# $\begin{array}{l} Review \ of \ approximations: \\ q_T \ resummation \ at \ NNLO+NNLL \ QCD \ with \ DYRes \end{array}$

**Giancarlo Ferrera** 

Milan University & INFN Milan





 $p_{\mathsf{T}_{\mathsf{Z}}}$  and  $p_{\mathsf{T}_{\mathsf{W}}}$  theory meeting – CERN – January 25 2018







Fixed-order perturbative expansion reliable only for  $q_T \sim M$ . When  $q_T \ll M$ :

$$\int_{0}^{q_{T}^{2}} d\bar{q}_{T}^{2} \frac{d\hat{\sigma}_{q\bar{q}}}{d\bar{q}_{T}^{2}} \sim 1 + \alpha_{S} \left[ c_{12}L_{q_{T}}^{2} + c_{11}L_{q_{T}} + \cdots \right] + \alpha_{S}^{2} \left[ c_{24}L_{q_{T}}^{4} + \cdots + c_{21}L_{q_{T}} + \cdots \right] + \mathcal{O}(\alpha_{S}^{3})$$

with  $\alpha_S^n L_{q_T}^m \equiv \alpha_S^n \log^m (M^2/q_T^2) \gg 1$ .

Resummation of logarithmic corrections needed.



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## NNLO QCD predictions at large $q_T$



 $Z q_T$  spectrum ( $q_T > 20 \ GeV$ ).

#### $\rightarrow$ see A. Huss talk

- ATLAS data ( $\sqrt{s} = 8 \text{ TeV}$ ) [1512.02192] (2.8% luminosity uncertainty not shown).
- NNLO (i.e. $\mathcal{O}(\alpha_5^3)$ ) QCD predictions [G.-DeRidder, Gehrmann, Glover, Huss, Morgan('16)]. NNLO correction positive (~6-8%) and reduce scale dependence (factor 2 around  $\mu = \sqrt{M^2 + q_T^2}$ ).
- Agreement between data and theory improves by considering normalized distributions.

In the small  $q_T$  region effects of soft-gluon resummation are essential At the LHC 90% of the  $W^{\pm}$  and  $Z^0$  are produced with  $q_T \lesssim 20 \text{ GeV}$ 

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Resummed (N)NLL/(N)NLO result at small  $q_T$  matched with fixed (N)LO (i.e.  $\alpha_S(\alpha_5^2)$ ) "finite" part at large  $q_T$ : uniform accuracy for  $q_T \ll M$  and  $q_T \sim M$ .

Giancarlo Ferrera – Milan University & INFN gT resummation at NNLO+NNLL QCD with DYRes

### $q_T$ spectrum of the Z boson



NLL+NLO and NNLL+NNLO Z  $q_T$  spectrum at the LHC at  $\sqrt{s} = 7/8$  TeV.

### q<sub>T</sub> spectrum of Z boson: theory vs ATLAS data



Left: NLL+NLO and NNLL+NNLO bands for  $Z/\gamma^* q_T$  spectrum compared with and ATLAS data (7 TeV).

Right Top: Ratios between ResBos predictions and ATLAS data.

Right Bottom: Ratios between various MC generators results and ATLAS data.

Giancarlo Ferrera – Milan University & INFN gT resummation at NNLO+NNLL QCD with DYRes

#### $\phi^*$ spectrum of Z boson: theory vs ATLAS data



NLL+NLO and NNLL+NNLO bands for  $Z/\gamma^* \phi^*$  spectrum compared with ATLAS data.



MC generators results and ATLAS data ratio to ResBos.

#### q<sub>T</sub> spectrum of W: theory vs ATLAS data



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## Lepton $p_T$ distributions from W decay



Ratios of the lepton  $p_T$  normalised distribution obtained using Powheg+Pythia 8 AZNLO, DYRES and Powheg MiNLO+Pythia 8 to the distribution obtained using PYTHIA 8 AZ.

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#### Transverse-mass distributions from W decay



Ratios of the  $m_T$  normalised distribution obtained using Powheg+Pythia 8 AZNLO, DYRES and Powheg MiNLO+Pythia 8 to the distribution obtained using PYTHIA 8 AZ.

## **PDF** uncertainties and **NP** effects



NNLL+NNLO result for  $Z q_T$  spectrum at the LHC. Perturbative scale dependence, PDF uncertainties and impact of NP effects.

- PDF uncertainty is smaller than the scale uncertainty and it is approximately independent on q<sub>T</sub> (around the 3% level).
- Non perturbative *intrinsic*  $k_T$  effects parametrized by a NP form factor  $S_{NP} = \exp\{-g_{NP}b^2\}$  with  $0 < g_{NP} < 1.2 \ GeV^2$ :

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- NP effects increase the hardness of the q<sub>T</sub> spectrum at small values of q<sub>T</sub>.
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Ratio of NNLL+NNLO and NLL+NLO results for  $W/Z q_T$  spectra at the LHC. Perturbative scale dependence.

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- Correlated (μ<sup>W</sup>/M<sub>W</sub> = μ<sup>Z</sup>/M<sub>Z</sub>) scale variations by factor 2 (avoiding ratios larger than 2) gives reasonable estimate of pert. uncertainty (nice overlap of scale variation bands for q<sub>T</sub> > 3 GeV).
- PDF uncertainty dominates at very small ( $q_T q_T \lesssim 5 \ GeV$ ).
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#### W/Z ratio $q_T$ spectrum: perturbative scale uncertainty



DYqT resummed predictions for the ratio of W/Z normalized  $q_T$  spectra. Uncorrelated perturbative scale variation band.

DYqT resummed predictions for the ratio of W/Z normalized  $q_T$  spectra. Correlated perturbative scale variation band.