Scattering Amplitudes with OpenLoops2

Federico Buccioni

in collaboration with

S. Pozzorini M. Zoller

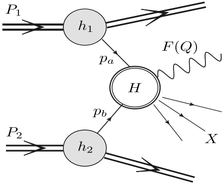
Universität Zürich[UZH]

FNSNF
FONDS NATIONAL SUISSE
SCHWEIZERISCHER NATIONALFONDS
FONDO NAZIONALE SVIZZERO
SWISS NATIONAL SCIENCE FOUNDATION

Physik-Institut, Universität Zürich, PhD Seminar 08.03.2018

Outline

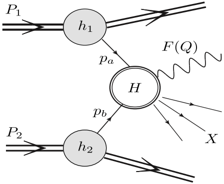
Cross Sections and Scattering Amplitudes



Hadronic hard-scattering: $h_1(P_1)\,h_2(P_2) \to F(q) + X$

$$\mathrm{d}\sigma = \sum_{a,b} \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \mathrm{d}\hat{\sigma} \left(Q, \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2\right)$$

Cross Sections and Scattering Amplitudes



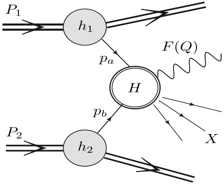
Hadronic hard-scattering: $h_1(P_1)\, h_2(P_2) \to F(q) + X$

$$d\sigma = \sum_{a,b} \int_0^1 dx_1 dx_2 f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) d\hat{\sigma}\left(Q, \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2\right)$$

Partonic Cross Section

$$d\hat{\sigma} = \frac{1}{F} \left|\mathcal{M}(p_a\, p_b \to p_f)\right|^2 \prod_f \frac{d\,\vec{p}_f}{(2\pi)^3 E_f} (2\pi)^4 \delta(p_a + p_b - \sum_f p_f)$$

Cross Sections and Scattering Amplitudes



Hadronic hard-scattering: $h_1(P_1)\, h_2(P_2) \to F(q) + X$

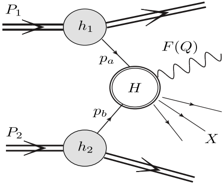
$$d\sigma = \sum_{a,b} \int_0^1 dx_1 dx_2 f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) d\hat{\sigma}\left(Q, \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2\right)$$

Partonic Cross Section

$$d\hat{\sigma} = \frac{1}{F} \left|\mathcal{M}(p_a\, p_b \to p_f)\right|^2 \prod_f \frac{d\,\vec{p}_f}{(2\pi)^3 E_f} (2\pi)^4 \delta(p_a + p_b - \sum_f p_f)$$

$$d\hat{\sigma} = d\hat{\sigma}_0 + \alpha_s d\hat{\sigma}_1 + \alpha_s^2 d\hat{\sigma}_2 + \alpha_s^3 d\hat{\sigma}_3 + \mathcal{O}(\alpha_s^4)$$

Cross Sections and Scattering Amplitudes



Hadronic hard-scattering: $h_1(P_1)\, h_2(P_2) \to F(q) + X$

$$d\sigma = \sum_{a,b} \int_0^1 dx_1 dx_2 f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) d\hat{\sigma}\left(Q, \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2\right)$$

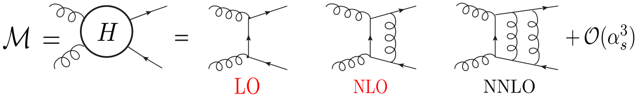
Partonic Cross Section

$$d\hat{\sigma} = \frac{1}{F} \left|\mathcal{M}(p_a\, p_b \to p_f)\right|^2 \prod_f \frac{d\,\vec{p}_f}{(2\pi)^3 E_f} (2\pi)^4 \delta(p_a + p_b - \sum_f p_f)$$

$$d\hat{\sigma} = d\hat{\sigma}_0 + \alpha_s d\hat{\sigma}_1 + \alpha_s^2 d\hat{\sigma}_2 + \alpha_s^3 d\hat{\sigma}_3 + \mathcal{O}(\alpha_s^4)$$

Scattering Amplitude

$\mathcal{M}$ itself is expanded in α_s. $\mathcal{M}$ is expressed in terms of **Feynman diagrams**



Need for Precision

Continuous improvements of statistics and experimental systematics at LHC

$\Rightarrow$ theoretical predictions in the SM must match the accuracy of the data

$\Rightarrow$ small deviations theory/data can open the door to BSM scenarios

Need for Precision

Continuous improvements of statistics and experimental systematics at LHC

$\Rightarrow$ theoretical predictions in the SM must match the accuracy of the data

$\Rightarrow$ small deviations theory/data can open the door to BSM scenarios

$\star$ Example: Higgs couplings relevant for BSM searches. Case study: $t\bar{t}H$ @ LHC.

$t\bar{t}H$ in $H \to b\bar{b}$ channel depends critically on theory uncertainty of $t\bar{t}b\bar{b}$ background.

ξ

Need for Precision

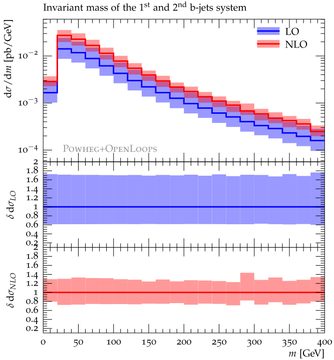
Continuous improvements of statistics and experimental systematics at LHC

⇒ theoretical predictions in the SM must match the accuracy of the data

⇒ small deviations theory/data can open the door to BSM scenarios

⋆ Example: Higgs couplings relevant for BSM searches. Case study: $t\bar{t}H$ @ LHC.

$t\bar{t}H$ in $H \to b\bar{b}$ channel depends critically on theory uncertainty of $t\bar{t}b\bar{b}$ background.

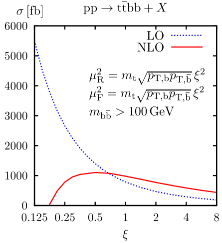


Plot by T. Jezo

$$d\sigma = d\sigma(Q, \mu_R, \mu_F) \qquad \sigma_{t\bar{t}b\bar{t}} \propto \alpha_s^4(\mu_R)$$

Theory uncertainty associated with scale variation

- 75-80% at LO
- 20-30% at NLO



Plot by S.Pozzorini

Need for Precision

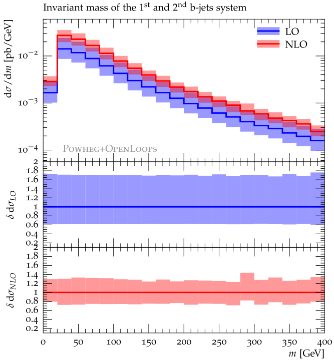
Continuous improvements of statistics and experimental systematics at LHC

⇒ theoretical predictions in the SM must match the accuracy of the data

⇒ small deviations theory/data can open the door to BSM scenarios

⋆ Example: Higgs couplings relevant for BSM searches. Case study: $t\bar{t}H$ @ LHC.

$t\bar{t}H$ in $H \to b\bar{b}$ channel depends critically on theory uncertainty of $t\bar{t}b\bar{b}$ background.



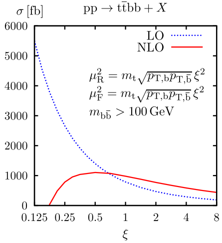
Plot by T. Jezo

$$d\sigma = d\sigma(Q, \mu_R, \mu_F) \qquad \sigma_{t\bar{t}b\bar{t}} \propto \alpha_s^4(\mu_R)$$

Theory uncertainty associated with scale variation

- 75-80% at LO
- 20-30% at NLO

Higher order theory predictions are crucial!



Plot by S.Pozzorini

Scattering Amplitudes at Higher orders

Accurate theory predictions $\equiv$ calculation of higher perturbative orders in α_s

$\Rightarrow$ LO is not enough!

$\Rightarrow$ NLO is absolutely necessary for reliable estimates of theory uncertainty.

$\Rightarrow$ NNLO required to match the exp. accuracy and test the convergence of the series

Already the complexity of 1-loop scattering amplitudes grows very fast

$\star$ e.g. $gg \to t\bar{t}b\bar{b}$ @ NLO $\sim \mathcal{O}(10^3)$ Feynman diagrams

Scattering Amplitudes at Higher orders

Accurate theory predictions $\equiv$ calculation of higher perturbative orders in α_s

$\Rightarrow$ LO is not enough!

$\Rightarrow$ NLO is absolutely necessary for reliable estimates of theory uncertainty.

$\Rightarrow$ NNLO required to match the exp. accuracy and test the convergence of the series

Already the complexity of 1-loop scattering amplitudes grows very fast

$\star$ e.g. $gg \to t\bar{t}b\bar{b}$ @ NLO $\sim \mathcal{O}(10^3)$ Feynman diagrams

Automation of 1-loop Amplitudes

opened the door to huge variety of scattering processes at 1-loop

Scattering Amplitudes at Higher orders

Accurate theory predictions ≡ calculation of higher perturbative orders in α_s

⇒ LO is not enough!

⇒ NLO is absolutely necessary for reliable estimates of theory uncertainty.

⇒ NNLO required to match the exp. accuracy and test the convergence of the series

Already the complexity of 1-loop scattering amplitudes grows very fast

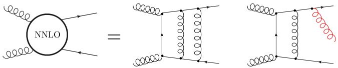
⋆ e.g. $gg \to t\bar{t}b\bar{b}$ @ NLO $\sim \mathcal{O}(10^3)$ Feynman diagrams

Automation of 1-loop Amplitudes

opened the door to huge variety of scattering processes at 1-loop

Yet there are challenges

- Technical limitations for high particle multiplicity. Need for efficient generators
- NLO amplitudes in NNLO calculation: real emission



numerical stability challenging in deep IR regions

Scattering Amplitudes at Higher orders

Accurate theory predictions $\equiv$ calculation of higher perturbative orders in α_s

$\Rightarrow$ LO is not enough!

$\Rightarrow$ NLO is absolutely necessary for reliable estimates of theory uncertainty.

$\Rightarrow$ NNLO required to match the exp. accuracy and test the convergence of the series

Already the complexity of 1-loop scattering amplitudes grows very fast

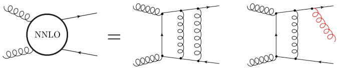
$\star$ e.g. $gg \to t\bar{t}b\bar{b}$ @ NLO $\sim \mathcal{O}(10^3)$ Feynman diagrams

Automation of 1-loop Amplitudes

opened the door to huge variety of scattering processes at 1-loop

Yet there are challenges

- Technical limitations for high particle multiplicity. Need for efficient generators
- NLO amplitudes in NNLO calculation: real emission



numerical stability challenging in deep IR regions

This Talk

OpenLoops

- Fully automated numerical algorithm for tree and one-loop amplitudes **[Cascioli, Lindert, Maierhöfer, Pozzorini]**
- hybrid tree-loop recursion $\Rightarrow$ very high speed
- NLO QCD and NLO EW corrections are fully implemented

• Mathematica for process generation and analytic/algebraic manipulation

• `Fortan` 90 for the numerical computation

• `C++` and `Phyton` for the interfaces

Successful applications:

- NLO calculations with up to $\mathcal{O}(10^5)$ loop diagrams/channel, e.g. $pp \to t\bar{t}+$ 3 jets
- used in several NNLO calculations e.g. $(p\,p \to V_1\,V_2,\; H\,H)$, $V_i = \gamma, Z, W$
- interfaced to Montecarlos: Sherpa, Powheg, Herwig, Whizard, Munich, Matrix

Publicly available at openloops.hepforge.org

OpenLoops: Tree Level Algorithm

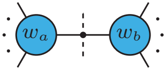
Tree level and one-loop amplitudes as sums of individual Feynman diagrams

$$\mathcal{M}_l = \sum_d \mathcal{M}_l^{(d)}, \quad l = 0, 1$$

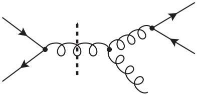
$\mathcal{M}_l^{(d)}$ factorizes into a colour factor and a colour stripped amplitude

$$\mathcal{M}_l^{(d)} = \mathcal{C}_l^{(d)} \, \mathcal{A}_l^{(d)}$$

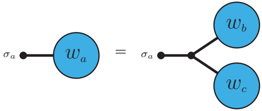
Each $\mathcal{A}_0^{(d)}$ is split into subtrees by cutting an internal line



for example



Numerical merging of subtrees performed **recursively** . $X^{\alpha}_{\beta\gamma}$ universal kernels



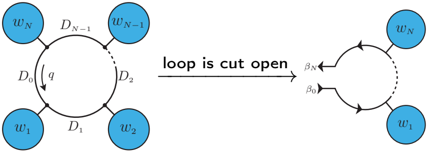
$$\Rightarrow w_a^{\alpha}(k_a, h_a) = \frac{X^{\alpha}_{\beta\gamma}(k_b, k_c)}{k_a^2 - m_a^2} w_b^{\beta}(k_b, h_b) w_c^{\gamma}(k_c, h_c)$$

OpenLoops: One Loop Algorithm

Colour stripped one-loop amplitudes $\mathcal{A}_1^{(d)}$: tensor coefficients $\times$ tensor integral

$$\mathcal{A}_1^d = \int \mathrm{d}^D q \frac{\mathcal{N}(\mathcal{I}_N; q)}{D_0 D_1 \cdots D_{N-1}} = \sum_{r=0}^{N} \mathcal{N}_{\mu_1 \cdots \mu_r} \underbrace{\int \mathrm{d}^D q \frac{q^{\mu_1} \cdots q^{\mu_r}}{D_0 D_1 \cdots D_{N-1}}}_{\text{fed to a tensor integral reduction library}}$$

The $\mathcal{N}_{\mu_1 \cdots \mu_r}$ are built numerically in recursive way:**hybrid tree-loop recursion**

$\mathcal{A}_1^d =$ 

Master Formula: $\mathcal{N}_\alpha^\beta(\mathcal{I}_N; q) = \mathcal{X}_{\gamma\delta}^\beta(q)\, w^\delta(i_N)\, \mathcal{N}_\alpha^\gamma(\mathcal{I}_{N-1}, q)$

- $\mathcal{X}_{\gamma\delta}^\beta(q) \sim$ Feynman rules of the theory (full SM implemented)
- q dependence fully retained $\Rightarrow$ **Model and process independent algorithm**

OpenLoops: One Loop Algorithm

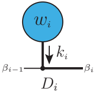
Observation: numerator $\mathcal{N}_{\alpha}^{\beta}(\mathcal{I}_N; q)$ **factorizes into segments** $\mathcal{S}(q,h)$ ($h \to$ helicity)

$$\left[\mathcal{S}_i(q,h_i)\right]_{\beta_{i-1}}^{\beta_i} \equiv \mathcal{X}_{\delta\beta_{i-1}}^{\beta_i}(q)\, w_i^{\delta}(h_i)$$

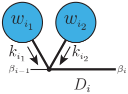
$\Rightarrow$ **segment**= external subtree + 1-loop vertex + propagator

$$\mathcal{N}_{\alpha}^{\beta}(\mathcal{I}_N;q;h) = \left[\prod_{i=i}^{N} \mathcal{S}_i(q,h_i)\right]_{\alpha}^{\beta} = \left[\mathcal{S}_1(q,h_1)\right]_{\beta_0}^{\beta} \left[\mathcal{S}_1(q,h_1)\right]_{\beta_1}^{\beta_0} \dots \left[\mathcal{S}_N(q,h_N)\right]_{\alpha}^{\beta_N}$$

e.g.: in the SM a **segment** is a $q-$polynomial of rank $r \leq 1$

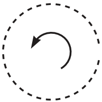
3-point segment $\left[\mathcal{S}_i(q,h_i)\right]_{\beta_{i-1}}^{\beta_i}$ 

$= \left[\mathcal{Y}_{\sigma_i} + Z_{\mu,\sigma_i} q^{\mu}\right]_{\beta_{i-1}}^{\beta_i} w_i^{\sigma_i}(k_i,h_i)$

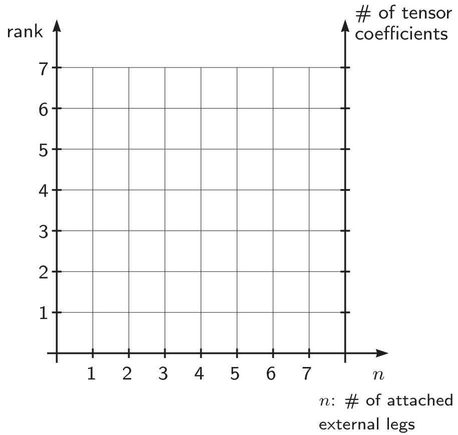
4-point segment $\left[\mathcal{S}_i(q,h_i)\right]_{\beta_{i-1}}^{\beta_i}$ 

$= \left[\mathcal{Y}_{\sigma_1,\sigma_2}\right]_{\beta_{i-1}}^{\beta_i} w_{i1}^{\sigma_1}(k_{i_1},h_{i_1}) w_{i_2}^{\sigma_2}(k_{i_2},h_{i_2})$

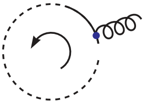
Flow of the Algorithm in OpenLoops1



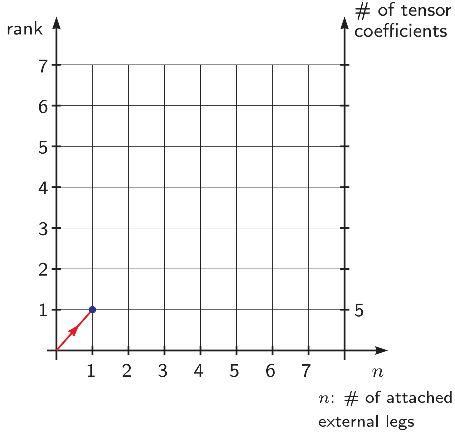
Example of a 7g diagram



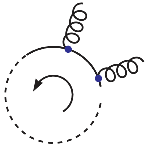
Flow of the Algorithm in OpenLoops1



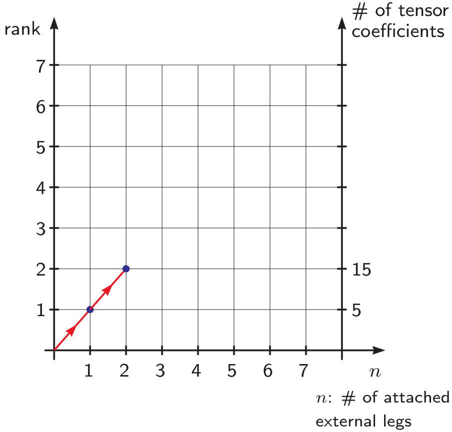
Example of a 7g diagram



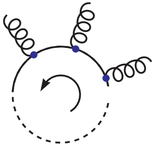
Flow of the Algorithm in OpenLoops1



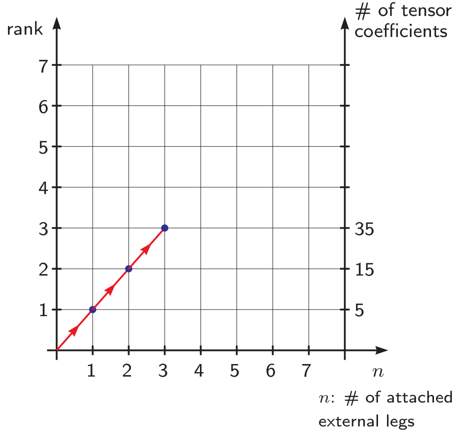
Example of a 7g diagram



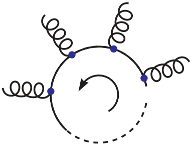
Flow of the Algorithm in OpenLoops1



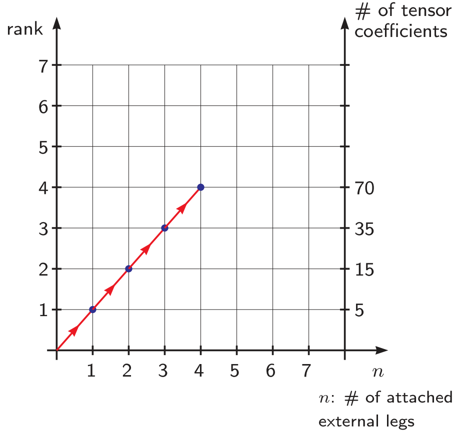
Example of a 7g diagram



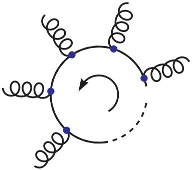
Flow of the Algorithm in OpenLoops1



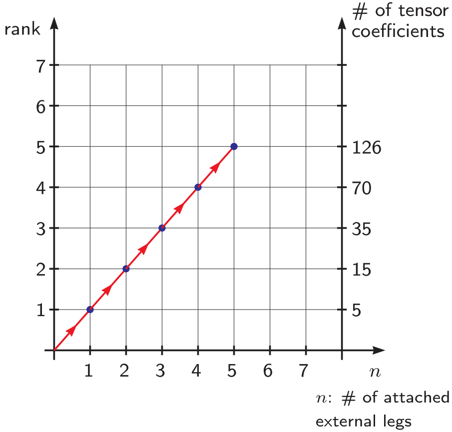
Example of a 7g diagram



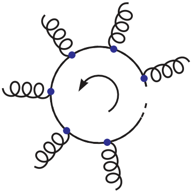
Flow of the Algorithm in OpenLoops1



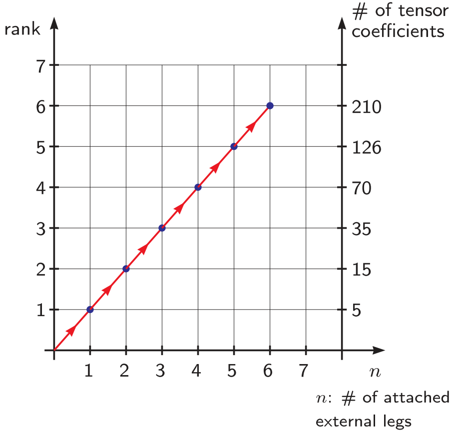
Example of a 7g diagram



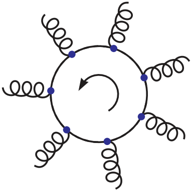
Flow of the Algorithm in OpenLoops1



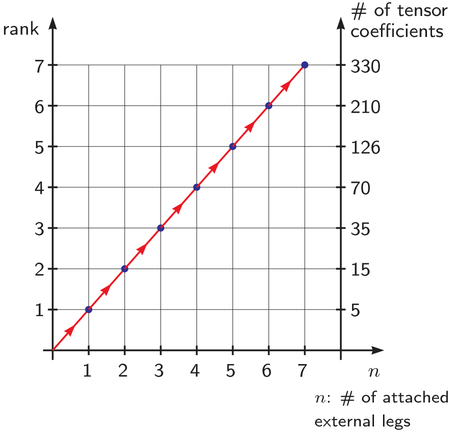
Example of a 7g diagram



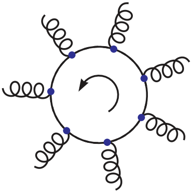
Flow of the Algorithm in OpenLoops1



Example of a 7g diagram

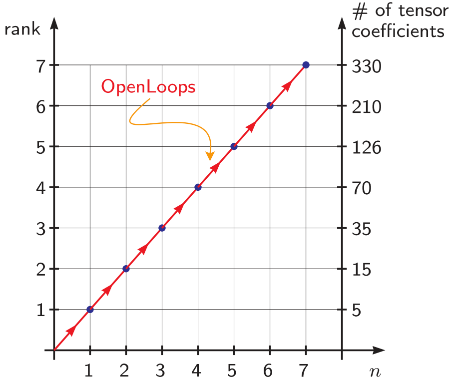


Flow of the Algorithm in OpenLoops1



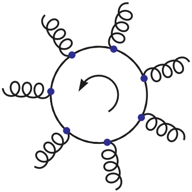
Example of a 7g diagram

complexity grows exponentially with tensor rank



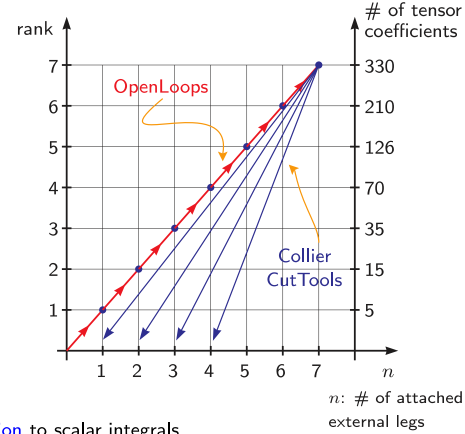
n: # of attached external legs

Flow of the Algorithm in OpenLoops1



Example of a 7g diagram

complexity grows exponentially with tensor rank



Numerical tensor integral reduction to scalar integrals

OpenLoops 2

on-the-fly Reduction

[F.B., Pozzorini, Zoller 1710.1145]

- **exploits factorization properties of OpenLoops recursion**
- **performs on-the-fly integrand reduction during amplitude construction**
- **keeps the rank ≤ 2 at any stage of the calculation**

On-the-fly reduction of open loops

We exploit the factorization of $\mathcal{N}(q)$ into segments $\rightarrow$ **integrand reduction**

$$\frac{\mathcal{N}(q)}{D_0 D_1 \dots D_{N_1}} = \frac{S_1(q) S_2(q)}{D_0 D_1 D_2 D_3} \times \frac{\prod_{i=3}^{N} S_i(q)}{D_4 \dots D_{N-1}}$$

indipendent of future segments

Valid for ≥ 3-point function of rank $r \geq 2$ **[del Aguila, Pittau '05]**

$$q^\mu q^\nu = A^{\mu\nu} + B^{\mu\nu}_\lambda \, q^\lambda = \left[A^{\mu\nu}_{-1} + A^{\mu\nu}_0 \mathbf{D_0} \right] + \left[B^{\mu\nu}_{-1,\lambda} + \sum_{k=0}^{3} B^{\mu\nu}_{k,\lambda} \mathbf{D_k} \right] q^\lambda$$

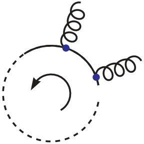
- $r = 2$ monomials are reduced to $r = 1$ on-the-fly, i.e. at any OL construction step

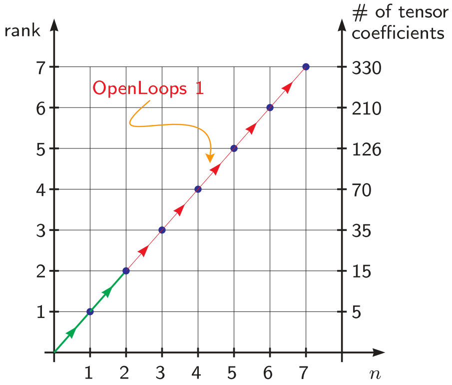
$\Rightarrow$ complexity associated with high tensor remains always low!

- $q-$dependence reconstructed in terms of denominators$\Rightarrow$ pinched subtopologies
- q^μ decomposed onto a basis of lightlike momenta l_i^μ, $i = 1, \dots, 4$

Flow of the Algorithm in OpenLoops2

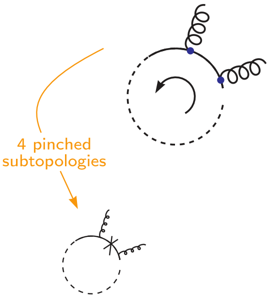
Example of a 7g diagram

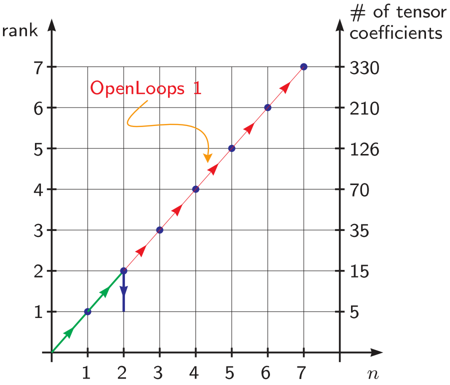




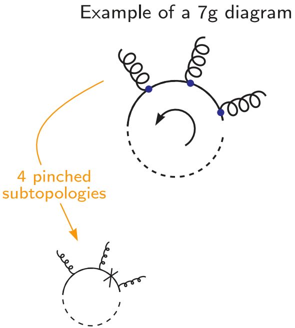
Flow of the Algorithm in OpenLoops2

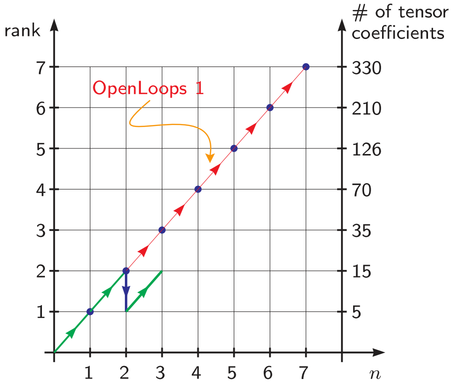
Example of a 7g diagram





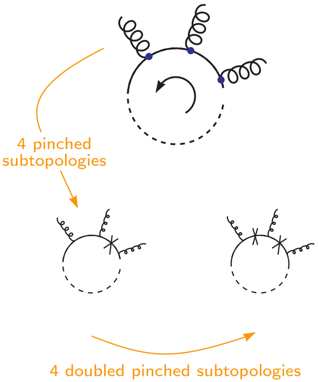
Flow of the Algorithm in OpenLoops2

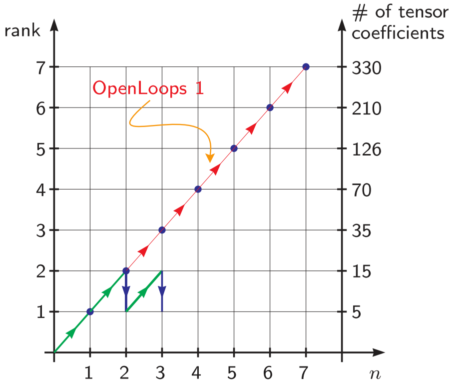




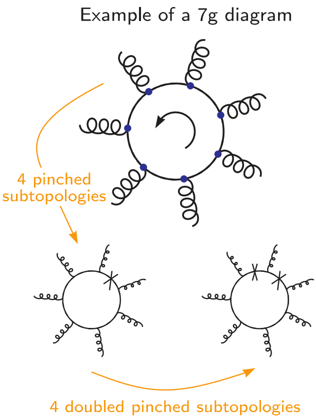
Flow of the Algorithm in OpenLoops2

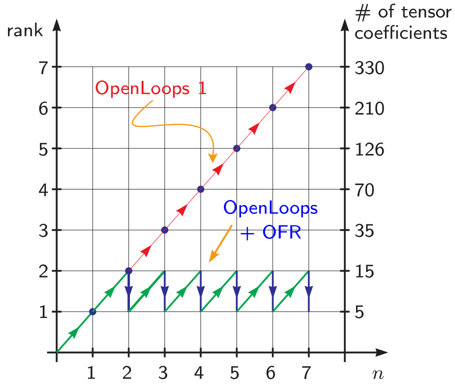
Example of a 7g diagram





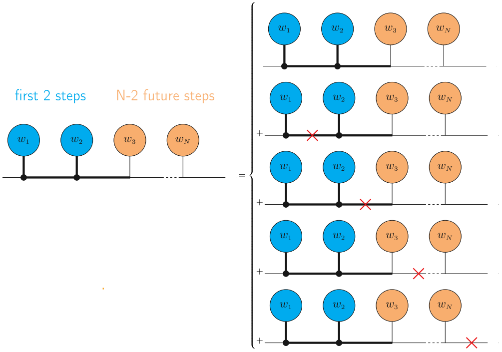
Flow of the Algorithm in OpenLoops2





complexity associated with tensor rank remains small!

Potential Problem n-1: Huge proliferation of subtopologies



naive reduction + recursion

potentially highly inefficient

D_1 pinched subtopology

future steps performed for (1+4) subtopologies

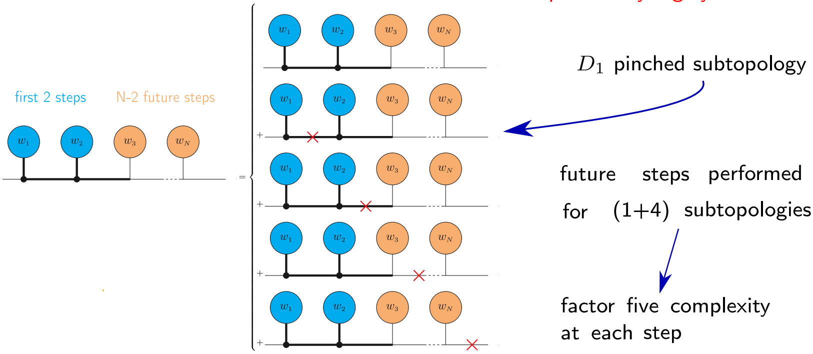
factor five complexity at each step

Example: start with a $r = 4$ pentagon $\Rightarrow$ end up with 689 subtopologies

Potential Problem n-1: Huge proliferation of subtopologies

naive reduction + recursion

potentially highly inefficient



future steps performed for (1+4) subtopologies

factor five complexity at each step

Example: start with a $r = 4$ pentagon $\Rightarrow$ end up with 689 subtopologies

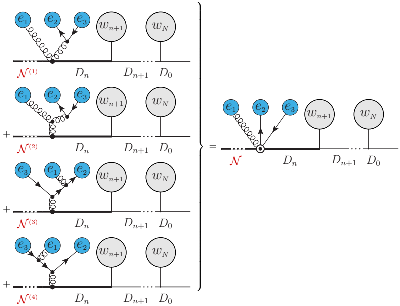
Solution: Diagrams Merging $\sim$ Sophisticated diagrams bookkeeping

The OpenLoops Diagrams Merging

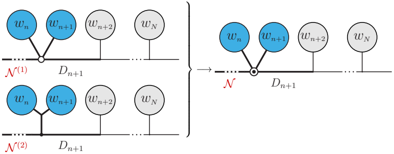
Partially constructed and helicity-summed open loops are merged. Criteria:

- same topology $\{D_0, \ldots, D_{N-1}\}$
- same future segments $S_{n+1}, \ldots, S_N$

Recursive steps for $S_{n+1}, \ldots, S_N$ only on the merged object $\Rightarrow$ # operations reduced



Merge all pinched and unpinched diagrams with same topology and future segments



- No extra cost for pinched topologies
- Significant efficiency improvement

Potential Problem n-2: Numerical Instabilities

$$q^\mu q^\nu = A_{-1}^{\mu\nu} + A_0^{\mu\nu}\mathbf{D_0} + \left[B_{-1,\lambda}^{\mu\nu} + \sum_{k=0}^{3} B_{k,\lambda}^{\mu\nu}\mathbf{D_k}\right] q^\lambda$$

q^μ decomposed onto a basis of light-like momenta l_i^μ computed out of external p_1^μ, p_2^μ

$$l_1^\mu = p_1^\mu - \alpha_1 p_2^\mu, \qquad l_3^\mu = \bar{v}(l_1)\gamma^\mu \left(\frac{1-\gamma^5}{2}\right) u(l_2), \qquad l_{1,2}\cdot l_{3,4} = 0$$

$$l_2^\mu = p_2^\mu - \alpha_2 p_1^\mu, \qquad l_4^\mu = \bar{v}(l_2)\gamma^\mu \left(\frac{1-\gamma^5}{2}\right) u(l_1), \qquad l_1\cdot l_2 = -\frac{l_3\cdot l_4}{4}$$

$$q^\mu = \frac{2}{\gamma}\left((q\cdot l_2)\, l_1^\mu + (q\cdot l_1)\, l_2^\mu\right) - \frac{1}{2\gamma}\left((q\cdot l_4)\, l_3^\mu + (q\cdot l_3)\, l_4^\mu\right), \quad \gamma \propto \Delta(p_1,p_2)$$

$$\Rightarrow A^{\mu\nu} = \gamma^{-1} a^{\mu\nu}, \quad B_{i,\lambda}^{\mu\nu} = \gamma^{-2}\left[b_{i,\lambda}^{(-2)}\right]^{\mu\nu} + \gamma^{-1}\left[b_{i,\lambda}^{(-1)}\right]^{\mu\nu}$$

Spurious singularities for $\Delta(p_1,p_2) \to 0 \Rightarrow$ Severe numerical instabilities!

Solution to numerical instabilities

Box Reduction: avoid small rank-two Gram Determinant via the permutation

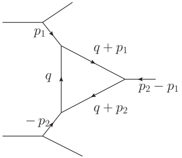
$$\{D_1, D_2, D_3\} \to \{D_{i_1}, D_{i_2}, D_{i_3}\}$$ basis built out of only two momenta!

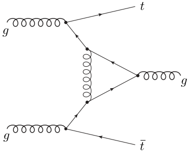
The criterion for the choice of the reduction basis is such that p_{i_1} and p_{i_2} give

$$\frac{|\Delta_{i_1 i_2}|}{Q^4_{i_1 i_2}} = \max\left\{\frac{|\Delta_{12}|}{Q^4_{12}}, \frac{|\Delta_{13}|}{Q^4_{13}}, \frac{|\Delta_{23}|}{Q^4_{23}}\right\}, \quad Q^2_{ij} = \max(|p_i \cdot p_j|),\ i,j = 1,2,3.$$

Triangle Reduction: excluding IR regions, small Δ_{12} arise from topology like

$$p_1^2 = -p^2,\ \ p_2^2 = -p^2(1+\delta)$$
$$(p_2 - p_1)^2 = 0$$
$$\Delta = \frac{p^4}{2}\delta^2,\ \gamma = -p^2\delta^2$$





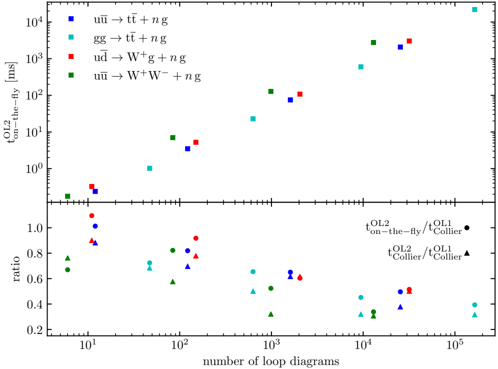
In this special configuration we use analytic integral reduction formulas.

If $\delta \ll 1$ perform dedicated "all-orders" δ-expansions.

Performances of OpenLoops2

Runtime per phase space point for the calculation of one-loop amplitudes.

t quarks and $W^{\pm}$ bosons are taken on-shell.



Speed has been measured on a single Intel i7-4790K core with gfortran-4.8.5.

# of Feynman diagrams ranges from $\mathcal{O}(1)$ to $\mathcal{O}(10^5)$

OpenLoops2 is up to a factor 3 faster than OpenLoops1!

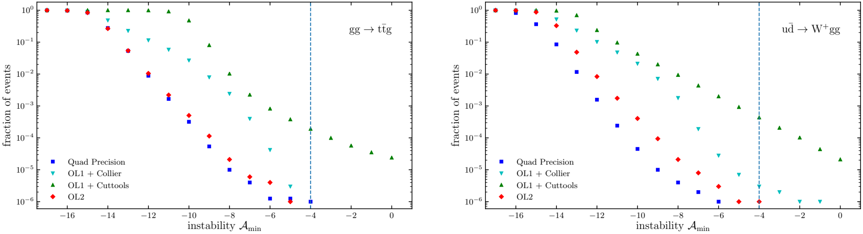
Numerical Stability of OpenLoops2

$$\mathcal{A}_{min} = \log_{10}\left|\frac{\mathcal{W}_{qp} - \mathcal{W}_{dp}}{\mathcal{W}_{min}}\right|, \qquad \mathcal{W}_{min} = \min\{|\mathcal{W}_{dp}|, |\mathcal{W}_{qp}|\}\ .$$

$\Rightarrow$ probability of relative accuracy $\mathcal{A}$ or less $\sim$ # of correct digits of dp evaluation

$\mathcal{W}_{qp}$ used as a benchmark has been obtained with $\mathrm{OpenLoops} + \mathrm{Cuttools}$.

$\star$ hard kinematics: $p_T > 50\, GeV$ and $\Delta R_{ij} > 0.5$ for final state QCD partons



- Orders of magnitude improvements wrt `Cuttools` dp. Very significant wrt `Collier`
- Behavior in the tails crucial for real life applications

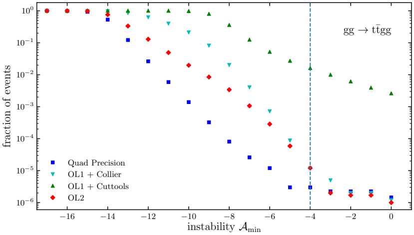
Numerical Stability of OpenLoops2

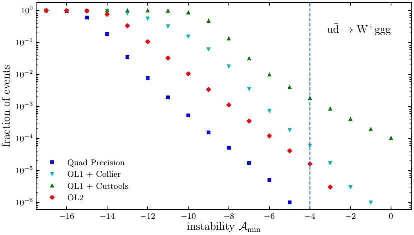
$$\mathcal{A}_{min} = \log_{10}\left|\frac{\mathcal{W}_{qp} - \mathcal{W}_{dp}}{\mathcal{W}_{min}}\right|, \qquad \mathcal{W}_{min} = \min\left\{|\mathcal{W}_{dp}|, |\mathcal{W}_{qp}|\right\} .$$

$\Rightarrow$ probability of relative accuracy $\mathcal{A}$ or less $\sim$ # of correct digits of dp evaluation

$\mathcal{W}_{qp}$ used as a benchmark has been obtained with $\mathrm{OpenLoops} + \mathrm{Cuttools}$.

$\star$ hard kinematics: $p_T > 50\,GeV$ and $\Delta R_{ij} > 0.5$ for final state QCD partons





- Orders of magnitude improvements wrt `Cuttools` dp. Very significant wrt `Collier`
- Behavior in the tails crucial for real life applications

Summary and Outlook

Summary

- new algorithm: construction + reduction of 1-loop amplitudes in one framework
- reduction of complexity at all stages of the calculation $r \leq 2$
- it makes possible to perform dedicated stability studies
  - simple targeted expansions
  - permutation tricks in the reduction $\Rightarrow$ excellent stability in hard regions
- code available in both double and quad precision. High speed up for the latter

Summary and Outlook

Summary

- new algorithm: construction + reduction of 1-loop amplitudes in one framework
- reduction of complexity at all stages of the calculation $r \leq 2$
- it makes possible to perform dedicated stability studies
  - simple targeted expansions
  - permutation tricks in the reduction $\Rightarrow$ excellent stability in hard regions
- code available in both double and quad precision. High speed up for the latter

Outlook

- **Coming soon**: new **OpenLoops2** code release. Stay tuned!
- Implementation of adaptive "all-orders" $\Delta-$expansions for full QP accuracy.
- first real life application to challenging pheno-projects: $2 \to 5$ @ NLO
- Investigations of IR/unresolved regions $\Rightarrow$ numerical stability very promising

$\Rightarrow$ crucial for NNLO applications