Estimation of T and CPT violation in $B^0 - \bar{B}^0$ mixing from time dependent CP asymmetry

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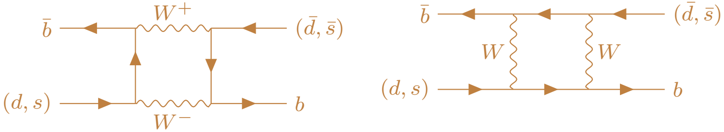
Introduction:

- CPT invariance $\rightarrow$ fundamental symmetry of nature.
- CPT violation leads to violation of Lorentz symmetry.

$\sim$ Greenberg [PRL 89, 231602 (2002)]

- CPT violation is expected to be a very weak effect.
- $B^0 - \bar{B}^0$ mixing is a good place to look for CPT violation.

$\sim$ Lavoura, Silva [PR D 60, 056003 (1999)]



- Most general mixing matrix includes T and CP violation.
- Entangled $(B^0 \bar{B}^0)$ states are not suitable for LHCb.
- We focus on T and CPT violation in mixing only.

$B^0 - \bar{B}^0$ mixing matrix:

- B^0 and $\bar{B}^0$ are *orthonormal* flavour eigenstates.
- $|B^0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|\bar{B}^0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
- $|\psi(t)\rangle = a(t)\,|B^0\rangle + b(t)\,|\bar{B}^0\rangle = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$
- Time evolution: $i\hbar \frac{\partial}{\partial t}|\psi(t)\rangle = \mathcal{H}\,|\psi(t)\rangle$
- If $\mathcal{H}$ were hermitian, $\langle\psi(t)|\psi(t)\rangle$ would have been 1.
  $\Rightarrow$ Probability conservation. $\Rightarrow$ No decay of B^0 or $\bar{B}^0$.
- Hence, $\mathcal{H}$ is *non-hermitian*.

$B^0 - \bar{B}^0$ mixing matrix:

- Any matrix can be written as sum of a hermitian and an anti-hermitian matrix.
- $\mathcal{H} = \boldsymbol{M} - \frac{i}{2}\boldsymbol{\Gamma}$; $\boldsymbol{M}$ and $\boldsymbol{\Gamma}$ are 2×2 hermitian matrices.
- $\boldsymbol{M}$ and $\boldsymbol{\Gamma}$ is related to the masses and decay width of B^0 and $\bar{B}^0$ respectively.
- ***Define:-***

  Eigenvectors of $\mathcal{H}$: $|B_L\rangle$ and $|B_H\rangle$

  Eigenvalues of $\mathcal{H}$: H_L and H_H

  $H_L = M_L - \frac{i}{2}\Gamma_L, \quad H_H = M_H - \frac{i}{2}\Gamma_H$

  $M = \frac{1}{2}(M_L + M_H), \quad \Gamma = \frac{1}{2}(\Gamma_L + \Gamma_H)$

  $\Delta M = (M_H - M_L), \quad \Delta\Gamma = (\Gamma_H - \Gamma_L)$
- As $\mathcal{H}$ is non-hermitian, $|B_L\rangle$ and $|B_H\rangle$ are not orthogonal in general.

Formalism:

~ *Book of Bigi, Sanda*

- $\mathcal{H} = E_1\sigma_1 + E_2\sigma_2 + E_3\sigma_3 - iDI_{2\times 2}$
- $E_1 = Re\,(M_{12}) - \frac{i}{2}Re\,(\Gamma_{12})$
  $E_2 = -Im\,(M_{12}) + \frac{i}{2}Im\,(\Gamma_{12})$
  $E_3 = \frac{1}{2}(M_{11} - M_{22}) - \frac{i}{4}Im\,(\Gamma_{11} - \Gamma_{22})$
  $D = \frac{i}{2}(M_{11} + M_{22}) + \frac{1}{4}Im\,(\Gamma_{11} + \Gamma_{22})$
- Spherical polar coordinates:
  $E_1 = E\sin\theta\cos\phi,\ E_2 = E\sin\theta\sin\phi,\ E_3 = E\cos\theta$
  E, θ, ϕ are complex numbers.
- $|B_L\rangle = p_1\,|B^0\rangle + q_1\,|\bar{B}^0\rangle,\ |B_H\rangle = p_2\,|B^0\rangle - q_2\,|\bar{B}^0\rangle$

  $p_1 = N_1\cos\frac{\theta}{2},\ q_1 = N_1 e^{i\phi}\sin\frac{\theta}{2}$
  $p_2 = N_2\sin\frac{\theta}{2},\ q_2 = N_2 e^{i\phi}\cos\frac{\theta}{2}$

  $H_L = E - iD,\ \ H_H = -E - iD$

T and CPT theorems:

$\sim$ ***Book of T. D. Lee***

1. CPT invariance (independent of T invariance) implies

$$\boldsymbol{M_{11} = M_{22}} \ \& \ \boldsymbol{\Gamma_{11} = \Gamma_{22}} \implies E_3 = 0 \Rightarrow \theta = \pi/2$$

2. T invariance (independent of CPT invariance) implies

$$\frac{\boldsymbol{\Gamma^*_{12}}}{\boldsymbol{M^*_{12}}} = \frac{\boldsymbol{\Gamma_{12}}}{\boldsymbol{M_{12}}} \implies Im(E_1 E_2^*) = 0 \Rightarrow Im(\phi) = 0$$

3. If CPT invariance holds, $\langle B_L | B_H \rangle$ is real.
If T invariance holds, $\langle B_L | B_H \rangle$ is imaginary.

$$\langle B_L | B_H \rangle = \cos\frac{\theta}{2}\sin\frac{\theta^*}{2} - e^{i(\phi-\phi^*)}\cos\frac{\theta^*}{2}\sin\frac{\theta}{2}$$

Obviously, $\theta = \pi/2$ makes $\langle B_L | B_H \rangle$ to be real
and $\phi = \phi^*$ forces it to be imaginary.

Mixing parameters:

- $Re(\theta) - \pi/2$ and $Im(\theta)$ are CPT violating parameters.
- $Im(\phi)$ is T violating parameter.
- Usually, it is said that absence of CP violation implies $|e^{i\phi}| = 1$. Strictly speaking, if it's due to absence of T violation. However, if CPT is conserved, both of the statements are right.
- If CPT and T are conserved, $\langle B_L | B_H \rangle = 0$.
- CPT and T violation $\Longrightarrow$ $\theta = \pi/2 + \epsilon_1 + i\epsilon_2$
  $\phi = -2\beta^{mix} + i\epsilon_3$

Measurements of mixing parameters:

- Belle and BaBar notation:
  $\Rightarrow \cos\theta \to -z,\ \sin\theta \to \sqrt{1-z^2},\ e^{i\phi} \to \frac{q}{p}.$
  $\Rightarrow \epsilon_1 = Re(z),\ \epsilon_2 = Im(z),\ \epsilon_3 = 1 - |\frac{q}{p}|$

- Belle: $\epsilon_1 = (1.9 \pm 3.7 \pm 3.3) \times 10^{-2}$
  $\epsilon_2 = (-5.7 \pm 3.3 \pm 3.3) \times 10^{-3}$ ~PRD 85 (2012) 071105

- BABAR: $\epsilon_1 \Delta\Gamma = (-7.1 \pm 3.9 \pm 2.0) \times 10^{-3}$ ps^{-1}
  $\epsilon_2 = (-13.9 \pm 7.3 \pm 3.2) \times 10^{-3}$ ~ PRL 114(8) (2015) 081801

- $y_d = \Delta\Gamma_d/(2\Gamma_d) = -0.003 \pm 0.015$
  $\Gamma_d = 1.520 \pm 0.004$ ps ~ PDG (2016)

- $|q/p| = 1.0010 \pm 0.0008 \implies \epsilon_3 = -(1.0 \pm 0.8) \times 10^{-3}$
  measured at the $\Upsilon(4S)$ using the same-sign dilepton asymmetry, assuming CPT conservation. ~ HFAG Collaboration (2016)

Time evolution:

- Time evolution operator, $U(t) = e^{-i\mathcal{H}t}$
- $|B_L(t)\rangle = e^{-iH_L t}\,|B_L\rangle$ and $|B_H(t)\rangle = e^{-iH_H t}\,|B_H\rangle$
- $|B^0\rangle = \left[\frac{q_2}{p_1 q_2 + p_2 q_1}\,|B_L\rangle + \frac{q_1}{p_1 q_2 + p_2 q_1}\,|B_H\rangle\right]$

  $|\bar{B}^0\rangle = \left[\frac{p_2}{p_1 q_2 + p_2 q_1}\,|B_L\rangle - \frac{p_1}{p_1 q_2 + p_2 q_1}\,|B_H\rangle\right]$
- $|B^0(t)\rangle = (g_+ + g_-\cos\theta)\,|B^0\rangle + e^{i\phi}\,g_-\sin\theta\,|\bar{B}^0\rangle$

  $|\bar{B}^0(t)\rangle = e^{-i\phi}\,g_-\sin\theta\,|B^0\rangle + (g_+ - g_-\cos\theta)\,|\bar{B}^0\rangle$

  where $g_\pm = (e^{-iH_L t} \pm e^{-iH_H t})/2$

$(B^0 - \bar{B}^0) \to f$ transition:

$$\frac{d\Gamma}{dt}(B^0(t) \to f) = \frac{e^{-\Gamma t}}{2}\Bigg[\cosh\left(\frac{\Delta\Gamma t}{2}\right)\Big\{|\mathcal{A}_f|^2 + |\cos\theta|^2|\mathcal{A}_f|^2 + |e^{i\phi}\sin\theta|^2|\bar{\mathcal{A}}_f|^2$$

$$+ 2\mathrm{Re}\Big(e^{i\phi}\cos\theta^*\sin\theta\mathcal{A}_f^*\bar{\mathcal{A}}_f\Big)\Big\} + \sinh\left(\frac{\Delta\Gamma t}{2}\right)\Big\{2\mathrm{Re}\Big(\cos\theta|\mathcal{A}_f|^2 + e^{i\phi}\sin\theta\mathcal{A}_f^*\bar{\mathcal{A}}_f\Big)\Big\}$$

$$+ \cos(\Delta M t)\Big\{|\mathcal{A}_f|^2 - |\cos\theta|^2|\mathcal{A}_f|^2 - |e^{i\phi}\sin\theta|^2|\bar{\mathcal{A}}_f|^2 - 2\mathrm{Re}\Big(e^{i\phi}\cos\theta^*\sin\theta\mathcal{A}_f^*\bar{\mathcal{A}}_f\Big)\Big\}$$

$$- \sin(\Delta M t)\Big\{2\mathrm{Im}\Big(\cos\theta|\mathcal{A}_f|^2 + e^{i\phi}\sin\theta\mathcal{A}_f^*\bar{\mathcal{A}}_f\Big)\Big\}\Bigg]$$

$$\frac{d\Gamma}{dt}(\bar{B}^0(t) \to f) = \frac{e^{-\Gamma t}}{2}\Bigg[\cosh\left(\frac{\Delta\Gamma t}{2}\right)\Big\{|\bar{\mathcal{A}}_f|^2 + |\cos\theta|^2|\bar{\mathcal{A}}_f|^2 + |e^{-i\phi}\sin\theta|^2|\mathcal{A}_f|^2$$

$$- 2\mathrm{Re}\Big(e^{i\phi^*}\cos\theta\sin\theta^*\mathcal{A}_f^*\bar{\mathcal{A}}_f\Big)\Big\} + \sinh\left(\frac{\Delta\Gamma t}{2}\right)\Big\{2\mathrm{Re}\Big(-\cos\theta^*|\bar{\mathcal{A}}_f|^2 + e^{i\phi^*}\sin\theta^*\mathcal{A}_f^*\bar{\mathcal{A}}_f\Big)\Big\}$$

$$+ \cos(\Delta M t)\Big\{|\bar{\mathcal{A}}_f|^2 - |\cos\theta|^2|\bar{\mathcal{A}}_f|^2 - |e^{-i\phi}\sin\theta|^2|\mathcal{A}_f|^2 + 2\mathrm{Re}\Big(e^{i\phi^*}\cos\theta\sin\theta^*\mathcal{A}_f^*\bar{\mathcal{A}}_f\Big)\Big\}$$

$$+ \sin(\Delta M t)\Big\{2\mathrm{Im}\Big(-\cos\theta^*|\bar{\mathcal{A}}_f|^2 + e^{i\phi^*}\sin\theta^*\mathcal{A}_f^*\bar{\mathcal{A}}_f\Big)\Big\}\Bigg]$$

$$\mathcal{A}_f = \langle f|H_D|B^0\rangle\,, \quad \bar{\mathcal{A}}_f = \langle f|H_D|\bar{B}^0\rangle$$

Time dependent indirect CP asymmetry:

- $$A^f_{CP}(t) = \frac{\frac{d\Gamma}{dt}(\bar{B}^0_d(t) \to f_{CP}) - \frac{d\Gamma}{dt}(B^0_d(t) \to f_{CP})}{\frac{d\Gamma}{dt}(\bar{B}^0_d(t) \to f_{CP}) + \frac{d\Gamma}{dt}(B^0_d(t) \to f_{CP})}$$
- In the absence of ϵ_j and $\Delta\Gamma$,
  $$A^f_{CP}(t) = S\sin(\Delta M t) - C\cos(\Delta M t)$$
  $$C = \frac{|\mathcal{A}_f|^2 - |\bar{\mathcal{A}}_f|^2}{|\mathcal{A}_f|^2 + |\bar{\mathcal{A}}_f|^2},\ S = \sqrt{1 - C^2}\ \sin\varphi,$$
  $$\varphi = -2\beta - \arg[\mathcal{A}_f] + \arg[\bar{\mathcal{A}}_f]$$
- No penguin pollution $\implies$ $C = 0\ \&\ \varphi = -2\beta$
- Taking ϵ_j and $\Delta\Gamma_d$ to be small,
  $$A^f_{CP/CPT}(t) \simeq c_0 + c_1\cos(\Delta M_d t) + c_2\cos(2\Delta M_d t) + s_1\sin(\Delta M_d t) + s_2\sin(2\Delta M_d t) + c_1'\,\Gamma_d\, t\cos(\Delta M_d t) + s_1'\,\Gamma_d\, t\sin(\Delta M_d t)$$

Observables:

- $c_0 = \epsilon_1 \cos\varphi + \epsilon_3 - \frac{1}{2}\epsilon_3 \sin^2\varphi$
- $c_1 = -C - \epsilon_3 - \epsilon_1 \cos\varphi - \epsilon_2 C \sin\varphi$
- $c_2 = \frac{1}{2}\epsilon_3 \sin^2\varphi + \epsilon_2 C \sin\varphi$
- $s_1 = \sqrt{1-C^2}\sin\varphi - \epsilon_2 \cos^2\varphi - \epsilon_3 C \sin\varphi$
- $s_2 = -\frac{1}{2}\epsilon_2 \sin^2\varphi + \epsilon_3 C \sin\varphi$
- $c_1' = C\, y_d \cos\varphi$ (here, $y_d = 2\Delta\Gamma_d/\Gamma_d$)
- $s_1' = -\frac{1}{2}\, y_d \sin 2\varphi$

Five unknown quantities $(\epsilon_1, \epsilon_2, \epsilon_3, C, \varphi)$.
We shall use first five equations.

Presence of c_0, c_2 and s_2 (i.e. constant piece, coefficient of $\cos(2\Delta M_d t)$ and coefficient of $\sin(2\Delta M_d t)$) will confirm the existence of T or CPT violation in mixing.

Solutions:

☞ $C = -(c_0 + c_1 + c_2)$

☞ $$\sin^4\varphi - 2\left[\frac{s_1 + 2s_2}{2 - C^2}\right]\sin^3\varphi + 4C\left[C + \frac{c_2}{2 - C^2}\right]\sin^2\varphi - 4\left[\frac{2C^2(s_1 + s_2) - s_2}{2 - C^2}\right]\sin\varphi - \left[\frac{8C\,c_2}{2 - C^2}\right] = 0$$

☞ $$\epsilon_1 = c_0 \sec\varphi - \frac{(2 - \sin^2\varphi)(c_2 \sin\varphi + 2C\,s_2)}{(4C^2 + \sin^2\varphi)\,\sin\varphi\,\cos\varphi}$$

☞ $$\epsilon_2 = \frac{2\,(2C\,c_2 - s_2\,\sin\varphi)}{(4C^2 + \sin^2\varphi)\,\sin\varphi}$$

☞ $$\epsilon_3 = \frac{2\,(c_2\,\sin\varphi + 2C\,s_2)}{(4C^2 + \sin^2\varphi)\,\sin\varphi}$$

One can solve for y_d also using c_1' or s_1'.

Even if y_d (or $\Delta\Gamma_d$) vanishes, ϵ_3 can still be extracted.

CPT conserving scenario (Example):

Taking $\epsilon_1 = \epsilon_2 = 0$;

$$c_0 = \epsilon_3 \left[1 - \frac{2 s_1^2}{(2 - c_1^2 + \epsilon_3^2)^2} \right],$$

$$c_2 = \frac{2 s_1^2 \, \epsilon_3}{(2 - c_1^2 + \epsilon_3^2)^2},$$

$$s_2 = -\frac{2 s_1 \, (c_1 + \epsilon_3) \, \epsilon_3}{(2 - c_1^2 + \epsilon_3^2)}.$$

With the existing values for ϵ_3 and c_1, s_1 for $J/\psi K_s$ mode, we get

$$c_0 = (-15.18 \pm 15.50) \times 10^{-4},$$

$$c_2 = (-4.31 \pm 4.41) \times 10^{-4},$$

$$s_2 = (0.29 \pm 0.43) \times 10^{-4}.$$

Significant deviations from these values will imply CPT violation in $B_d^0 - \bar{B}_d^0$ mixing.

Comments:

- $A^f_{CP}(t)$ contains enough information to extract T and CPT violation in mixing along with CP violating weak phases.
- For B^0_s system, $\Delta\Gamma_s$ is not very small. So, one should keep more terms in the expansion of $\sinh(\Delta\Gamma t/2)$ and $\cosh(\Delta\Gamma t/2)$ and proceed in the same way.
- Entangled states are not required.
- Penguin pollutions need not be neglected.
- It is also possible to extract these parameters by fitting the data for mode and conjugate mode separately following same way.