Impact of low-x resummation on QCD analysis of HERA data

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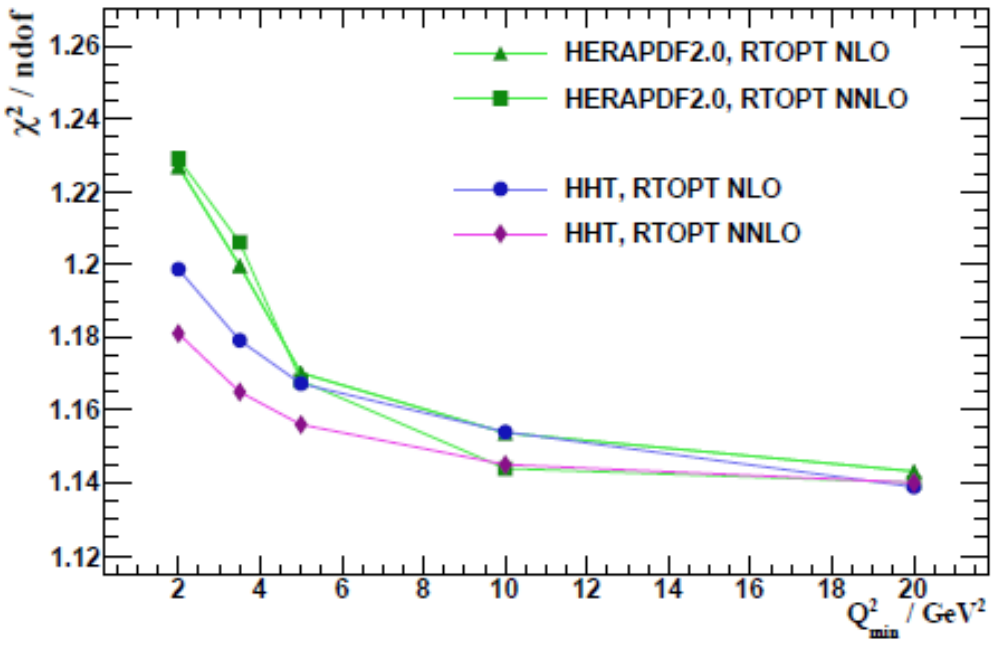
Low-x resummation:

- Improves the description of HERA data at low-x, low-Q2, high-y, without need for further parameters (as for example when adding higher twist terms)
- Results in a rising low-x gluon, which is always larger than the total Sea

The χ2/ndof of the HERAPDF2.0 NLO and NNLO fits deteriorate as the minimum value of Q^2 for data entering the fit is lowered



One way to improve this is to add higher twist terms - HHT analysis 1604.02299

But low Q2 is also low x and we have long suspected that the low-x region could require BFKL– ln(1/x) resummation. This does not require any extra parameters to fit the data (as higher twist does). It requires extending the DGLAP formalism to include ln(1/x) resummation. The tool to do this is the HELL code (Bonvini et al)

HELL has been implemented in xFitter

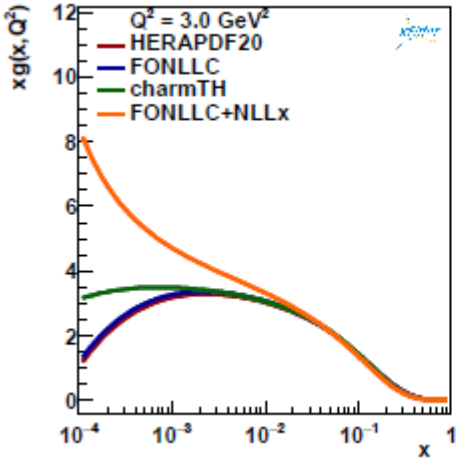
Here we explore consequences for a HERAPDF style fit

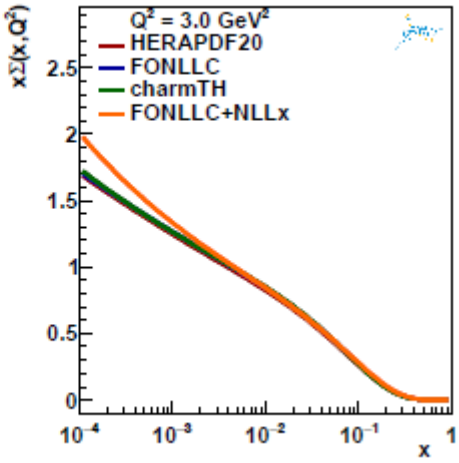
HELL implements resummation corrections to the fixed order splitting functions and coefficient functions up to NLL accuracy in ln(1/x), denoted as NLLx. The fixed order quantities are calculated by APFEL within the FONLL variable flavour number scheme. Thus we must use FONLL for the HERAPDF fit

The computation of ln(1/x) resummation is unreliable at low scales due to the large value of α_S thus the starting scale is raised to Q^2_0=2.56GeV^2 rather than the usual HERAPDF value of Q^2_0=1.9GeV^2. Consequently the charm quark threshold, μ_c, must be displaced above Q_0 while keeping the charm mass, m_c, fixed. (see 1707.05343)

Finally NLLx resummation can be applied

HERAPDF2.0 NNLO	FONLL-C	Move Q^2_0 and charm threshold	include NLLx resummation
HERA χ^2/d.o.f			
1363/1131	1387/1131	1389/1131	1316/1131





The χ2 for the NNLO fit improves dramatically at this final step

The shape of the gluon is also changed dramatically from flattening/turning over at low-x to singular at low-x

Technical aside

HERAPDF2.0 NNLO	FONLL-C	Move Q_0^2 and charm threshold	include NLLx resummation
1363/1131	1387/1131	1389/1131	1316/1131

The increase in χ2 for FONLLC is due to the treatment of FL, for which terms up to d$\mathcal{O}(\alpha_S^3)$are included for RTOPT, but terms up to di$\mathcal{O}(\alpha_S^2)$re included for FONLLC

The slight change in shape of the gluon when the charm threshold is displaced is Ppartly due to the PDF matching conditions . Moving up the charm threshold using fixed order NNLO depresses the charm PDF at larger scales. Since charm is generated by gluon splitting this requires a slightly larger gluon to compensate. However PDF matching conditions are also affected by logs of 1/x, which are resummed in HELL such that the spread cause by matching is significantly reduced for NNLO+NLLx.

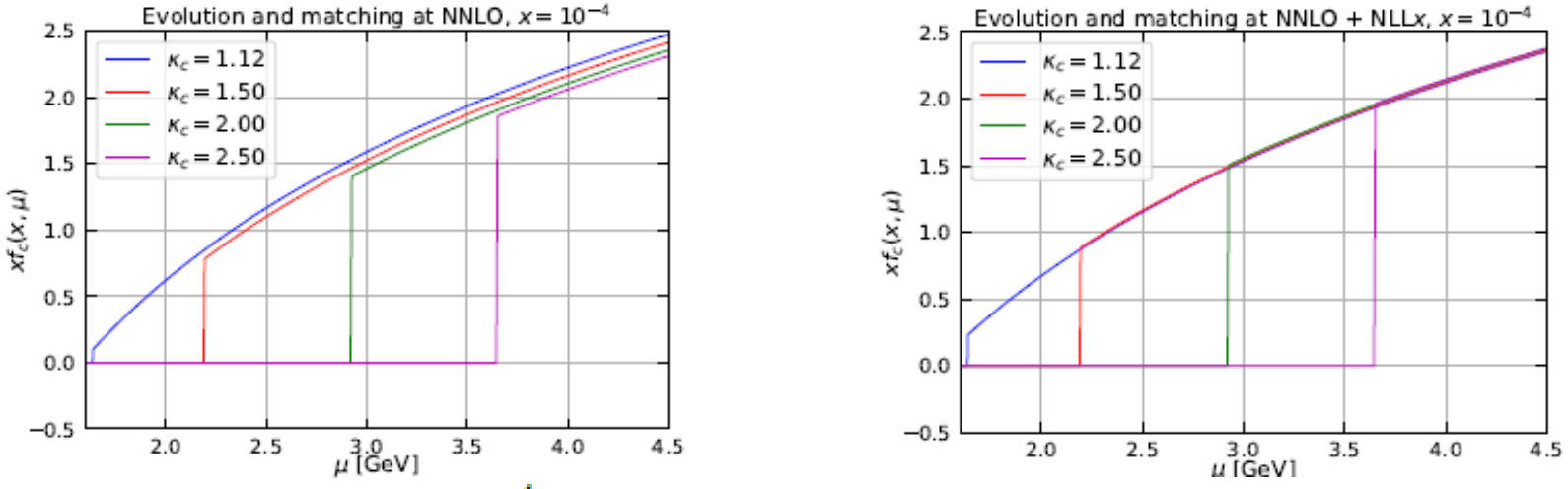


Figure 1 The charm PDF at $x = 10^{-4}$ as a function of the factorisation scale μ for different values of the charm threshold $\mu_c = \kappa_c m_c$, with $\kappa_c = 1.12, 1.5, 2, 2.5$. The plots show the effect of the matching at NNLO (upper plot) and at NNLO+NLLx (lower plot).

Further considerations:

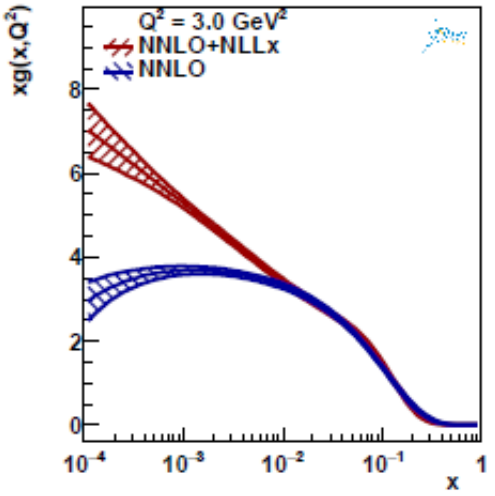
Since we have change the heavy quark scheme the charm and beauty masses used may not be optimal for the new scheme. Thus charm and beauty data from HERA are included in the fit and charm and beauty mass scans are performed to determine new values m_c=1.46 and m_b=4.5GeV. Only mc differs from that of the HERAPDF and the charm threshold is move to μ_c=1.64 correspondingly. We retain charm data in the main fit since it is potentially sensitive to low x resummation.

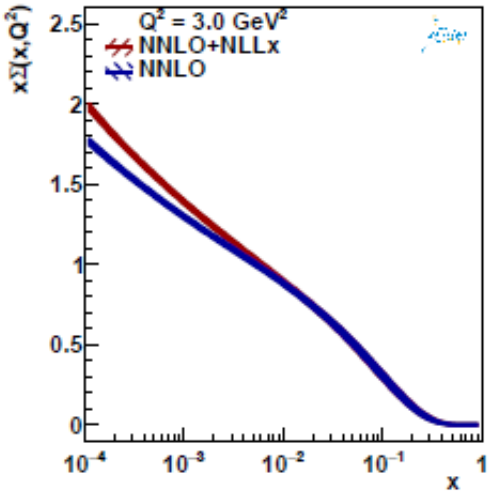
Since we have a very different shape of the gluon PDF a parametrisation scan is performed at NNLO+NLLx to determine if the HERAPDF parametrisation is adequate. The form of the parametrisation is confirmed, however the negative term in the gluon is now small ~3σ from zero. In fact this is also the case for the NNLO fit due to the raised starting scale Q^2_0=2.56GeV^2
Nevertheless the resulting gluon shapes are very different.
The form of the common parametrisation used for both NNLO and NLLO+NLLx is

$$
\begin{aligned}
xg(x) &= A_g x^{B_g}(1-x)^{C_g} - A'_g x^{B'_g}(1-x)^{C'_g}, \\
xu_v(x) &= A_{u_v} x^{B_{u_v}}(1-x)^{C_{u_v}}\left(1+E_{u_v}x^2\right), \\
xd_v(x) &= A_{d_v} x^{B_{d_v}}(1-x)^{C_{d_v}}, \\
x\bar{U}(x) &= A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}}\left(1+D_{\bar{U}}x\right), \\
x\bar{D}(x) &= A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}}.
\end{aligned}
$$

Note it is not the negative term which makes the gluon turn over at low-x for NNLO, the main term can also have a valence like shape if Bg is positive.

After these adjustments – and adding experimental uncertainties we have



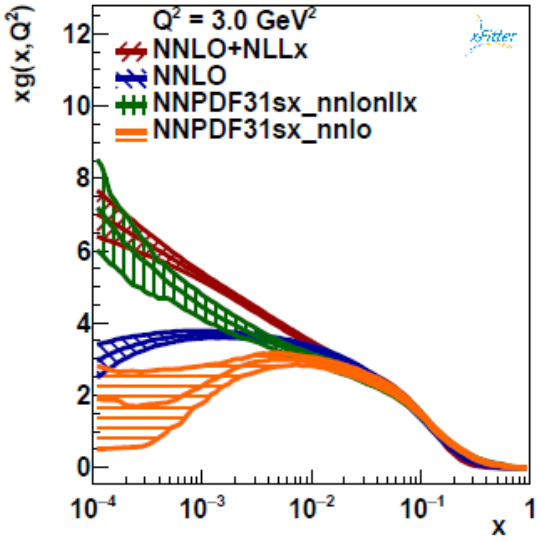


	NNLO fit with new settings	NNLO+NLLx fit with new settings
Total χ^2/d.o.f	1446/1178	1373/1178
subset NC 920 χ^2/n.d.p	446/377	413/377
subset NC 820 χ^2/n.d.p	70/70	65/70
subset charm χ^2/n.d.p	48/47	49/47
correlated shifts inclusive	102	77
correlated shifts charm	15	11
log term inclusive	20	−3
log term charm	−2	−1

A decrease in χ2 of 73
Largely due to the NC e+p 920 data
But also less need for shifts of systematic uncertainties

$$\chi^2 = \sum_i \frac{\left[D_i - T_i\left(1 - \sum_j \gamma_j^i b_j\right)\right]^2}{\delta_{i,\mathrm{unc}}^2 T_i^2 + \delta_{i,\mathrm{stat}}^2 D_i T_i} + \sum_j b_j^2$$

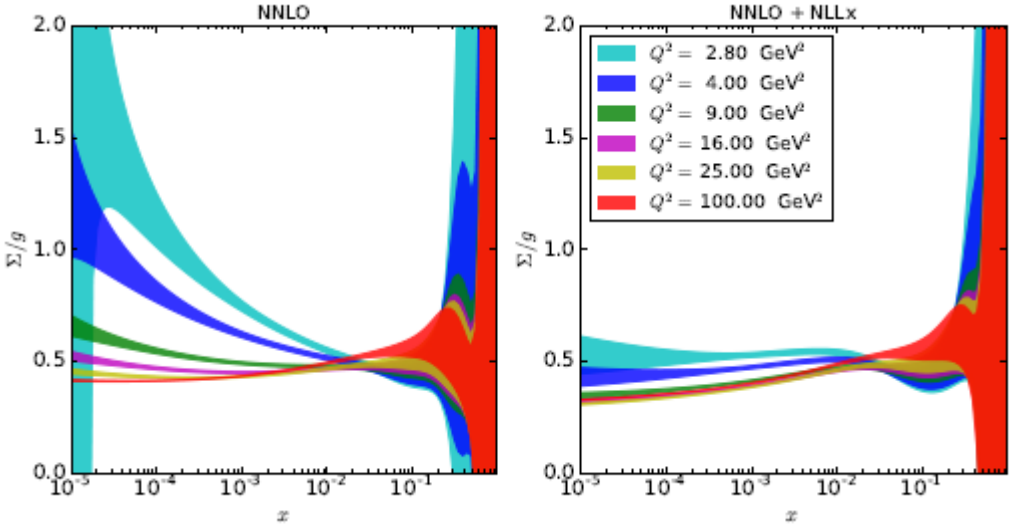
$$+ \sum_i \ln \frac{\delta_{i,\mathrm{unc}}^2 T_i^2 + \delta_{i,\mathrm{stat}}^2 D_i T_i}{\delta_{i,\mathrm{unc}}^2 D_i^2 + \delta_{i,\mathrm{stat}}^2 D_i^2},$$



NNPDF: arXiv:1710.05935, see similar trends in the gluon shape and improvements in χ2

NOTE: these PDFs including low x resummation will be much more suitable for use in MC generators

Further study of the shapes of the gluon and the sea



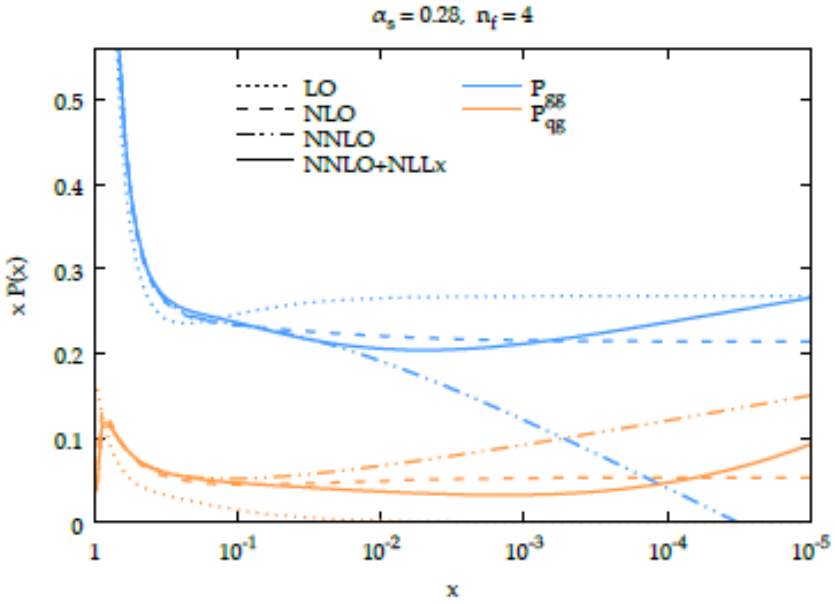
In fixed order fits at NNLO the gluon dips below the sea at low-x as Q^2 is reduced.
This is a general feature of NNLO fits- not just of HERAPDF.

These plots show the ratio of total sea Σ to gluon vs x for various Q2 for NNLO and NNLO+NLLx fits.
The ratio is much more stable with Low x resummation

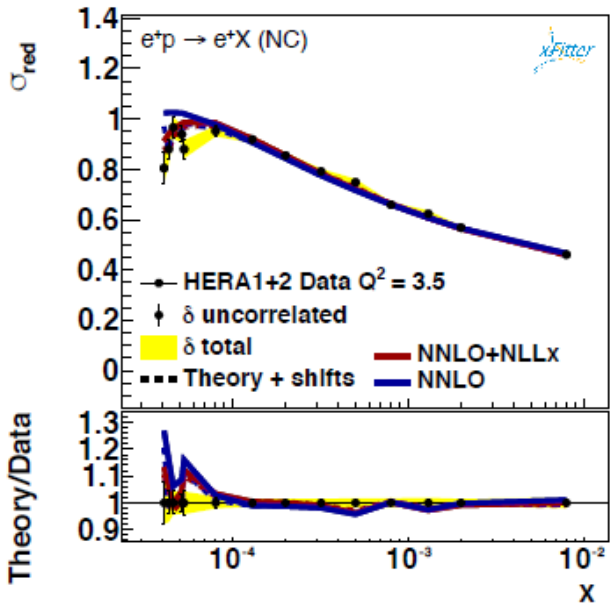
This arises from the behaviour of the P_{qg} and P_{gg} splitting functions.
At NNLO $xP_{qg}(x) > xP_{gg}(x)$ for $x \lesssim 10^{-3}$,

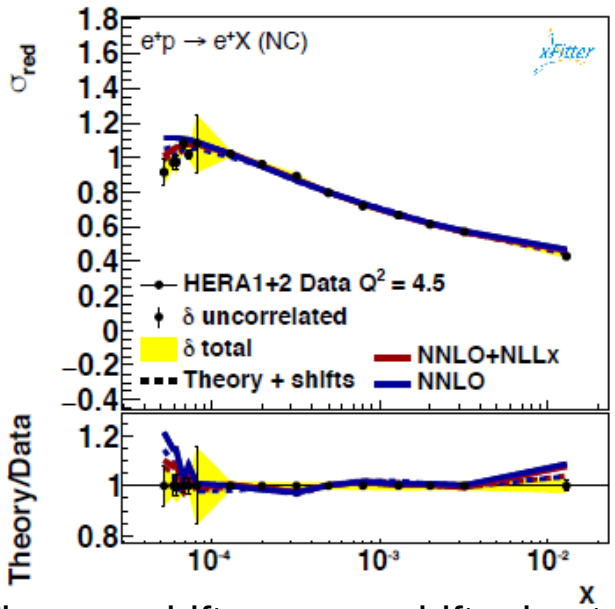
Whereas for NNLO+NLLx

$$xP_{qg}(x) < xP_{gg}(x)$$



Comparison to data



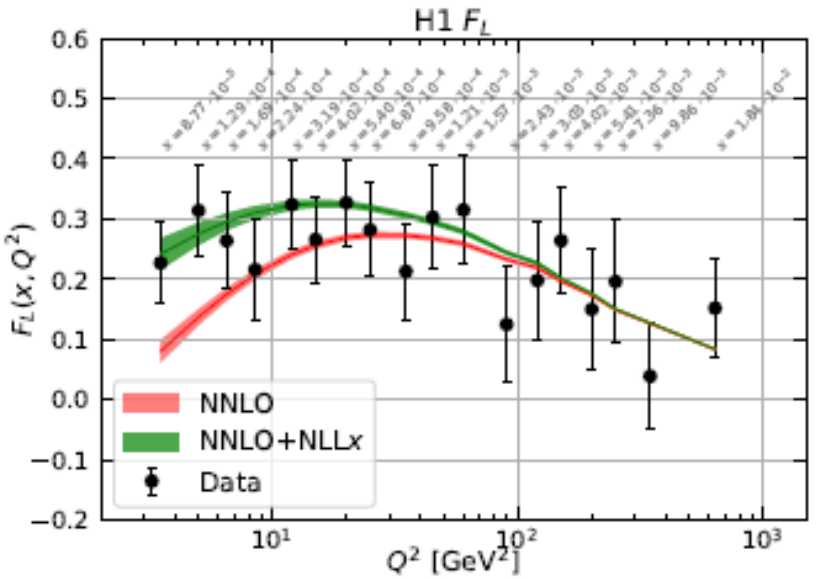


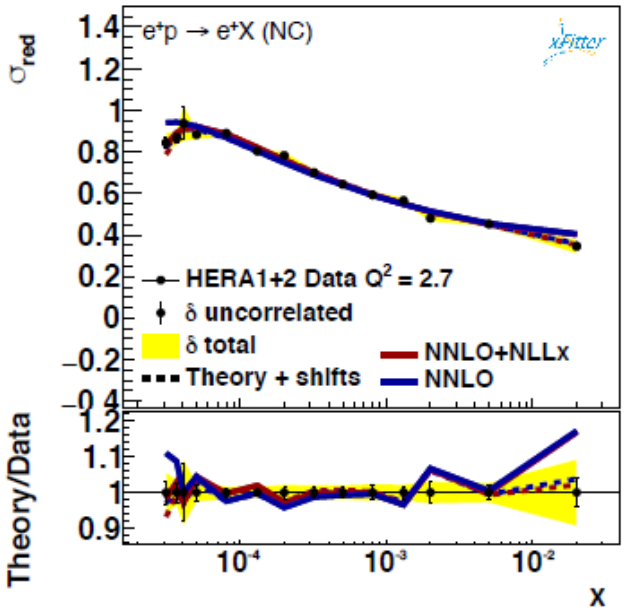
Theory +shifts means shifts due to experimental systematics

Comparison to data in the lowest Q^2 bins shows that the fit with low x resummation is much better able to follow the turn over of the data that happens at low-x, low Q^2, high-y due to the F_L term in the reduced cross section

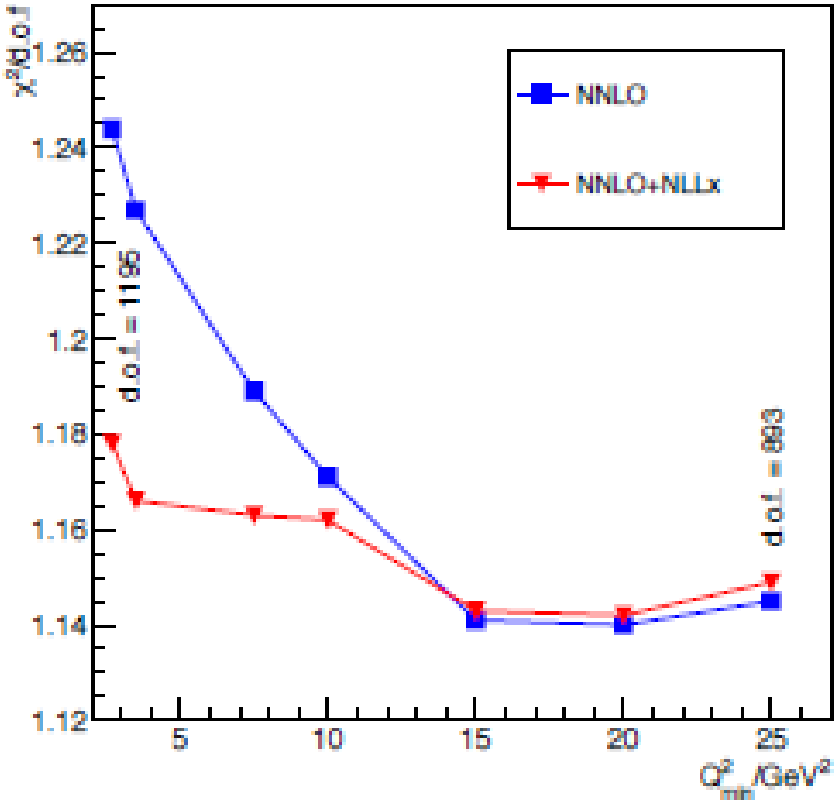
$$\sigma_{\text{red}} = F_2 - \frac{y^2}{Y_+} F_L$$

Looking at H1 F_L data directly shows that FL is larger at low Q^2/x for the NLLx fit

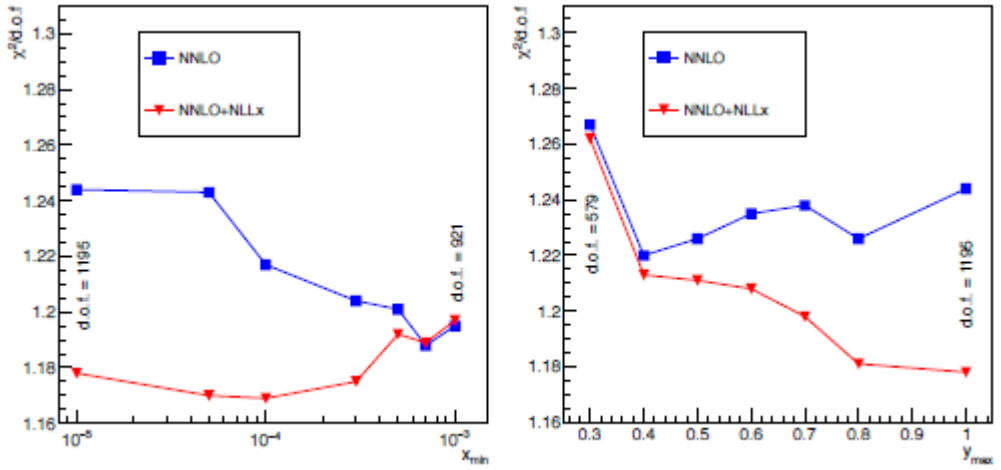




We also extend below the usual low Q2 cut –off for the data, down to Q2=2.7GeV2.
The NNLO+NLLx fit including these data gives very similar PDFs to the standard fit



To dileneate the kinematic region for which NLLX resummation improves the fits we perform a χ2 scan in Q^2_{min} for NNLO and NNLO+NLLx fits
This shows an improvement at low Q^2, $Q^2_{min} < 15$ GeV^2



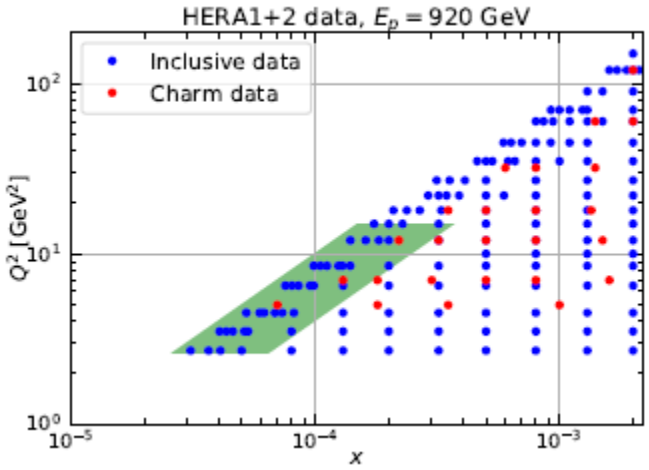
We also scan vs x_{min} seeing improvement for $x_{min} < 5.10^{-4}$

And against y^{max} seeing improvement for $y_{max}>0.4$

This emphasizes the importance of low x resummation at high-y for the DIS data because of the role of the FL term

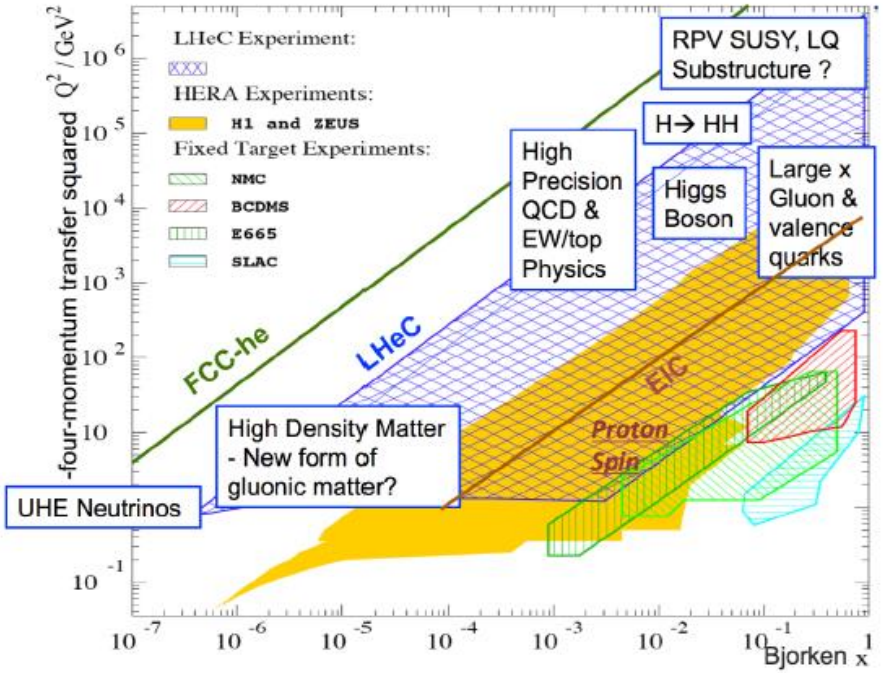
$$\sigma_{\text{red}} = F_2 - \frac{y^2}{Y_+} F_L$$

The scans shown here were done refitting the PDFs at each step—thus they dileneate a region where the fixed order calculation is poor –even with refitting –as illustrated on the x, Q^2 plane here.



$$H = \ln(1/x)/\ln Q^2/\Lambda^2 \text{ with } \Lambda = 88 \text{ MeV}.$$

LHeC and FCC-eh



LHeC kinematic reach:

Q^2 up to 10^6 GeV^2

x down to 10^{-6}

FCC-eh extends further,

Q^2 to 10^7 GeV^2, x to 10^{-7}

- **outline of this talk:**
- PDFs at FCC-eh
- strong coupling (α_s)

The relevance of this kinematic region to the LHeC is obvious

Summary

Low-x resummation:

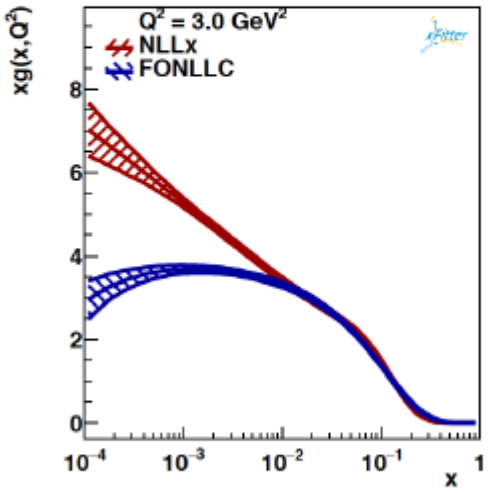
- Improves the description of HERA data at low-x, low-Q2, high-y, without need for further parameters (as for example when adding higher twist terms)
- Results in a rising low-x gluon, which is always larger than the total Sea

Back-up

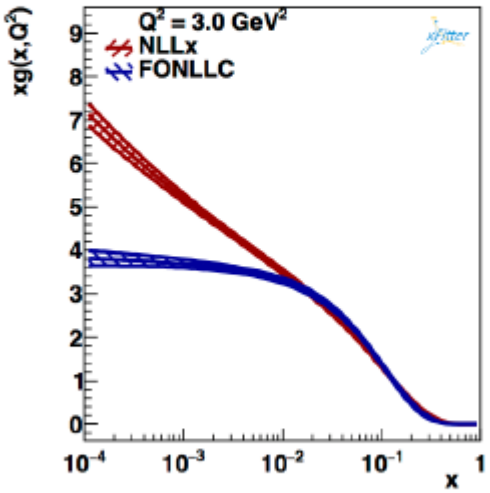
Adding the negative gluon term

Do we really need the negative term of gluon? → We produced a version of the **final NNLO+NLLx and NNLO fits without the negative term** just to check this

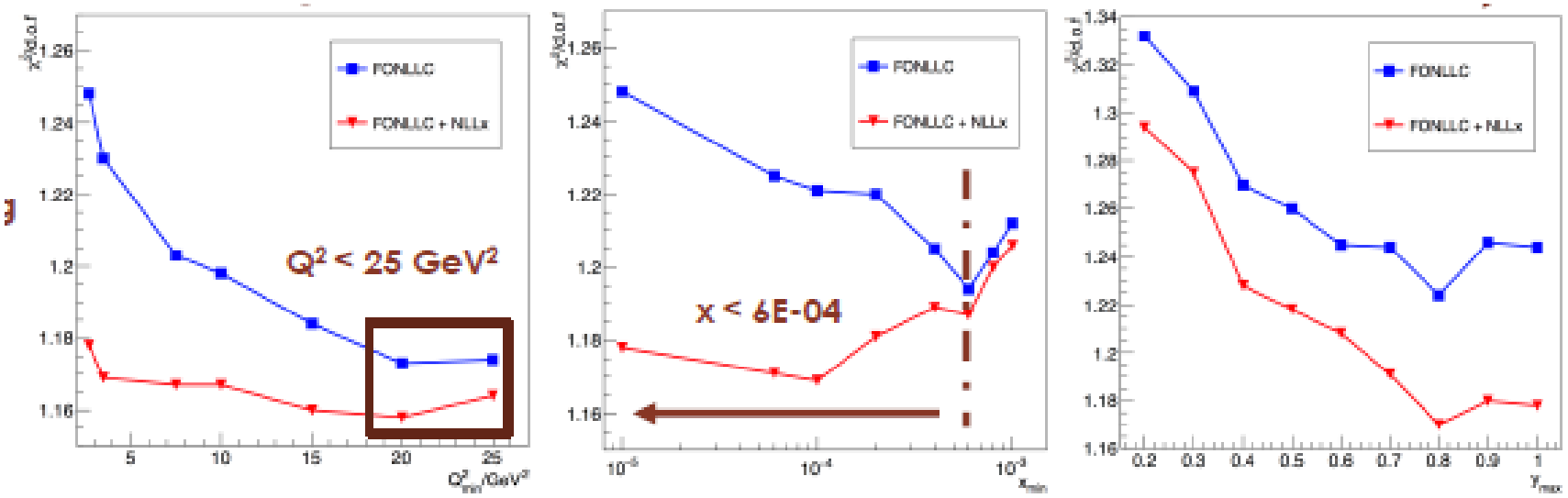
NNLO+NLLx (standard)



NNLO+NLLx (w/o neg term gluon)



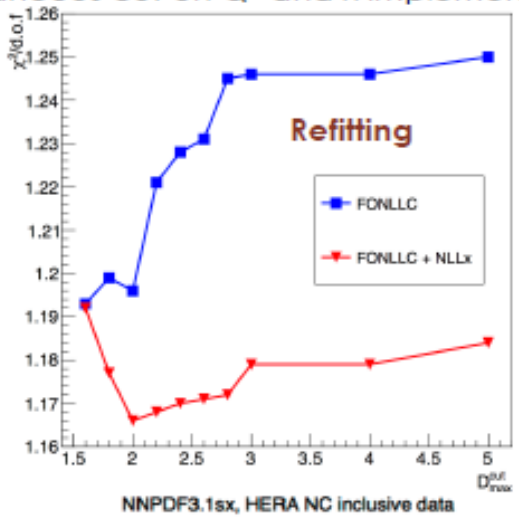
The point is that even without the negative term the gluon for NLLO likes to take a flattish shape at low-x, whereas for NNLO+NLLx it takes a singular shape

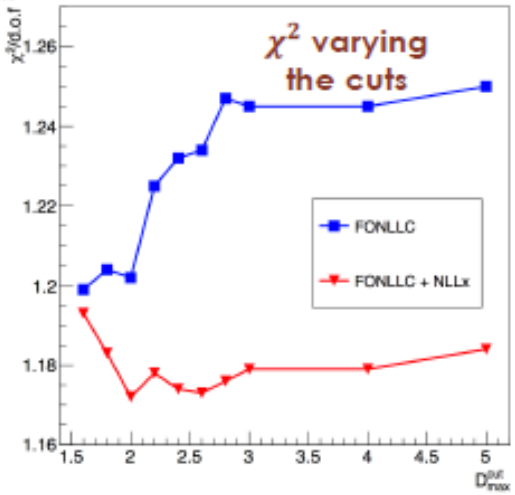


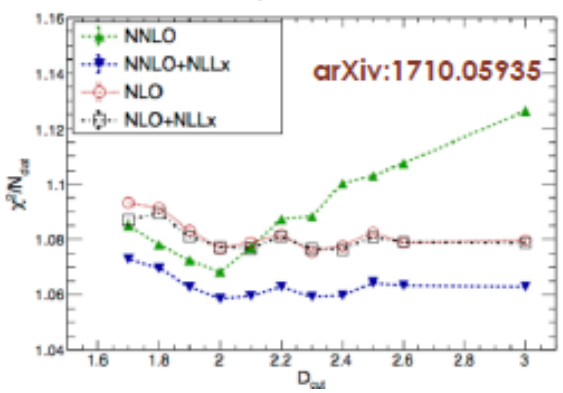
IF we do not refit at every step of the scan but we use the same PDFs as determined for the minimal cuts, we get these results.
The improvement of NLLx is still at low-x and low Q2 but is not so obviously concentrated at high-y. The refitting takes into account the need to fit DIS data for which the effect of F_{IL} is concentrated at high-y

$$\sigma_{\mathrm{red}} = F_2 - \frac{y^2}{Y_+} F_L$$

Simultaneous cut on Q^2 and x implemented: $\ln\frac{1}{x} \geq \beta_0 H_{\rm cut} \ln\frac{Q^2}{\Lambda^2}$ $\Lambda \simeq 88$ MeV $\beta_0 \simeq 0.61$







Consistent with what has been found in the NNPDF paper:

- $D_{cut} > 2$ defines the region where resummation is important
- Flat-ish χ^2 distribution for NNLO+NLLx
- Above $D_{cut} = 3$ few data points added even if with huge steps

We could also perform a cut jointly in x and Q^2 which follows the dependence of resummation terms on $\alpha_S(Q^2)$ ln(1/x) ie on ln(1/x)/ln(Q^2)

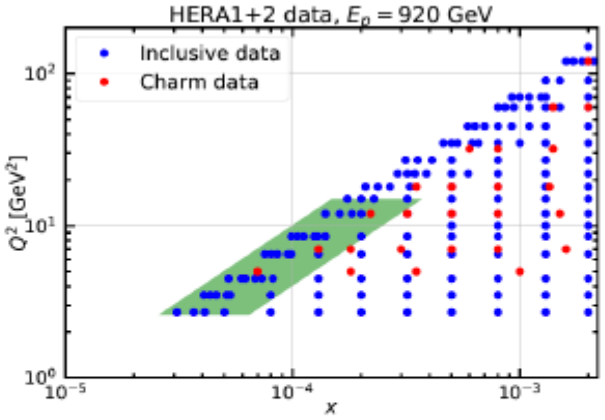


Figure 11 Scatter plot of the low-x and low-Q^2 kinematic region covered by the HERA1+2 inclusive data and charm data at E_p = 920 GeV. The green shaded area indicates the region in which $\ln(1/x)$ resummation has a significant effect.

Defined by:
- $x < 5 \cdot 10^{-4}$
- $2.7\ GeV^2 < Q^2 < 15\ GeV^2$
- $0.4 < y < 1.0$

Since the χ^2 scans have been obtained independently from one another, one may wonder whether our estimate is reliable.

In order to check this, we have performed two additional fits, one with and one without resummation, excluding only the data points for which $Q^2 < 15$ GeV^2 and $y > 0.4$.

--- NNLO+NLLx ---

```
After minimisation      1249.201064 1.174

Partial chi2s
395.95( +3.95)    354  HERA1+2 NCep 920
 51.32( -0.64)     56  HERA1+2 NCep 820
179.52( -1.09)    214  HERA1+2 NCep 575
179.12( -2.25)    170  HERA1+2 NCep 460
222.78( -0.82)    159  HERA1+2 NCem
 45.59( +0.57)     39  HERA1+2 CCep
 53.88( -2.45)     42  HERA1+2 CCem
 44.53( -1.11)     44  Charm cross section

Correlated Chi2      80.329061352348674
Log penalty Chi2     -3.8395890369565198
```

--- NNLO ---

```
After minimisation      1264.22  1064 1.188

Partial chi2s
402.82( +7.25)    354  HERA1+2 NCep 920
 52.23( -0.10)     56  HERA1+2 NCep 820
177.53( +1.15)    214  HERA1+2 NCep 575
176.67( -0.31)    170  HERA1+2 NCep 460
215.44( +1.04)    159  HERA1+2 NCem
 44.30( +0.35)     39  HERA1+2 CCep
 54.93( -1.58)     42  HERA1+2 CCem
 45.39( -1.31)     44  Charm cross

Correlated Chi2      88.418716117383113
Log penalty Chi2      6.4854418695532452
```

- The total χ^2's of these fits differ by around 15 units in favour of the resummed fit, mostly due to the correlated and logarithmic terms, to be compared to the 73 units of when the shaded area is instead included.
- This confirms that, the context of DIS, the shaded area in Fig. 11 does provide a reliable estimate of the kinematic region in which resummation works significantly better than fixed order.