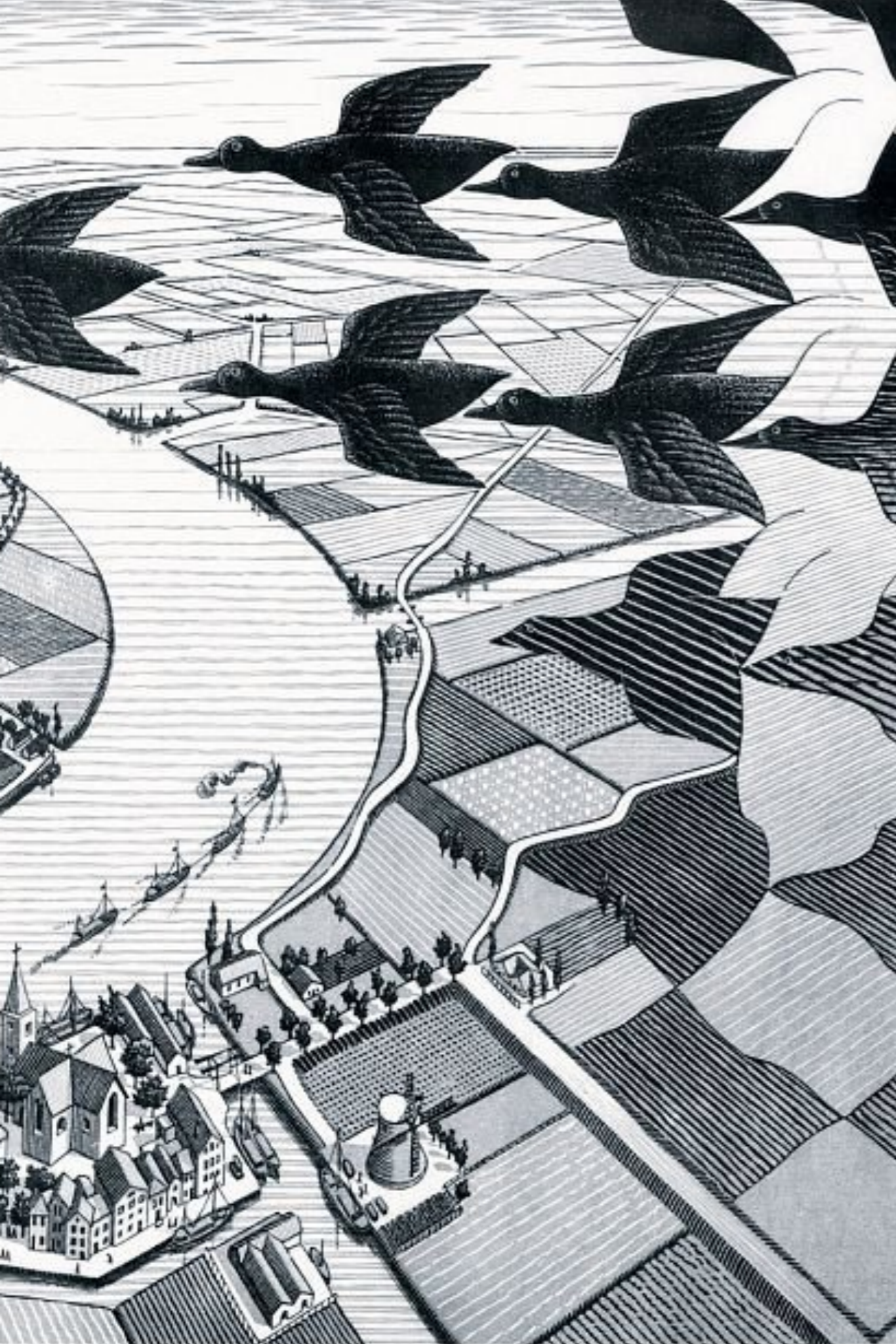




HEAVY QUARK PRODUCTION AT THE LHC

Zurich PhD Seminars, 09.03.18

Simone Devoto
Advisor: Massimiliano Grazzini



CONTENTS

- Motivations
- MATRIX
- My PhD Project
- Conclusions

HEAVY QUARK PRODUCTION AT THE LHC

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- **Heavy quark** → Top quark
Third family quark, heaviest particle of the SM

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HEAVY QUARK PRODUCTION **AT THE LHC**

- **Heavy quark** → Top quark
Third family quark, heaviest particle of the SM
- **Production** → Pair production
 $t\bar{t}$ production is the main source of top quark events in the SM
- **At the LHC** → Large Hadron Collider
The world's largest and most powerful particle collider

WHY TOP QUARK?

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- Heaviest elementary particle known so far ($m_t \approx 173 \text{ GeV}$)

Strong coupling with the Higgs Boson

Study of $t\bar{t}$ production can shed light on electroweak symmetry breaking mechanism

WHY TOP QUARK?

- Heaviest elementary particle known so far ($m_t \approx 173 \text{ GeV}$)
Strong coupling with the Higgs Boson
Study of $t\bar{t}$ production can shed light on electroweak symmetry breaking mechanism
- Top quarks are abundantly produced at the LHC
Its production is an important background both for NP model and SM precision measurements
Experimental measurements require reliable predictions of $t\bar{t}$ production

MOST RECENT ATLAS PAPERS



arXiv id	Observable	$t\bar{t}$ background?
1802.08168	Missing Transverse Momentum	✓
1802.09572	$t\bar{t}$ production	✓
1802.06572	$H \rightarrow cc$	✓
1802.03388	$H \rightarrow ZX/XX \rightarrow 4\ell$	✓
1802.03158	Supersymmetry	✓
1802.01840	Tetraquark	✗
1802.04146	$H \rightarrow \gamma\gamma$	✓
1801.08769	$q\bar{q} + \gamma$ or jet	✓
1801.07893	$W' \rightarrow tb$	✓
1801.06992	$X \rightarrow \tau\nu$	✓
1801.02052	$t\bar{t}$ production	✓
1712.08891	$pp \rightarrow t\bar{t}H$	✓

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11/12 REQUIRE THEORETICAL

PREDICTION OF $t\bar{t}$ PRODUCTION!

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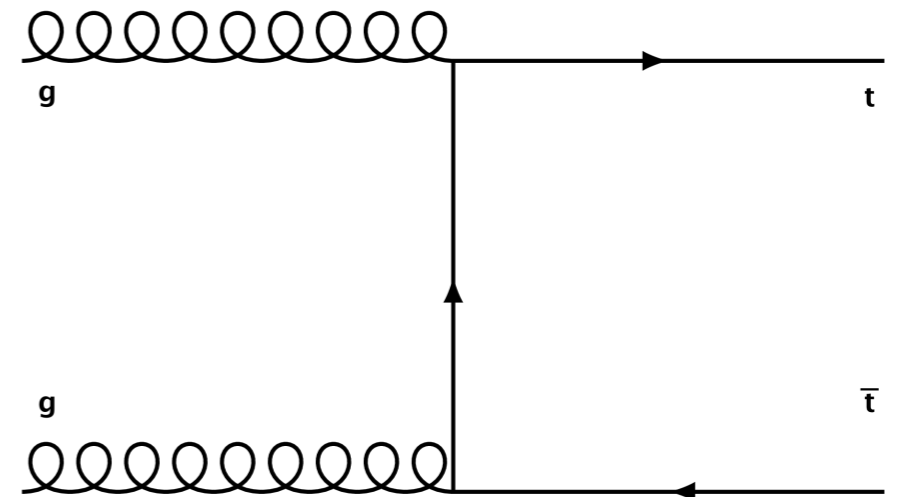
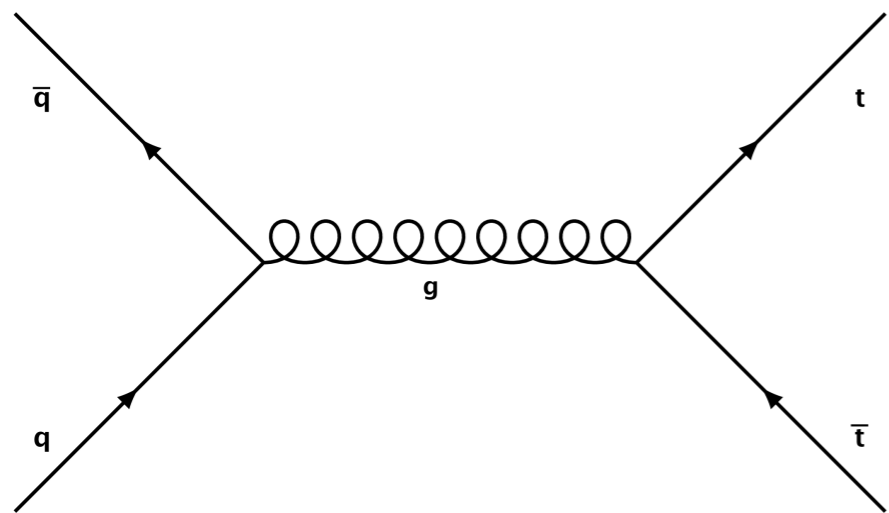
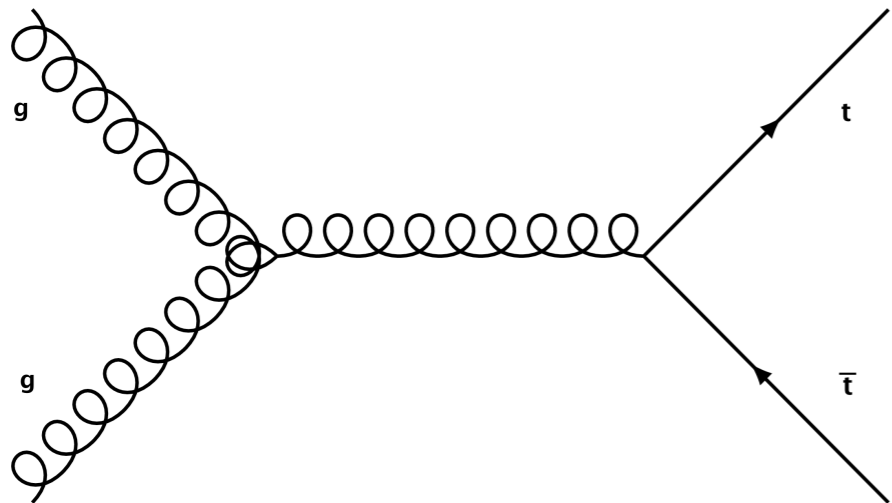
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- Perturbation theory → Feynman diagrams

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► Perturbation theory → Feynman diagrams

Leading Order (LO)



LO → order of magnitude prediction

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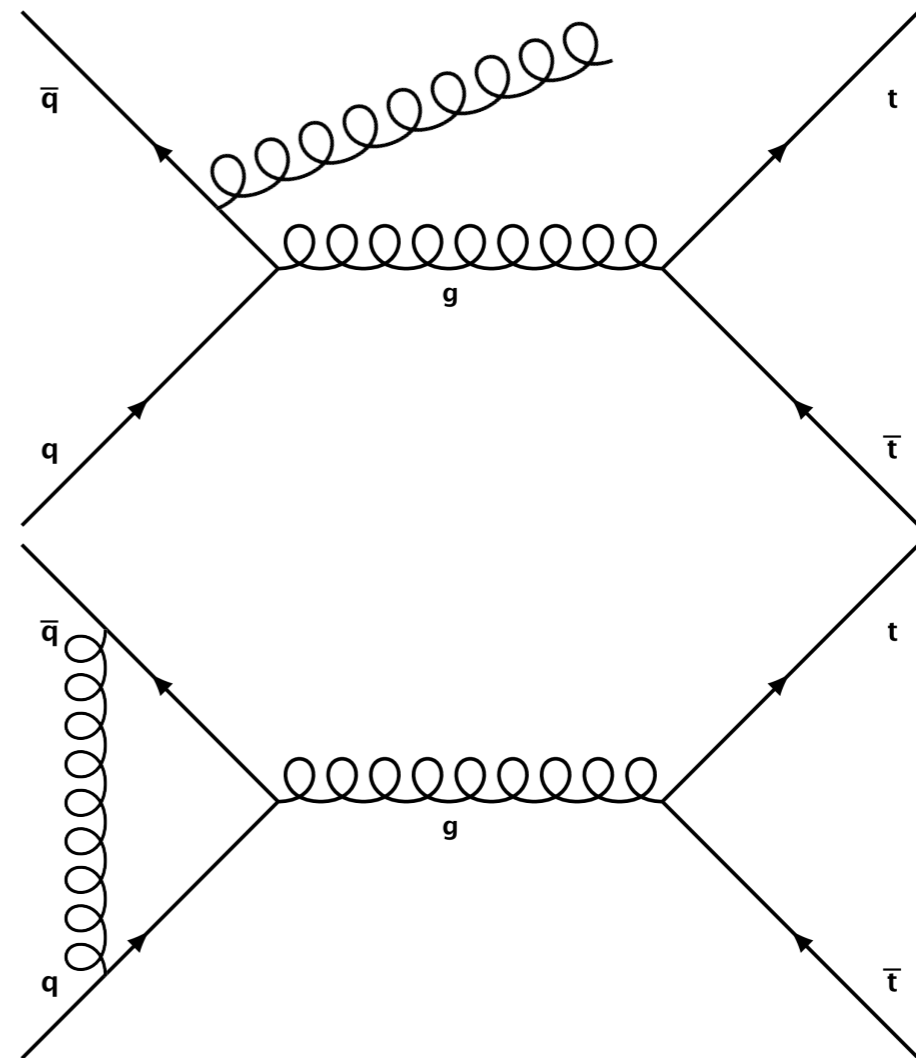
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Next to Leading Order (NLO)

Two types of corrections:

- Real

- Virtual



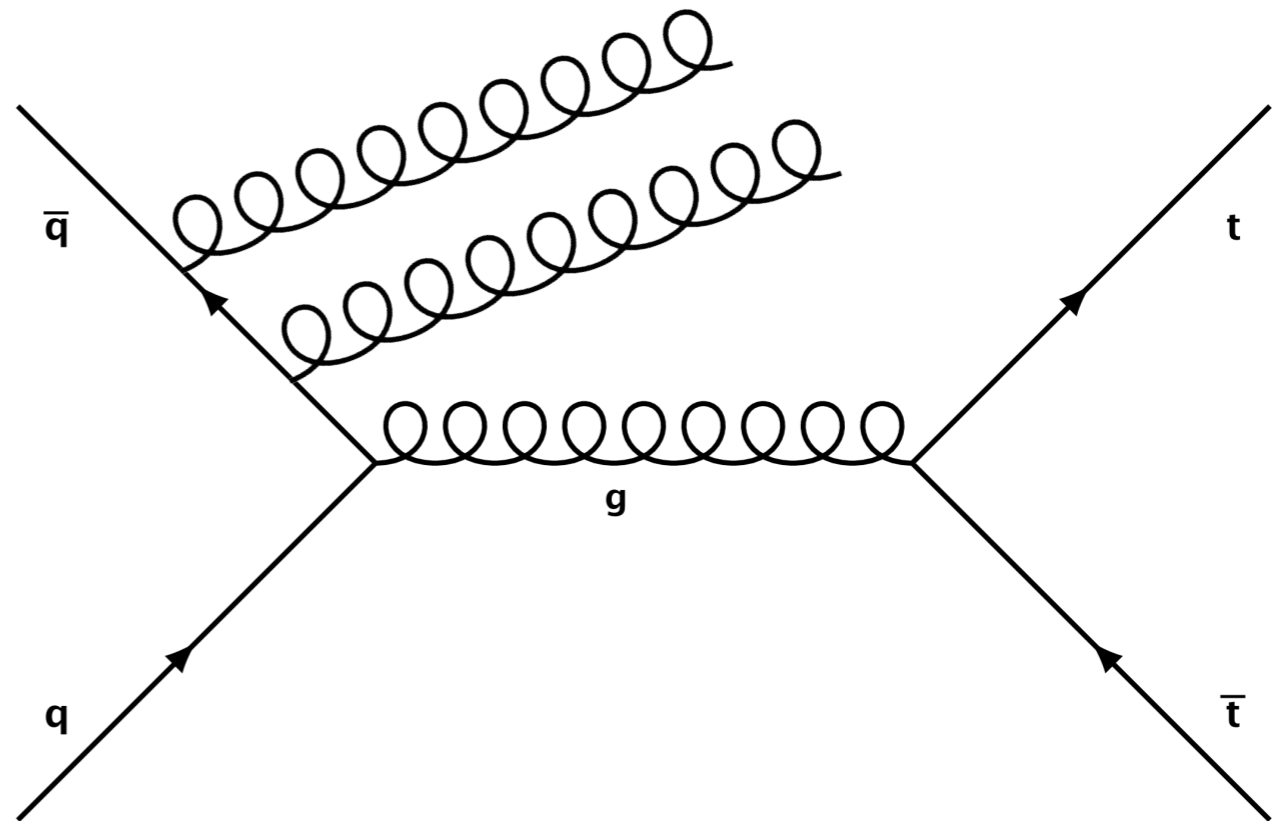
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Next to Next to Leading Order (NNLO)

Three types of corrections:

- ▶ Double real
- ▶ Single real at 1 loop
- ▶ 2 loop Virtual



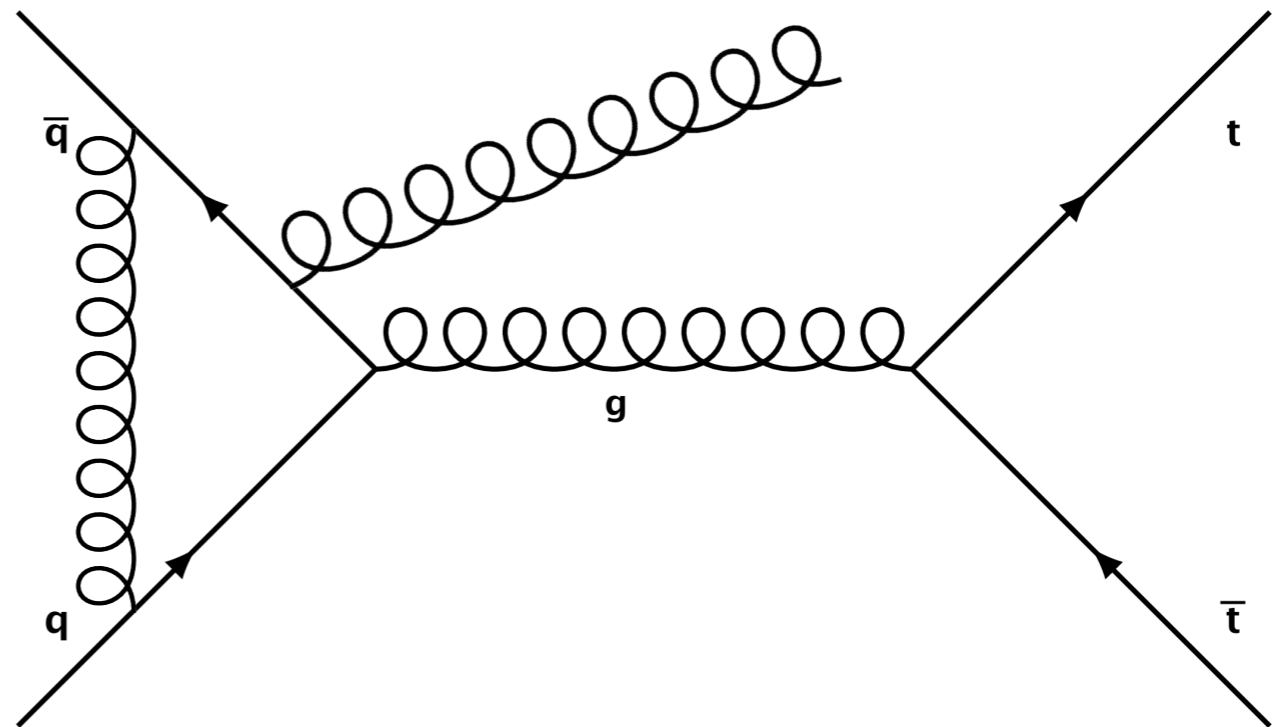
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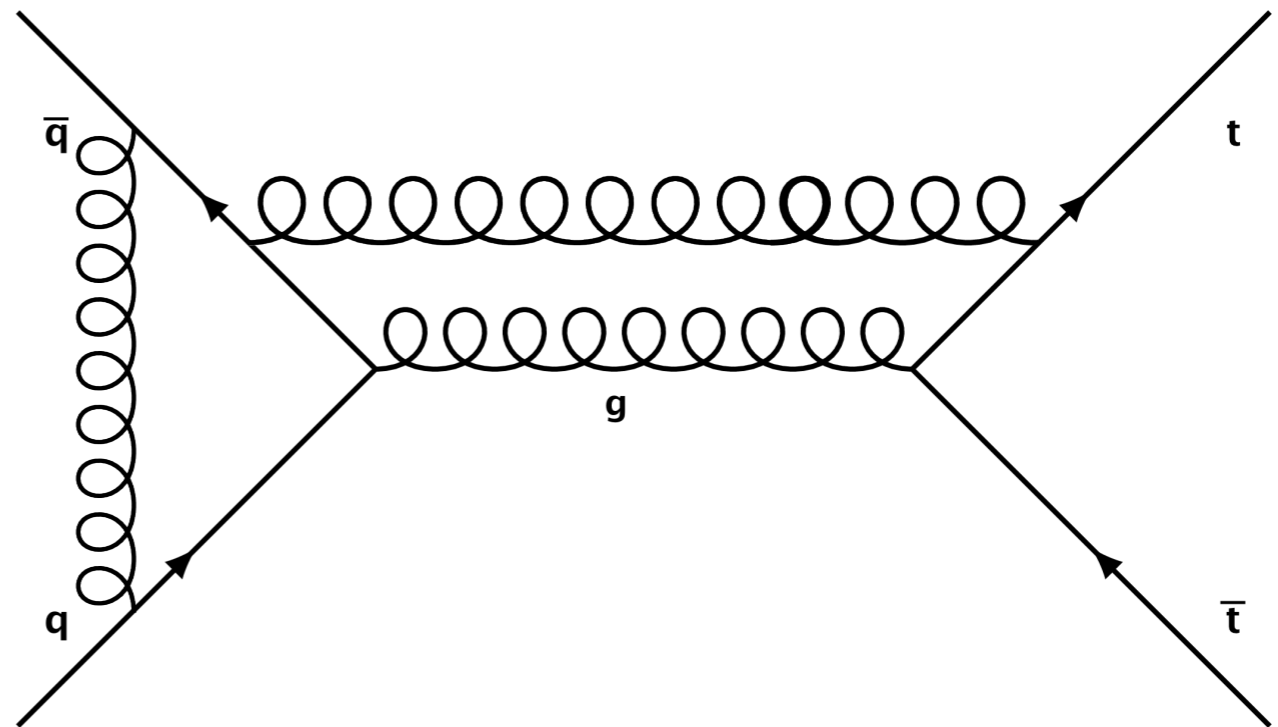
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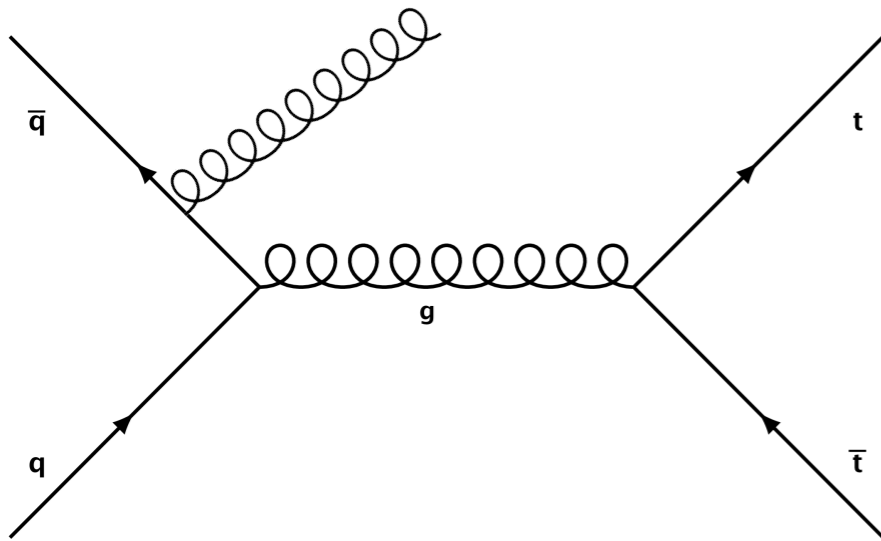


QCD CORRECTIONS

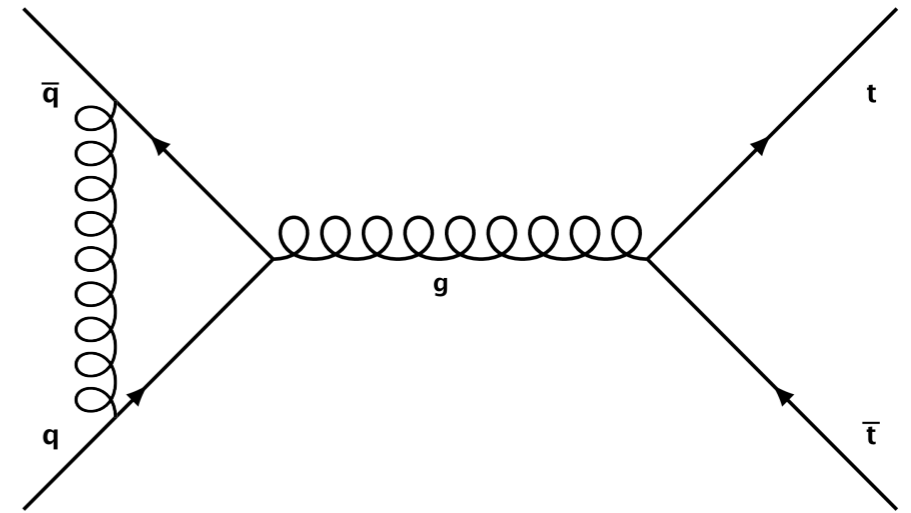
They are challenging because of IR divergences!

WHY ARE QCD CORRECTIONS CHALLENGING?

Real

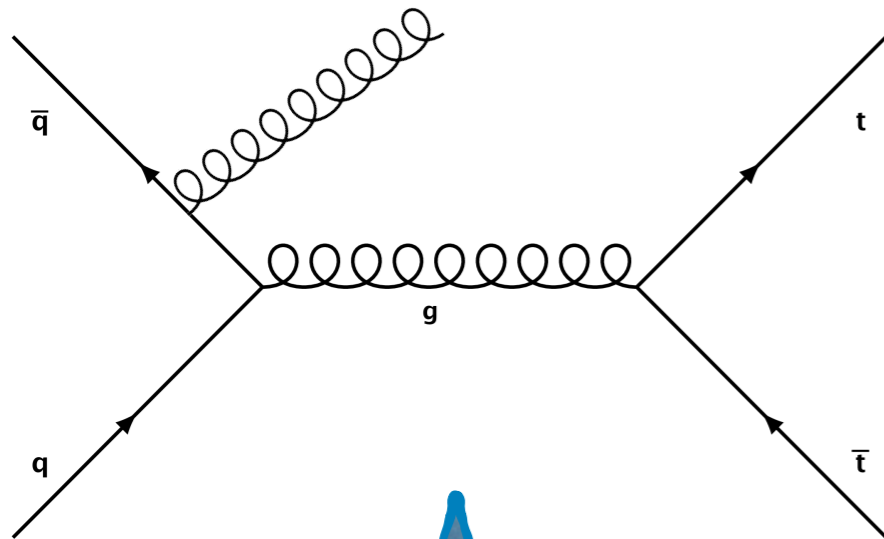


Virtual



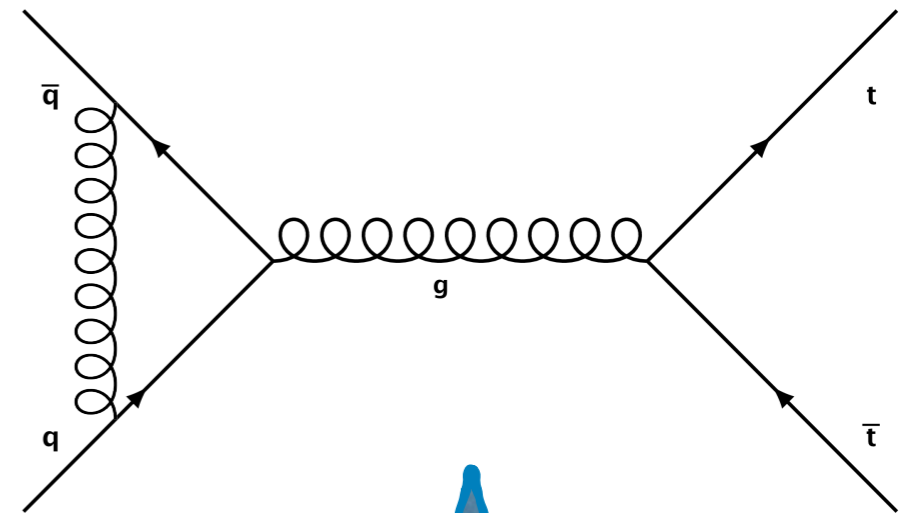
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IR Divergent

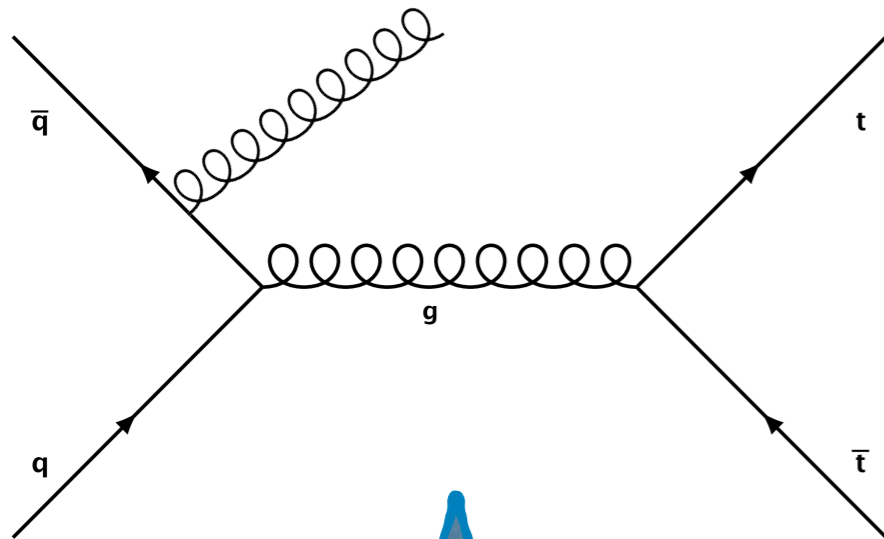
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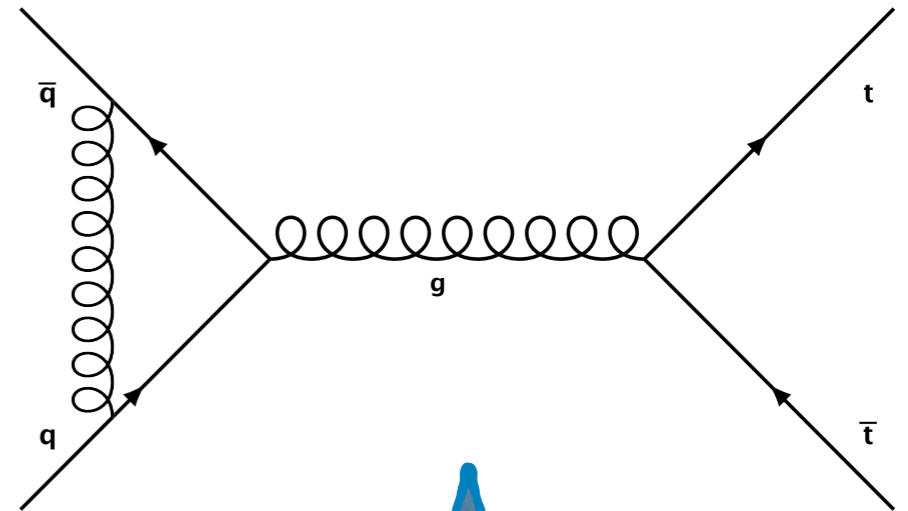
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IR Divergent

IR divergences are guaranteed to cancel out for inclusive observables after summing real and virtual contributions (KLN Theorem)

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Presence of IR divergences at intermediate steps of the computation of QCD higher order corrections does not allow a straightforward implementation of numerical techniques.

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SUBTRACTION METHODS

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SUBTRACTION METHODS

$$\sigma^{NLO} = \int d\sigma^{NLO} = \int_{m+1} d\sigma^R \quad + \quad \int_m d\sigma^V$$

Divergent *Divergent*

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$$\sigma^{NLO} = \int d\sigma^{NLO} = \int_{m+1} [d\sigma^R - d\sigma^{CT}] + \int_m \left[d\sigma^V + \int_1 d\sigma^{CT} \right]$$

Divergent

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~~Divergent~~

Convergent!

~~Divergent~~

Convergent!

WHY ARE QCD CORRECTIONS CHALLENGING?

Subtraction methods:

► **NLO:**

- *Catani-Seymour dipole subtraction* [S. Catani, M. Seymour (1996)]
- *FKS subtraction* [S. Frixione, Z. Kunszt, A. Signer (1996)]

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- *Stripper formalism* [M. Czakon (2010); Boughezal et al (2011)]
- *q_T subtraction formalism* [S. Catani, M. Grazzini]
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[devoto:/mnt/runs2/devoto/MATRIX_v1.0.0] ./matrix

MATRIX

[arXiv 1711.06631]




```
[devoto:/mnt/runs2/devoto/MATRIX_v1.0.0] ./matrix
```



Version: 1.0.0
Reference: arXiv:1711.06631
Nov 2017

Munich -- the MULTI-chaNnel Integrator at swiss (CH) precision --
Automates q_T -subtraction and Resummation to Integrate X-sections



M. Grazzini (grazzini@physik.uzh.ch)
S. Kallweit (stefan.kallweit@cern.ch)
M. Wiesemann (maris.wiesemann@cern.ch)

MATRIX is based on a number of different computations and tools
from various people and groups. Please acknowledge their efforts
by citing the list of references which is created with every run.

```
<<MATRIX-MAKE>> This is the MATRIX process compilation.  
<<MATRIX-READ>> Type process_id to be compiled and created. Type "list" to show  
available processes. Try pressing TAB for auto-completion. Type  
"exit" or "quit" to stop.  
|=====|>> list
```

process_id	process	description
pph21	p p --> H	on-shell Higgs production
ppz01	p p --> Z	on-shell Z production
ppw01	p p --> W^-	on-shell W- production with CKM
ppwx01	p p --> W^+	on-shell W+ production with CKM
ppeex02	p p --> e^- e^+	Z production with decay
ppnenex02	p p --> v_e^- v_e^+	Z production with decay
ppenex02	p p --> e^- v_e^+	W- production with decay and CKM
ppexne02	p p --> e^+ v_e^-	W+ production with decay and CKM
ppaa02	p p --> gamma gamma	gamma gamma production
ppeexa03	p p --> e^- e^+ gamma	Z gamma production with decay
ppnenexa03	p p --> v_e^- v_e^+ gamma	Z gamma production with decay
ppenexa03	p p --> e^- v_e^+ gamma	W- gamma production with decay
ppexnea03	p p --> e^+ v_e^- gamma	W+ gamma production with decay
ppzz02	p p --> Z Z	on-shell ZZ production
ppwxw02	p p --> W^+ W^-	on-shell WW production
ppemexmx04	p p --> e^- mu^- e^+ mu^+	ZZ production with decay
ppeeexex04	p p --> e^- e^- e^+ e^+	ZZ production with decay
ppeexnmnm04	p p --> e^- e^+ v_mu^- v_mu^+	ZZ production with decay
ppemxnmnex04	p p --> e^- mu^+ v_mu^- v_e^+	WW production with decay
ppeexnenex04	p p --> e^- e^+ v_e^- v_e^+	ZZ/WW production with decay
ppemxnm04	p p --> e^- mu^- e^+ v_mu^+	W-Z production with decay
ppeeexnex04	p p --> e^- e^- e^+ v_e^+	W-Z production with decay
ppeexmxnm04	p p --> e^- e^+ mu^+ v_mu^-	W+Z production with decay
ppeeexexne04	p p --> e^- e^+ e^+ v_e^-	W+Z production with decay

MATRIX [arXiv 1711.06631]

.....

*Computational framework
which allows us to evaluate
fully differential cross
sections for a wide class of
processes at hadron colliders
where the final state is a
colour singlet in next-to-
next-to-leading order
(NNLO) QCD by using q_T
subtraction.*


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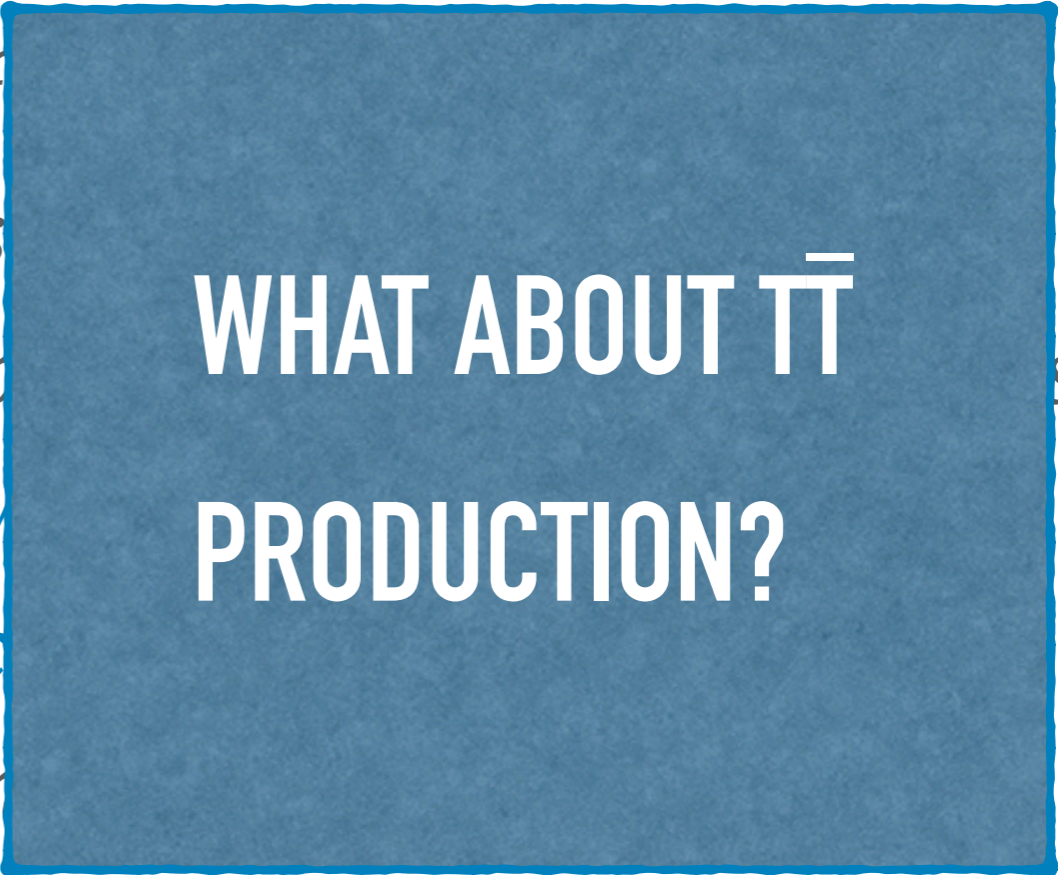
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ppwxw02    >> p p --> W^+ W^-  >> on-shell WW production
ppemexmx04 >> p p --> e^- mu^- e^+ mu^+ >> ZZ production with decay
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ppeeexnmnx04 >> p p --> e^- e^+ v_mu^- v_mu^+ >> ZZ production with decay
ppemxnmnex04 >> p p --> e^- mu^+ v_mu^- v_e^+ >> WW production with decay
ppeeexnenex04 >> p p --> e^- e^+ v_e^- v_e^+ >> ZZ/WW production with decay
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MATRIX

[arXiv 1711.06631]

Computational framework
which allows us to evaluate



(NNLO) QCD by using qT
subtraction.

MY PHD PROJECT

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To compute the missing ingredient to implement $t\bar{t}$ production at NNLO in MATRIX

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To compute the missing ingredient to implement $t\bar{t}$ production at NNLO in MATRIX

Coloured final state



QCD corrections also from the final state

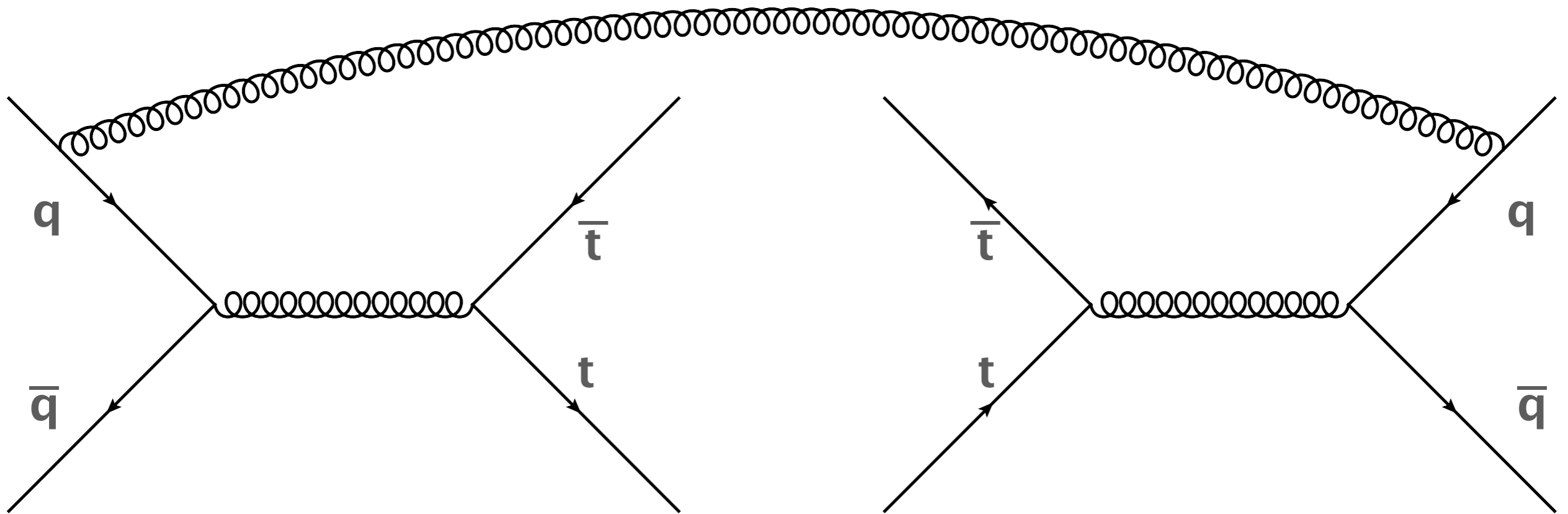
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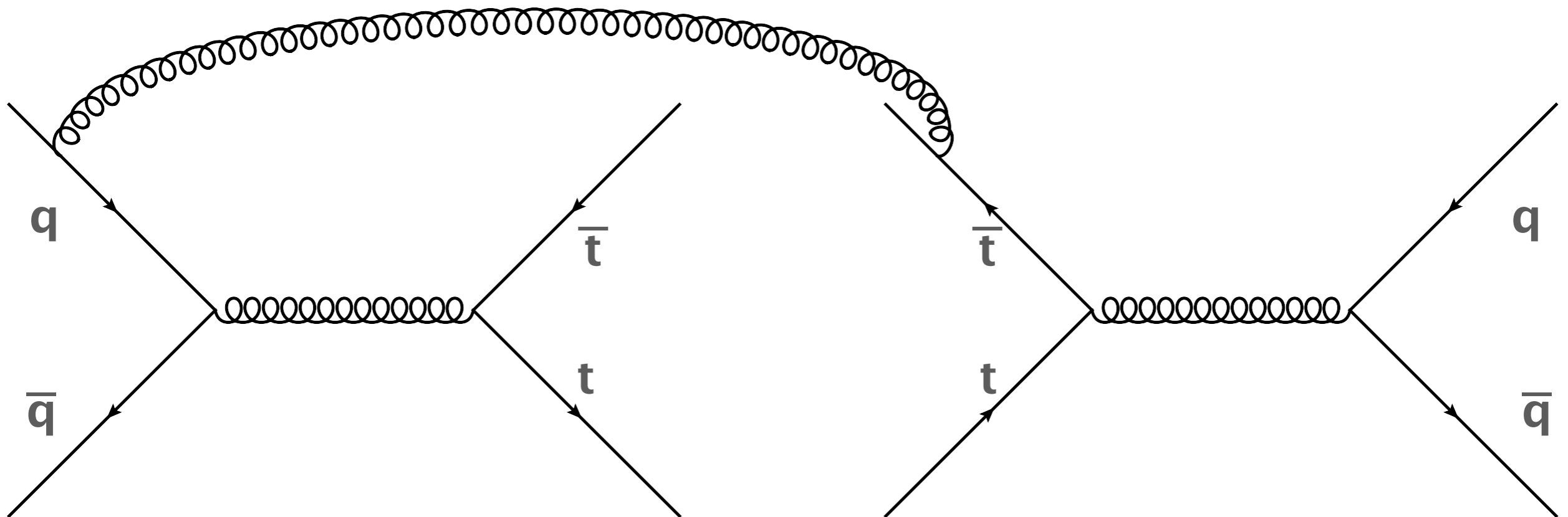
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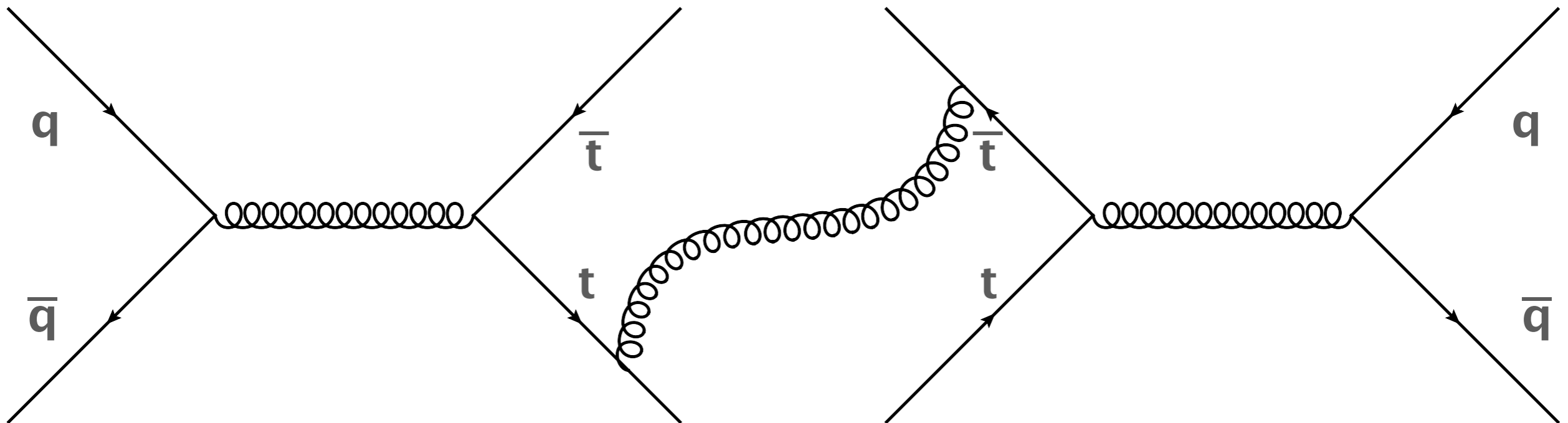
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QCD corrections also from the final state



WHAT IS q_T SUBTRACTION?

q_T subtraction exploits the fact that the behaviour of the q_T distribution at small q_T has a universal structure known from transverse momentum resummation formalism to construct a process independent counterterm.

$$d\sigma_{NNLO}^{Q\bar{Q}} = \mathcal{H}_{NNLO}^{Q\bar{Q}} \otimes d\sigma_{LO}^{Q\bar{Q}} + \left[d\sigma_{NLO}^{Q\bar{Q}+\text{jet}} - d\sigma_{NNLO}^{CT} \right]$$

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[arXiv:1408.4564; arXiv:1508.03585]

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HARD VIRTUAL
COEFFICIENT

Needs to be computed!

Can be computed with NLO
subtraction techniques

IR behaviour known from studies in q_t resummation
[arXiv:1408.4564; arXiv:1508.03585]

WHAT DO I NEED TO COMPUTE?

Hard Virtual Coefficient

To obtain this goal, one has to:

- Integrate the NNLO matrix elements for the real contribution in the soft limit (subtraction operator).
- Add them to the virtual contribution.
- Check the cancellation of the IR poles, keep the finite part.

WHAT DO I NEED TO COMPUTE?

Computation of the soft emission



Integration of the NNLO soft currents (eikonal currents)

We can distinguish between two classes of contribution:

WHAT DO I NEED TO COMPUTE?

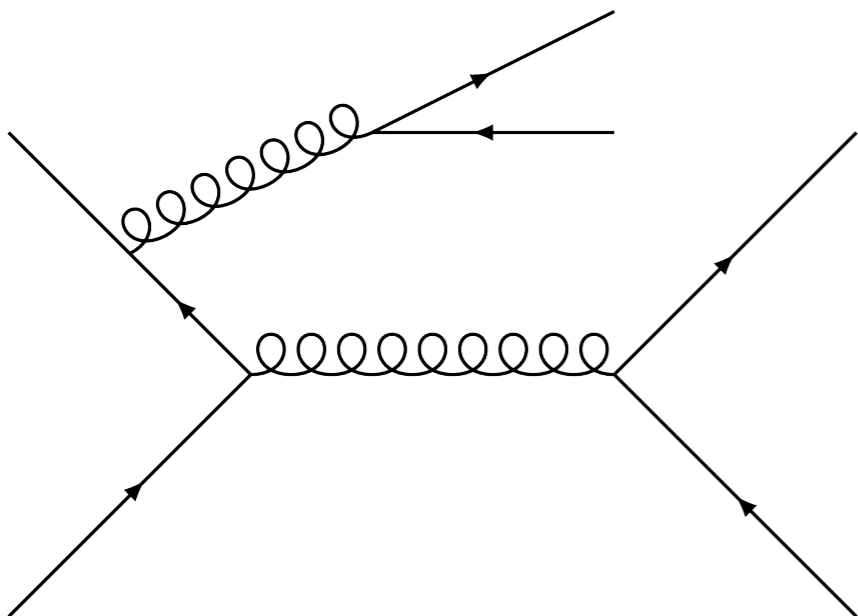
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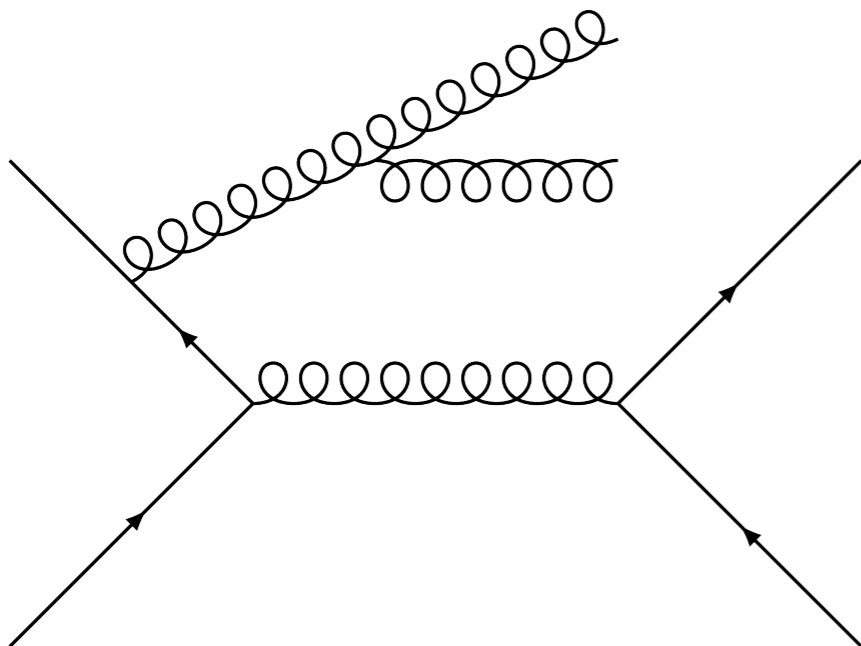
Computation of the soft emission



Integration of the NNLO soft currents (eikonal currents)

We can distinguish between two classes of contribution:

- proportional to the number of light quark flavours n_f ;
- **not** proportional to the number of light quark flavours n_f .



N_f CONTRIBUTION

One has to consider:

- Soft quark pair production;
- NNLO contribution to single gluon emission;
- 2 loop contribution.

N_f CONTRIBUTION

One has to consider:

- Soft quark pair production;
- NNLO contribution to single gluon emission;
- 2 loop contribution.

We computed the missing terms and combined them together.

We observed a complete cancellation of the poles and we extracted the finite part.

Full result for the n_f contribution!

N_f CONTRIBUTION

Most tricky part: soft quark pair production.

Process:



$$a_1(p_1^\mu) a_2(p_2^\mu) \rightarrow Q(p_3^\mu) \bar{Q}(p_4^\mu) [g \rightarrow q(q_1^\mu) \bar{q}(q_2^\mu)]$$

Soft Limit

$$|\mathcal{M}_{a_1 a_2 \rightarrow Q \bar{Q} q \bar{q}}|^2 = (\alpha_0 \mu_0^{2\epsilon})^q (4\pi \alpha_0 \mu_0^{2\epsilon})^2 \left\langle \mathcal{M}^{(0)} \left| J_\mu(k) \Pi^{\mu\nu}(q_1, q_2) J_\nu(k) \right| \mathcal{M}^{(0)} \right\rangle$$

$$k = q_1 + q_2 \quad J^\mu = T_i \frac{p_i^\mu}{p_i \cdot q} \quad \Pi^{\mu\nu}(q_1, q_2) = \frac{T_R}{(q_1 \cdot q_2)^2} (-g^{\mu\nu} q_1 \cdot q_2 + q_1^\mu q_2^\nu + q_1^\nu q_2^\mu)$$

Need to compute:



$$\int d^n q_1 \int d^n q_2 J_\mu(k) \Pi^{\mu\nu}(q_1, q_2) J_\nu(k)$$

DOUBLE GLUON EMISSION

One has to consider:

- Double real contribution;
- Real - virtual contribution;
- 2 loop virtual contribution.

DOUBLE GLUON EMISSION

One has to consider:

- Double real contribution;
- Real - virtual contribution;
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Current status:

We started the computation of the most tricky part, double real contribution (double gluon emission).

DOUBLE GLUON EMISSION

Process:



$$a_1(p_1^\mu) a_2(p_2^\mu) \rightarrow Q(p_3^\mu) \bar{Q}(p_4^\mu) g(q_1^\mu) g(q_2^\mu)$$

Soft Limit

$$J_{\mu\nu}^{a_1 a_2}(q_1, q_2) g^{\sigma\mu} g^{\rho\nu} J_{\sigma\rho}^{a_1 a_2}(q_1, q_2) = \frac{1}{2} \{ \mathbf{J}^2(q_1), \mathbf{J}^2(q_2) \} - C_A \sum_{i,j=1}^n \mathbf{T}_i \cdot \mathbf{T}_j \mathcal{S}_{ij}(q_1, q_2)$$

$$\mathcal{S}_{ij}(q_1, q_2) = \mathcal{S}_{ij}^{m=0}(q_1, q_2) + \left(m_i^2 \mathcal{S}_{ij}^{m \neq 0}(q_1, q_2) + m_j^2 \mathcal{S}_{ji}^{m \neq 0}(q_1, q_2) \right)$$

$$|\mathcal{M}_{a_1 a_2 \rightarrow Q \bar{Q} g g}|^2 = (\alpha_0 \mu_0^{2\epsilon})^q (4\pi \alpha_0 \mu_0^{2\epsilon}) \left\langle \mathcal{M}^{(0)} \left| J_{\mu\nu}^{a_1 a_2}(q_1, q_2) g^{\sigma\mu} g^{\rho\nu} J_{\sigma\rho}^{a_1 a_2}(q_1, q_2) \right| \mathcal{M}^{(0)} \right\rangle$$

Need to compute:

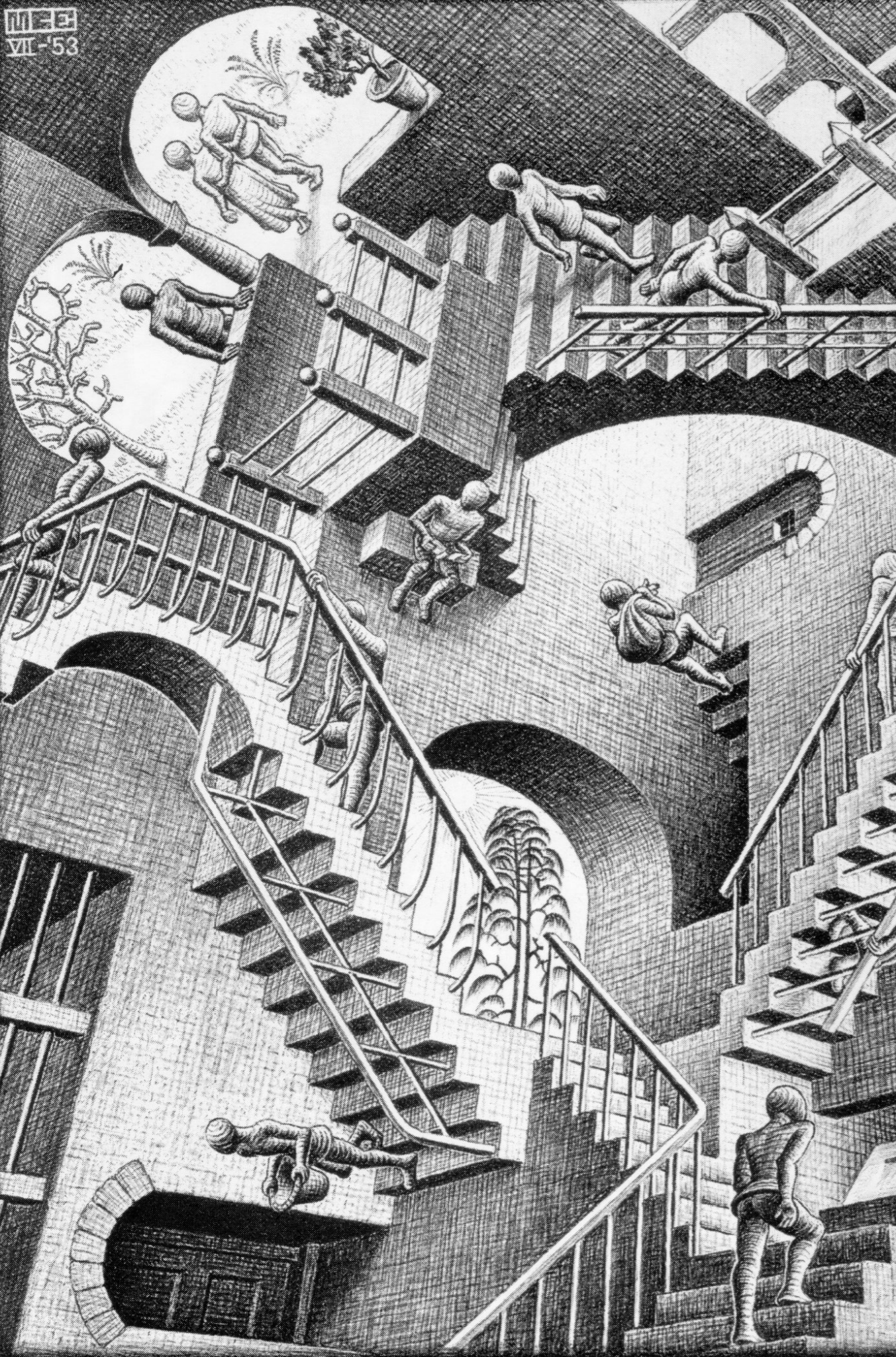


$$\int d^n q_1 d^n q_2 J_{\mu\nu}^{a_1 a_2}(q_1, q_2) g^{\sigma\mu} g^{\rho\nu} J_{\sigma\rho}^{a_1 a_2}(q_1, q_2)$$

DOUBLE GLUON EMISSION

$$\begin{aligned}
 \mathcal{S}_{ij}^{m=0}(q_1, q_2) = & \frac{(1 - \epsilon)}{(q_1 \cdot q_2)^2} \frac{p_i \cdot q_1 p_j \cdot q_2 + p_i \cdot q_2 p_j \cdot q_1}{p_i \cdot (q_1 + q_2) p_j \cdot (q_1 + q_2)} \\
 & - \frac{(p_i \cdot p_j)^2}{2 p_i \cdot q_1 p_j \cdot q_2 p_i \cdot q_2 p_j \cdot q_1} \left[2 - \frac{p_i \cdot q_1 p_j \cdot q_2 + p_i \cdot q_2 p_j \cdot q_1}{p_i \cdot (q_1 + q_2) p_j \cdot (q_1 + q_2)} \right] \\
 & + \frac{p_i \cdot p_j}{2 q_1 \cdot q_2} \left[\frac{2}{p_i \cdot q_1 p_j \cdot q_2} + \frac{2}{p_j \cdot q_1 p_i \cdot q_2} - \frac{1}{p_i \cdot (q_1 + q_2) p_j \cdot (q_1 + q_2)} \right. \\
 & \left. \times \left(4 + \frac{(p_i \cdot q_1 p_j \cdot q_2 + p_i \cdot q_2 p_j \cdot q_1)^2}{p_i \cdot q_1 p_j \cdot q_2 p_i \cdot q_2 p_j \cdot q_1} \right) \right]
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{S}_{ij}^{m \neq 0}(q_1, q_2) = & \frac{p_i \cdot p_j p_j \cdot (q_1 + q_2)}{2 p_i \cdot q_1 p_j \cdot q_2 p_i \cdot q_2 p_j \cdot q_1 p_i \cdot (q_1 + q_2)} \\
 & - \frac{1}{2 q_1 \cdot q_2 p_i \cdot (q_1 + q_2) p_j \cdot (q_1 + q_2)} \left(\frac{(p_j \cdot q_1)^2}{p_i \cdot q_1 p_j \cdot q_2} + \frac{(p_j \cdot q_2)^2}{p_i \cdot q_2 p_j \cdot q_1} \right)
 \end{aligned}$$



CONCLUSIONS

➤ What?

Computation of the hard virtual coefficient for $t\bar{t}$ production.

➤ Why?

To implement q_t subtraction for coloured final state.

➤ Done:

Computation of the n_f contribution

➤ To do:

Complete the computation for the double gluon emission contribution