A solution of the fermionic mass hierarchy of the SM

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Based on
arXiv:1712.08052 [hep-ph], G. Abbas

The fermionic mass hierarchy

- The fermionic mass hierarchy in the standard model (SM) is a very complex problem, and can be divided into three hierarchies.
- The first is the mass hierarchy among fermionic families, i.e. $m_\tau >> m_\mu >> m_e$, $m_b >> m_s >> m_d$ and $m_t >> m_c >> m_u$.
- The second hierarchy is within the family, i.e., $m_d > m_u$, $m_c >> m_s$, $m_t >> m_b$.
- The third hierarchy resides in the quark-mixing angles, i.e. $\sin\theta_{12} >> \sin\theta_{23} >> \sin\theta_{13}$ where $\sin\theta_{12} = 0.23$, $\sin\theta_{23} = 0.041$ and $\sin\theta_{13} = 0.0035$.
- Serious efforts have been made to address this problem within the framework of extended gauge or flavour symmetries. Froggatt et al 79,Georgi 1983, Balakrishna 1987, Babu et al 1990, 2000

The fermionic mass hierarchy

Apparently, a solution of the fermionic mass hierarchy is the last wish of Steven Weinberg as expressed in a recent interview with CERN COURIER.

.. asked what single mystery, if he could choose, he would like to see solved in his lifetime, Weinberg does not have to think for long: he wants to be able to explain the observed pattern of quark and lepton masses.

Model physicist, Matthew Chalmers, CERN Courier, Oct 13, 2017

Model

- The central idea of this work comes from the observation that in the SM the mass generation of the charged fermions of the three families is achieved by ad hoc insertion of the Yukawa Lagrangian of dimension 4.
  arXiv:1712.08052 G. Abbas

- This Lagrangian is indeed a selection in the sense that for recovering masses of fermions one can also choose the next operator which is of dimension 5 provided the Yukawa operator is forbidden by some symmetry.

- We propose that masses of all fermions are generated through diemnsion-5 operators instead of the Yukawa Lagrangian.

- Having chosen this selection rule, we investigate the impact of this rule on the fermionic mass spectrum of the SM.

Model

- We extend the SM symmetry by discrete symmetries $\mathcal{Z}_2$ and $\mathcal{Z}_2'$.
  arXiv:1712.08052 G. Abbas

- We first discuss fermionic mass heirarchy among the three families.

- For this purpose, we add three real singlet scalar fields क$_1$, क$_2$ and क$_3$ to the SM whose transformations under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ are the following

$$\text{क}_1 : (1, 1, 0),\ \text{क}_2 : (1, 1, 0),\ \text{क}_3 : (1, 1, 0). \quad (1)$$

- Under CP Higgs doublet φ and the additional singlets transform as,

$$(CP)\varphi(CP)^{\dagger} = \varphi^{\dagger T},\ (CP)\,\text{क}_i(CP)^{\dagger} = \text{क}_i \quad (2)$$

Model

Symmetries $\mathcal{Z}_2$ and $\mathcal{Z}_2'$ are imposed on the right handed fermions of each family, scalar fields क$_1$, क$_2$, क$_3$ as shown in Table 1.

Fields	$\mathcal{Z}_2$	$\mathcal{Z}_2'$
ψ_R^1	+	-
क$_1$	+	-
ψ_R^2	-	-
क$_2$	-	-
ψ_R^3	-	+
क$_3$	-	+

Table: The charges of right-handed fermions of three families of the SM and singlet scalar fields under $\mathcal{Z}_2$ and $\mathcal{Z}_2'$ symmetries. Here, superscript denotes a family number.

Fermionic mass hierarchy among the three families

- The masses of fermions of three families are now recovered from dimension-5 operators given by the following equation:

$$\begin{aligned} \mathcal{L}_{mass} &= \frac{1}{\Lambda}\Big[\Gamma_1 \bar{\psi}^1_L \varphi \psi^1_R \,\text{क}_1 + \Gamma_2 \bar{\psi}^2_L \varphi \psi^2_R \,\text{क}_2 + \Gamma_3 \bar{\psi}^3_L \varphi \psi^3_R \,\text{क}_3 \\ &+ \Gamma'_1 \bar{\psi}^1_L \tilde{\varphi} \psi^1_R \,\text{क}_1 + \Gamma'_2 \bar{\psi}^2_L \tilde{\varphi} \psi^2_R \,\text{क}_2 + \Gamma'_3 \bar{\psi}^3_L \tilde{\varphi} \psi^3_R \,\text{क}_3\Big] + \frac{c}{\Lambda} \bar{l}^c_L \tilde{\varphi}^* \tilde{\varphi}^\dagger l_L + \text{H.c.} \end{aligned} \tag{3}$$

- The pattern $\langle \text{क}_3 \rangle >> \langle \text{क}_2 \rangle >> \langle \text{क}_1 \rangle$ explains why top quark is so heavy and electron is so light.

Fermionic mass hierarchy among and within the three families

- It is natural to ask if a simultaneous natural explanation could be obtained for observed pattern of charged fermions among the three families as well as within the family.
- This is a very difficult and bigger problem. Hence, the cost of its solution is also bigger.
- The price to pay is to add six real singlet scalar fields $क_i : (1, 1, 0)$ where $i = 1 - 6$ to the SM.
- We extend the SM symmetry by discrete symmetries $\mathcal{Z}_2$, $\mathcal{Z}_2'$ and $\mathcal{Z}_2''$.

Fermionic mass hierarchy among and within the three families

Six real singlet scalar fields क$_i$: $(1, 1, 0)$ have charges under discrete symmetries $\mathcal{Z}_2$, $\mathcal{Z}_2'$ and $\mathcal{Z}_2''$ as given in Table 2.

Fields	$\mathcal{Z}_2$	$\mathcal{Z}_2'$	$\mathcal{Z}_2''$
ψ_R^u	+	+	-
क$_1$	+	+	-
$\psi_R^{d,e}$	-	-	+
क$_2$	-	-	+
ψ_R^c	+	-	-
क$_3$	+	-	-
$\psi_R^{s,\mu}$	+	-	+
क$_4$	+	-	+
ψ_R^t	-	+	-
क$_5$	-	+	-
$\psi_R^{b,\tau}$	-	+	+
क$_6$	-	+	+

Table: The charges of right-handed fermions of three families of the SM and singlet scalar fields under $\mathcal{Z}_2$, $\mathcal{Z}_2'$ and $\mathcal{Z}_2''$ symmetries. We show flavour of right-handed fermion by superscript.

Fermionic mass hierarchy among and within the three families

- The mass Lagrangian now reads,

$$\begin{aligned}\mathcal{L}_{mass} &= \frac{1}{\Lambda}\Big[\Gamma_1\bar{\psi}_L^1\tilde{\varphi}u_R\,\text{क}_1 + \Gamma_2\bar{\psi}_L^2\tilde{\varphi}c_R\,\text{क}_3 + \Gamma_3\bar{\psi}_L^3\tilde{\varphi}t_R\,\text{क}_5 \\ &+ \Gamma_1'\bar{\psi}_L^1\varphi d_R\,\text{क}_2 + \Gamma_2'\bar{\psi}_L^2\varphi s_R\,\text{क}_4 + \Gamma_3'\bar{\psi}_L^3\varphi b_R\,\text{क}_6 \\ &+ \Gamma_4\bar{\psi}_L^1\varphi e_R\,\text{क}_2 + \Gamma_5\bar{\psi}_L^2\varphi \mu_R\,\text{क}_4 + \Gamma_6\bar{\psi}_L^3\varphi \tau_R\,\text{क}_6\Big] + \frac{c}{\Lambda}\bar{l}_L^c\tilde{\varphi}^*\tilde{\varphi}^\dagger l_L + \text{H.c.}.\end{aligned} \tag{4}$$

- Six real singlet scalar fields क_i acquire VEVs in such a way that $\langle\text{क}_2\rangle > \langle\text{क}_1\rangle$, $\langle\text{क}_3\rangle >> \langle\text{क}_4\rangle$, $\langle\text{क}_5\rangle >> \langle\text{क}_6\rangle$, $\langle\text{क}_6\rangle >> \langle\text{क}_4\rangle >> \langle\text{क}_2\rangle$, and $\langle\text{क}_5\rangle >> \langle\text{क}_3\rangle >> \langle\text{क}_1\rangle$.

- Thus, this VEVs pattern explains naturally why $m_d > m_u$, $m_c >> m_s$, $m_t >> m_b$, $m_\tau >> m_\mu >> m_e$, $m_b >> m_s >> m_d$, and $m_t >> m_c >> m_u$.

CKM matrix: hierarchy among the quark mixing angles

The Cabibbo-Kobayashi-Maskawa matrix is given by the three mixing angles in the standard parameterization,

$$\begin{aligned} \tan\theta_{12} &= \frac{v_2|C_1^d|}{v_4|C_2^d|} + \mathcal{O}(\frac{v_1}{v_5}), \;\; \tan\theta_{23} = \frac{v_4|C_2^d|}{v_6|C_3^d|} + \mathcal{O}(\frac{v_1}{v_5}), \\ \tan\theta_{13} &= \frac{v_2 v_4^2|C_1^d||C_2^d|^2}{v_6^3|C_3^d|^3} + \mathcal{O}(\frac{v_1}{v_5}). \end{aligned} \tag{5}$$

From the above results, it is remarkably obvious that we have obtained $\sin\theta_{12} >> \sin\theta_{23} >> \sin\theta_{13}$ in the limit $v_2 << v_4 << v_6$.

Scalar potential

We discuss now how to avoid artificial fine-tuning of parameters in the scalar potential in this model and keep hierarchies among different VEVs.

Fields	$\mathcal{Z}_3$	$\mathcal{Z}_3'$	$\mathcal{Z}_3''$
u_R, d_R, e_R	ω^2	1	1
क$_1$, क$_2$	ω	1	1
c_R, s_R, μ_R	1	ω^2	1
क$_3$, क$_4$	1	ω	1
t_R, b_R, τ_R	1	1	ω^2
क$_5$, क$_6$	1	1	ω

Table: The charges of right-handed fermions of three families of the SM and singlet scalar fields under $\mathcal{Z}_3$, $\mathcal{Z}_3'$ and $\mathcal{Z}_3''$ symmetries. Here, ω is the cube root of unity.

$$V = \mu\varphi^\dagger\varphi + \lambda_5(\varphi^\dagger\varphi)^2 + \sum_{i=1}^{6}\mu_i \text{क}_i^2 + \varphi^\dagger\varphi\sum_{i=1}^{6}\rho_i \text{क}_i. \quad (6)$$

This potential is already studied in the context of strong electroweak phase transition. Saeedhosini et al 2017

Ultra-violet completion

- A UV completion of models described in tables 1 and 2 can be achieved by introducing one vector-like isosinglet fermions.
- Their transformations under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ are given by,

$$\begin{aligned} Q &= U_{L,R} : (3, 1, \frac{4}{3}))_{+,+,+}, D_{L,R} : (3, 1, -\frac{2}{3}))_{+,+,+}, \\ L &= E_{L,R} : (1, 1, -2))_{+,+,+}; N_{L,R} : (1, 1, 0))_{+,+,+}, \end{aligned} \tag{7}$$

where subscript denotes the charges under under $\mathcal{Z}_2$, $\mathcal{Z}_2'$ and $\mathcal{Z}_2''$ symmetries.

Summary

- The main idea of the models discussed in this work is to assume that masses of all fermions are generated by the dimensions-5 operators instead of the Yukawa operators of dimension-4.
- This selection rule provides a dynamical origin of the fermionic mass hierarchy among and within the three fermionic families of the SM along with the quark-mixing pattern.
- We stress that a simultaneous explanation of these three hierarchies in terms of a dynamical mechanism is rare in literature.
- Phenomenological investigation is under progress.
- We remark that an explanation of the fermionic mass hierarchy among and within the three families can also come from a complete realization of the Froggatt-Nielson mechanism discussed in Ref. arX: 1807.05683 G. Abbas.