Connecting B decay anomalies with neutrino mass, dark matter and flavor violation

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Talk based on **arXiv:1806.10146**

With **Chandan Hati, Ana Teixeira, and Jean Orloff**

Introduction

The Standard Model (SM) is a remarkably successful theory of subatomic world but is not a complete description

A partial list of deficiencies of the SM includes:

Neutrino oscillations

Matter-antimatter Asymmetry

Dark Matter candidate

Many theoretical motivations…

Introduction

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Many theoretical motivations…

→ Need to look for beyond the SM Physics

No direct evidence of new particle at LHC so far

Several intriguing signs of NP in semileptonic B decays!

LFU violation in B decays ?

In the SM, couplings of gauge bosons to leptons are independent of lepton favour

Define lepton flavor universal observable: $$R_{K^{(*)}} = \frac{\mathrm{BR}(B \to K^{(*)}\mu^+\mu^-)}{\mathrm{BR}(B \to K^{(*)}e^+e^-)}$$

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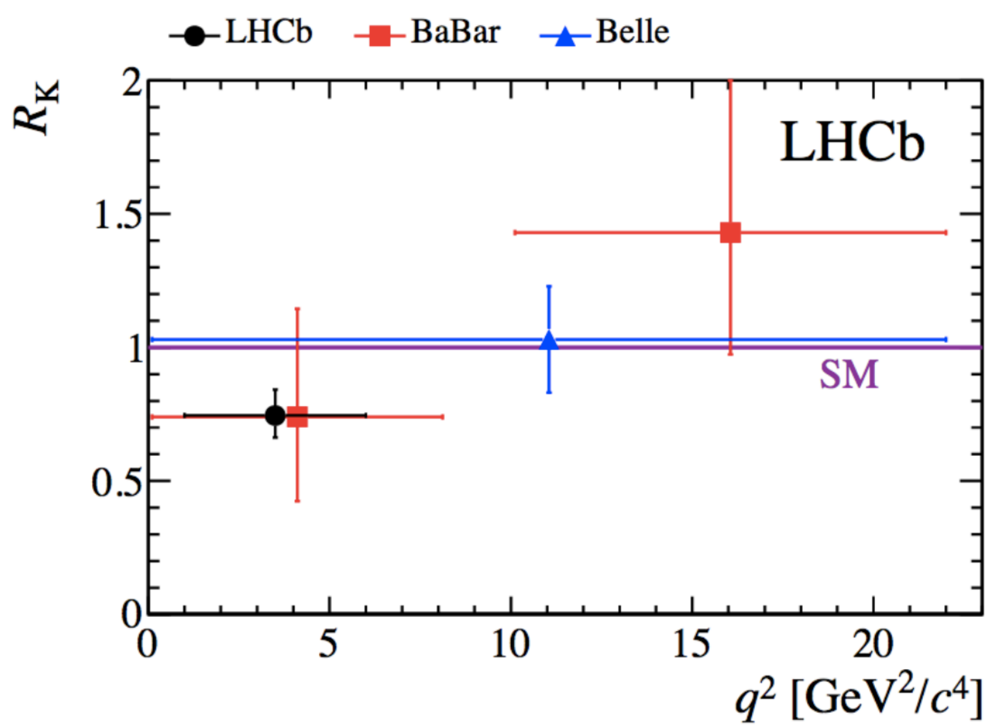
~ 2.5 sigma tension in lepton flavour universality ratio in

$$R_{K,\,[1-6]\text{ GeV}^2} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

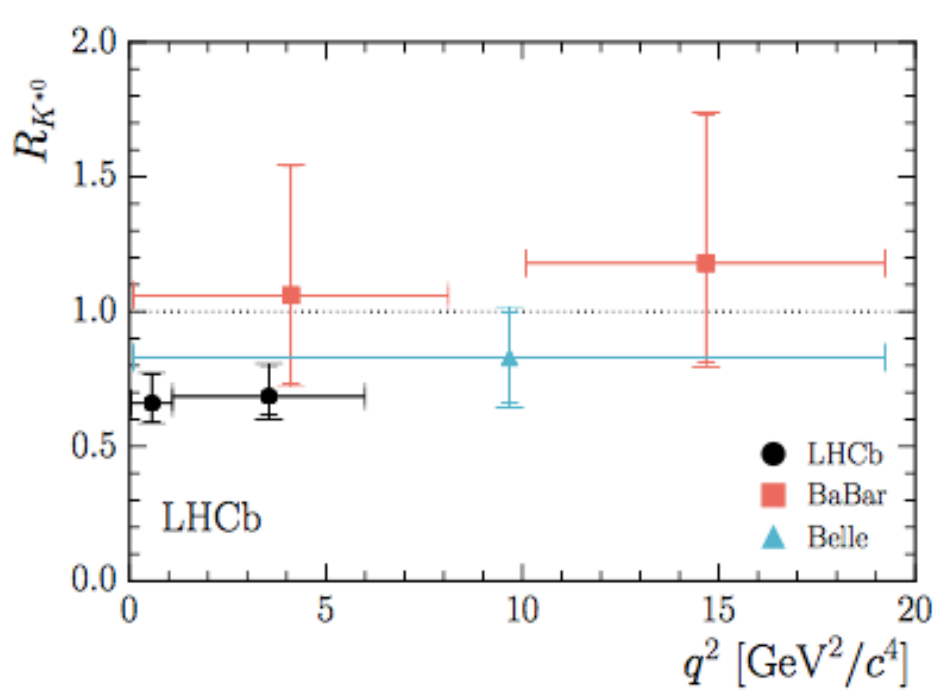
$$R_{K^*[1.1,6]} = 0.69^{+0.11}_{-0.07} \pm 0.05,$$

$$R_{K^*[0.045,1.1]} = 0.66^{+0.11}_{-0.07} \pm 0.03$$

LHCb PRL 113 (2014) 151601



LHCb, JHEP 1708, 055 (2017)



Hints of New Physics ?

Claims of NP are further strengthened by other data on $b \to s\mu^+\mu^-$

$B \to K^*\mu^+\mu^-$: Measurement of form factor free observables

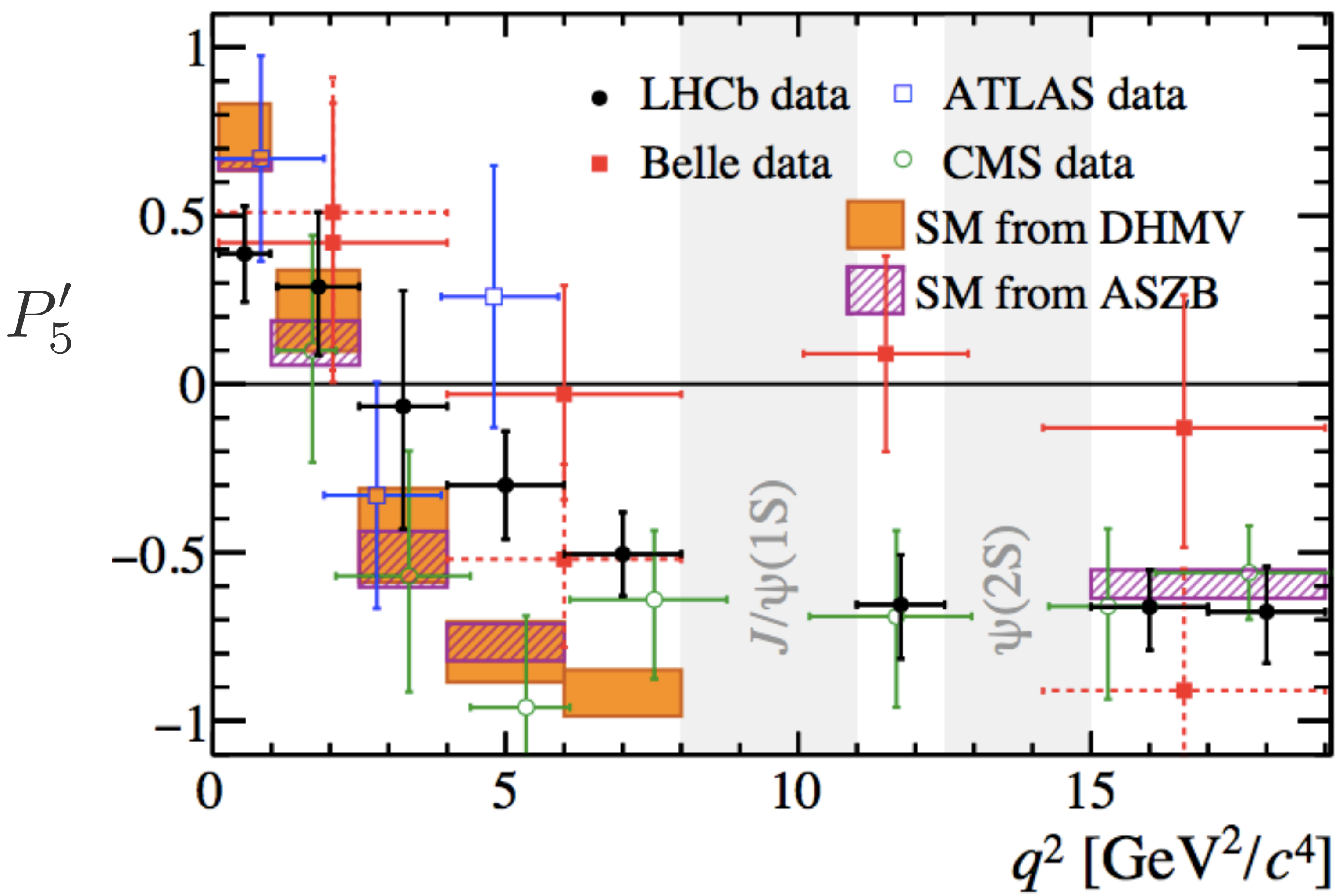


Fig. from T. Gershan's talk, Moriond 2017

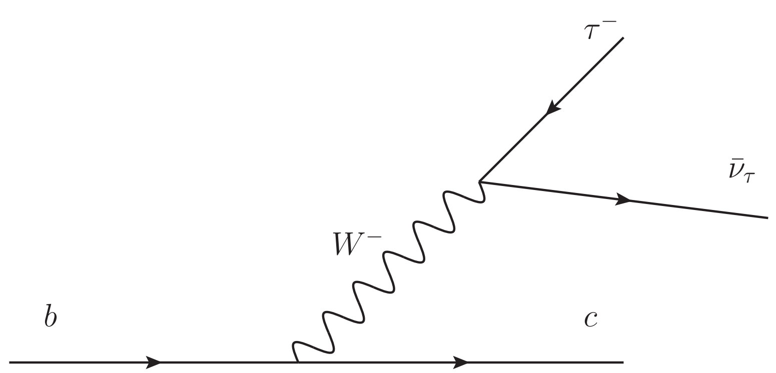
In BR of $B \to \phi\,\mu^+\mu^-$, a deviation of 3.5 sigma with respect to SM

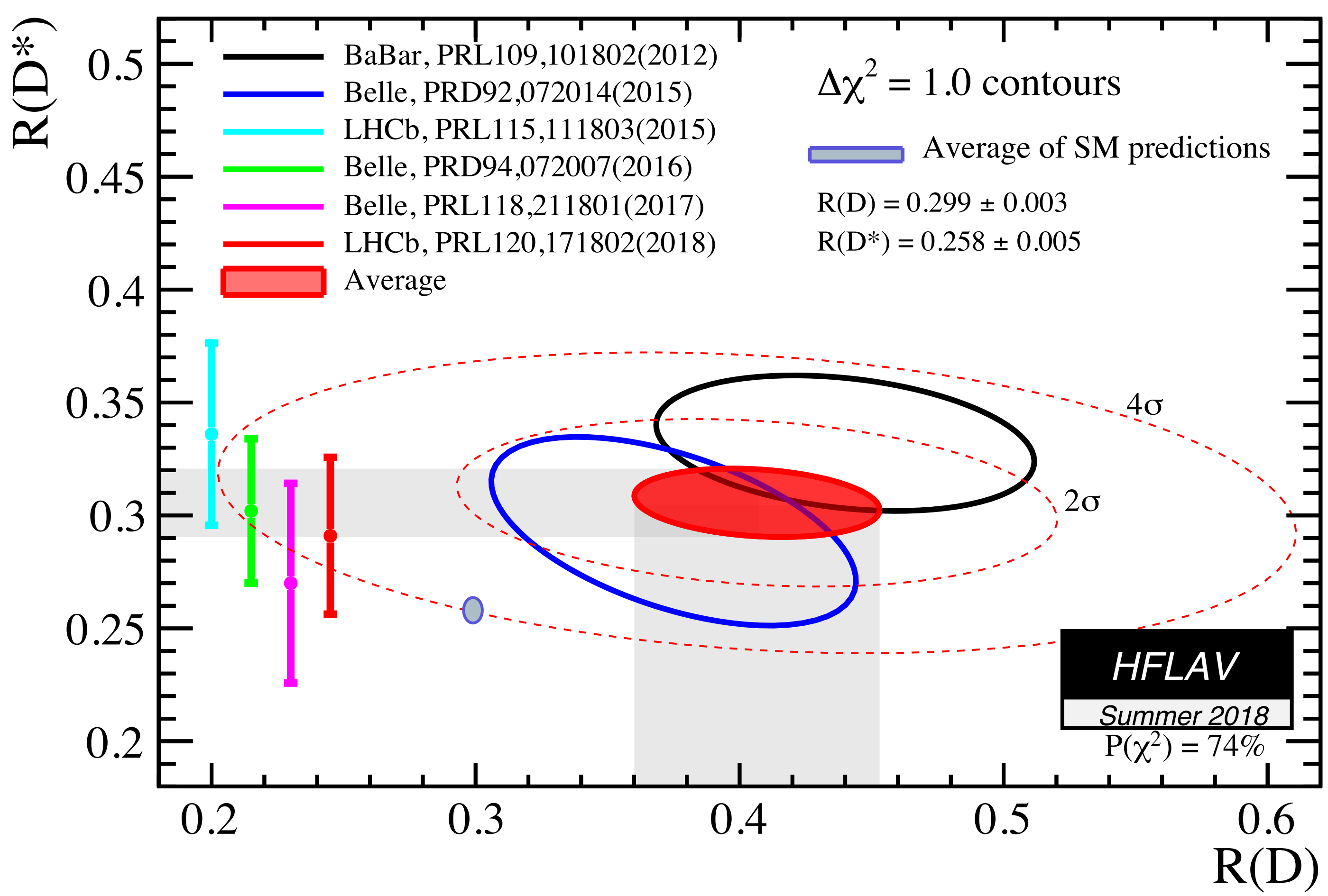
LHCb, JHEP 09 (2015) 179

LFU violation in B decays ?

~4 sigma deviation from the SM in the ratio

$$R_{D^{(*)}} = \frac{\mathrm{BR}(\bar{B} \to D^{(*)}\tau\bar{\nu})}{\mathrm{BR}(\bar{B} \to D^{(*)}\ell\bar{\nu})}; \quad \ell = e, \mu$$





(Heavy Flavor Averaging group, HFLAV)

Also, in $B_c^+ \to J/\psi\tau^+\nu_\tau$

$$R_{J/\psi} = 0.71 \pm 0.17 \pm 0.18$$

About 2σ above the SM

PRL 120, 121801 (2018)

The Model

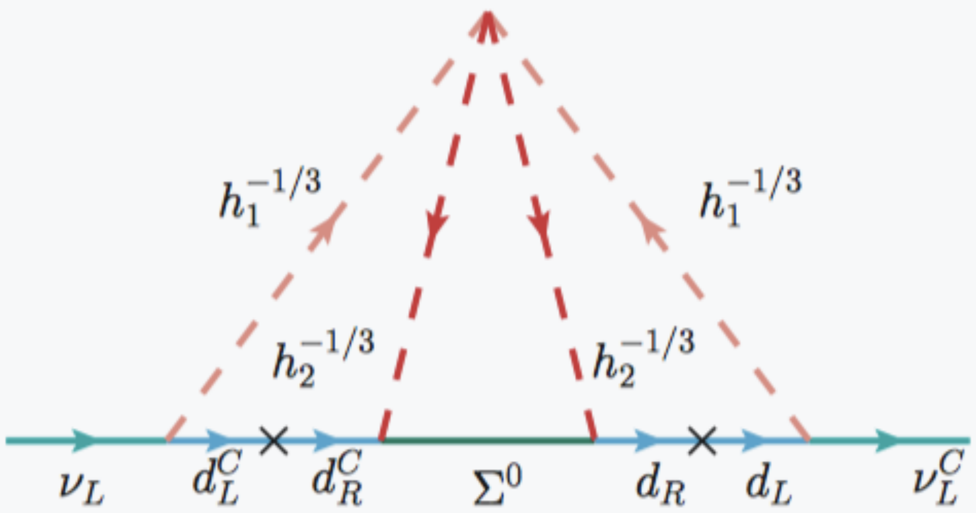
	Field	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Z_2
Fermions	$Q_L \equiv (u, d)_L^T$	$(\mathbf{3}, \mathbf{2}, 1/6)$	1
	u_R	$(\mathbf{3}, \mathbf{1}, 2/3)$	1
	d_R	$(\mathbf{3}, \mathbf{1}, -1/3)$	1
	$\ell_L \equiv (\nu,\ e)_L^T$	$(\mathbf{1}, \mathbf{2}, -1/2)$	1
	e_R	$(\mathbf{1}, \mathbf{1}, -1)$	1
	Σ_R	$(\mathbf{1}, \mathbf{3}, 0)$	-1
Scalars	H	$(\mathbf{1}, \mathbf{2}, 1/2)$	1
	h_1	$(\bar{\mathbf{3}}, \mathbf{3}, -1/3)$	1
	h_2	$(\bar{\mathbf{3}}, \mathbf{3}, -1/3)$	-1

Reinforced by a Z_2 symmetry : Forbids Type III seesaw, provides DM candidate

$$\mathscr{L}_{\rm int}^{h,\Sigma} = y_{ij}\, \bar{Q}_L^{C\,i}\, \epsilon\, (\vec{\tau}\,.\,\overrightarrow{h}_1)\, L_L^j + \tilde{y}_{ij}\, \overline{(\vec{\tau}\,.\,\overrightarrow{\Sigma})_R}^{C\,i,ab}\, [\epsilon\, (\vec{\tau}\,.\,\overrightarrow{h}_2)\, \epsilon^T]^{ba}\, d_R^j$$

$$-\frac{1}{2}\, \overline{\Sigma^C}^i\, M_{ij}^{\Sigma}\, \Sigma^j - V_{\rm scalar}^{H,h} + {\rm h.c.}$$

Neutrino mass generation

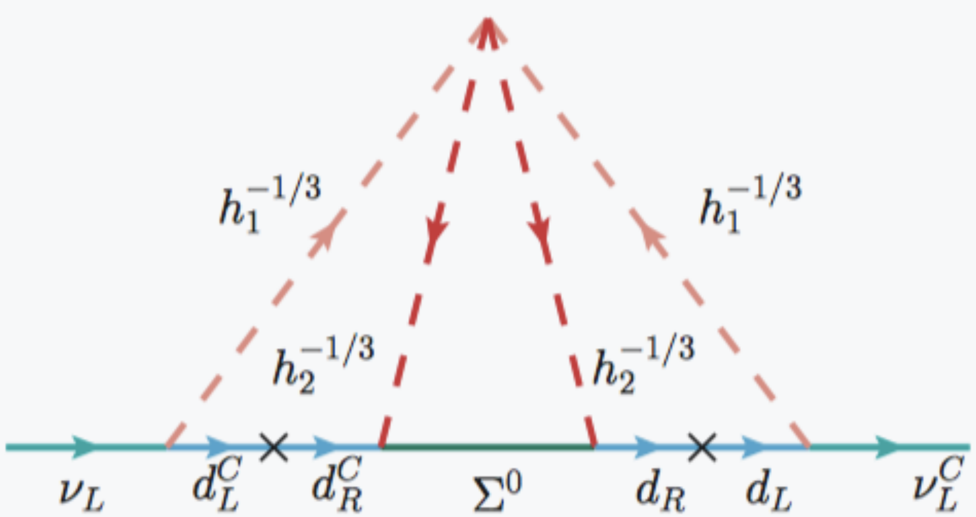


+ additional diagrams

$$(m_\nu)_{\alpha\beta} = -\,30\,\frac{\lambda_h}{\left(4\pi^2\right)^3\, m_{h_2}}\, y^T_{\alpha i}\, m_{D_i}\, \tilde{y}^T_{ij}\, G\left(\frac{m^2_{\Sigma_j}}{m^2_{h_2}}, \frac{m^2_{h_1}}{m^2_{h_2}}\right) \tilde{y}_{jk}\, m_{D_k}\, y_{k\beta}$$

Krauss, Nasri, Trodden, PRD 2003 (with ordinary scalars with singlet DM in the loop)

Neutrino mass generation



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Krauss, Nasri, Trodden, PRD 2003 (with ordinary scalars with singlet DM in the loop)

Modified Casas-Ibarra parametrisation to write $\tilde{y}$ in terms of y :

Casas, Ibarra Nucl. Phys. B 618, 171 (2001)

$$m_\nu^{\rm diag} = U^T\, y^T\, m_D\, \tilde{y}^T\, F(\lambda_h, m_\Sigma, m_{h_{1,2}})\, \tilde{y}\, m_D\, y\, U$$

U : PMNS matrix

$\mathscr{R}$: Complex Orthogonal matrix

$$(\sqrt{m_\nu^{\rm diag}}^{-1}\, U^T\, y^T\, m_D\, \tilde{y}^T\, \sqrt{F})(\sqrt{F}\, \tilde{y}\, m_D\, y\, U\, \sqrt{m_\nu^{\rm diag}}^{-1}) = I = \mathscr{R}^T\, \mathscr{R}$$

$$\tilde{y} = F^{-1/2}\, \mathscr{R}\, \sqrt{m_\nu^{\rm diag}}\, U^\dagger y^{-1} m_d^{-1}$$

In agreement with Cheung at al, PRD 2017

A Viable Dark Matter Candidate

Recall: Σ_R is odd under Z2 symmetry

EW radiative corrections : $m_{\Sigma^\pm} - m_{\Sigma^0} \sim 166$ MeV

Cirelli, Fornengo, Strumia
Nucl.Phys.B753, 2006

This renders Σ_0 the lightest stable particle among new states

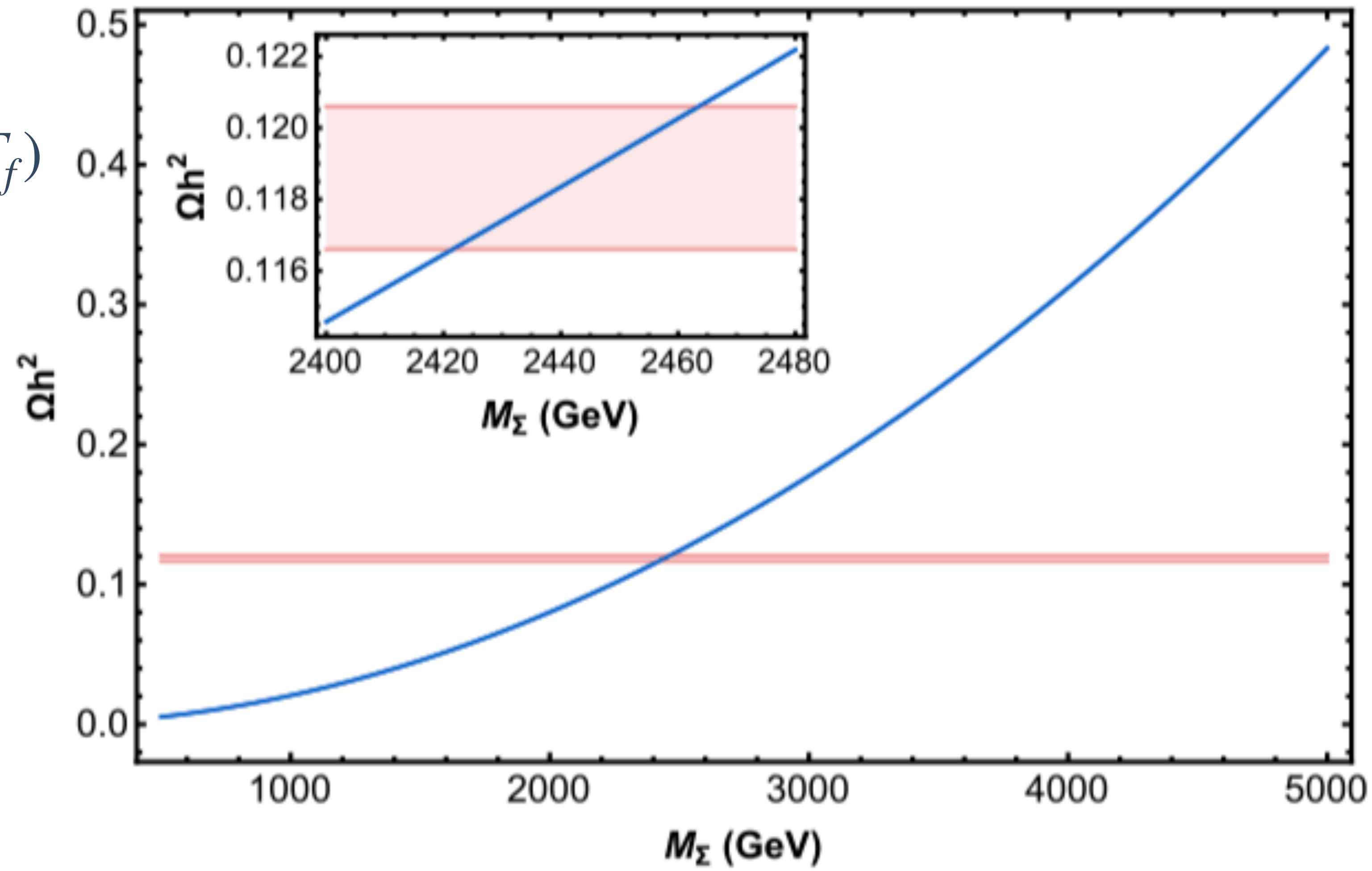
Σ_R coannihilation primarily via gauge interactions

Freeze-out temperature $x_f\,(\equiv m_\Sigma/T_f)$ is calculated using

$$x_f = \ln\left(\frac{0.038\, g_{\rm eff}\, M_{\rm Pl}\, m_\Sigma\, \langle \sigma_{\rm eff} |\bar{v}| \rangle}{g_*^{1/2} x_f^{1/2}}\right)$$

$$\Omega h^2 = \frac{1.07\times 10^9\, x_f}{g_*^{1/2}\, M_{\rm Pl}({\rm GeV})\, I_a}$$

$$I_a = x_f \int_{x_f}^{\infty} x^{-2} a_{\rm eff}\, dx$$



$\mathbf{b \to s\, \ell^+\ell^-}$: $\mathbf{R_K}$ and $\mathbf{R_{K^*}}$

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}}\lambda_t \sum_i C_i^\ell O_i^\ell + \sum \frac{\mathcal{C}^{\rm NP}}{\Lambda^2_{\rm NP}}\mathcal{O}^{\rm NP}$$

$$O_9^\ell \propto (\bar{s}_L\gamma^\mu b_L)(\bar{\ell}\gamma_\mu \ell), \qquad O_{10}^\ell \propto (\bar{s}_L\gamma^\mu b_L)(\bar{\ell}\gamma_\mu\gamma_5 \ell)$$

In SM

$$C_9 \simeq -C_{10} \simeq -4.1$$

In this model, we obtain

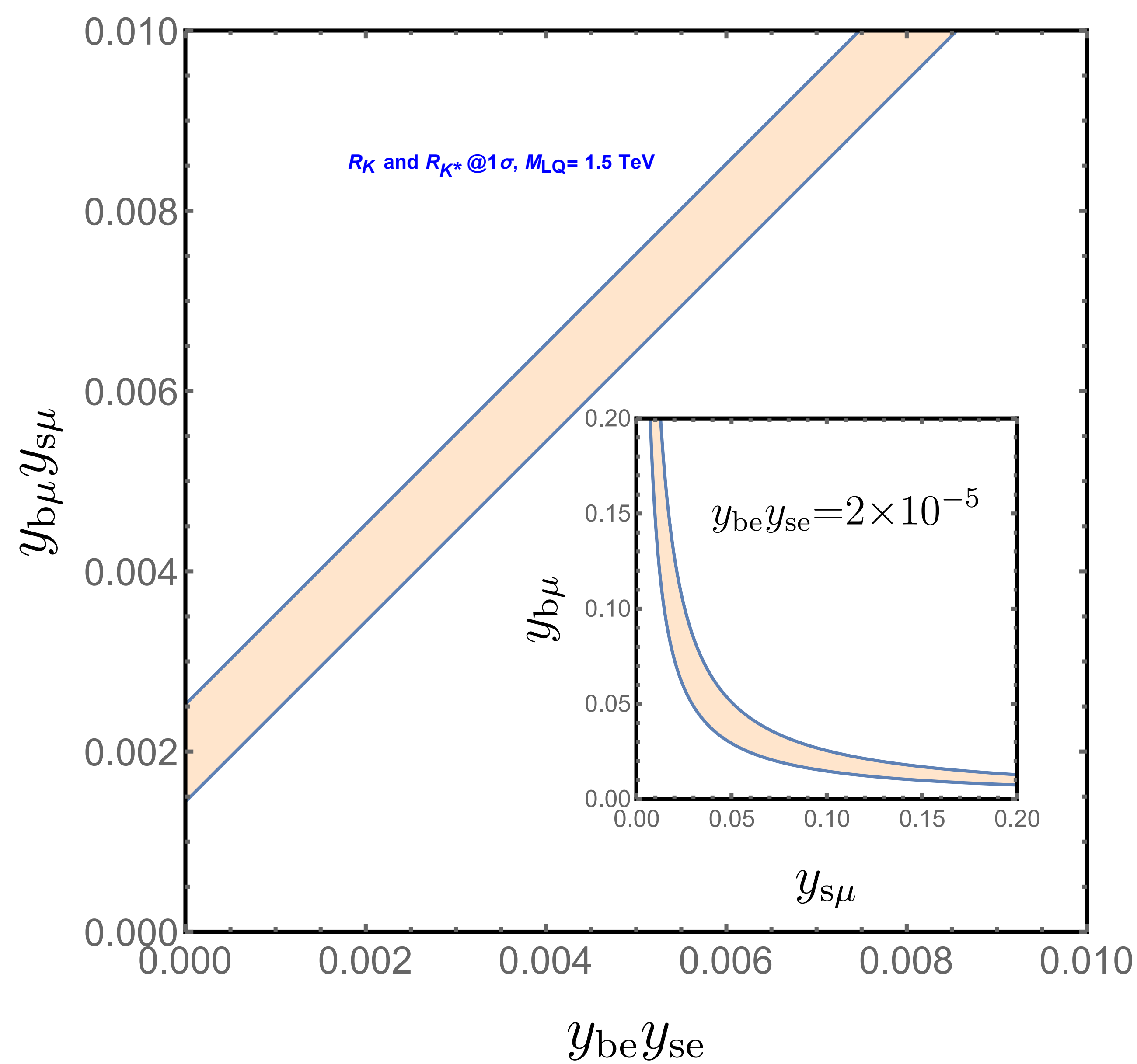
$$C_9^{\ell\ell'} = -\,C_{10}^{\ell\ell'} = \frac{\pi\, v^2}{\alpha_e\, V_{tb}\, V_{ts}^*}\,\frac{y_{b\ell'}\, y_{s\ell}^*}{m_{h_1}^2}$$

Neutral current anomalies can be explained by satisfying:

$$-1.4 \lesssim 2\,\mathrm{Re}[C_{9,\rm NP}^{\mu\mu} - C_{9,\rm NP}^{ee}] \lesssim -0.8$$

@ 1σ

Hiller et al 1707.05444, Matias et al 1704.05340



$\mathbf{b \to c\,\tau\nu}$: $\mathbf{R_D}$ and $\mathbf{R_{D^*}}$

$$\mathcal{H}_{\rm eff} = \frac{4\,G_F}{\sqrt{2}} V_{jk} \left(U_{\ell i} - \frac{v^2}{4\,V_{cb}\,m_{h_1}^2} (yU)_{ki} (Vy^*)_{j\ell} \right) \left(\bar{u}_j \gamma^\mu P_L d_k\right) \left(\bar{\ell}\,\gamma_\mu P_L \nu_i\right) + \text{h.c.}$$

Define

$$\frac{R_{D^{(*)}}}{R_{D^{(*)},\,\rm SM}} = \frac{1 - 2\,{\rm Re}\left(x_{c\tau}\, y_{b\tau}\right)}{1 - 2\,{\rm Re}\left(x_{c\mu}\, y_{b\mu}\right)}$$

with

$$x_{j\ell} = (Vy^*)_{j\ell}\left(v^2/4V_{cb}m_{h_1}^2\right)$$

After taking all the relevant flavor constraints into account in our model

$b \to c\tau\nu$ is SM-like

We satisfy muon to electron LFU ratio

$$\frac{R^{\mu/e}_{D^{(*)}}}{R^{\mu/e}_{D^{(*)},\,\rm SM}} = \frac{1\ -\ 2\,{\rm Re}\left(x_{c\mu}\, y_{b\mu}\right)}{1\ -\ 2\,{\rm Re}\left(x_{ce}\, y_{be}\right)}$$

Belle Collab: Glattauer 2015, Abdesselam 2017

$$R_D^{\mu/e,\rm exp} = 0.995 \pm 0.022 \pm 0.039$$

$$R_{D^*}^{e/\mu,\rm exp} = 1.04 \pm 0.05 \pm 0.01$$

A way out if anomalies persist : add another scalar LQ (3, 2, 7/6) ?

See Bečirevic et al, 1806.05689

Identifying textures of scalar triplet LQ Yukawa

We take phenomenological approach

Parametrize $y_{ij} = a_{ij} \odot \epsilon^{n_{ij}}$

$$y \sim \begin{pmatrix} \epsilon^{n_{11}} & \epsilon^{n_{12}} & \epsilon^{n_{13}} \\ \epsilon^{n_{21}} & \epsilon^{n_{22}} & \epsilon^{n_{23}} \\ \epsilon^{n_{31}} & \epsilon^{n_{32}} & \epsilon^{n_{33}} \end{pmatrix}$$

Requirement of saturating the B -meson anomalies [$R_{K^{(*)}}$] at 1σ for $m_{h_1} = 1.5$ TeV

$$y_{22}\, y_{32} \approx 2.1555 \times 10^{-3} \sim \epsilon^{n_{22}+n_{32}} \quad \rightarrow \quad \epsilon^4 \sim 2.1555 \times 10^{-3} \Leftrightarrow \epsilon \approx 0.215$$

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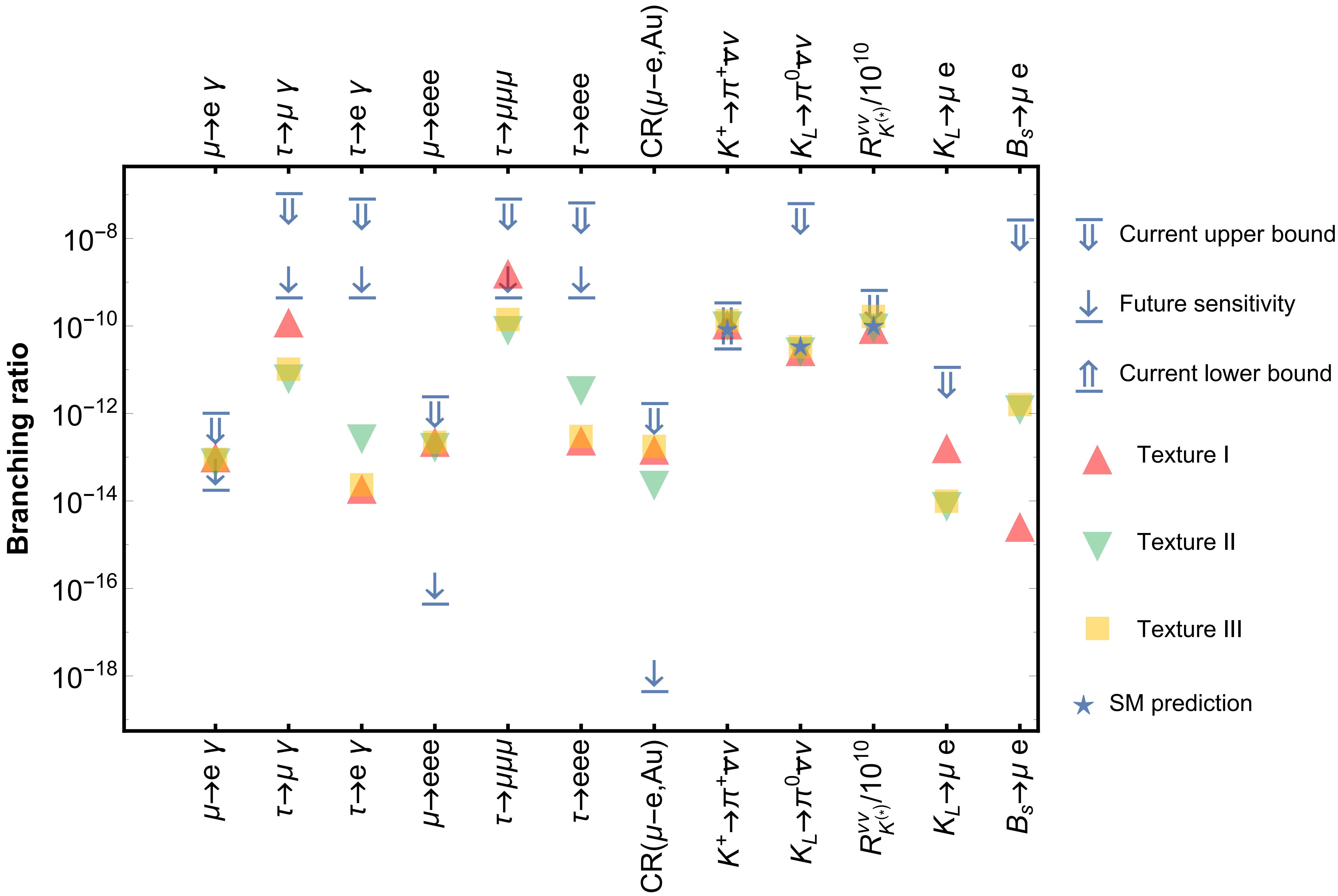
We find the following allowed textures consistent with flavor violation processes

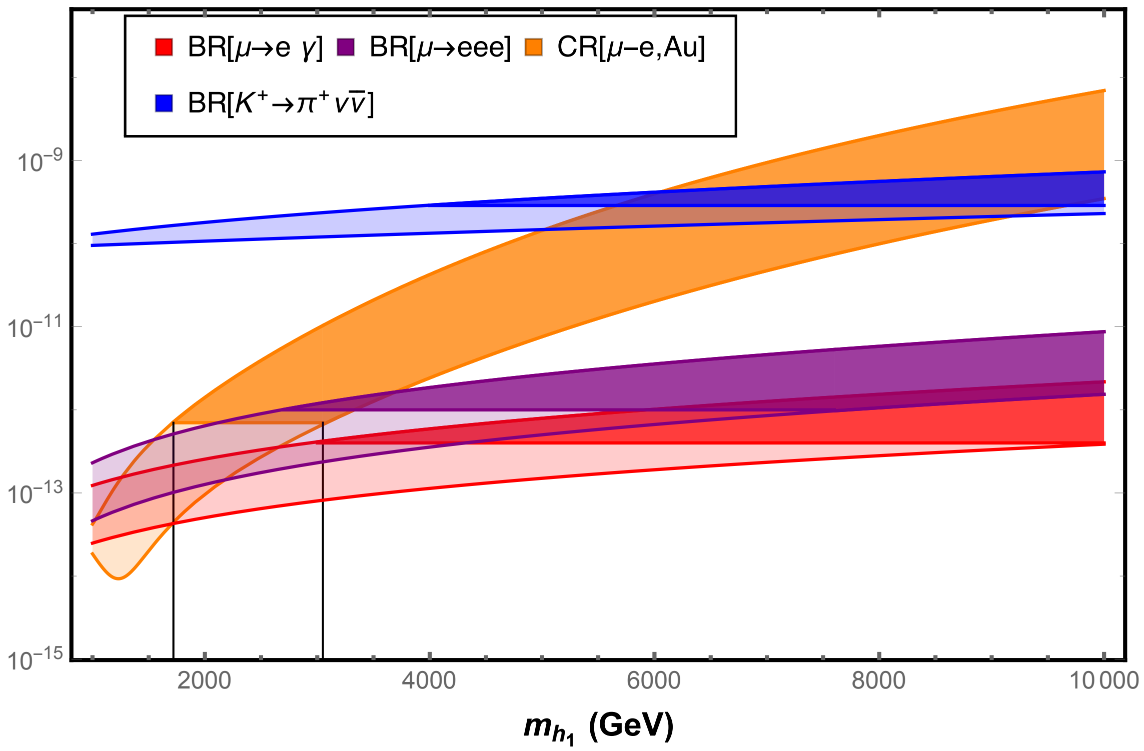
	Type I	Type II	Type III
y	$\begin{pmatrix} \times & \times & \times \\ \times & \epsilon^3 & \times \\ \times & \epsilon & \times \end{pmatrix}$	$\begin{pmatrix} \times & \times & \times \\ \times & \epsilon^2 & \times \\ \times & \epsilon^2 & \times \end{pmatrix}$	$\begin{pmatrix} \times & \times & \times \\ \times & \epsilon & \times \\ \times & \epsilon^3 & \times \end{pmatrix}$
Generic allowed textures	$\begin{pmatrix} \epsilon^4 & \epsilon^{\geq 5} & \epsilon^{\geq 2} \\ \epsilon^{\geq 3} & \epsilon^3 & \epsilon^{\geq 4} \\ \epsilon^{\geq 4} & \epsilon & \epsilon^{\geq 1} \end{pmatrix}$	$\begin{pmatrix} \epsilon^6 & \epsilon^{\geq 4} & \epsilon^{\geq 3} \\ \epsilon^{\geq 5} & \epsilon^2 & \epsilon^{\geq 3} \\ \epsilon^{\geq 3} & \epsilon^2 & \epsilon^{\geq 1} \end{pmatrix}$	$\begin{pmatrix} \epsilon^5 & \epsilon^{\geq 5} & \epsilon^{\geq 4} \\ \epsilon^4 & \epsilon & \epsilon^{\geq 2} \\ \epsilon^{\geq 4} & \epsilon^3 & \epsilon^{\geq 1} \end{pmatrix}$

Contributions to various flavor violating processes

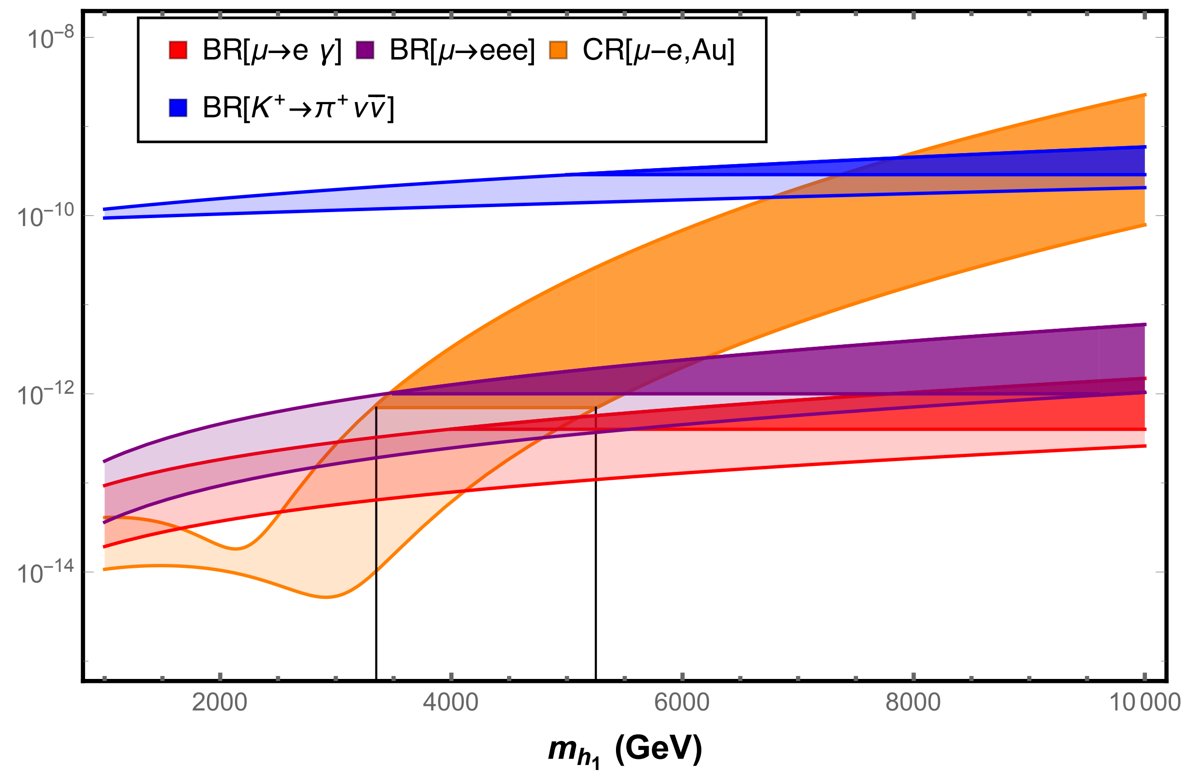


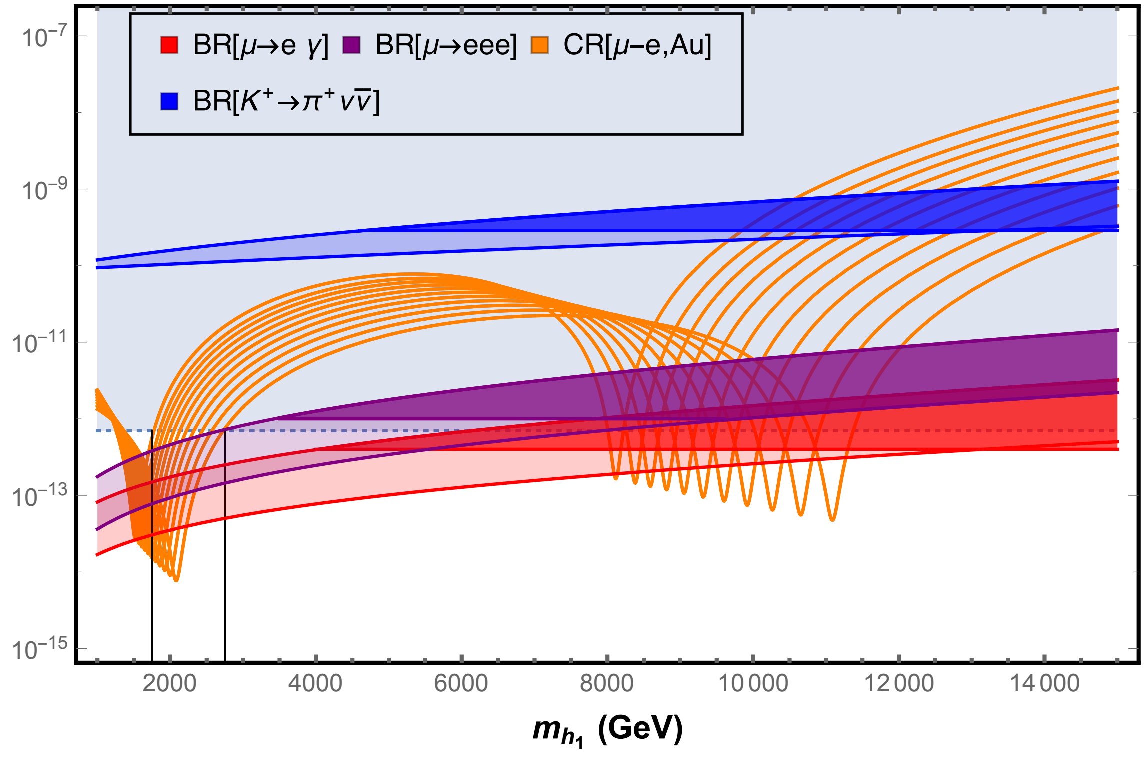
Constraints on LQ masses from flavor violation

Type I texture



Type II texture





Type III texture

Constraints from neutrino oscillation data

$$\tilde{y} = F^{-1/2} \mathscr{R} \sqrt{m_\nu^{\text{diag}}}\, U^\dagger y^{-1} m_d^{-1}$$

$m_{\Sigma^{1,2,3}} \sim 2.45,\ 3.5,\ 4.5$ TeV

$m_{h_1} \sim 1.5$ TeV $\quad$ $m_{h_2} \sim 2.6$ TeV

Lightest neutrino mass ~ 0.001 eV
Global best fit values for other parameters

Scan for $\tilde{y}$ complying with flavor constraints

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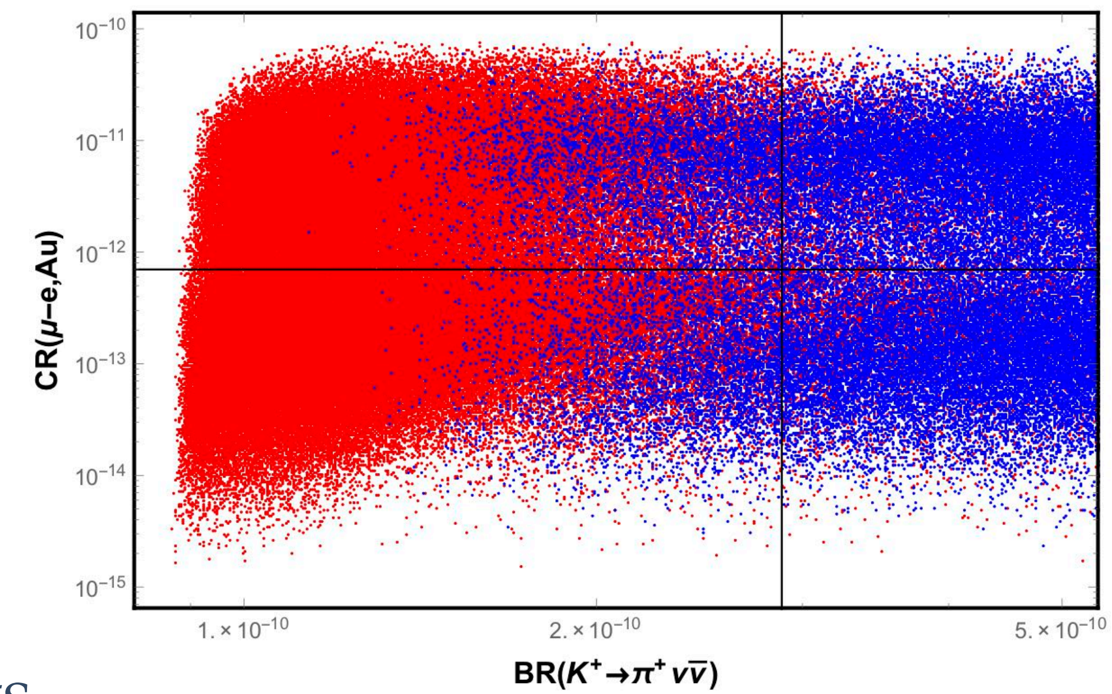

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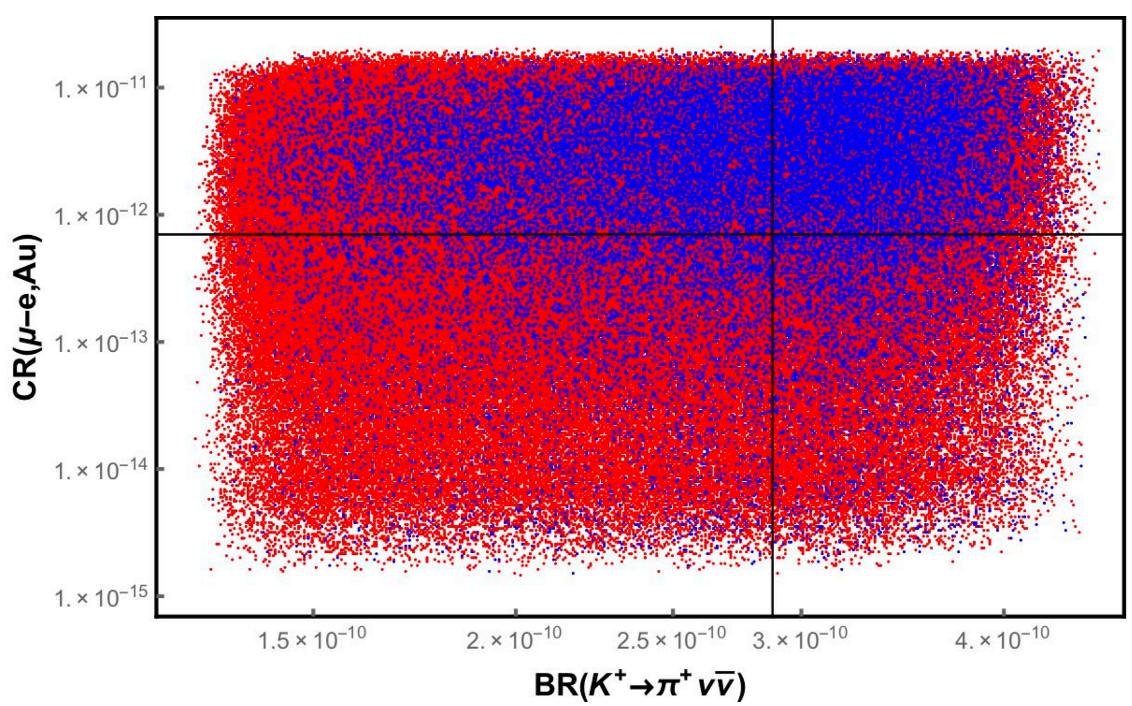

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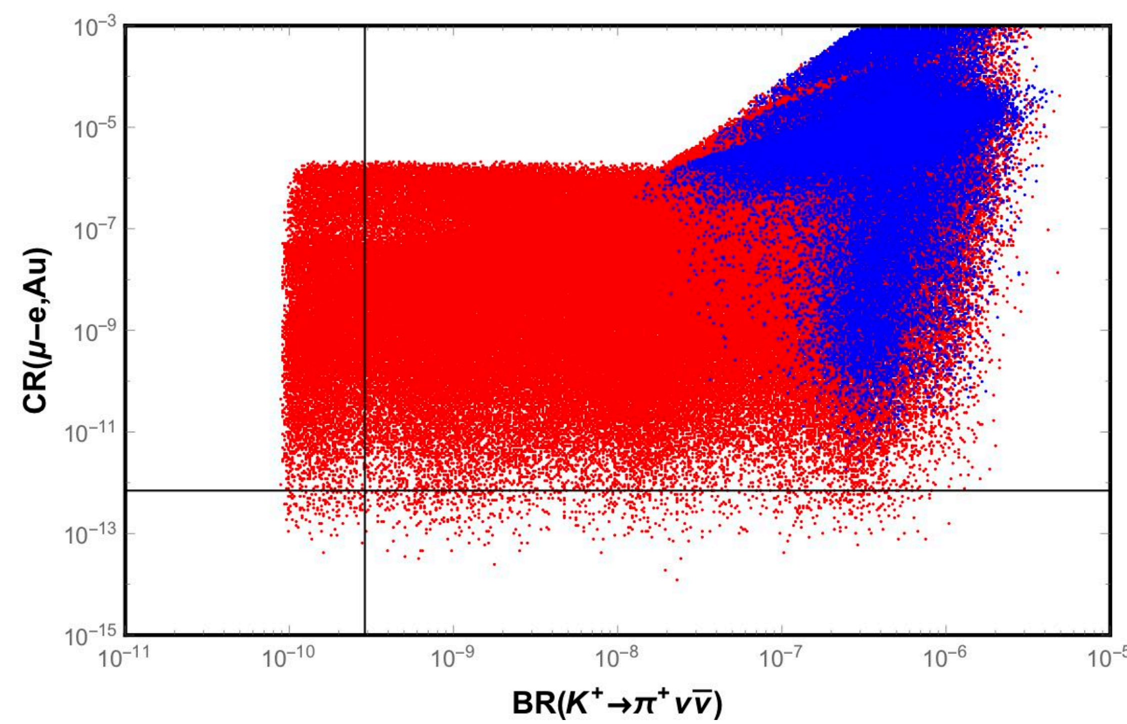

Conclusions

The SM extension via (2) scalar LQ + (3) Triplet Majorana fermions

✓ Addresses neutrino mass generation

✓ Explains anomalies in R_K and R_{K^*}

✓ Has a viable dark matter candidate accounting for correct relic density

Satisfy all relevant flavor constraints : meson decays, meson-antimeson oscillations, cLFV decays

$\mu - e$ and $K \to \pi\nu\bar{\nu}$ are the most constraining observables for this class of models

Does not account for anomalies in charged current $b \to c$; possibility of accommodating these via extended the LQ sector

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Thank you

Scalar potential : Backup

$$
\begin{aligned}
V(H,h_1,h_2) = & \mu_H^2 H^\dagger H + \frac{1}{2}\lambda_H\,|H^\dagger H|^2 + \mu_{h_1}^2 \mathrm{Tr}[h_1^\dagger h_1] + \mu_{h_2}^2 \mathrm{Tr}[h_2^\dagger h_2] + \\
& + \frac{1}{8}\lambda_{h_1}\,[\mathrm{Tr}(h_1^\dagger h_1)]^2 + \frac{1}{8}\lambda_{h_2}\,[\mathrm{Tr}(h_2^\dagger h_2)]^2 + \frac{1}{4}\lambda'_{h_1}\,\mathrm{Tr}[(h_1^\dagger h_1)]^2 + \frac{1}{4}\lambda'_{h_2}\,\mathrm{Tr}[(h_2^\dagger h_2)]^2 + \\
& + \frac{1}{2}\lambda_{Hh_1}\,(H^\dagger H)\,\mathrm{Tr}[h_1^\dagger h_1] + \frac{1}{2}\lambda'_{Hh_1}\,\sum_{i=1}^{3}(H^\dagger\,\tau_i\,H)\,\mathrm{Tr}[h_1^\dagger\,\tau_i\,h_1] + \\
& + \frac{1}{2}\lambda_{Hh_2}\,(H^\dagger H)\,\mathrm{Tr}[h_2^\dagger h_2] + \frac{1}{2}\lambda'_{Hh_2}\,\sum_{i=1}^{3}(H^\dagger\,\tau_i\,H)\,\mathrm{Tr}[h_2^\dagger\,\tau_i\,h_2] + \\
& + \frac{1}{4}\lambda_h\,\mathrm{Tr}[h_1^\dagger\,h_2]^2 + \frac{1}{8}\lambda'_h\,[\mathrm{Tr}\,(h_1^\dagger\,h_2)]^2 + \frac{1}{4}\lambda''_h\,\mathrm{Tr}[h_1^\dagger\,h_1]\,\mathrm{Tr}[h_2^\dagger\,h_2] + \mathrm{H.c.}
\end{aligned}
$$

$$
\frac{\lambda_h}{4}\,\mathrm{Tr}(h_1^\dagger\,h_2\,h_1^\dagger\,h_2) = \frac{\lambda_h}{2}\,h_1^{-1/3}\,h_2^{1/3}\,h_1^{-1/3}\,h_2^{1/3} - \lambda_h\,h_1^{-1/3}\,h_2^{-2/3}\,h_1^{-1/3}\,h_2^{4/3}
$$

Interaction Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\rm int}^{h,\Sigma} = & - y_{ij}\,\bar{d}_L^{C\,i}\,h_1^{1/3}\,\nu_L^j - \sqrt{2}\,y_{ij}\,\bar{d}_L^{C\,i}\,h_1^{4/3}\,e_L^j + \sqrt{2}\,y_{ij}\,\bar{u}_L^{C\,i}\,h_1^{-2/3}\,\nu_L^j - y_{ij}\,\bar{u}_L^{C\,i}\,h_1^{1/3}\,e_L^j \\
& - 2\,\tilde{y}_{ij}\,\overline{\Sigma^0}_R^{C\,i}\,h_2^{1/3}\,d_R^j - 2\,\tilde{y}_{ij}\,\overline{\Sigma^+}_R^{C\,i}\,h_2^{-2/3}\,d_R^j - 2\,\tilde{y}_{ij}\,\overline{\Sigma^-}_R^{C\,i}\,h_2^{4/3}\,d_R^j + \mathrm{H.c.}
\end{aligned}
$$

Flavor constraints : Backup

Process	Observables	SM Prediction	Experimental data
$K^+ \to \pi^+ \nu\bar{\nu}$	$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu})$	$(8.4 \pm 1.0) \times 10^{-11}$ [Buras et.al. 2015]	$17.3^{+11.5}_{-10.5} \times 10^{-11}$ [E949] $< 11 \times 10^{-10}$ [Na62]
$K_L \to \pi^0 \nu\bar{\nu}$	$\mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu})$	$(3.4 \pm 0.6) \times 10^{-11}$ ["]	$\leq 2.6 \times 10^{-8}$ [E391a]
$B \to K^{(*)} \nu\bar{\nu}$	$R_K^{\nu\nu}$, $R_{K^*}^{\nu\nu}$	$R_{K^{(*)}}^{\nu\nu} = 1$	$R_K^{\nu\nu} < 3.9$ [Belle] $R_{K^*}^{\nu\nu} < 2.7$ [Belle]
$B_s^0 - \bar{B}_s^0$ oscillation	mixing parameters	$C_{B_s} = 1$ $\phi_{B_s} = 0$	$C_{B_s} = 1.070 \pm 0.088$ [UTfit] $\phi_{B_s} = (0.054 \pm 0.951)^\circ$ [UTfit]
$K_L \to \mu e$	$\mathrm{Br}(K_L \to \mu e)$	—	$< 4.7 \times 10^{-12}$ [PDG]
$B_s \to \mu e$	$\mathrm{Br}(B_s \to \mu e)$	—	$< 1.1 \times 10^{-8}$ [PDG]

This list is not exhaustive

cLFV processes : Backup

LFV Process	Current experimental bound bound (Experiment)	Future sensitivity sensitivity (Experiment)
$\mathrm{Br}(\mu \to e\gamma)$ $\mathrm{Br}(\tau \to \mu\gamma)$ $\mathrm{Br}(\tau \to e\gamma)$	$< 4.2 \times 10^{-13}$ (MEG) $< 4.4 \times 10^{-8}$ (BaBar) $< 3.3 \times 10^{-8}$ (BaBar)	4×10^{-14} (MEG II) 10^{-9} (Super B) 10^{-9} (Super B)
$\mathrm{Br}(\mu \to 3e)$ $\mathrm{Br}(\tau \to 3\mu)$	$< 1.0 \times 10^{-12}$ (SINDRUM) $< 4.4 \times 10^{-8}$ (BaBar)	$10^{-15(16)}$ (Mu3e) 10^{-9} (Super B)
$\mathrm{Cr}(\mu \to e\,, N)$	$< 7 \times 10^{-13}$ (Au) (SINDRUM)	10^{-14} (DeeMe) $10^{-15(-17)}$ (COMET) 3×10^{-17} (Mu2e) 10^{-18} (PRISM/PRIME)

Dark Matter : Backup

s-channel

$$\Sigma^0\Sigma^\pm \to W^\pm \to W^\pm W^0, W^\pm H, \bar{f}f'$$

$$\Sigma^+\Sigma^- \to W^0 \to W^+W^-, W^0 H, \bar{f}f$$

t-channel

$$\Sigma^0\Sigma^0 \to W^+W^- \quad \Sigma^0\Sigma^\pm \to W^\pm W^0$$

$$\Sigma^\pm\Sigma^\pm \to W^\pm W^\pm$$

$$\Sigma^+\Sigma^- \to W^0W^0 (W^+W^-)$$

Thermally averaged eff. cross section

$$\langle\sigma_{\rm eff}|\bar{v}|\rangle = \frac{g_0^2}{g_{\rm eff}^2}\,\sigma(\Sigma^0\Sigma^0)|\bar{v}| + 4\,\frac{g_0\,g_\pm}{g_{\rm eff}^2}\,\sigma(\Sigma^0\Sigma^\pm)|\bar{v}|\left(1+\frac{\Delta_{m_\Sigma}}{m_\Sigma}\right)^{3/2}\exp\left(-\frac{\Delta_{m_\Sigma}}{m_\Sigma}\,x_f\right) + \\ + \frac{g_\pm^2}{g_{\rm eff}^2}\left[2\,\sigma(\Sigma^+\Sigma^-)|\bar{v}| + 2\,\sigma(\Sigma^\pm\Sigma^\pm)|\bar{v}|\right]\left(1+\frac{\Delta_{m_\Sigma}}{m_\Sigma}\right)^3\exp\left(-2\frac{\Delta_{m_\Sigma}}{m_\Sigma}x_f\right)$$

$$x_f = \ln\left(\frac{0.038\,g_{\rm eff}\,M_{\rm Pl}\,m_\Sigma\,\langle\sigma_{\rm eff}|\bar{v}|\rangle}{g_*^{1/2}x_f^{1/2}}\right)$$

$$g_{\rm eff} = g_0 + 2\,g_\pm\left(1+\frac{\Delta_{m_\Sigma}}{m_\Sigma}\right)^{3/2}\exp\left(-\frac{\Delta_{m_\Sigma}}{m_\Sigma}\,x_f\right)$$

$$\Omega\,h^2 = \frac{1.07\times 10^9\,x_f}{g_*^{1/2}\,M_{\rm Pl}({\rm GeV})\,I_a}$$