PAUL SCHERRER INSTITUT

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Physik-Institut

NLO QCD corrections to Higgs boson pair production via gluon fusion

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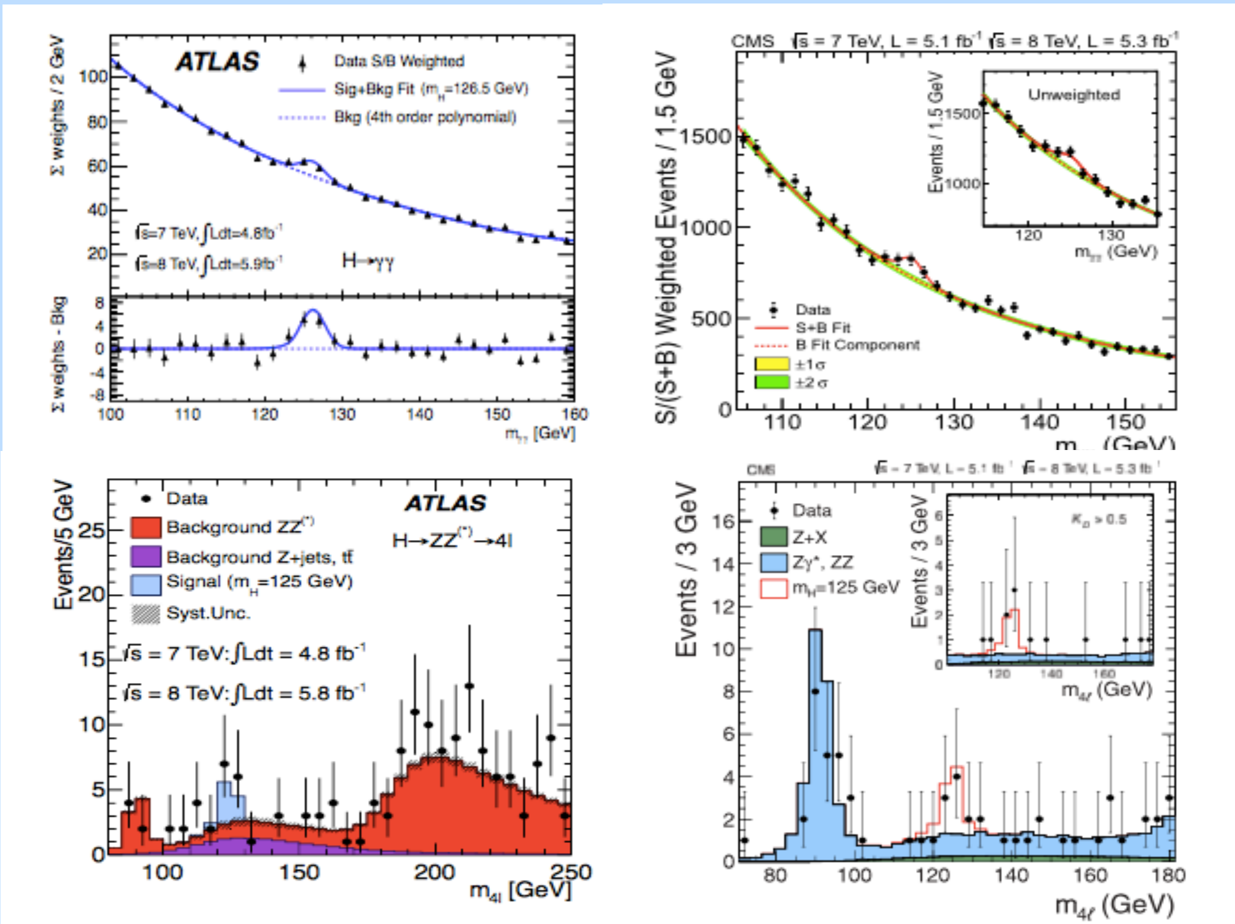
Outline

- Motivation
- Objective
- Previous work
- NLO Cross section
  - Virtual Corrections
  - Real Corrections
- Numerical analysis
- Next steps
- Conclusions

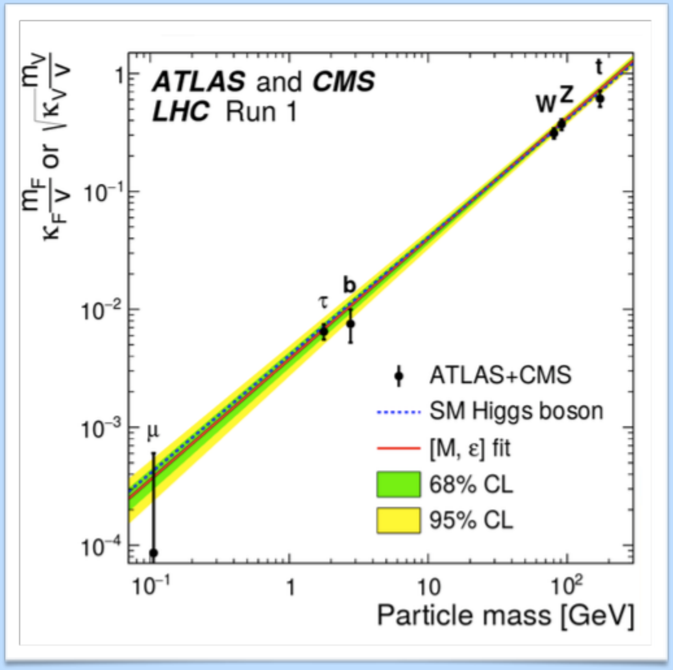
Motivation

Detection of a Higgs boson with a mass ~ 125 GeV



Motivation

- Higgs mass, coupling strengths, spin and CP already determined
- self-coupling strength still unknown



$$V(\phi) = \frac{\lambda}{2}\left\{|\phi|^2 - \frac{v^2}{2}\right\}^2$$

$$\lambda = \frac{M_H^2}{2v^2}$$

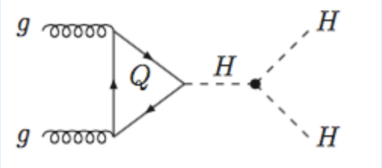
$$\lambda_{H^3} = 3\frac{M_H^2}{v}$$

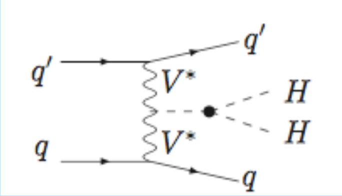
$$\lambda_{H^4} = 3\frac{M_H^2}{v^2}$$

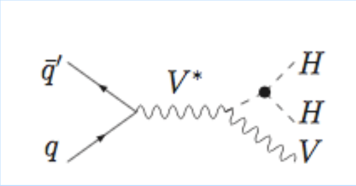
Motivation

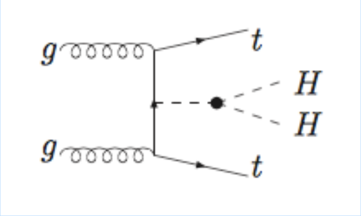
Higgs boson pair production

Production channels

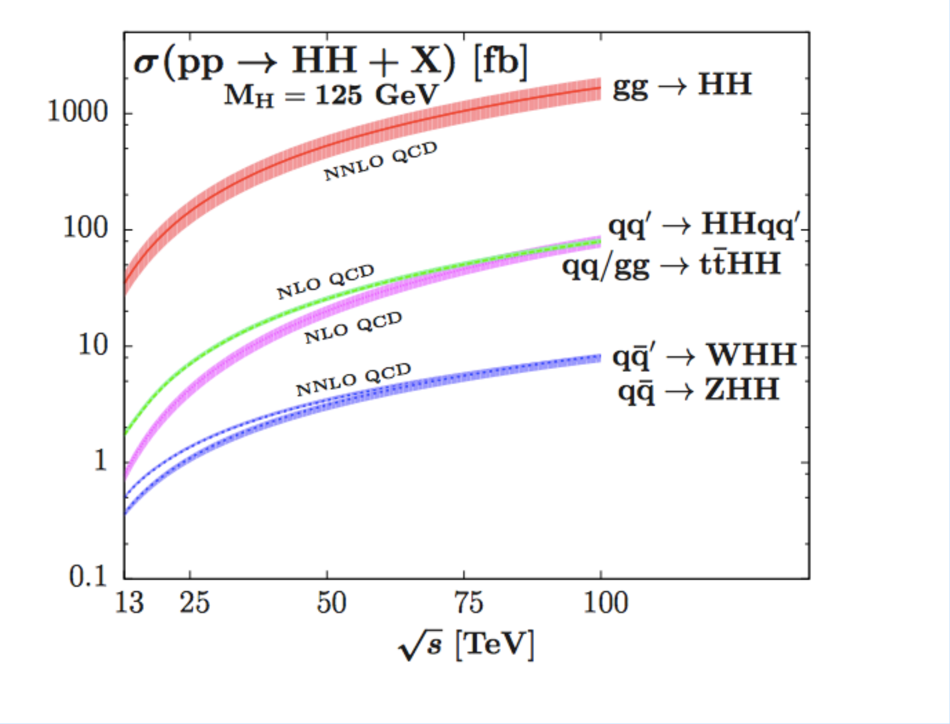








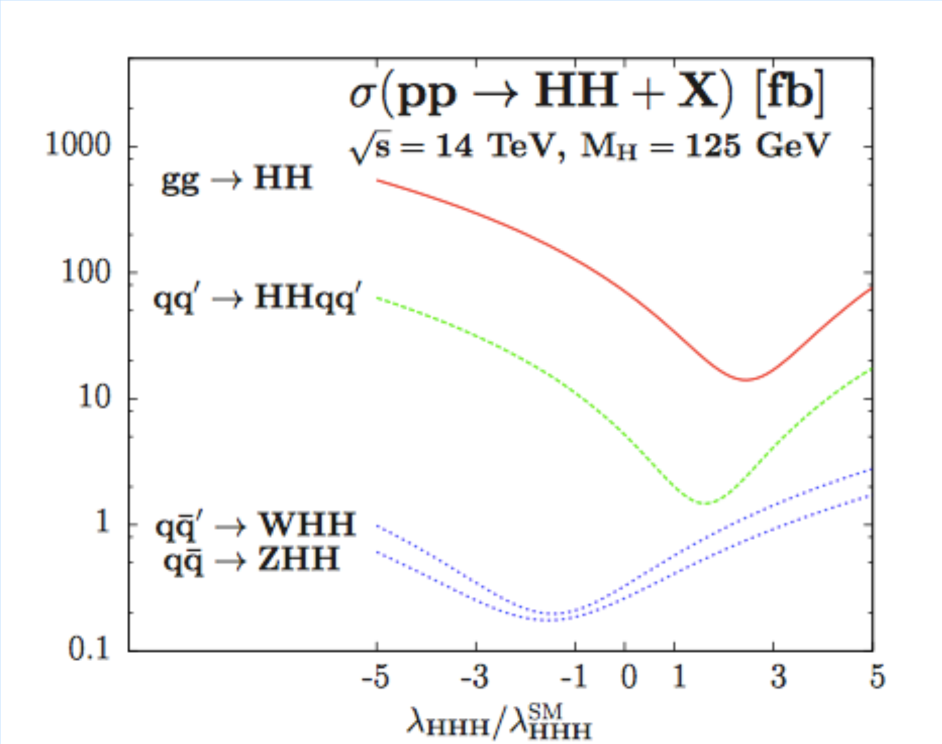
Cross sections



Baglio, Djouadi, Quevillon

Motivation

Uncertainties:



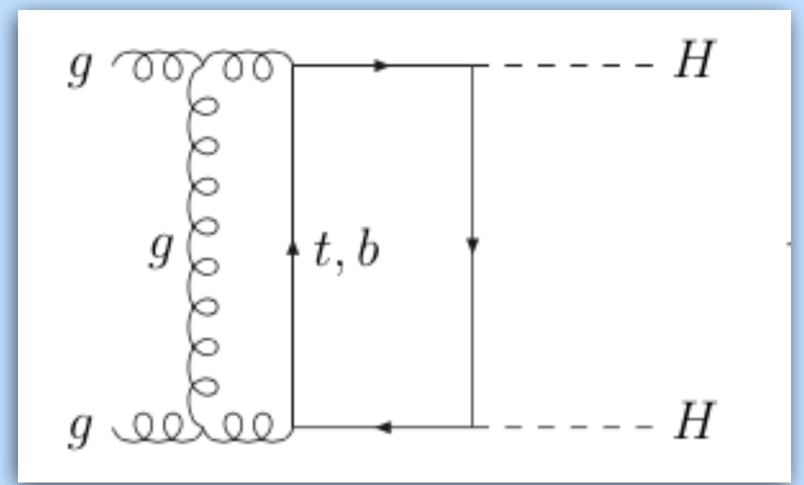
$$gg \to HH : \frac{\Delta\sigma}{\sigma} \sim -\frac{\Delta\lambda}{\lambda}$$

Baglio, Djouadi, Gröber,
Mühlleitner, Quevillon, Spira

Objective

Gluon fusion gg → HH: loop induced

Complete calculation of the NLO QCD corrections (2-loop) considering the top- and bottom mass dependences in the context of the Standard Model → 2-loop integrals with 3 kinematical parameter ratios



Previous work

- Virtual & real (N)NLO QCD corrections in large top mass limit: ~100%

Dawson,Dittmaier,Spira
de Florian,Mazzitelli
Grigo,Melnikov,Steinhauser

- Large top mass expansion: ~ ±10%

$$\sigma = \sigma_0 + \frac{\sigma_1}{m_t^2} + \cdots + \frac{\sigma_4}{m_t^8}$$

Grigo, Hoff, Melnikov, Steinhauser

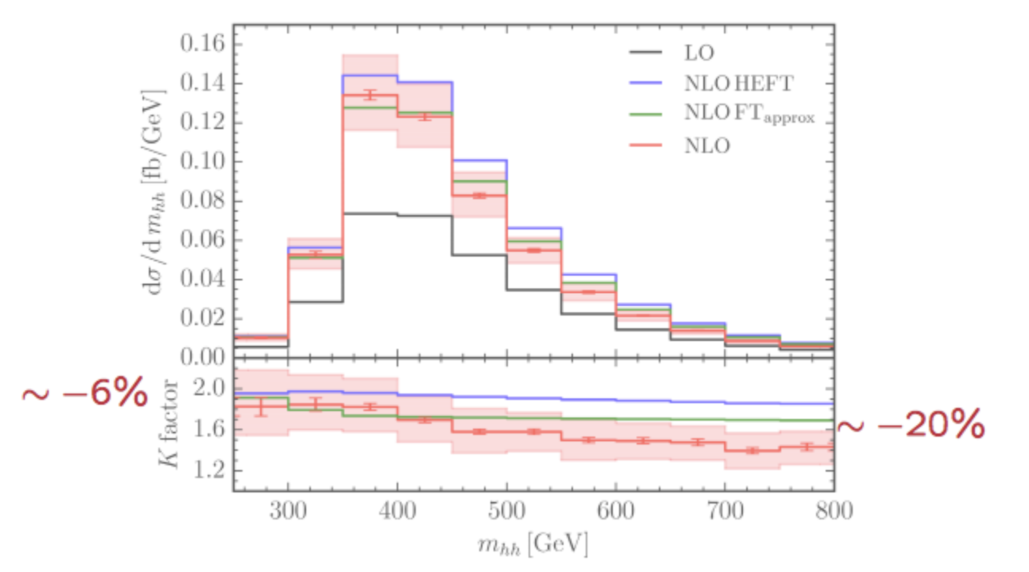
- NLO mass effects of the real NLO correction alone

~ -10 %

Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro

Previous work

- NLO QCD corrections including the full top mass dependence



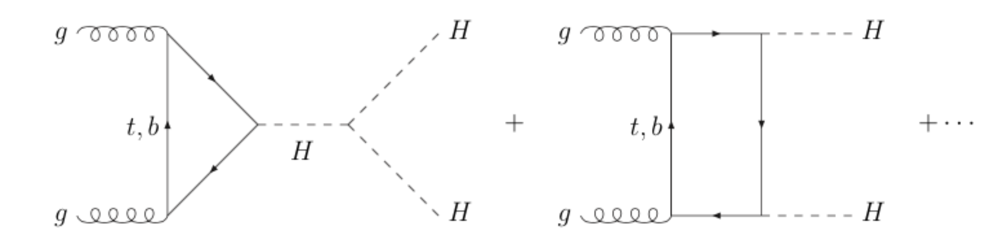
Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke

→ - 13.7 % mass effects

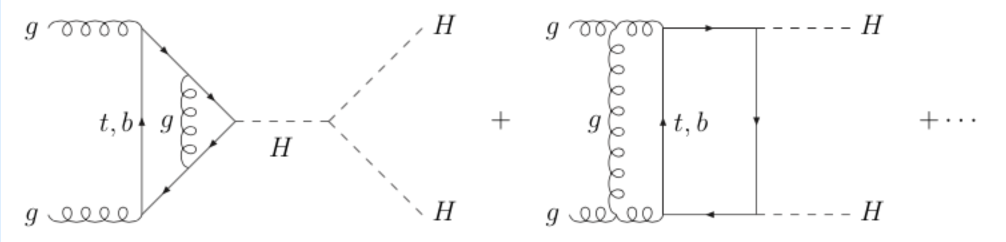
NLO Corrections

$$\sigma_{\text{NLO}}(pp \to HH + X) = \sigma_{\text{LO}} + \Delta\sigma_{\text{virt}} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}},$$

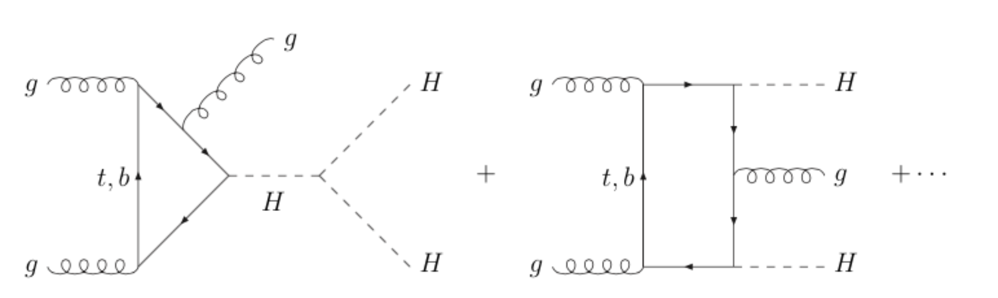
σ_{LO}:



$\Delta\sigma_{\text{virt}}$:



$\Delta\sigma_{\text{ij}}$:



NLO Corrections

$$\sigma_{\mathrm{NLO}}(pp \to HH + X) = \sigma_{\mathrm{LO}} + \Delta\sigma_{\mathrm{virt}} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}},$$

$$\begin{aligned}
\sigma_{\mathrm{LO}} &= \int_{\tau_0}^{1} d\tau \, \frac{d\mathcal{L}^{gg}}{d\tau} \, \hat{\sigma}_{\mathrm{LO}}(Q^2 = \tau s) \\
\Delta\sigma_{\mathrm{virt}} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \, \frac{d\mathcal{L}^{gg}}{d\tau} \, \hat{\sigma}_{\mathrm{LO}}(Q^2 = \tau s) \; C \\
\Delta\sigma_{gg} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \, \frac{d\mathcal{L}^{gg}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z} \, \hat{\sigma}_{\mathrm{LO}}(Q^2 = z\tau s) \left\{ -z P_{gg}(z) \log \frac{M^2}{\tau s} \right. \\
&\qquad \left. + d_{gg}(z) + 6[1 + z^4 + (1-z)^4] \left(\frac{\log(1-z)}{1-z} \right)_+ \right\} \\
\Delta\sigma_{gq} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \sum_{q,\bar{q}} \frac{d\mathcal{L}^{gq}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z} \, \hat{\sigma}_{\mathrm{LO}}(Q^2 = z\tau s) \left\{ -\frac{z}{2} P_{gq}(z) \log \frac{M^2}{\tau s (1-z)^2} \right. \\
&\qquad \left. + d_{gq}(z) \right\} \\
\Delta\sigma_{q\bar{q}} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \sum_{q} \frac{d\mathcal{L}^{q\bar{q}}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z} \, \hat{\sigma}_{\mathrm{LO}}(Q^2 = z\tau s) \; d_{q\bar{q}}(z)
\end{aligned}$$

$$C \to \pi^2 + \frac{11}{2} + C_{\triangle\triangle}, \qquad d_{gg} \to -\frac{11}{2}(1-z)^3, \qquad d_{gq} \to \frac{2}{3}z^2 - (1-z)^2, \qquad d_{q\bar{q}} \to \frac{32}{27}(1-z)^3$$

Virtual Corrections

Set-up

- Associate the 47 two-loop box diagrams to similar topologies
- Two independent calculations in the collaboration
- Use dimensional regularisation: $D = 4 - 2\varepsilon$
- Perform Feynman parametrisation → 6-dimensional integrals

Divergences

- Extraction of the ultraviolet divergences of the matrix elements using endpoint subtractions of the 6-dimensional Feynman integrals

$$\int_0^1 dx\, \frac{f(x)}{(1-x)^{1-\epsilon}} = \int_0^1 dx\, \frac{f(1)}{(1-x)^{1-\epsilon}} + \int_0^1 dx\, \frac{f(x)-f(1)}{(1-x)^{1-\epsilon}}$$

$$= \frac{f(1)}{\epsilon} + \int_0^1 dx\, \frac{f(x)-f(1)}{1-x} + \mathcal{O}(\epsilon)$$

- Extraction of the infrared and collinear divergences using a proper subtraction of the integrand

- Integration by parts due to numerical instabilities at the thresholds

$M_{HH}^2 > 4m_Q^2 \Rightarrow m_Q^2 \to m_Q^2(1-i\bar{\epsilon})$ wi $\bar{\epsilon} \ll 1$

$$\int_0^1 dx \frac{f(x)}{(a+bx)^3} = \frac{f(0)}{2a^2} - \frac{f(1)}{2(a+b)^2} + \int_0^1 dx \frac{f'(x)}{2(a+bx)^2}$$

different $\bar{\epsilon} \longrightarrow$ Richardson extrapolation $\longrightarrow \bar{\epsilon} = 0$ (NWA)

Virtual Corrections

Renormalisation

α_s and m_q need to be renormalised

→ α_s in $\overline{MS}$ with $N_F = 5$

→ m_q on shell

$$\delta\sigma = \delta\alpha_s \frac{\delta\sigma_{LO}}{\delta\alpha_s} + \delta m_t \frac{\delta\sigma_{LO}}{\delta m_t}$$

Subtraction of the heavy-top limit → virtual mass effects only (infrared finite)

$$\Delta C_{mass} = C^0 - C^0_{HTL}$$

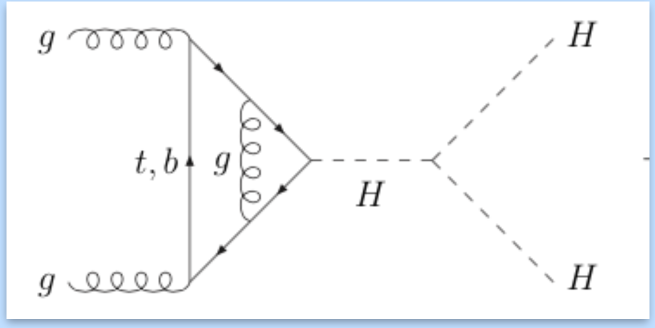
Adding back the results of HPAIR (heavy-top limit)

$$C = \underbrace{C_{HTL}}_{\text{HPAIR}} + \Delta C_{mass}$$

Remaining Diagrams

Triangular diagrams

← single Higgs case

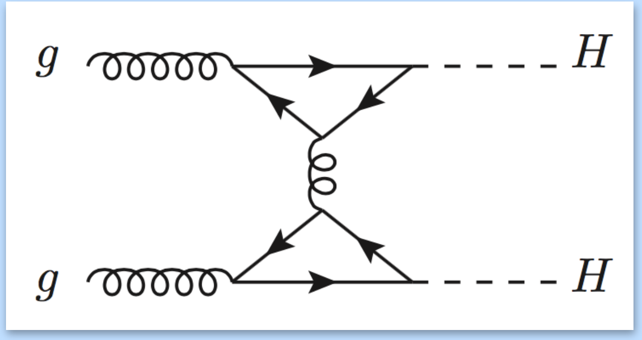


One-particle reducible diagrams

→ analytical results for $C_{\triangle\triangle}$

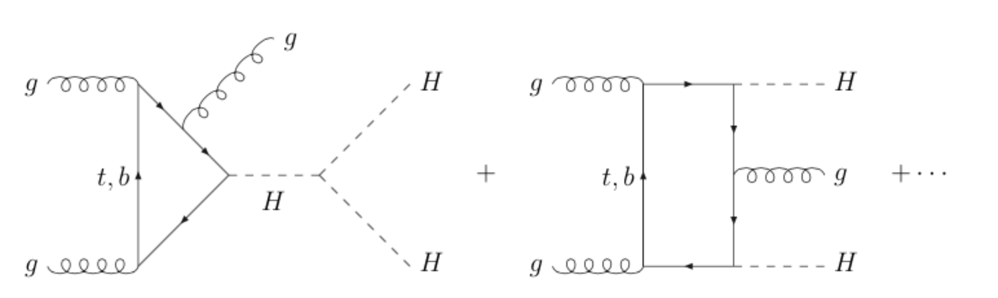
$(H \to Z\gamma)$

see e.g. Degrassi, Giardino, Gröber



Real Corrections

Processes: gg,$q\bar{q}$ → HHg; gq → HHq



- Full matrix elements generated with LoopTools
- Matrix elements in the heavy-top limit rescaled locally by massive LO matrix elements → subtracted → free of divergences
- Adding back the results of HPAIR (heavy-top limit)
- Integration of real corrections straight forward

Next steps

Grid for differential cross section $\frac{d\sigma}{dQ^2}$

- first with 50 GeV steps in Q (up to about 1 – 1.5 TeV)
- gradual refinement of the grid

Cross checks

- two independent codes using different Feynman parametrisations
- cancellation of divergencies
- comparison with heavy top limit
- comparison with full top mass dependent results of Munich group
- comparison with previously available real corrections

(Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro)

Conclusions

- Managed to calculate two-loop integral with three free parameter ratios without reduction to master integrals
- Full NLO calculation close to finalisation
- Only matter of CPU time until we get differential cross sections
- NLO mass effects expected in the 10-20% range
- Compared to Munich group:
  - completely different methods
  - no fixed masses $\longrightarrow$ easy to implement arbitrary top and Higgs masses
  - bottom-loops (presently checking stability)

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Virtual Corrections

Divergences

- Extraction of the ultraviolet divergences of the matrix elements using endpoint subtractions of the 6-dimensional Feynman integrals

$$\int_0^1 dx\, \frac{f(x)}{(1-x)^{1-\epsilon}} = \int_0^1 dx\, \frac{f(1)}{(1-x)^{1-\epsilon}} + \int_0^1 dx\, \frac{f(x)-f(1)}{(1-x)^{1-\epsilon}}$$

$$= \frac{f(1)}{\epsilon} + \int_0^1 dx\, \frac{f(x)-f(1)}{1-x} + \mathcal{O}(\epsilon)$$

- Extraction of the infrared and collinear divergences using a proper subtraction of the integrand

denominator: $N = ar^2 + br + c$ $\quad a = \mathcal{O}(\rho)$ $\quad \rho_s = \frac{\hat{s}}{m_Q^2}$

$N_0 = br + c$ $\quad b = 1 + \mathcal{O}(\rho)$

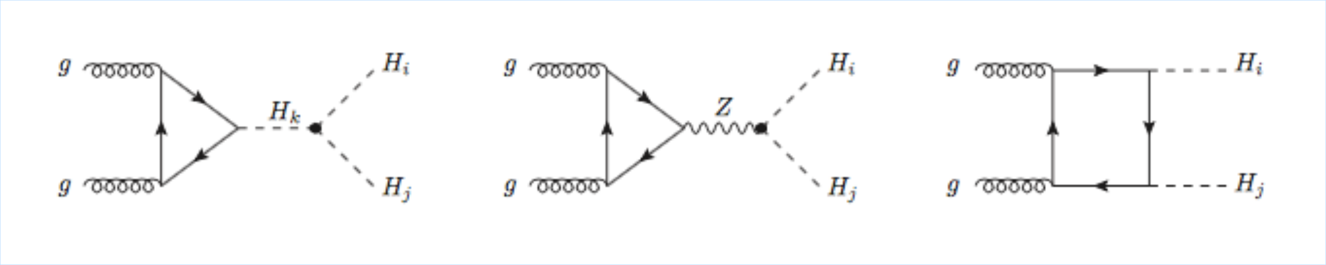
$c = -\rho_s x(1-x)(1-s)t$

$$\int_0^1 \mathrm{d}\vec{x}\mathrm{d}r \frac{rH(\vec{x},r)}{N^{3+2\epsilon}} = \int_0^1 \mathrm{d}\vec{x}\mathrm{d}r \Big\{ \underbrace{\Big(\frac{rH(\vec{x},r)}{N^{3+2\epsilon}} - \frac{rH(\vec{x},0)}{N_0^{3+2\epsilon}} \Big)}_{\text{Taylor expansion in } \epsilon} + \underbrace{\frac{rH(\vec{x},0)}{N_0^{3+2\epsilon}}}_{\text{analytical r-integration}} \Big\}$$

Next steps

Extension to BSM-Higgs scenarios

- Example 2: 2HDM $\rightarrow$ h,H,A
  pairs of different Higgs bosons
  mixing $\rightarrow$ modified Yukawa couplings
  pseudoscalar: treatment of γ_5 (e.g. 't Hooft-Veltman, Larin, Kreimer)
  resonant contributions: H $\rightarrow$ hh, AA



Numerical Analysis

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- Use Vegas for numerical integration P. Lepage
- Calculation of differential cross section $\frac{d\sigma}{dQ^2}$, $(Q^2 = m_{HH}^2)$

$$Q^2 \frac{d\Delta\sigma_{virt}}{dQ^2} = \tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{virt}(Q^2)\Big|_{\tau = \frac{Q^2}{s}}$$

partonic cross section

(seven-dimensional integrals)

- Thresholds: $m_Q^2 \to m_Q^2(1 - i\bar{\epsilon})$

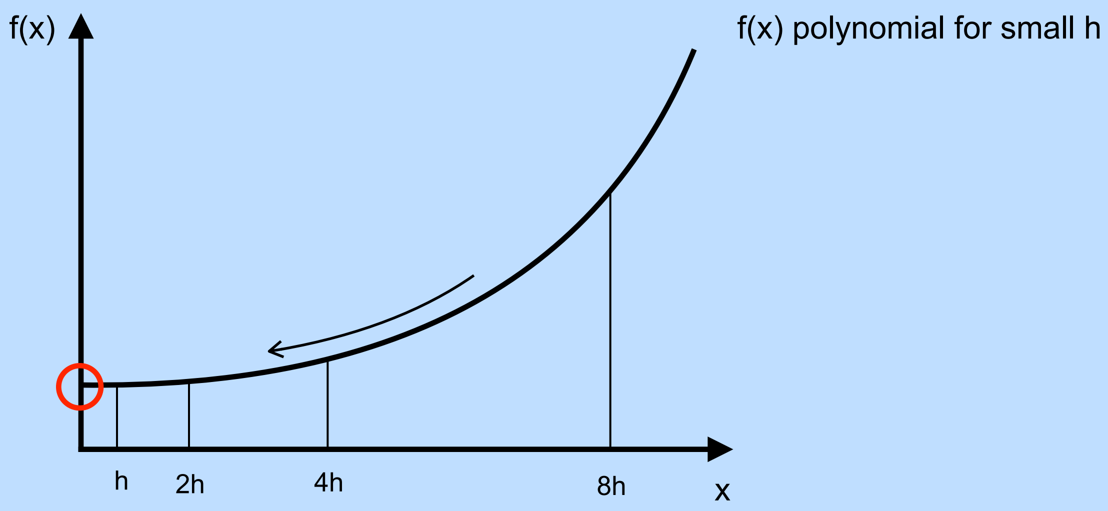
  different $\bar{\epsilon} \longrightarrow$ Richardson extrapolation $\longrightarrow \bar{\epsilon} = 0$ (NWA)
- Integration of real corrections straight forward

Richardson Extrapolation

$$M_2[f(h), f(2h)] = 2f(h) - f(2h) \qquad = f(0) + \mathcal{O}(h^2)$$

$$M_4[f(h), f(2h), f(4h)] = (8f(h) - 6f(2h) + f(4h))/3 \qquad = f(0) + \mathcal{O}(h^3)$$

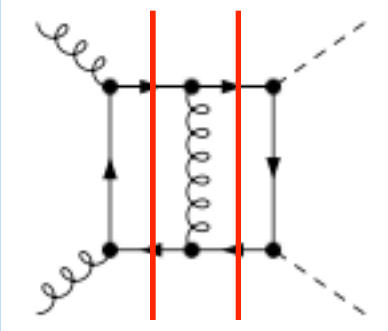
$$M_8[f(h), f(2h), f(4h), f(8h)] = (64f(h) - 56f(2h) + 14f(4h) - f(8h))/21 \quad = f(0) + \mathcal{O}(h^4)$$

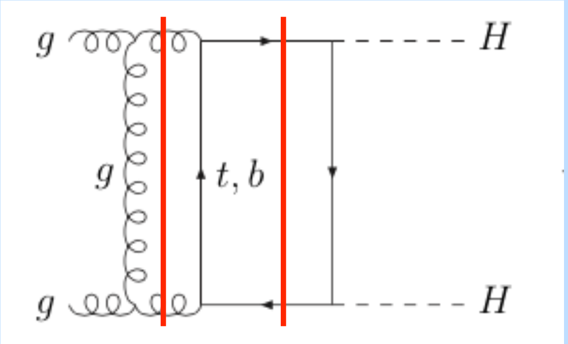


Virtual Corrections

Divergences

- gluon rescattering: threshold for $M_{HH}^2 > 0$





→ tried contour deformation

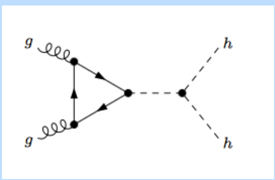
→ several ‘proper’ subtractions of the integrand

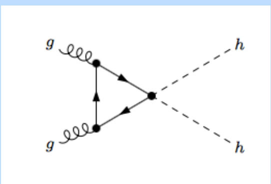
Next steps

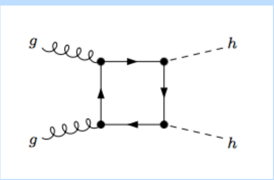
Extension to BSM-Higgs scenarios

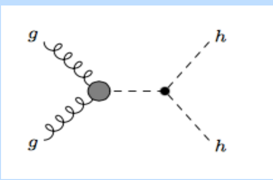
- Example 1: dim6

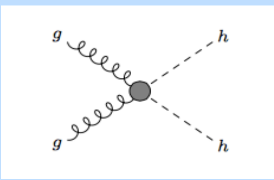
$$\mathcal{L}_{eff} = -m_t \bar{t} t \left(c_t \frac{h}{v} + c_{tt} \frac{h^2}{2v^2} \right) - c_3 \frac{1}{6} \left(\frac{3M_h^2}{v} \right) h^3 + \frac{\alpha_s}{\pi} G^{a\,\mu\nu} G^a_{\mu\nu} \left(c_g \frac{h}{v} + c_{gg} \frac{h^2}{2v^2} \right)$$

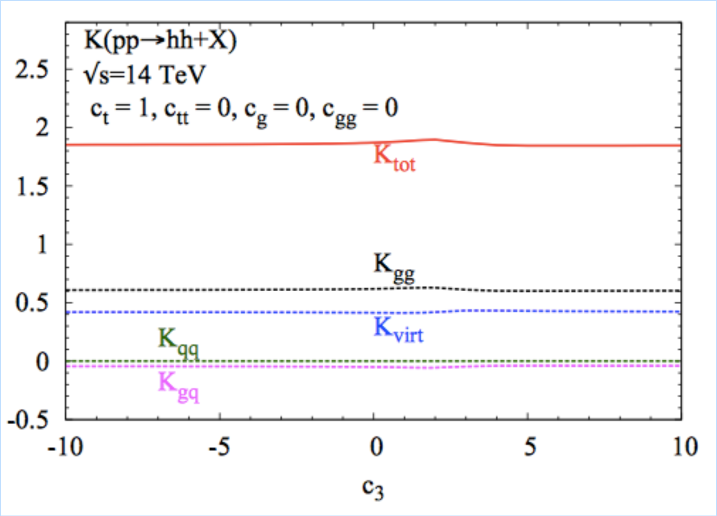


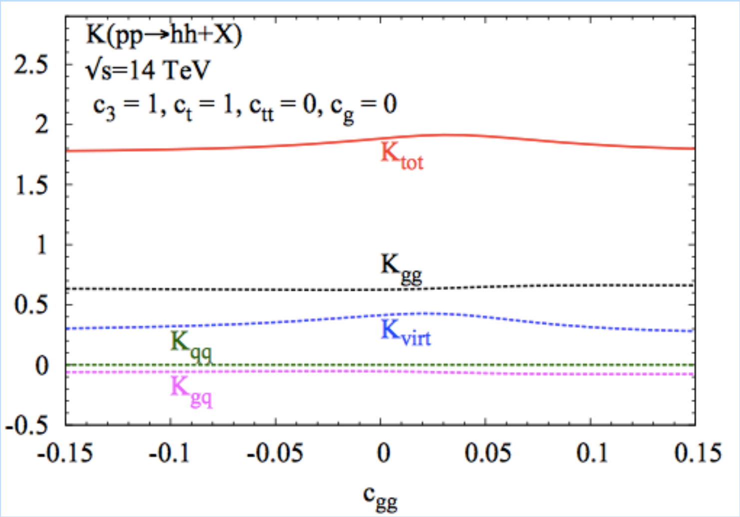










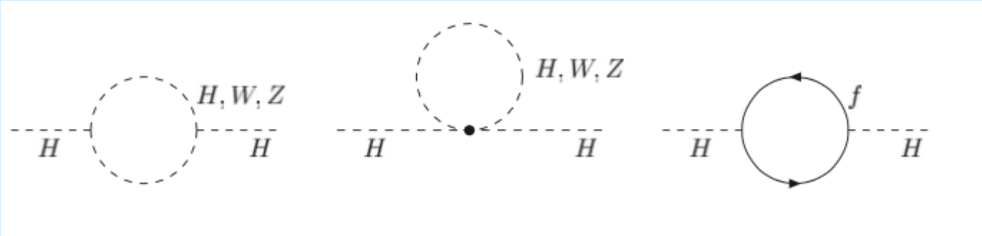


Gröber, Mühlleitner, Spira, Streicher

Higgs boson

Hierarchy problem:

Quantum corrections to Higgs mass:



$$\Delta M_H^2 \sim \Lambda^2 \sim \mathcal{O}(M_{GUT}^2) \gg M_H^2$$

mass difference can be absorbed in counter term:

$$M_H^2 \to M_H^2 + \Delta M_H^2 - \delta M_H^2$$

fine tuning of ~ 28 digits