize the variance;
  - upon convergence, an optimal tabulation of the pdf is obtained.
- Event generation:
  - an "exact" generation from $f(\lambda)$ is then obtained easily from the tabulated $f_0(\lambda)$ the acceptance-rejection method
- The pdf depends on photon energy E (and target nature through $F(q^2)$)
  - the integration step is time consuming (few seconds)
  - generation of batches of zillions events with same energy easy.

VEGAS-free version

- 5 variables x_i, related to the λ_i, taken flat.

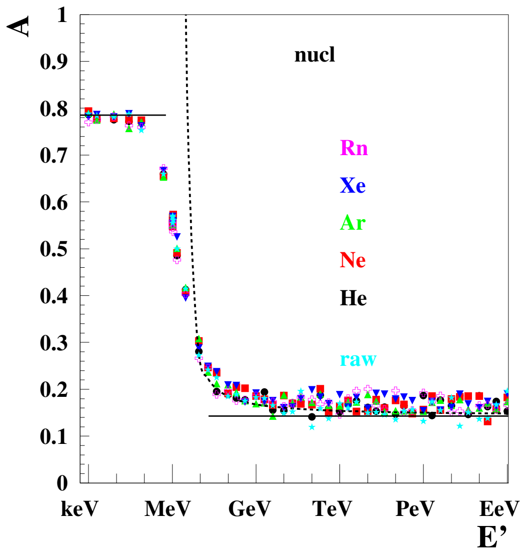
i		Jacobian	x_i range
1	$\cos\theta = \frac{y-1}{1+y}$, $y = \exp(x_1)$	$\frac{y}{(1+y)^2}$	$[x_{1l}, x_{1u}]$
2	$\mu = \mu_{\rm min} \times (\mu_{\rm r})^{x_2^2}$	$2x_2 \log(\mu_{\rm r})\mu$	$[0, 1]$
3	$\cos\theta_\ell = x_3$	$\lvert\sin\theta_\ell\rvert$	$[0, \pi]$
4	$\phi_\ell = x_4$	1	$[-\pi, \pi]$
5	$\phi = x_5$	1	$[-\pi, \pi]$

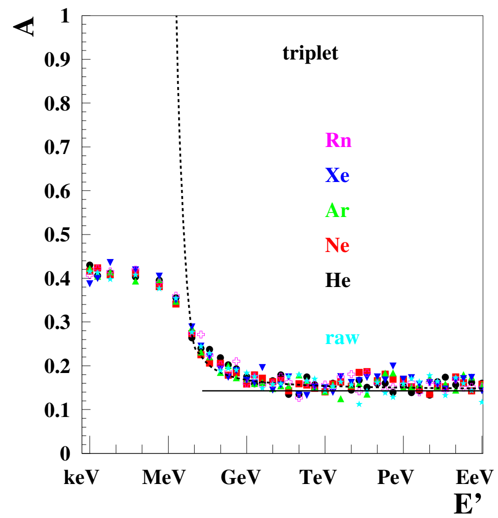
  - that defines an approximate version p_0 of true pdf p

- Bethe-Heitler pdf p generated with acceptance-rejection method

Validation .. Validation .. again

- Polarisation asymmetry as a function of E-threshold.

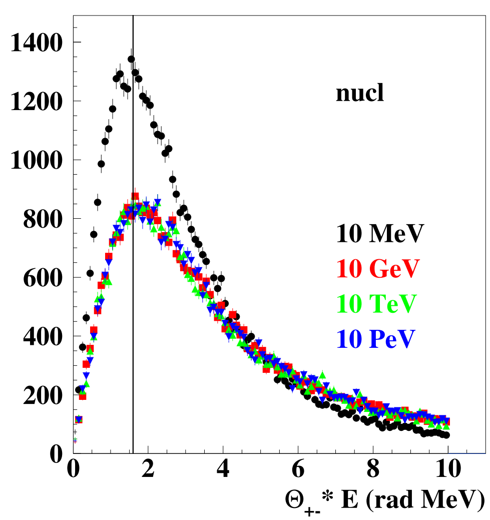


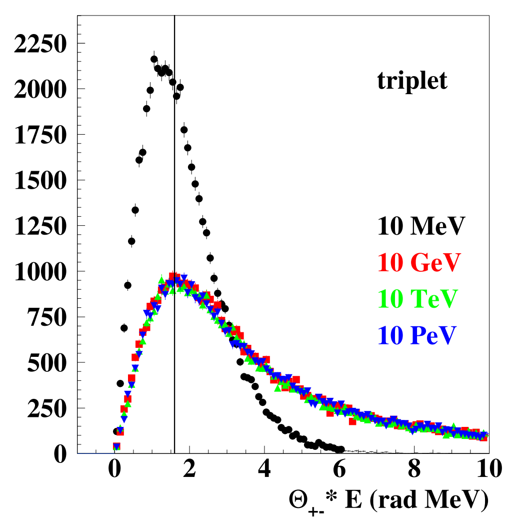


Asymptotes

- low E, $\pi/4$
- high E, $1/7$

- (pair opening angle) $\times$ (photon energy), Argon





Vertical line: high energy most probable value

$$\frac{1.6 \text{ rad} \cdot \text{MeV}}{E}$$

H. Olsen, Phys. Rev. 131, 406 (1963).

Work in progress :
Fortran $\rightarrow$ C++ $\rightarrow$ Geant4 Physics Model Candidate

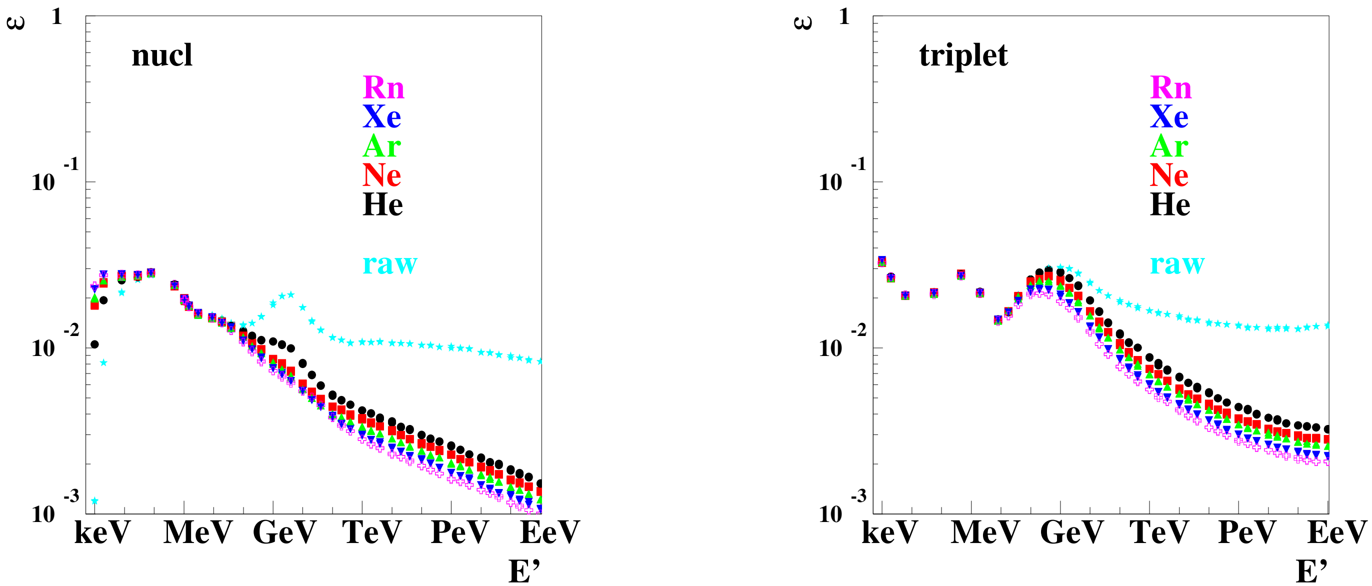
- geant4/source/processes/electromagnetic/standard/
- G4BetheHeitler5DModel.cc "inspired" from G4BetheHeitlerModel.cc
- At the moment, word-for-word translation fortran $\rightarrow$ "C++", done.
- At the moment, stand-alone "nucleus", no input from G4 (geometry, polarisation direction), no output to G4. Yet.
- Very minimal validation done. Yet.

Foreseen issues

- I am just able to take a final state at random after a differential cross section, "SampleSecondaries"
- What about total cross sections ?
  - nuclear: inheritate from G4BetheHeitlerModel;
  - triplet ?
- Energy range ?
  - Presently validated (1 keV above threshold) $\rightarrow$ (1 EeV) (fortran, quadruple precision)
  - technical validation only : I do sample the BH differential cross section, up to 1 EeV
  - any scientific interest ? up to what energy ?
  - Xsections above G4BetheHeitlerModel limit ?
  - LPM ?

Foreseen issues: Sampling efficiency

- efficiency = (average number of tries per event)$^{-1}$



- not good
- worse at high energy
- improves only by a factor of $2 - 3$ when x_1 and x_2 taken from their (1D, target and energy dependent ..) pdf : Correlations !

  CPU time/event ($100\,\mathrm{MeV}$ γ-ray nuclear on Argon, on a DELL Precision M4600)

$0.22\,\mathrm{ms}$	(fortran REAL*8)
$6.6\,\mathrm{ms}$	(fortran REAL*16)

- might want to keep this 5D model for γ conversion studies, not for full EM shower simulation

Design considerations

- We agreed to implement the 5D model as a G4VEMModel:
  - time consuming model
  - to be activated by region only
    - ie, à la G4-DNA
- The model samples the final state "only"
  - hence the total cross-section has to be provided by "someone" else
  - proposed to inherit from G4BetheHeitlerModel for this
    - what cross-section is it ?
      - any low/high energy approximation ?
      - inclusion of triplet ?
- Gamma conversion final state of the 5D model is not only "e+e-"
  - the model can provide the momentum of the recoil
  - or make this recoil as a local energy deposit, in the PostStepDoIt
  - any difficulty to provide such quantities in the context of a G4VEMModel ?
  - control this production with the general production cuts scheme ?
  - Sampling of the nucleus in mixtures (E-field screening form-factor is Z dependent)
    - does G4BetheHeitlerModel show this ?

Integration with other physics models & example

- 5D model provides linear gamma_pol $\rightarrow e^+e^-$
  - no polarization transferred to e^+ and e^-
  - rest of event (EM shower) with standard EM (should be) fine.
- Setup of physics will be provided by a physics constructor
  - help on this appreciated
  - eg : is activation per region comes for free ?
  - do we need to extend G4EMParameters ?
  - example of parameters to control the 5D model:
    - region, energy max, switch on/off recoil production, etc.
- Proposed test and/or example
  - reproduce a small number of distributions of key variables (e.g. pair opening angle, recoil momentum, positron energy fraction, polarization asymmetry) in a given configuration (e.g. 20 MeV, nuclear, argon)
  - check usual G4 behaviour
    - TestEm2 - shower development in an homogeneous material : longitudinal and lateral profiles
    - TestEm5 - how to study transmission, absorption and reflection of particles through a single, thin or thick, layer.
  - should it appear in extended/electromagnetic

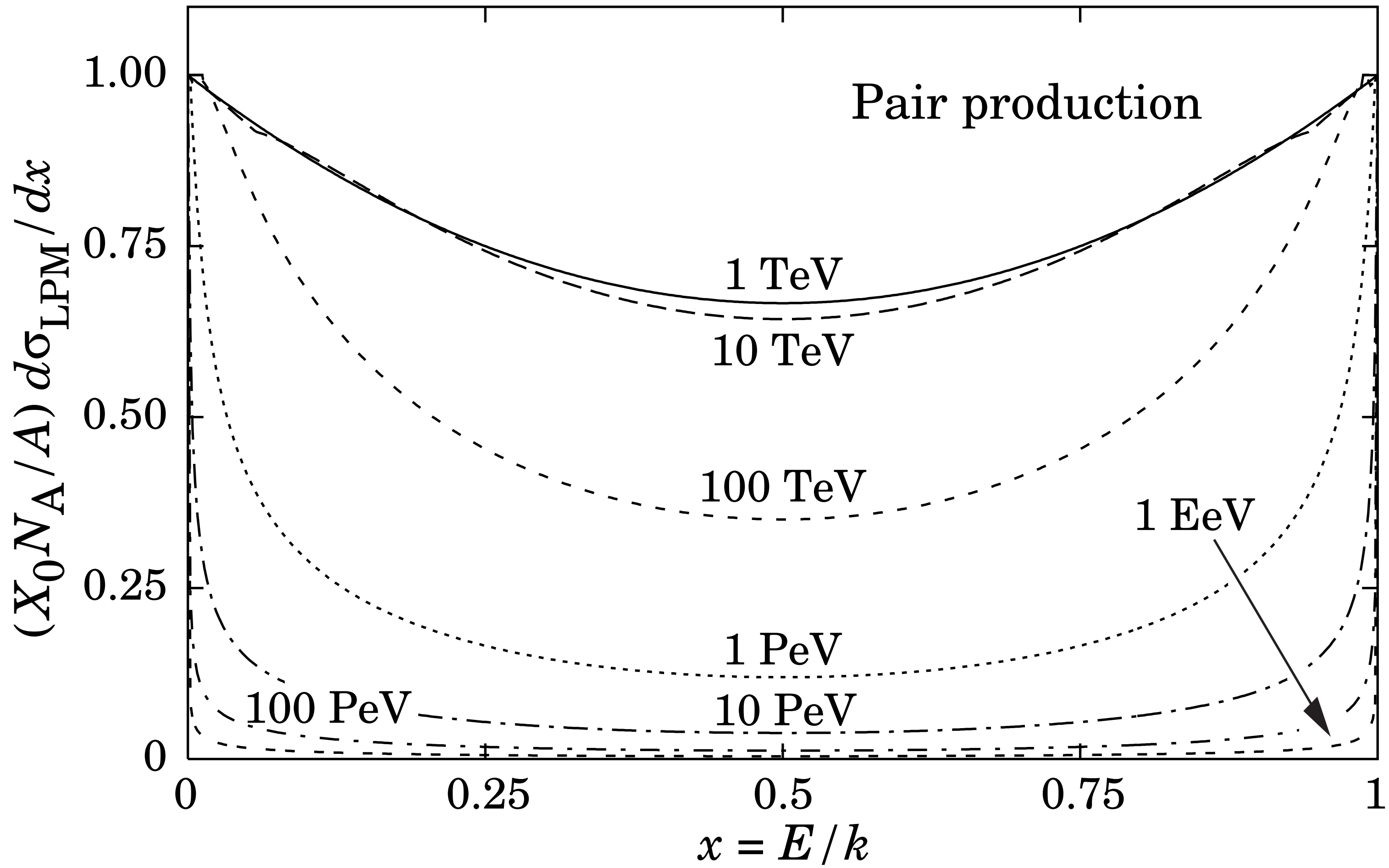
Conclusion

- VEGAS-free version of the code finalised and fully tested (fortran)
- C++ "prof of concept" version done.
- Move from 10.3.3 to 10.4.0 soon.
- To be integrated as a G4 physics model;
- Acknowledgements, gratitude to

  Vladimir Ivantchenko & Marc Verderi, The Geant4 Collaboration

Back-up slides

LPM



Pair production
1.00
0.75
0.50
0.25
0
$(X_0 N_A/A)\, d\sigma_{LPM}/dx$
1 TeV
10 TeV
100 TeV
1 EeV
1 PeV
100 PeV
10 PeV
0
0.25
0.5
0.75
1
$x = E/k$