Rare kaon decays from lattice QCD

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Summary of lattice study on rare kaon decays

Proposal to study rare kaon decays using lattice QCD

- Isidori, Martinelli, Turchetti, hep-lat/0506026 - PLB 2006

Method paper and first calculation of $K \to \pi\ell^+\ell^-$ @ $m_\pi = 430$ MeV

- Christ, XF, Portelli, Sachrajda, 1507.03094 - PRD 2015
- Christ, XF, Jüttner, Lawson, Portelli, Sachrajda, 1608.07585 - PRD 2016

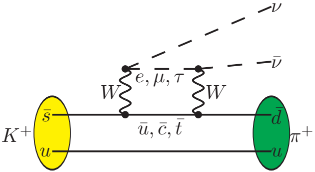
Method paper and first calculation of $K^+ \to \pi^+\nu\bar{\nu}$ @ $m_\pi = 420$ MeV

- Christ, XF, Portelli, Sachrajda, 1605.04442 - PRD 2016
- Bai, Christ, XF, Lawson, Portelli, Sachrajda, 1701.02858 - PRL 2017
  1806.11520 - submit to PRD

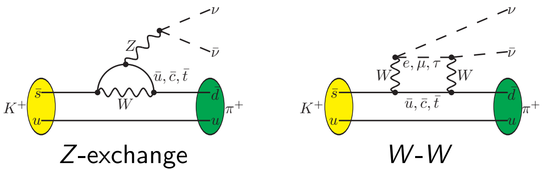
$K^+ \to \pi^+\nu\bar{\nu}$, $32^3 \times 64$, $m_\pi = 170$ MeV, $m_c^{\overline{\mathrm{MS}}}(2\text{ GeV}) = 750$ MeV

$K^+ \to \pi^+\nu\bar{\nu}$, $64^3 \times 128$, $m_\pi = 140$ MeV, $m_c^{\overline{\mathrm{MS}}}(2\text{ GeV}) = 1.2$ GeV

$K^+ \to \pi^+ \nu\bar{\nu}$



$K^+ \to \pi^+ \nu\bar{\nu}$: Experiment vs Standard model



$K^+ \to \pi^+ \nu\bar{\nu}$: largest contribution from top quark loop, thus theoretically clean

$$\mathcal{H}_{eff} \sim \frac{G_F}{\sqrt{2}} \cdot \underbrace{\frac{\alpha_{\rm EM}}{2\pi \sin^2\theta_W} \lambda_t X_t(x_t)}_{\mathcal{N} \sim 2\times 10^{-5}} \cdot (\bar{s}d)_{V-A} (\bar{\nu}\nu)_{V-A}$$

Probe the new physics at scales of $\mathcal{N}^{-\frac{1}{2}} M_W = O(10\text{ TeV})$

Past experimental measurement is 2 times larger than SM prediction

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\rm exp} = 1.73^{+1.15}_{-1.05} \times 10^{-10} \quad \text{[BNL E949, '08]}$$
$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\rm SM} = 9.11 \pm 0.72 \times 10^{-11} \quad \text{[Buras et. al., '15]}$$

but still consistent with $> 60\%$ exp. error

New experiments

New generation of experiment: NA62 at CERN [talk by J. Engelfried]

- aims at observation of $O(100)$ events [2016-2018, 2021-2023]
- 10%-precision measurement of $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$

NA62 timeline

- 2016 run: 2×10^{11} K^+ decays
- 2017 run: 6.7×10^{11} K^+ decays
- 2018 run: starting from April 9, 210 days run

Analysis of 2016 data $\Rightarrow$ 1 candidate event is identified

$\boldsymbol{K_L \to \pi^0 \nu\bar{\nu}}$ - KOTO experiment at J-PARC [talk by K. Nakagiri]

$K^+ \to \pi^+ \nu\bar{\nu}$ in the Standard Model

Branching ratio for $K^+ \to \pi^+ \nu\bar{\nu}$ [Buras, Buttazzo, Girrbach-Noe, Knegjens, '15]

$$\mathrm{Br} = \kappa_+ (1 + \Delta_{\mathrm{EM}}) \cdot \Big[\big(\underbrace{\frac{\mathrm{Im}\,\lambda_t}{\lambda^5} X(x_t)}_{0.270 \times 1.481(9)} \big)^2 + \big(\underbrace{\frac{\mathrm{Re}\,\lambda_c}{\lambda} P_c}_{-0.974 \times 0.405(23)} + \underbrace{\frac{\mathrm{Re}\,\lambda_t}{\lambda^5} X(x_t)}_{-0.533 \times 1.481(9)} \big)^2 \Big]$$

- $X(x_t)$: top quark contribution; P_c: charm and LD contribution

> Without P_c, branching ratio is 50% smaller

Uncertainty budget

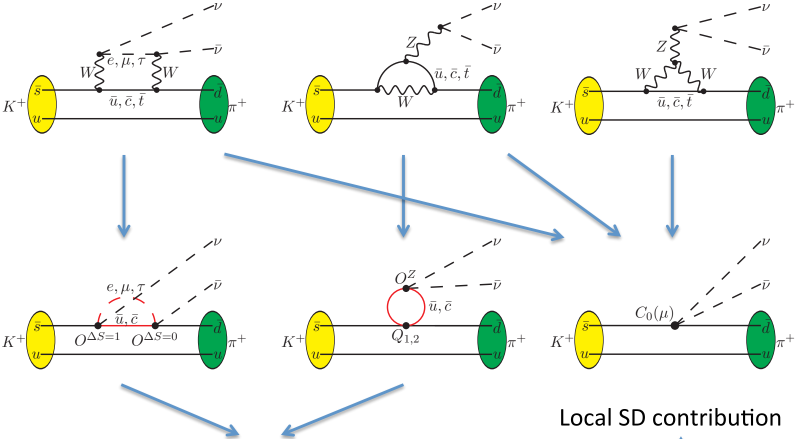
- dominant uncertainty from CKM factor λ_t
- once fixing CKM factor, then P_c dominates the uncertainty
    - P_c's uncertainty mainly come from LD

> Important to determine the LD contribution to P_c accurately

Current estimate $\delta P_{c,u} = 0.04(2)$ [Isidori, Mescia, Smith, '05]

- OPE+χPT, estimate LD correction by including dim-8 operators

OPE: integrate out the heavy fields, Z, W, t, ⋯



Local SD contribution

Bilocal LD contribution

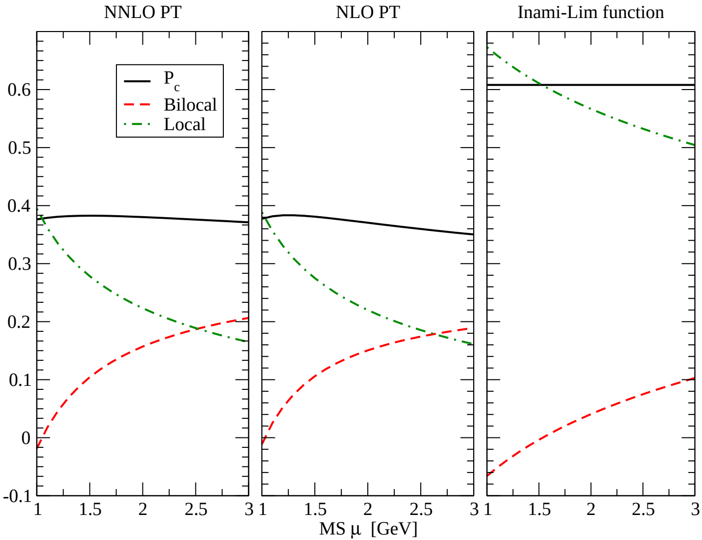
Hadronic part known: $\langle \pi^+ | V_\mu | K^+ \rangle$

$\langle \pi^+ \nu\bar{\nu} | Q_A(x) Q_B(0) | K^+ \rangle$: need lattice QCD

Bilocal contribution vs local contribution

Bilocal $C_A^{\overline{\mathrm{MS}}}(\mu) C_B^{\overline{\mathrm{MS}}}(\mu) r_{AB}^{\overline{\mathrm{MS}}}(\mu)$ vs Local $C_0^{\overline{\mathrm{MS}}}(\mu)$

[Buras, Gorbahn, Haisch, Nierste, '06]



At $\mu = 2.5$ GeV, 50% charm quark contribution from bilocal term

Exponential contamination at large Euclidean time

Hadronic matrix element for the 2nd weak interaction

$$\int_{-T}^{T} dt\, \langle \pi^+ \nu\bar{\nu} | T\left[Q_A(t) Q_B(0)\right] | K^+ \rangle$$
$$= \sum_n \left\{ \frac{\langle \pi^+ \nu\bar{\nu} | Q_A | n \rangle \langle n | Q_B | K^+ \rangle}{M_K - E_n} + \frac{\langle \pi^+ \nu\bar{\nu} | Q_B | n \rangle \langle n | Q_A | K^+ \rangle}{M_K - E_n} \right\} \left(1 - e^{(M_K - E_n)T}\right)$$

- For $E_n > M_K$, the exponential terms exponentially vanish at large T
- For $E_n < M_K$, the exponentially growing terms must be removed
- $\sum_n$: principal part of the integral replaced by finite-volume summation
  - possible large finite volume correction when $E_n \to M_K$

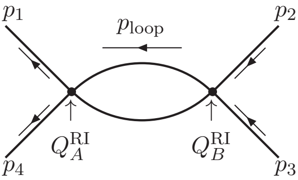
[Christ, XF, Martinelli, Sachrajda, '15]

Christ, XF, Portelli, Sachrajda, PRD 93 (2016) 114517

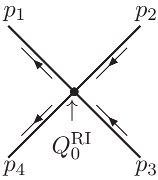
New SD divergence appears in $Q_A(x)Q_B(0)$ when $x \to 0$

- Introduce a counter term $X \cdot Q_0$ to remove the SD divergence

$\langle\{Q_A Q_B\}^{\mathrm{RI}}\rangle\Big|_{p_i^2=\mu_0^2} =$



$- \; X(\mu_0, a)\times$



$= \; 0$

The coefficient X is determined in the RI/(S)MOM scheme

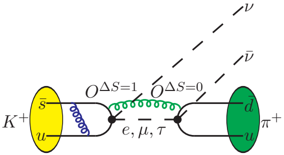
- The bilocal operator in the $\overline{\mathrm{MS}}$ scheme can be written as

$$\left\{\int d^4x\, T[Q_A^{\overline{\mathrm{MS}}}(x)\,Q_B^{\overline{\mathrm{MS}}}(0)]\right\}^{\overline{\mathrm{MS}}}$$

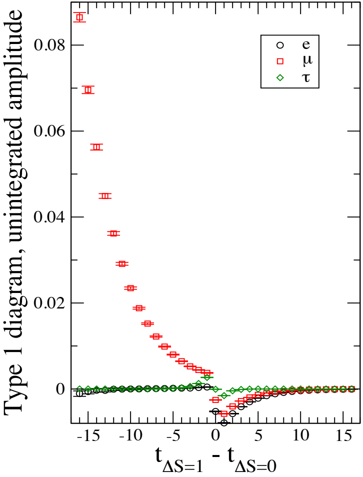
$$= Z_A Z_B \left\{\int d^4x\, T[Q_A^{\mathrm{lat}}\,Q_B^{\mathrm{lat}}]\right\}^{\mathrm{lat}} + \left(-X^{\mathrm{lat}\to\mathrm{RI}} + Y^{\mathrm{RI}\to\overline{\mathrm{MS}}}\right) Q_0(0)$$

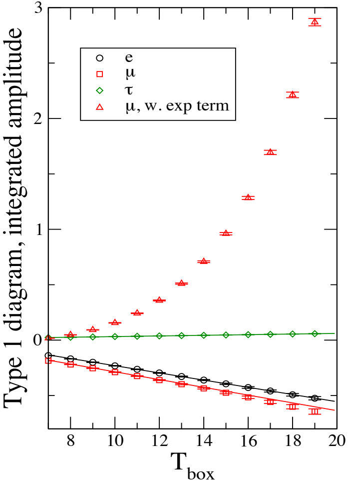
  - $X^{\mathrm{lat}\to\mathrm{RI}}$ is calculated using NPR and $Y^{\mathrm{RI}\to\overline{\mathrm{MS}}}$ calculated using PT

W-W diagram, type 1

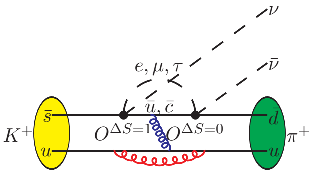


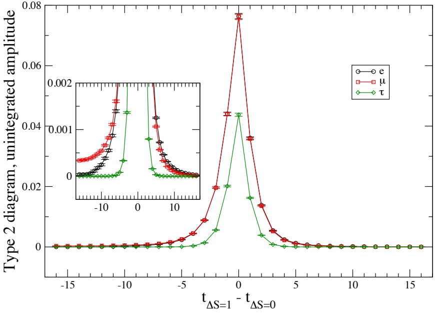
F_{WW}	Type 1
e	$-1.685(47) \times 10^{-2}$
μ	$-1.818(40) \times 10^{-2}$
τ	$1.491(36) \times 10^{-3}$



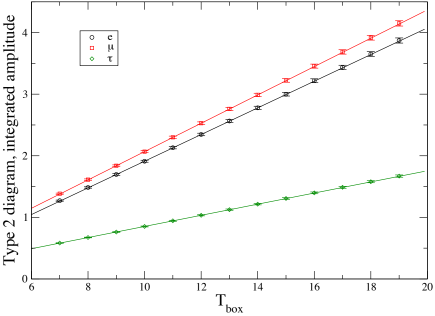


W-W diagram, type 2

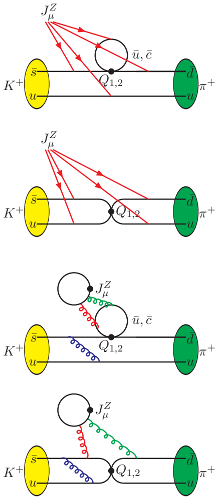


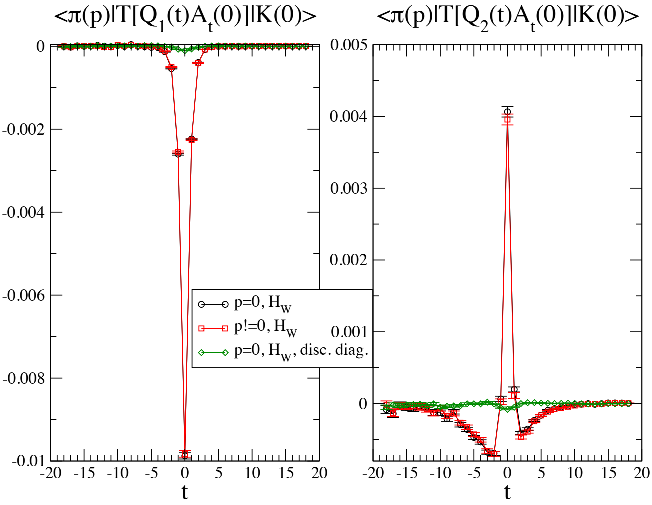


F_{WW}	Type 2
e	$1.123(17) \times 10^{-1}$
μ	$1.194(18) \times 10^{-1}$
τ	$4.690(77) \times 10^{-2}$

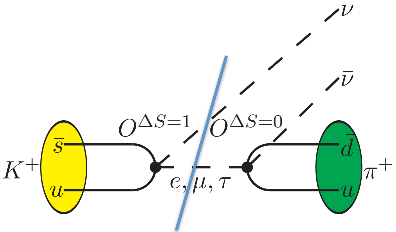


Z-exchange diagram

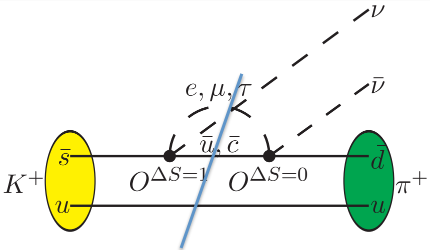




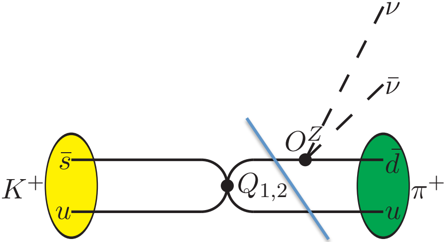
Low lying intermediate states



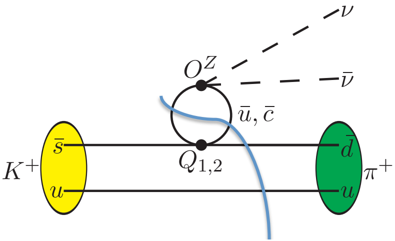
$K^+ \to \ell^+ \nu \quad \& \quad \ell^+ \to \pi^+ \bar{\nu}$



$K^+ \to \pi^0 \ell^+ \nu \quad \& \quad \pi^0 \ell^+ \to \pi^+ \bar{\nu}$



$K^+ \xrightarrow{H_W} \pi^+ \quad \& \quad \pi^+ \xrightarrow{V_\mu} \pi^+$



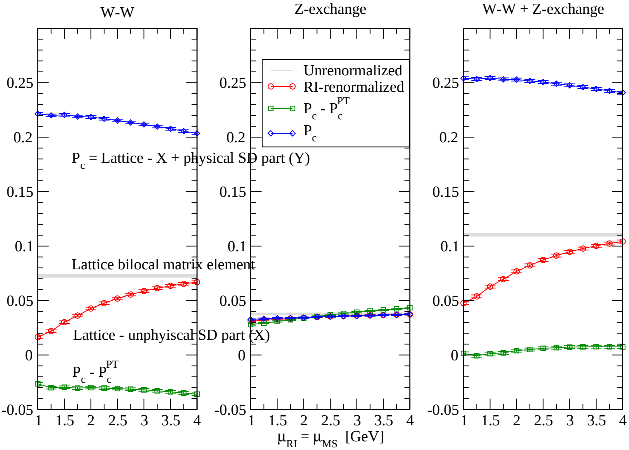
$K^+ \xrightarrow{H_W} \pi^+ \pi^0 \quad \& \quad \pi^+ \pi^0 \xrightarrow{A_\mu} \pi^+$

Lattice results

Published results @ $m_\pi = 420$ MeV, $m_c = 860$ MeV

[Bai, Christ, XF, Lawson, Portelli, Schrajda, PRL 118 (2017) 252001]

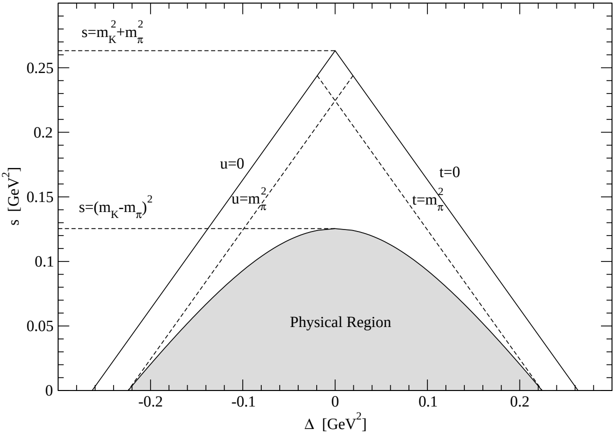
$$P_c = 0.2529(\pm 13)_{\mathrm{stat}}(\pm 32)_{\mathrm{scale}}(-45)_{\mathrm{FV}}$$



Lattice QCD is now capable of first-principles calculation of rare kaon decay

- The remaining task is to control various systematic effects

Momentum choice



- Two Lorentz invariant variables

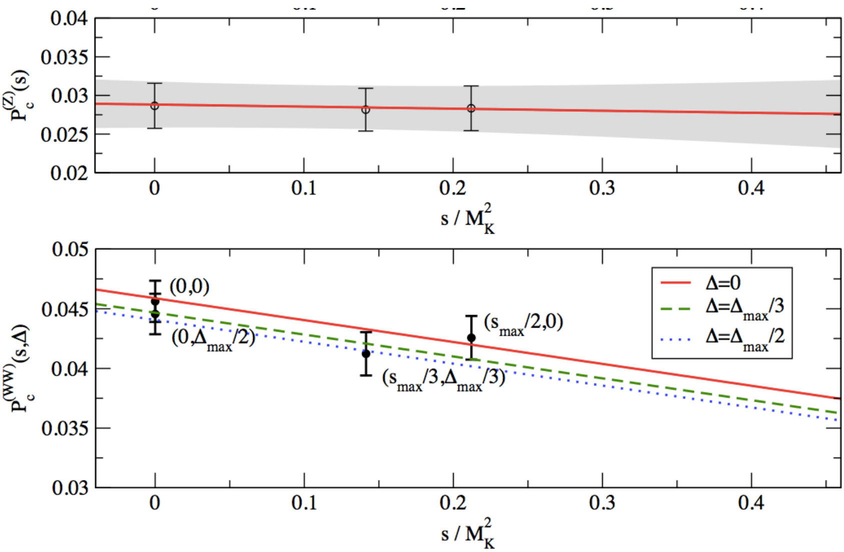
$$s = (p_K - p_\pi)^2, \quad \Delta = (p_K - p_\nu)^2 - (p_K - p_{\bar{\nu}})^2$$

- $s_{\max} = (M_K - M_\pi)^2, \quad \Delta_{\max} = M_K^2 - M_\pi^2$
- Momentum choice

$$(s, \Delta) = (0,0),\ (s_{\max}/2, 0),\ (0, \Delta_{\max}),\ (s_{\max}/3, \Delta_{\max}/3)$$

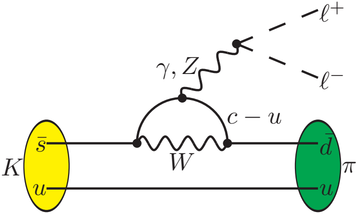
Momentum dependence

calculation @ m_π = 170 MeV, $L^3 \times T = 32^3 \times 64$



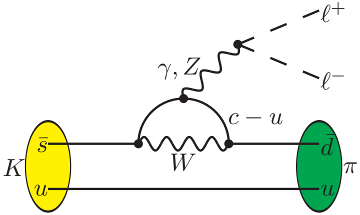
Momentum dependence is mild at near-physical pion mass

$K \to \pi\ell^+\ell^-$



CP conserving decay: $K^+ \to \pi^+\ell^+\ell^-$ and $K_S \to \pi^0\ell^+\ell^-$

- Involve both γ- and Z-exchange diagram, but γ-exchange is much larger



- Unlike Z-exchange, the γ-exchange diagram is LD dominated
  - By power counting, loop integral is quadratically UV divergent
  - EM gauge invariance reduces divergence to logarithmic
  - $c - u$ GIM cancellation further reduces log divergence to be UV finite

Lattice calculation strategy

Focus on γ-exchange

- Hadronic part of decay amplitude is described by a form factor

$$\begin{aligned} T^{\mu}_{+,S}(p_K, p_\pi) &= \int d^4x\, e^{iqx} \langle \pi(p_\pi) | T\{J^{\mu}_{em}(x) \mathcal{H}^{\Delta S=1}(0)\} | K^+/K_S(p_K)\rangle \\ &= \frac{G_F M_K^2}{(4\pi)^2} V_{+,S}(z) \left[z(k+p)^\mu - (1-r_\pi^2) q^\mu \right] \end{aligned}$$

with $q = p_K - p_\pi$, $z = q^2/M_K^2$, $r_\pi = M_\pi/M_K$

The target for lattice QCD is to calculate the form factor $V_{+,S}(z)$

- Lattice calculation strategy: [RBC-UKQCD, PRD92 (2015) 094512]
  - Use conserved vector current to protect the EM gauge invariance
  - Use charm as an active quark flavor to maintain GIM cancellation

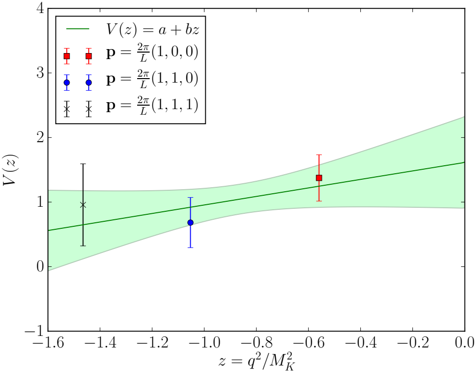
First exploratory calculation on $K^+ \to \pi^+ \ell^+ \ell^-$

Use $24^3 \times 64$ ensemble, $N_{\rm conf} = 128$
[RBC-UKQCD, PRD94 (2016) 114516]

$a^{-1} = 1.78$ GeV, $m_\pi = 430$ MeV
$m_K = 625$ MeV, $m_c = 530$ MeV

Momentum dependence of $V_+(z)$

$$V_+(z) = a_+ + b_+ z$$
$$\Rightarrow \quad a_+ = 1.6(7),\ b_+ = 0.7(8)$$



$K^+ \to \pi^+ e^+ e^-$ data + phenomenological analysis: $a_+ = -0.58(2)$, $b_+ = -0.78(7)$
[Cirigliano, et. al., Rev. Mod. Phys. 84 (2012) 399]

$$V_j(z) = a_j + b_j z + \underbrace{\frac{\alpha_j r_\pi^2 + \beta_j (z - z_0)}{G_F M_K^2 r_\pi^4}}_{K\to\pi\pi\pi} \underbrace{\left[1 + \frac{z}{r_V^2}\right]}_{F_V(z)} \underbrace{\left[\phi(z/r_\pi^2) + \frac{1}{6}\right]}_{\text{loop}}, \quad j = +, S$$

- Experimental data only provide $\frac{d\Gamma}{dz} \Rightarrow$ square of form factor $|V_+(z)|^2$
- Need phenomenological knowledge to determine the sign for a_+, b_+

$K_L \to \pi^0 \ell^+ \ell^-$ decay contains important *CPV* information

- Indirect *CPV*: $K_L \xrightarrow{\epsilon} K_+^0 \to \pi^0 \gamma^* \to \pi^0 \ell^+ \ell^-$
- Direct + indirect *CPV* contribution to branching ratio

  [Cirigliano et. al., Rev. Mod. Phys. 84 (2012) 399]

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-)_{CPV} = 10^{-12} \times \left[15.7 |a_S|^2 \pm 6.2 |a_S| \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}} \right) + 2.4 \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}} \right)^2 \right]$$

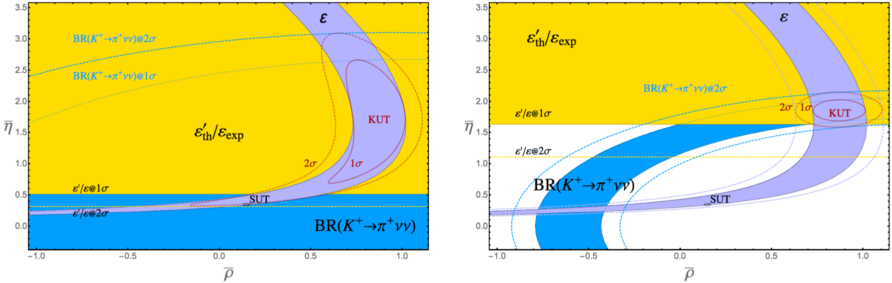
  - $\mathrm{Im}\,\lambda_t$-term from direct *CPV*, $\lambda_t \approx 1.35 \times 10^{-4}$
  - $|a_S|$-term from indirect *CPV*, $a_S = V_S(0)$
  - $\pm$ arises due to the unknown sign of a_S

Even a determination of the sign of a_S from lattice is desirable

[C. Lehner, E. Lunghi, A. Soni, PLB759 (2016) 82]

Advances in experiments + lattice QCD simulations

$\Rightarrow$ construction of a unitarity triangle purely from Kaon physics?



Current status	Future scenario
$\delta[A_2] = 12\%$	$\delta[A_2] = 5\%$
$\delta[A_0] = 86\%$	$\delta[A_0] = 18\%$
$\delta[\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})] = 64\%$	$\delta[\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})] = 10\%$

When will the future scenario become true?

Conclusion

- We present the lattice calculation on
  - $K^+ \to \pi^+\nu\bar{\nu}$ @ $m_\pi = 420$ MeV, $m_c = 860$ MeV
    @ $m_\pi = 170$ MeV, $m_c = 750$ MeV
  - $K^+ \to \pi^+\ell^+\ell^-$ @ $m_\pi = 430$ MeV, $m_c = 530$ MeV

  Calculation at physical point is timely since NA62 is underway
  - $K^+ \to \pi^+\nu\bar{\nu}$ @ $m_\pi = 140$ MeV, $m_c = 1.2$ GeV $\Rightarrow$ a few configurations analyzed
- Other interesting rare kaon modes
  - $K_L \to \mu^+\mu^-$
  - $K_S \to \pi^0\ell^+\ell^-$
  - $K_L \to \pi^0\ell^+\ell^-$

Backup slides