NNLOPS for Drell-Yan production

Emanuele Re

CERN & LAPTh Annecy

pTZ and pTW theory meeting
CERN, 25 January 2018

towards NNLO+PS

- what do we need and what do we already have?

	V (inclusive)	V+j (inclusive)	V+2j (inclusive)
V @ NLOPS	NLO	LO	shower
VJ @ NLOPS	/	NLO	LO
V-VJ @ NLOPS	NLO	NLO	LO
V @ NNLOPS	NNLO	NLO	LO

☞ a merged V-VJ generator is almost OK

towards NNLO+PS

- what do we need and what do we already have?

	V (inclusive)	V+j (inclusive)	V+2j (inclusive)
V @ NLOPS	NLO	LO	shower
VJ @ NLOPS	/	NLO	LO
V-VJ @ NLOPS	NLO	NLO	LO
V @ NNLOPS	NNLO	NLO	LO

☞ a merged V-VJ generator is almost OK

- many of the multijet NLO+PS merging approaches work by combining 2 (or more) NLO+PS generators, introducing a merging scale
- POWHEG + MiNLO: no need of merging scale: it extends the validity of an NLO computation with jets in the final state in regions where jets become unresolved

MiNLO

Multiscale Improved NLO

[Hamilton,Nason,Zanderighi, 1206.3572]

- original goal: method to a-priori choose scales in multijet NLO computation
- non-trivial task: hierarchy among scales can spoil accuracy (large logs can appear, without being resummed)
- how: correct weights of different NLO terms with CKKW-inspired approach (without spoiling formal NLO accuracy)

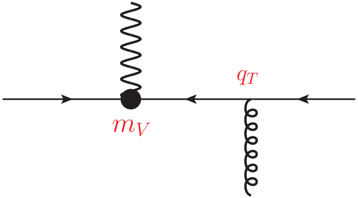
MiNLO

Multiscale Improved NLO

[Hamilton,Nason,Zanderighi, 1206.3572]

- original goal: method to a-priori choose scales in multijet NLO computation
- non-trivial task: hierarchy among scales can spoil accuracy (large logs can appear, without being resummed)
- how: correct weights of different NLO terms with CKKW-inspired approach (without spoiling formal NLO accuracy)

$$\bar{B}_{\rm NLO} = \alpha_{\rm S}(\mu_R)\Big[B + \alpha_{\rm S}^{\rm (NLO)} V(\mu_R) + \alpha_{\rm S}^{\rm (NLO)} \int d\Phi_{\rm r} R\Big]$$



MiNLO

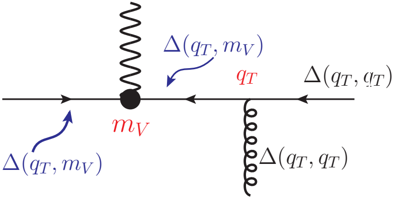
Multiscale Improved NLO

[Hamilton,Nason,Zanderighi, 1206.3572]

- original goal: method to a-priori choose scales in multijet NLO computation
- non-trivial task: hierarchy among scales can spoil accuracy (large logs can appear, without being resummed)
- how: correct weights of different NLO terms with CKKW-inspired approach (without spoiling formal NLO accuracy)

$$\bar{B}_{\rm NLO} = \alpha_{\rm S}(\mu_R)\Big[B + \alpha_{\rm S}^{\rm (NLO)} V(\mu_R) + \alpha_{\rm S}^{\rm (NLO)} \int d\Phi_{\rm r} R\Big]$$

$$\bar{B}_{\rm MiNLO} = \alpha_{\rm S}(q_T)\Delta_q^2(q_T, m_V)\Big[B\left(1 - 2\Delta_q^{(1)}(q_T, m_V)\right) + \alpha_{\rm S}^{\rm (NLO)} V(\bar{\mu}_R) + \alpha_{\rm S}^{\rm (NLO)} \int d\Phi_{\rm r} R\Big]$$



MiNLO

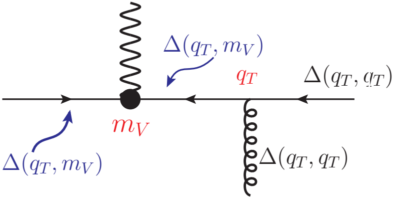
Multiscale Improved NLO

[Hamilton,Nason,Zanderighi, 1206.3572]

- original goal: method to a-priori choose scales in multijet NLO computation
- non-trivial task: hierarchy among scales can spoil accuracy (large logs can appear, without being resummed)
- how: correct weights of different NLO terms with CKKW-inspired approach (without spoiling formal NLO accuracy)

$$\bar{B}_{\rm NLO} = \alpha_{\rm S}(\mu_R)\Big[B + \alpha_{\rm S}^{(\rm NLO)} V(\mu_R) + \alpha_{\rm S}^{(\rm NLO)} \int d\Phi_{\rm r} R\Big]$$

$$\bar{B}_{\rm MiNLO} = \alpha_{\rm S}(q_T)\Delta_q^2(q_T, m_V)\Big[B\left(1 - 2\Delta_q^{(1)}(q_T, m_V)\right) + \alpha_{\rm S}^{(\rm NLO)} V(\bar{\mu}_R) + \alpha_{\rm S}^{(\rm NLO)} \int d\Phi_{\rm r} R\Big]$$



- $\bar{\mu}_R = q_T$
- $\log \Delta_{\rm f}(q_T, m_V) = -\int_{q_T^2}^{m_V^2} \frac{dq^2}{q^2} \frac{\alpha_{\rm S}(q^2)}{2\pi}\left[A_f \log\frac{m_V^2}{q^2} + B_f\right]$
- $\Delta_{\rm f}^{(1)}(q_T, m_V) = -\frac{\alpha_{\rm S}^{(\rm NLO)}}{2\pi}\left[\frac{1}{2}A_{1,\rm f}\log^2\frac{m_V^2}{q_T^2} + B_{1,\rm f}\log\frac{m_V^2}{q_T^2}\right]$
- $\mu_F = q_T$

MiNLO

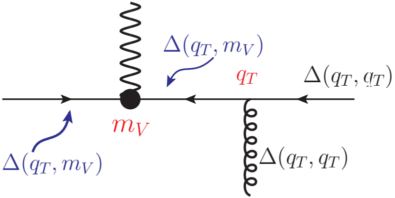
Multiscale Improved NLO

[Hamilton,Nason,Zanderighi, 1206.3572]

- original goal: method to a-priori choose scales in multijet NLO computation
- non-trivial task: hierarchy among scales can spoil accuracy (large logs can appear, without being resummed)
- how: correct weights of different NLO terms with CKKW-inspired approach (without spoiling formal NLO accuracy)

$$\bar{B}_{\rm NLO} = \alpha_{\rm S}(\mu_R)\Big[B + \alpha_{\rm S}^{(\rm NLO)} V(\mu_R) + \alpha_{\rm S}^{(\rm NLO)} \int d\Phi_{\rm r} R\Big]$$

$$\bar{B}_{\rm MiNLO} = \alpha_{\rm S}(q_T)\Delta_q^2(q_T, m_V)\Big[B\left(1 - 2\Delta_q^{(1)}(q_T, m_V)\right) + \alpha_{\rm S}^{(\rm NLO)} V(\bar{\mu}_R) + \alpha_{\rm S}^{(\rm NLO)} \int d\Phi_{\rm r} R\Big]$$



☞ Sudakov FF included on $V+j$ Born kinematics

- MiNLO-improved VJ yields finite results also when 1st jet is unresolved ($q_T \to 0$)
- $\bar{B}_{\rm MiNLO}$ ideal to extend validity of VJ−POWHEG [called “VJ−MiNLO” hereafter]

"Improved" MiNLO & NLOPS merging

- formal accuracy of `VJ-MiNLO` for inclusive observables carefully investigated

  [Hamilton et al., 1212.4504]

- `VJ-MiNLO` describes inclusive observables at order α_S
- to reach genuine NLO when fully inclusive (NLO$^{(0)}$), "spurious" terms must be of relative order α_S^2, *i.e.*

$$O_{\mathrm{VJ-MiNLO}} = O_{\mathrm{V@NLO}} + \mathcal{O}(\alpha_S^2) \qquad \text{if } O \text{ is inclusive}$$

- "Original `MiNLO`" contains ambiguous "$\mathcal{O}(\alpha_S^{1.5})$" terms

---

"Improved" MiNLO & NLOPS merging

- formal accuracy of VJ−MiNLO for inclusive observables carefully investigated

  [Hamilton et al., 1212.4504]

- VJ−MiNLO describes inclusive observables at order α_S
- to reach genuine NLO when fully inclusive (NLO$^{(0)}$), "spurious" terms must be of relative order α_S^2, *i.e.*

$$O_{\mathrm{VJ-MiNLO}} = O_{\mathrm{V@NLO}} + \mathcal{O}(\alpha_S^2) \qquad \text{if } O \text{ is inclusive}$$

- "Original MiNLO" contains ambiguous "$\mathcal{O}(\alpha_S^{1.5})$" terms

---

- Possible to improve VJ−MiNLO such that inclusive NLO is recovered (NLO$^{(0)}$), without spoiling NLO accuracy of V+j (NLO$^{(1)}$).
- accurate control of subleading small-p_T logarithms is needed
  (scaling in low-p_T region is $\alpha_S L^2 \sim 1$, *i.e.* $L \sim 1/\sqrt{\alpha_S}$!)

"Improved" MiNLO & NLOPS merging

- formal accuracy of `VJ-MiNLO` for inclusive observables carefully investigated
[Hamilton et al., 1212.4504]
- `VJ-MiNLO` describes inclusive observables at order α_S
- to reach genuine NLO when fully inclusive (NLO$^{(0)}$), "spurious" terms must be of relative order α_S^2, *i.e.*

$$O_{\mathrm{VJ-MiNLO}} = O_{\mathrm{V@NLO}} + \mathcal{O}(\alpha_S^2) \qquad \text{if } O \text{ is inclusive}$$

- "Original `MiNLO`" contains ambiguous "$\mathcal{O}(\alpha_S^{1.5})$" terms

---

- Possible to improve `VJ-MiNLO` such that inclusive NLO is recovered (NLO$^{(0)}$), without spoiling NLO accuracy of $V+j$ (NLO$^{(1)}$).
- accurate control of subleading small-p_T logarithms is needed
(scaling in low-p_T region is $\alpha_S L^2 \sim 1$, *i.e.* $L \sim 1/\sqrt{\alpha_S}$!)

Effectively as if we merged NLO$^{(0)}$ and NLO$^{(1)}$ samples, without merging different samples (no merging scale used: there is just one sample).

Drell-Yan at NNLO+PS

- VJ−MiNLO+POWHEG generator gives V-VJ @ NLOPS

	V (inclusive)	V+j (inclusive)	V+2j (inclusive)
✓ V-VJ @ NLOPS	NLO	NLO	LO
V @ NNLOPS	NNLO	NLO	LO

Drell-Yan at NNLO+PS

- VJ−MiNLO+POWHEG generator gives V-VJ @ NLOPS

	V (inclusive)	V+j (inclusive)	V+2j (inclusive)
✓ V-VJ @ NLOPS	NLO	NLO	LO
V @ NNLOPS	NNLO	NLO	LO

- reweighting (differential on Φ_B) of "MiNLO-generated" events:

$$W(\Phi_B) = \frac{\left(\frac{d\sigma}{d\Phi_B}\right)_{\mathrm{NNLO}}}{\left(\frac{d\sigma}{d\Phi_B}\right)_{\mathrm{VJ-MiNLO}}}$$

  - by construction NNLO accuracy on fully inclusive observables ($\sigma_{\mathrm{tot}}, y_V, M_V, ...$) [✓]
  - to reach NNLOPS accuracy, need to be sure that the reweighting doesn't spoil the NLO accuracy of VJ−MiNLO in 1-jet region [ ]

Drell-Yan at NNLO+PS

- VJ−MiNLO+POWHEG generator gives V-VJ @ NLOPS

	V (inclusive)	V+j (inclusive)	V+2j (inclusive)
✓ V-VJ @ NLOPS	NLO	NLO	LO
✓ V @ NNLOPS	NNLO	NLO	LO

- reweighting (differential on Φ_B) of "MiNLO-generated" events:

$$W(\Phi_B) = \frac{\left(\frac{d\sigma}{d\Phi_B}\right)_{\rm NNLO}}{\left(\frac{d\sigma}{d\Phi_B}\right)_{\rm VJ-MiNLO}} = \frac{c_0 + c_1\alpha_S + c_2\alpha_S^2}{c_0 + c_1\alpha_S + d_2\alpha_S^2} \simeq 1 + \frac{c_2 - d_2}{c_0}\alpha_S^2 + \mathcal{O}(\alpha_S^3)$$

  - by construction NNLO accuracy on fully inclusive observables ($\sigma_{\rm tot}, y_V, M_V, ...$) [✓]
  - to reach NNLOPS accuracy, need to be sure that the reweighting doesn't spoil the NLO accuracy of VJ−MiNLO in 1-jet region [✓]

Drell-Yan at NNLO+PS

- `VJ-MiNLO+POWHEG` generator gives V-VJ @ NLOPS

	V (inclusive)	V+j (inclusive)	V+2j (inclusive)
✓ V-VJ @ NLOPS	NLO	NLO	LO
✓ V @ NNLOPS	NNLO	NLO	LO

- reweighting (differential on Φ_B) of "`MiNLO`-generated" events:

$$W(\Phi_B) = \frac{\left(\frac{d\sigma}{d\Phi_B}\right)_{\rm NNLO}}{\left(\frac{d\sigma}{d\Phi_B}\right)_{\rm VJ-MiNLO}} = \frac{c_0 + c_1\alpha_S + c_2\alpha_S^2}{c_0 + c_1\alpha_S + d_2\alpha_S^2} \simeq 1 + \frac{c_2 - d_2}{c_0}\alpha_S^2 + \mathcal{O}(\alpha_S^3)$$

  - by construction NNLO accuracy on fully inclusive observables ($\sigma_{\rm tot}, y_V, M_V, ...$) [✓]
  - to reach NNLOPS accuracy, need to be sure that the reweighting doesn't spoil the NLO accuracy of `VJ-MiNLO` in 1-jet region [✓]
  - notice: formally works because no spurious $\mathcal{O}(\alpha_S^{3/2})$ terms in V-VJ @ NLOPS

Drell-Yan at NNLO+PS

- `VJ-MiNLO+POWHEG` generator gives V-VJ @ NLOPS

	V (inclusive)	V+j (inclusive)	V+2j (inclusive)
✓ V-VJ @ NLOPS	NLO	NLO	LO
✓ V @ NNLOPS	NNLO	NLO	LO

- reweighting (differential on Φ_B) of "`MiNLO`-generated" events:

$$W(\Phi_B) = \frac{\left(\frac{d\sigma}{d\Phi_B}\right)_{\rm NNLO}}{\left(\frac{d\sigma}{d\Phi_B}\right)_{\rm VJ-MiNLO}} = \frac{c_0 + c_1\alpha_S + c_2\alpha_S^2}{c_0 + c_1\alpha_S + d_2\alpha_S^2} \simeq 1 + \frac{c_2 - d_2}{c_0}\alpha_S^2 + \mathcal{O}(\alpha_S^3)$$

  - by construction NNLO accuracy on fully inclusive observables $(\sigma_{\rm tot}, y_V, M_V, \ldots)$ [✓]
  - to reach NNLOPS accuracy, need to be sure that the reweighting doesn't spoil the NLO accuracy of `VJ-MiNLO` in 1-jet region [✓]
  - notice: formally works because no spurious $\mathcal{O}(\alpha_S^{3/2})$ terms in V-VJ @ NLOPS
- Variants for reweighting $(W(\Phi_B, p_T))$ are also possible:
  - freedom to distribute "NNLO/NLO K-factor" only over medium-small p_T region

NNLO+PS I

- For Drell-Yan, needs to use variables specifing the Born process $pp \to \ell\bar{\ell}$
  $\hookrightarrow$ also need variable to take into account spin-correlation in vector-boson decay products
- we need a 3-d differential distribution, and there is some freedom in choosing the 3 variables
  $\hookrightarrow$ Useful to make choices such that bins in multidimensional distribution are $\sim$ uniformly populatad
- we have chosen:
  - V-boson rapidity: y_V
  - variable for dilepton invariant mass: $\arctan((m_{\ell\ell}^2 - M_V^2)/(\Gamma_V M_V))$
  - angle between electron and beam in frame where $p_V^z = 0$

NNLO+PS I

- For Drell-Yan, needs to use variables specifing the Born process $pp \to \ell\bar{\ell}$
  $\hookrightarrow$ also need variable to take into account spin-correlation in vector-boson decay products
- we need a 3-d differential distribution, and there is some freedom in choosing the 3 variables
  $\hookrightarrow$ Useful to make choices such that bins in multidimensional distribution are $\sim$ uniformly populatad
- we have chosen:
  - V-boson rapidity: y_V
  - variable for dilepton invariant mass: $\arctan((m_{\ell\ell}^2 - M_V^2)/(\Gamma_V M_V))$
  - angle between electron and beam in frame where $p_V^z = 0$

- Variants for W are possible:

$$W(\Phi_B, p_T) = h(p_T)\frac{\int d\sigma_A^{\mathrm{NNLO}}\delta(\Phi_B - \Phi_B(\mathbf{\Phi}))}{\int d\sigma_A^{\mathrm{MiNLO}}\delta(\Phi_B - \Phi_B(\mathbf{\Phi}))} + (1 - h(p_T))$$

$$d\sigma_A = d\sigma\ h(p_T), \qquad d\sigma_B = d\sigma\ (1 - h(p_T)), \qquad h = \frac{(\beta m_V)^2}{(\beta m_V)^2 + p_T^2}$$

- $h(p_T)$ controls where the NNLO/NLO K-factor is distributed
  (in the high-k_{T} region, there is no improvement in including it)

- β cannot be too small, otherwise resummation spoiled

NNLO+PS II

In 1309.0017, and for DY too, we use

$$W(\Phi_B, p_T) = h(p_T)\frac{\int d\sigma^{\rm NNLO}\delta(\Phi_B - \Phi_B(\mathbf{\Phi})) - \int d\sigma_B^{\rm MiNLO}\delta(\Phi_B - \Phi_B(\mathbf{\Phi}))}{\int d\sigma_A^{\rm MiNLO}\delta(\Phi_B - \Phi_B(\mathbf{\Phi}))} + (1 - h(p_T))$$

$$d\sigma_A = d\sigma\; h(p_T), \qquad d\sigma_B = d\sigma\;(1 - h(p_T)), \qquad h = \frac{(\beta m_V)^2}{(\beta m_V)^2 + p_T^2}$$

- one gets exactly $(d\sigma/d\Phi_B)_{\rm NNLO}$ (no α_S^3 terms)
- we used $h(p_T^{j_1})$, and $\beta = 1$

---

inputs for following plots:

- scale choices: NNLO input with $\mu = m_V$, VJ-MiNLO has its own scale
- PDF: everywhere MSTW2008 NNLO
- NNLO from DYNNLO [Catani,Cieri,Ferrera et al.]
  (3pts scale variation, but 7pts in pure NNLO plots)
- MiNLO: 7pts scale variation (using POWHEG BOX-V2 machinery)
- events reweighted at the LH level: 21-pts scale variation $(7_{\rm Mi} \times 3_{\rm NN})$

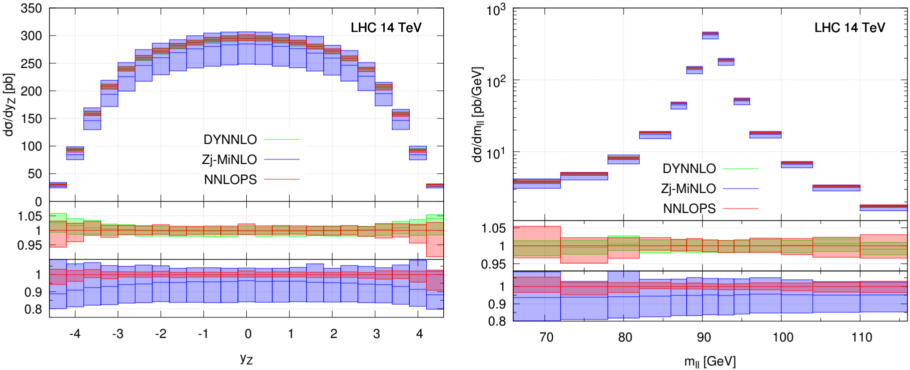
settings for plots shown

inputs for following plots:

- used p_T-dependent reweighting ($W(\Phi_B, p_T)$), smoothly approaching 1 at $p_T \gtrsim m_V$

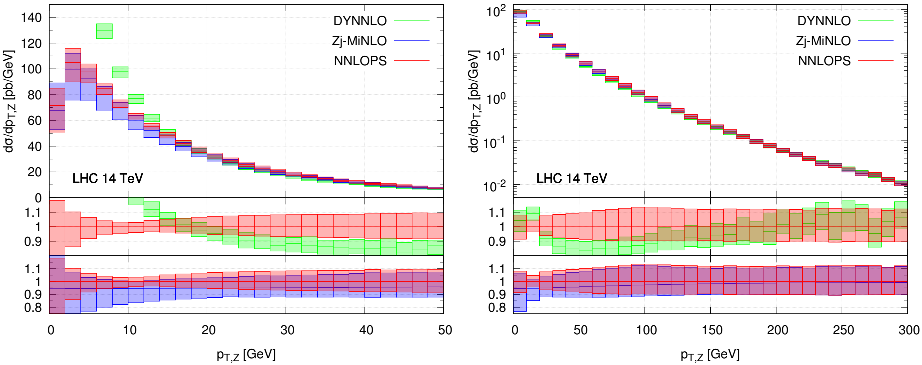
- scale choices: NNLO input with $\mu = m_V$, VJ-MiNLO has its own scale
- PDF: everywhere MSTW2008 NNLO
- NNLO from DYNNLO [Catani,Cieri,Ferrera et al., '09]
  (3pts scale variation, but 7pts in pure NNLO plots)
- MiNLO: 7pts scale variation (using POWHEG BOX-V2 machinery)
- events reweighted at the LH level: 21-pts scale variation $(7_{\mathrm{Mi}} \times 3_{\mathrm{NN}})$
- tunes: Pythia6: "Perugia P12-M8LO" , Pythia8: "Monash 2013"

Z@NNLOPS, PS level



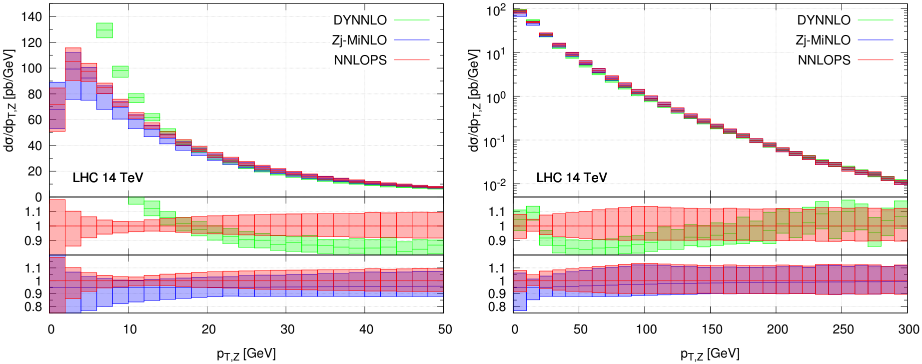
- $(7_{\mathrm{Mi}} \times 3_{\mathrm{NN}})$ pts scale var. in `NNLOPS`, 7pts in NNLO
- agreement with `DYNNLO`
- scale uncertainty reduction wrt `ZJ-MiNLO`

Z@NNLOPS, PS level



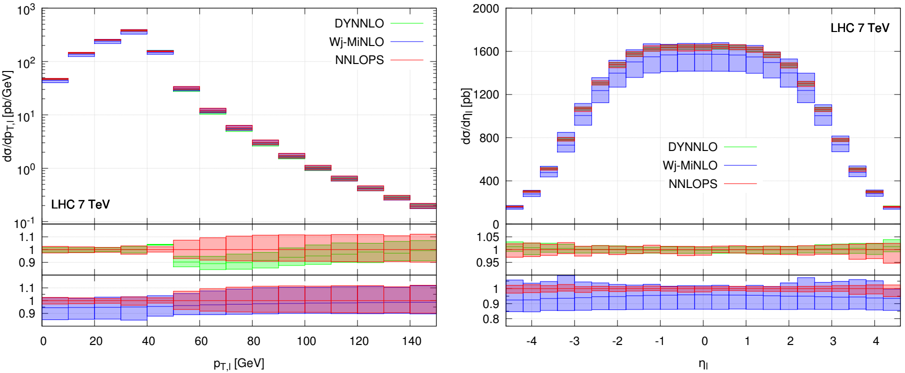
- NNLOPS: smooth behaviour at small k_{T}, where NNLO diverges
- at high p_T, all computations are comparable (band size similar)
- at very high p_T, DYNNLO and ZJ-MiNLO (and hence NNLOPS) use different scales !

Z@NNLOPS, PS level



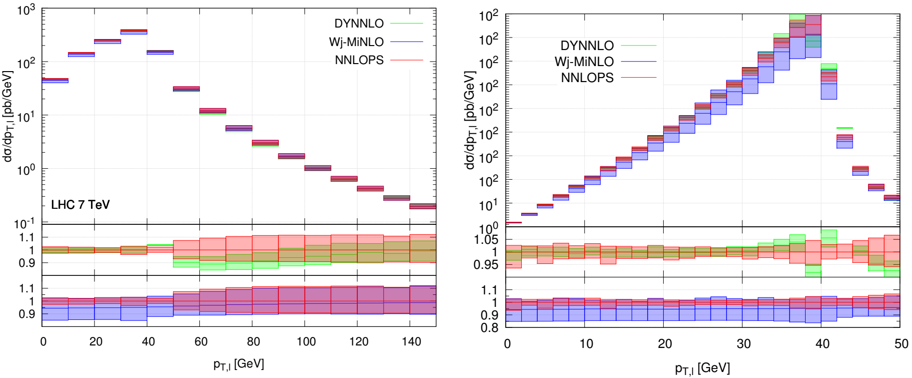
- NNLO envelope shrinks at ~ 10 GeV; NNLOPS inherits it
- notice that in Sudakov region, NNLO rescaling doesn't alter shape from MiNLO

W@NNLOPS, PS level



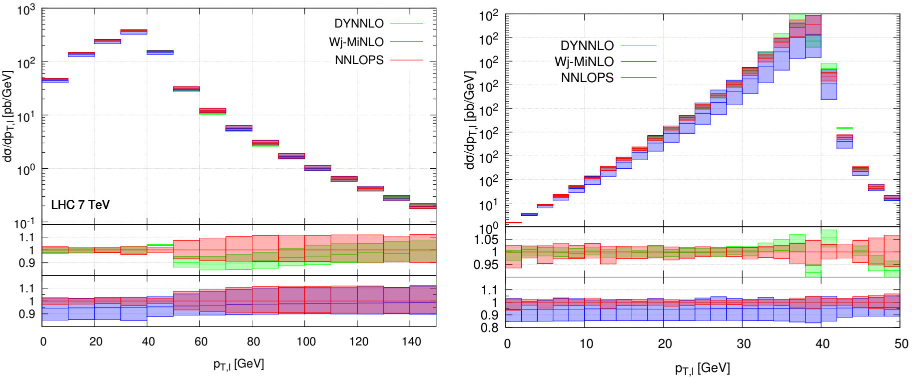
- **not** the observables we are using to do the NNLO reweighting
  - observe exactly what we expect:
    $p_{T,\ell}$ has NNLO uncertainty if $p_T < M_W/2$, NLO if $p_T > M_W/2$
  - η_ℓ is NNLO everywhere

W@NNLOPS, PS level



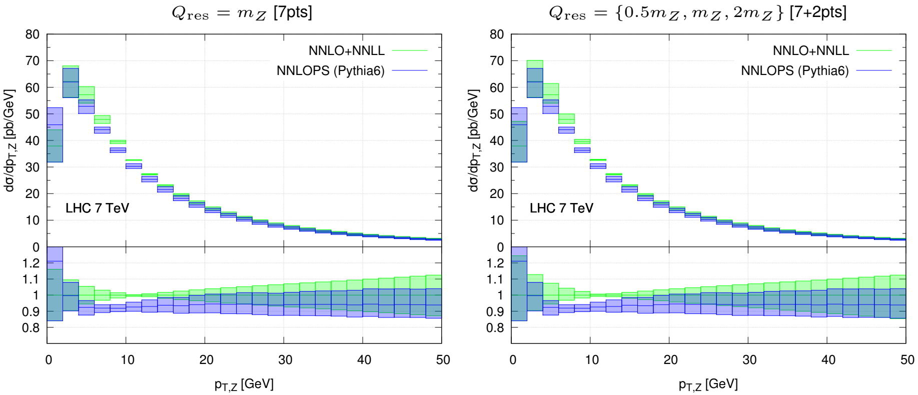
- **not** the observables we are using to do the NNLO reweighting
  - observe exactly what we expect:
    $p_{T,\ell}$ has NNLO uncertainty if $p_T < M_W/2$, NLO if $p_T > M_W/2$
  - smooth behaviour when close to Jacobian peak (also with small bins)
    (due to resummation of logs at small $p_{T,V}$)

W@NNLOPS, PS level



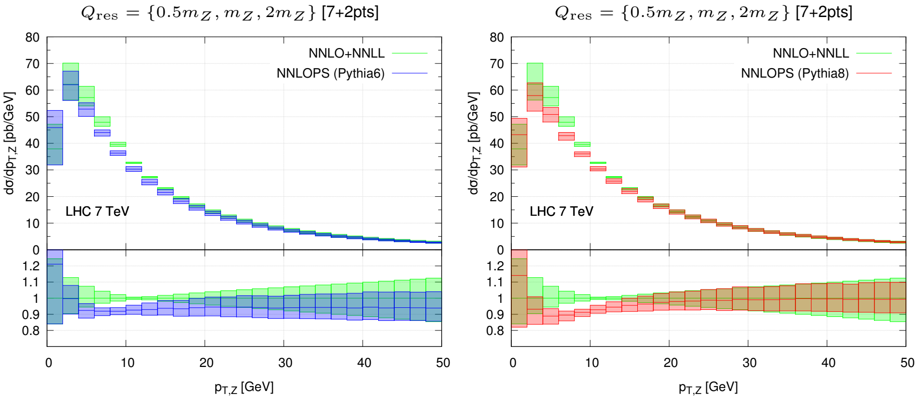
- not the observables we are using to do the NNLO reweighting
  - observe exactly what we expect:
    $p_{T,\ell}$ has NNLO uncertainty if $p_T < M_W/2$, NLO if $p_T > M_W/2$
  - smooth behaviour when close to Jacobian peak (also with small bins)
    (due to resummation of logs at small $p_{T,V}$)
- just above peak, DYNNLO uses $\mu = M_W$, WJ-MiNLO uses $\mu = p_{T,W}$
  - here $0 \lesssim p_{T,W} \lesssim M_W$ (so resummation region does contribute)

Vector boson p_T: resummation



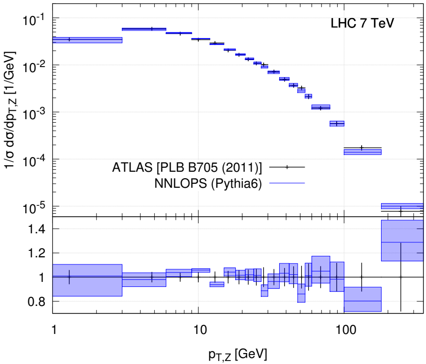
- DyQT: NNLL+NNLO [Bozzi,Catani,Ferrera, et al., '10]
  $\mu_R = \mu_F = m_Z$ [7pts], $Q_{\rm res} = m_Z$ [+ $Q_{\rm res} = 2m_Z, m_Z/2$]
- agreement with resummation good (PS only), but not perfect
  - formal accuracy not the same!
  - shrinking of bands at 10 GeV makes it looking perhaps "worse" than what it is...
  - at 30-50 GeV, bands similar to DyQT

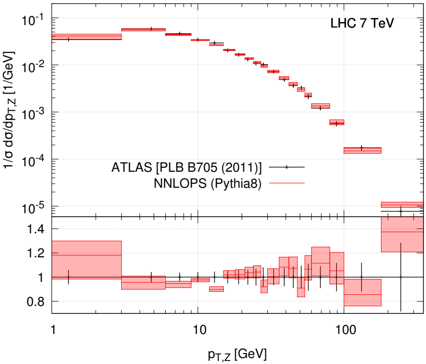
Vector boson p_T: resummation



- similar pattern, although some differences visible between `Pythia6` and `Pythia8`
- NP/tune effects are not negligible

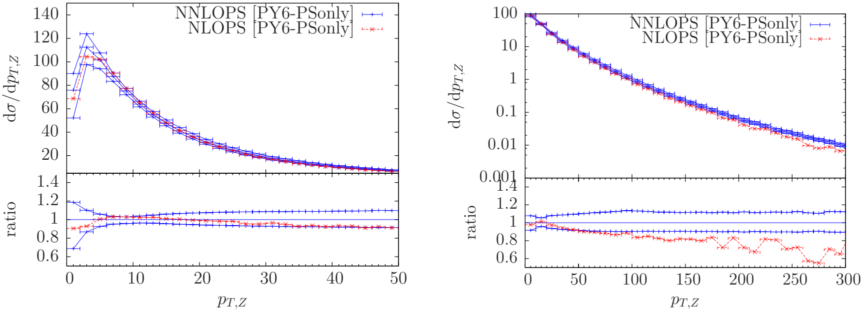
Vector boson: comparison with data ($p_{T,Z}$)





- good agreement with data (PS+hadronisation+MPI)
- band shrinking at ~ 10 GeV
- `Pythia8` is slightly harder at large p_T, and in less good agreement at small p_T
  - part of this can be considered a genuine uncertainty (different shower)
  - specific tune likely to have an impact at small p_T

NNLOPS vs. NLOPS



- different terms in Sudakov, although both contain NLL terms in momentum space
  - in NLOPS: α_S in radiation scheme; in NNLOPS: `MiNLO` Sudakov
- formally they have the same logarithmic accuracy (as supported by above plot)
- at large p_T, difference as expected