Resummation with RadISH

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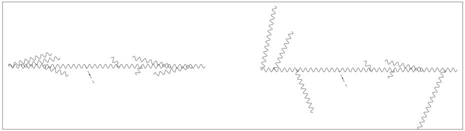
pTZ and pTW theory meeting
CERN, 25 January 2018

small p_{T} resummation in momentum space

- logarithmic accuracy usually defined at the level of the logarithm of the cumulative cross section Σ

$$\Sigma(p_{\mathrm{T}}) = \int_0^{p_{\mathrm{T}}} dp'_{\mathrm{T}} \frac{d\sigma}{dp'_{\mathrm{T}}} \sim \exp\{\alpha_{\mathrm{S}}^n L^{n+1} + \alpha_{\mathrm{S}}^n L^n + \alpha_{\mathrm{S}}^n L^{n-1} + \alpha_{\mathrm{S}}^n L^{n-2} + ...\}$$

  for LL, NLL, NNLL, N3LL, where $L = \log(M/p_{\mathrm{T}})$
- as p_{T} absorbs the recoil of all emissions k_{ti}, when $p_{\mathrm{T}} \to 0$, two mechanism compete:
  - Sudakov (exponential) suppression when $k_{ti} \sim p_{\mathrm{T}}$
  - azimuthal cancellations when $k_{ti} \gg p_{\mathrm{T}}$



- the latter mechanism is dominant when $p_{\mathrm{T}} \to 0$: $\Sigma(p_{\mathrm{T}}) \sim p_{\mathrm{T}}^2$ [Parisi,Petronzio '79]

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- just an hierarchy in $\log(M/p_{\mathrm{T}})$ doesn't work, as neglected effects actually dominate the limit. It's impossible to recover power behaviour at any given order in L.
- Moreover, at any log order in $L = \log(M/p_{\mathrm{T}})$, resummation in direct space cannot be, at the same time, free of subleading terms and of spurious singularities at finite p_{T} [Frixione,Nason,Ridolfi '98]
- when going in b-space, the vectorial nature of azimuthal cancellations is taken care by a Fourier transform

$$\delta^{(2)}(\vec{p_{\mathrm{T}}} - (\vec{k}_{t1} + ... + \vec{k}_{tn})) = \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_t} \prod_{i=1}^{n} e^{-i\vec{b}\cdot\vec{k}_{ti}}$$

$\to$ The information of the radiation is, by construction, lost.

small $p_{T,H}$ resummation in momentum space (I)

Our approach:

- Write all-order cross-section for $v = p_T$ $(V(\{\tilde{p}\}, k_1 \dots k_n) = |\vec{k}_{t1} + ... + \vec{k}_{tn}|)$

$$\Sigma(v) = \int d\Phi_B \mathcal{V}(\Phi_B) \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} [dk_i] |M(\tilde{p}_1, \tilde{p}_2, k_1, \dots, k_n)|^2 \, \Theta\left(v - V(\{\tilde{p}\}, k_1 \dots k_n)\right)$$

$\mathcal{V}$: all-order form factor $\qquad$ $|M(\tilde{p}_1, \tilde{p}_2, k_1, \dots, k_n)|^2$: real emissions

- re-organize multiple-emission squared amplitudes into "n-particle-correlated blocks".
  - the rIRC safety of the observable guarantees a hierarchy between the different blocks (n-particle $\to$ one higher log-order than $n-1$-particle)
  - for *inclusive observables*, can integrate the blocks before evaluating the observable.

$$|M(\tilde{p}_1, \tilde{p}_2, k_1, \dots, k_n)|^2 \longrightarrow |M_B(\tilde{p}_1, \tilde{p}_2)|^2$$
$$\times \frac{1}{n!} \left\{ \prod_{i=1}^{n} \left(|M(k_i)|^2 + \int [dk_a][dk_b] |\tilde{M}(k_a, k_b)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} - \vec{k}_{ti}) \delta(Y_{ab} - Y_i) \right.\right.$$
$$\left.\left. + \int [dk_a][dk_b][dk_c] |\tilde{M}(k_a, k_b, k_c)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} + \vec{k}_{tc} - \vec{k}_{ti}) \delta(Y_{abc} - Y_i) + \dots \right) \right\}$$

  - each $|\tilde{M}|$ has a perturbative expansion

- LL : $|M^{(0)}(k_i)|^2$; NLL : $\int |\tilde{M}^{(0)}(k_a, k_b)|^2$, $|M^{(1)}(k_i)|^2$; ...

cancellation of singularities

- need subtraction of IRC poles between $\mathcal{V}$ and $\int \prod_{i=1}^{n} [dk_i] |M(\tilde{p}_1, \tilde{p}_2, k_1, \ldots, k_n)|^2$
- introduce a resolution scale ϵk_{t1} (not $\epsilon\, p_{\mathrm{T,H}}$)
  - emissions with $k_{ti} < \epsilon k_{t1}$ are unresolved. They don't contribute to the observable, and upon integration they regularise virtual corrections leaving a Sudakov factor

$$e^{-R(\epsilon k_{t1})} = e^{-R(k_{t1}) - \log(1/\epsilon) R'(k_{t1}) + \ldots}$$

  - emissions above ϵk_{t1} are resolved, hence are treated exclusively (they are used to compute the observable!). This is done through a MC.
  - ϵ dependence in the resolved emissions cancel against the one in the Sudakov!
- Resolved k_{ti} are not necessarily $\sim p_{\mathrm{T}}$: all kinematics properly covered, without assumptions on the hierarcy between k_{ti} and p_{T}.
- $k_{ti} \gg p_{\mathrm{T}}$ included. This also removes the spurious singularities at finite p_{T} and gives the correct power behaviour at $p_{\mathrm{T}} \to 0$

small $p_{\mathrm{T,H}}$ resummation in momentum space (II)

the role of subleading terms

- logarithmic counting is defined in terms of $\log(m_{\mathrm{H}}/k_{ti})$.
- in the Sudakov limit, the hierarchy in $\log(m_{\mathrm{H}}/p_{\mathrm{T,H}})$ makes sense, one has $k_{ti} \sim p_{\mathrm{T,H}} \sim 0$.
  - same as resummation of $\log(m_{\mathrm{H}}/p_{\mathrm{T,H}})$, i.e. log accuracy in $\log(m_{\mathrm{H}}/k_{ti})$ translates into the same accuracy in $\log(m_{\mathrm{H}}/p_{\mathrm{T,H}})$, plus subleading terms.
- similar conclusions were found by Ebert and Tackmann, '16.

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some advantages with respect to b-space

- closer connections to a parton-shower formalism
- if observables have the same LL as p_{T}, then we can keep using the same resolution scale ϵk_{t1}, and compute all of them at the same time.
- might allow joint resummation (the k_{ti} are not integrated over).

comparison with b-space and implementation

$$\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathcal{C}_1} \frac{dN_1}{2\pi i} \int_{\mathcal{C}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1 c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0) \hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) \mathbf{f}_{N_2}(\mu_0)$$

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi}$$

$$\times\, e^{-\mathbf{R}(\epsilon k_{t1})} \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

$$\sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi} \mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$

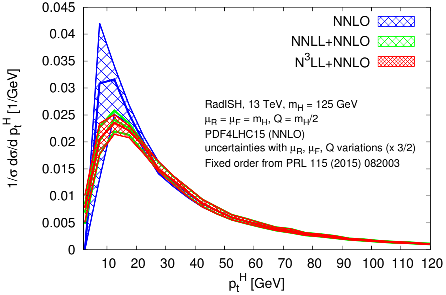
$$\times\, \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right)$$

$\zeta_i = k_{ti}/k_{t1}$ $\quad$ Γ and $\Gamma^{(C)}$ anomalous dimensions of PDFs and coeff. function

- from above expression, proven equivalence with b-space formula, and extracted necessary term to add to A_4, B_3 and $H^{(2)}$ (or $C^{(2)}$)
- as the transverse momenta of the resolved reals are of the same order, we can expand the integrand of the resolved radiation about k_{t1} up to the desired accuracy
- a code (named `RadISH`), performing all of the above, also for Drell-Yan, will be released soon.

...we are finishing the DY case; for now, I show just a result for Higgs production...

$p_{\mathrm{T,H}}$ at N3LL+NNLO



- NNLO matching ($\sigma^{\mathrm{N3LO}}_{pp\to H}$, $d\sigma^{\mathrm{NNLO}}_{pp\to Hj}/dp_{\mathrm{T}}$)
- N3LO from Anastasiou et al., '15
- $pp \to Hj$ at NNLO from Boughezal, Caola, et al., '15
- anomalous dimension from Li, Zhu '16, Vladimirov '16

+ resummation: relevant below 30 GeV
+ medium-high p_{T}: matching to differential NNLO matters (as expected): + 10 % wrt NLO, reduced uncertainty bands.

- N3LL+NNLO corrections: few percent at peak, more sizeable below
- after matching at NNLO, only moderate reduction in uncertainty from NNLL to N3LL. Precise quantitative statement needs very stable NNLO distributions below peak.
- phenomenology: with this precision, perturbative uncertainty from resummation seems to saturate; including quark mass effects will be relevant to improve further.

[Melnikov,Penin '16; Melnikov et al. '16; Lindert et al. '17]

Multiplicative vs Additive Matching

$$\Sigma(p_T, \Phi_B) = \int_0^{p_T} dp'_T \frac{d\sigma}{dp'_T d\Phi_B} \qquad \begin{cases} \to \Sigma_{\text{res}} & \text{if } p_T \ll M_B \\ \to \Sigma_{\text{F.O.}} & \text{if } p_T \gtrsim M_B \end{cases}$$

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additive matching

$$\Sigma^{add}_{\text{matched}}(p_T) = \Sigma_{\text{res}}(p_T) + \Sigma_{\text{F.O.}}(p_T) - \Sigma_{\text{res,exp}}(p_T)$$

multiplicative matching

$$\Sigma^{mult}_{\text{matched}}(p_T) = \Sigma_{\text{res}}(p_T) \frac{\Sigma_{\text{F.O.}}(p_T)}{\Sigma_{\text{res,exp}}(p_T)}$$

- there's no rigorous theory argument to favour a prescription over the other

- additive: probably the more natural choice, simpler and clear
- numerically delicate when $p_T \to 0$ (F.O. result needs to be extremely stable)

- multiplicative: numerically more stable, as physical suppression at small p_T fixes potentially unstable F.O. results
- allows to include constant terms from F.O.

Multiplicative vs Additive Matching

- for $p_{\mathrm{T,H}}$ at N3LL, used mult. matching: constant terms at $\mathcal{O}(\alpha_{\mathrm{S}}^3)$ recovered *without* the need of knowing analytically coefficient and hard functions.

$$\Sigma_{\mathrm{F.O.}} = \sigma^{\mathrm{N3LO}}_{pp\to H} - \int_{p_{\mathrm{T}}} dp'_{\mathrm{T}} \frac{d\sigma^{\mathrm{NNLO}}_{pp\to Hj}}{dp'_{\mathrm{T}}}$$

- in additive matching, one would instead need $C^{(3)}$ and $H^{(3)}$ in effective luminosity $\mathcal{L}_{\mathrm{N^3LL}}$

- to estimate higher-order logarithmic corrections, introduce resummation scale Q:

$$L \equiv \ln \frac{M}{k_{\mathrm{T},1}} = \ln \frac{Q}{k_{\mathrm{T},1}} - \ln \frac{Q}{M}$$

  and then vary Q, making sure that the first term is larger than the second, as we are in fact expanding about $\ln(Q/k_{\mathrm{T},1})$.

- in resummation formula, use replacement above in Sudakov and parton densities. Expand about $\ln Q/k_{\mathrm{T},1}$ and reabsorb $\ln Q/M$ in H and C functions, entering the generalized luminosities

$$H^{(1)}(\mu_R) \to \tilde{H}^{(1)}(\mu_R, x_Q) = H^{(1)}(\mu_R) + \left(-\frac{1}{2}A^{(1)} \ln x_Q^2 + B^{(1)}\right) \ln x_Q^2, \quad x_Q = Q/M.$$

$$C^{(1)}_{ij}(z) \to \tilde{C}^{(1)}_{ij}(z, \mu_F, x_Q) = C^{(1)}_{ij}(z) + \hat{P}^{(0)}_{ij}(z) \ln \frac{x_Q^2 M^2}{\mu_F^2}$$

Outlook

- we are now studying different matching scheme for DY ($p_{\mathrm{T,Z}}$ and ϕ^*).
- $pp \to Zj$ at NNLO from NNLOJET, Gehrmann-De Ridder et al., '16
- we are comparing different schemes, with their uncertainty band, and against data as well.