GENERATION OF COSMOLOGICAL GRAVITATIONAL WAVES

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INTRODUCTION AND MOTIVATION

- Generation of Gravitational Waves from MHD turbulence (EW phase trans.)
- First order phase transitions can produce helical magnetic fields
- Astrophysical observations suggest possible existence of primordial magnetic fields
- Possible detection of primordial gravitational waves with space detector, e.g. LISA

GRAVITATIONAL WAVE EQUATION

- Small tensor-mode perturbations h_{ij} above FLRW metric leads to the GW equation:

$$\frac{\partial^2 h_{ij}}{\partial t^2} = -3H\frac{\partial h_{ij}}{\partial t} + \frac{c^2}{a^2}\nabla^2 h_{ij} + \frac{16\,\pi G}{c^2} S_{ij}$$

For small time scales, $\tau_T \ll H_*^{-1}$, we can neglect the effects of the expansion of the Universe

- T_{ij}^T is the stress-energy tensor and acts as a source of GWs.
- For a perfect fluid:

$$T_{ij}\,(\boldsymbol{x},t) = (p+\rho)u_i u_j - p\delta_{ij}$$

- For magnetic fields:

$$T_{ij}\,(\boldsymbol{x},t) = -B_i B_j + \frac{1}{2}\delta_{ij}B^2$$

2 D.O.F are left for a spatial, symmetric, traceless and transverse tensor:

$$h^+, h^\times, T^T, T^\times$$

- Transverse and traceless gauge GWs are given by

$$\tilde{S}_{ij} = \left(P_{il}P_{jm} - \frac{1}{2}P_{ij}P_{lm}\right)\tilde{T}_{ij}$$

where $P_{ij} = \delta_{ij} - \hat{k}_i\hat{k}_j$ is the projection operator

SPECTRAL DEFINITIONS

- Two spectra functions can be define for Gaussian stochastic tensor fields: $S(k), A(k)$
  - Symmetric spectrum $S(k)$
  - Antisymmetric/helical spectrum $\mathrm{A}(k)$
- Gravitational waves energy density:

$$\mathcal{E}_{GW} = \frac{\mathrm{c}^2}{32\pi \mathrm{G}} \int_0^\infty S_{\dot{h}}\,(k) dk, \quad E_{GW} = \frac{c^2}{32\pi G} S_{\dot{h}}(k)$$

  - Degree of polarization:

$$\mathcal{P} = \frac{\mathcal{H}_{GW}}{\mathcal{E}_{GW}} \in [-1, 1]$$

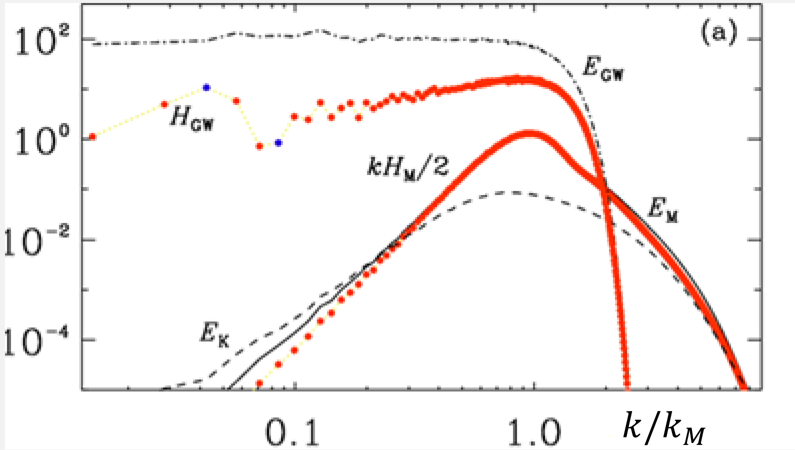
- Antisymmetric contribution:

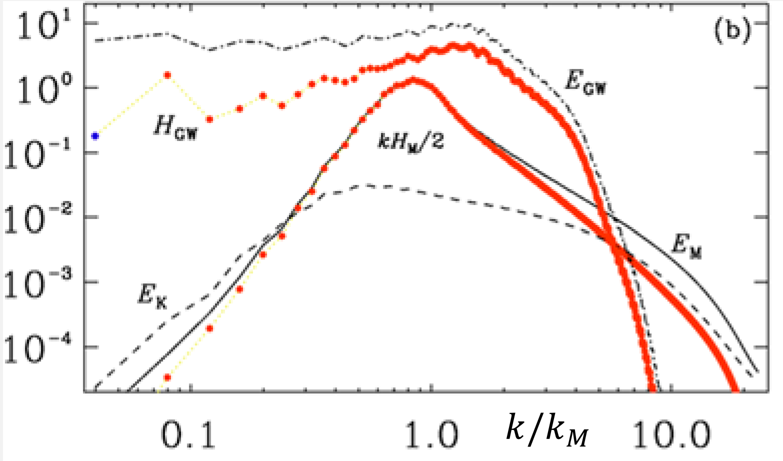
$$\mathcal{H}_{GW} = \frac{c^2}{32\pi \mathrm{G}} \int_0^\infty A_{\dot{h}}\,(k) dk, \quad H_{GW} = \frac{c^2}{32\pi \mathrm{G}} A_{\dot{h}}$$

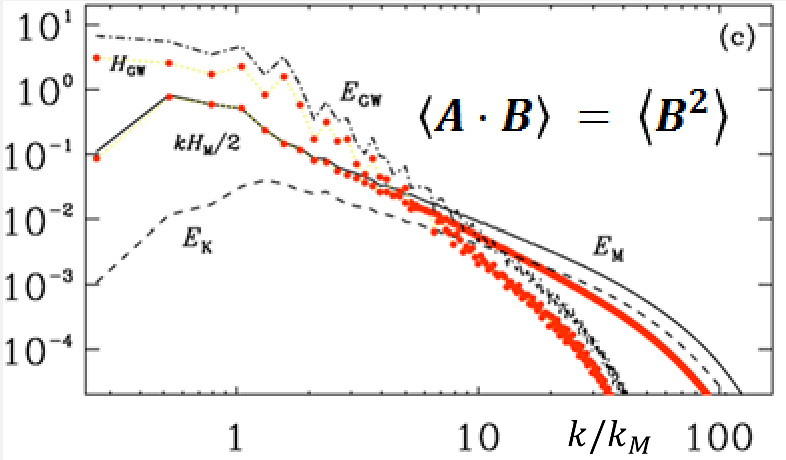
- Root mean square value

$$h_{rms}^2 = \int_0^\infty \hat{h}_{rms}^2 \; d\,(\ln k), \quad \hat{h}_{rms}(k) = \sqrt{k S_h(k)}$$

Decaying turbulence with initial stochastic and fully helical magnetic field







$k_M/k_H = 300$

$k_M/k_H = 60$

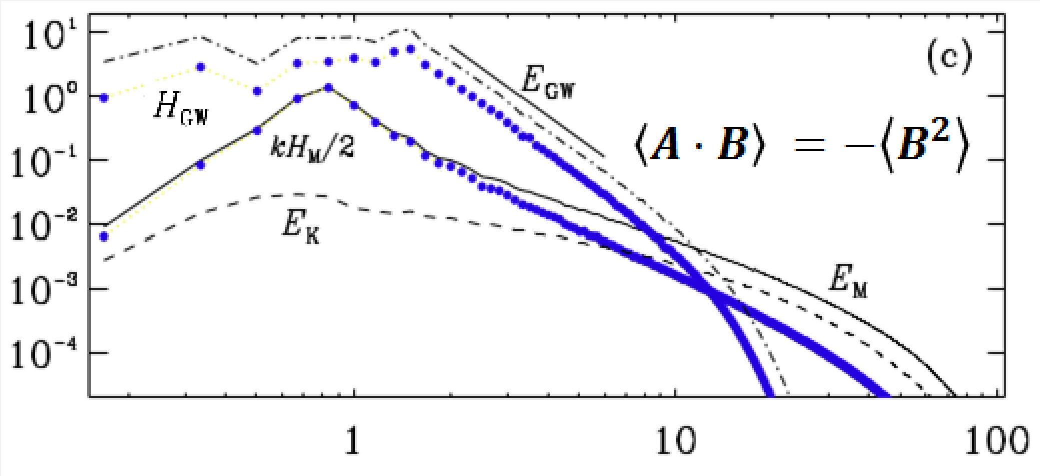
$k_M/k_H = 2$

PRELIMINARY RESULTS NOT PUBLISHED

k_M is the peak of the initial MF

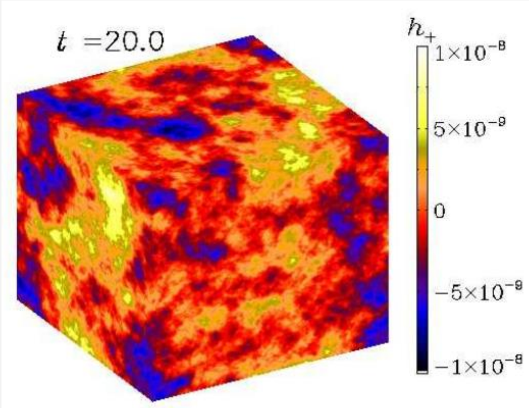
$k_H = c/H$ is the Hubble scale

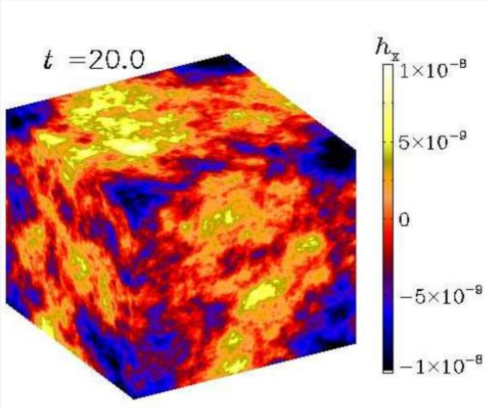
- Numerical simulations using the PENCIL CODE
- Spectra are normalized with their initial mean values
- Grid points: 1152x1152x1152



- Red dots indicate positive polarization/ helicity
- Blue dots indicate positive polarization/ helicity

$k_M/k_H = 6$

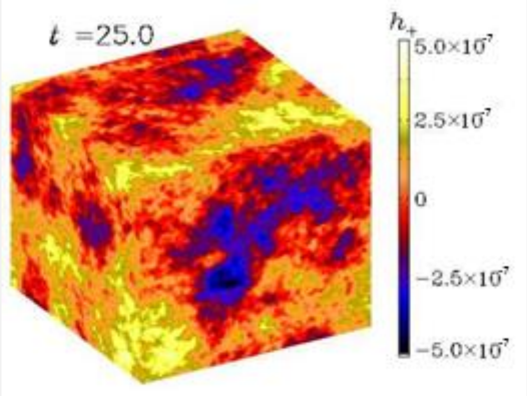


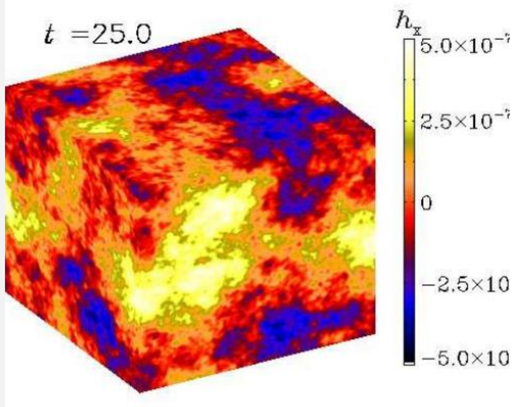


$k_M/k_H = 300$

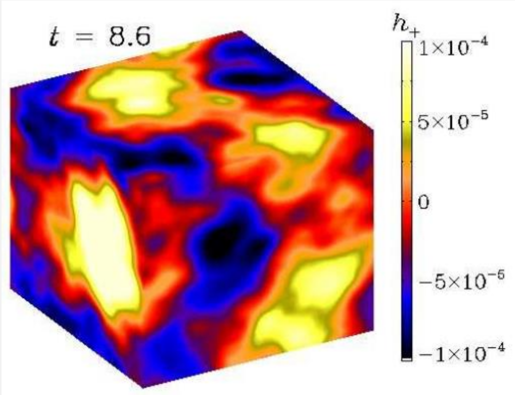
PRELIMINARY RESULTS NOT PUBLISHED

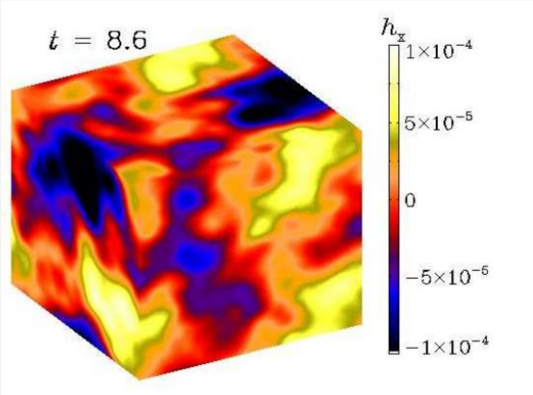
- 300 → 2: field becomes smoother
- $h^+, h^\times$ appear phase-shifted
- Indicates helicity





$k_M/k_H = 60$





$k_M/k_H = 2$

ILLUSTRATION; 1D HELICAL FIELD

- 1D Beltrami magnetic field: $\boldsymbol{B}(z) = B_0(\sin k_0 z, \cos k_0 z)$, i.e., $\boldsymbol{B}(z) \cdot \left(\nabla \times \boldsymbol{B}(z)\right) = \mathrm{sgn}(k_0)$
- TT stress-energy tensor is

$$T_{ij} = B_i B_j - \frac{1}{2}\delta_{ij}B^2 = \frac{1}{2}B_0^2 \begin{pmatrix} -\cos 2k_0 z & \sin 2k_0 z & 0 \\ \sin 2k_0 z & \cos 2k_0 z & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- Solution to the GW equation is

$$h^+ = \frac{2\pi G}{c^4 k_0^2} B_0^2 \cos 2k_{0x} \,(1 \,- \cos 2k_0 c\, t), \qquad h^\times = \frac{2\pi G}{c^4 k_0^2} B_0^2 \sin 2k_0 x \,(1 \,- \cos 2k_0 c\, t)$$

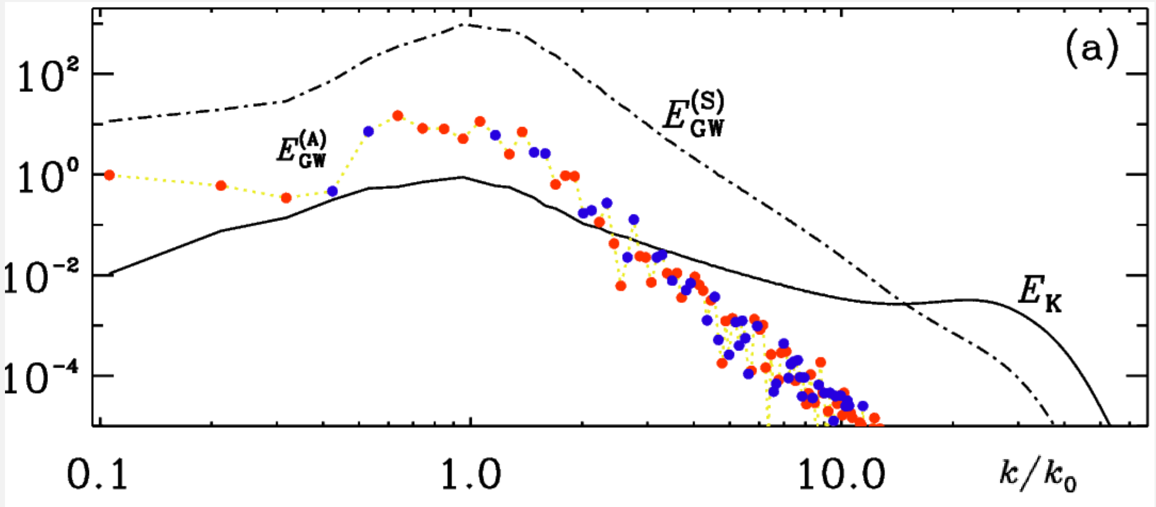
$$\tilde{\mathcal{E}}_{GW} = \frac{\pi G}{c^4 k_0^2} \mathcal{E}_M^2 \left(\delta(k + 2k_0) + \delta(k - 2k_0)\right), \qquad \widetilde{\mathcal{H}}_{GW} = \frac{\pi G}{c^4 k_0^2} \mathcal{E}_M^2 (\delta(k - 2k_0) - \delta(k + 2k_0))$$

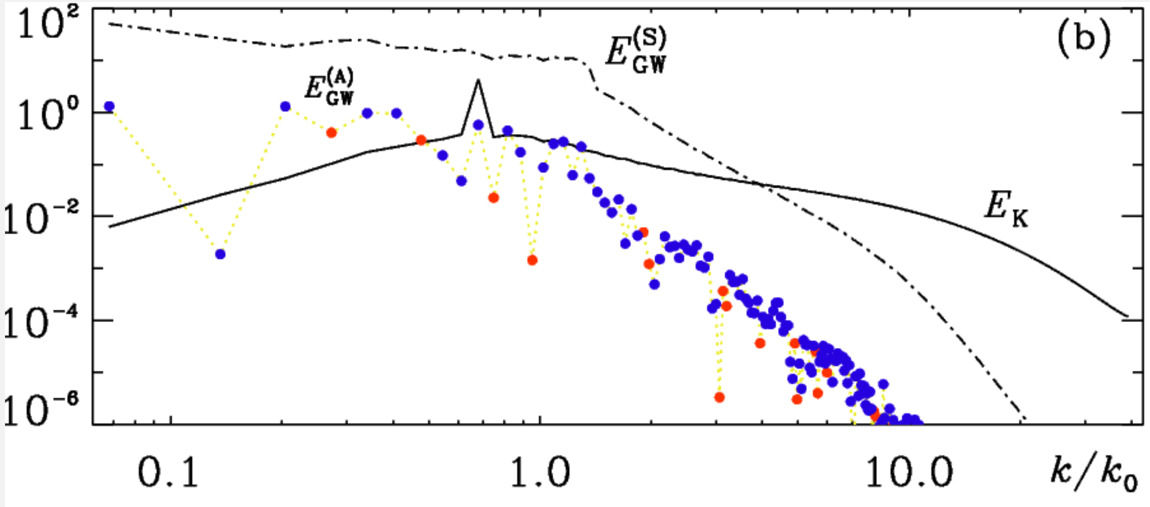
$$\mathcal{P}_{GW} = \frac{\widetilde{\mathcal{H}}_{GW}}{\tilde{\mathcal{E}}_{GW}} = \mathrm{sgn}(\mathrm{k}_0)$$

RESULTS FOR FORCED HYDRODYNAMIC TURBULENCE

- Acoustic turbulence
- $\nabla \cdot \boldsymbol{u} \ll \sqrt{k^2 \boldsymbol{u}^2}$

- Rotational turbulence
- $\nabla \times \boldsymbol{u} \ll \sqrt{k^2 \boldsymbol{u}^2}$

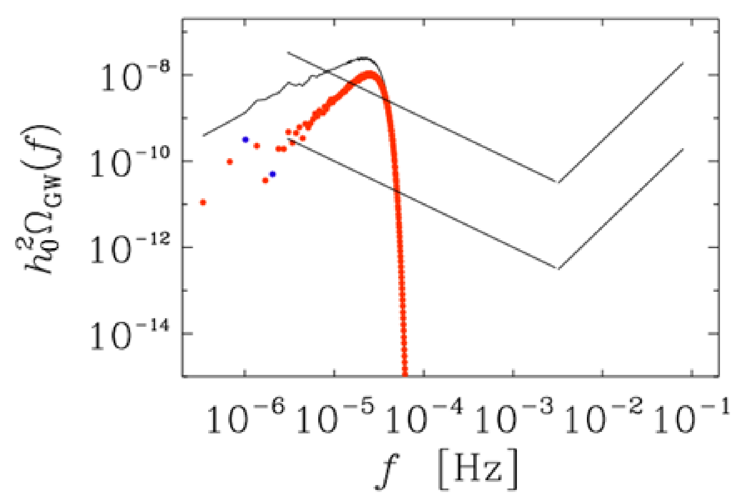




- Blue dots indicate negative polarization
- Red dots indicate positive polarization

PRELIMINARY RESULTS NOT PUBLISHED

Decaying turbulence with initial stochastic and fully helical magnetic field PRELIMINARY RESULTS NOT PUBLISHED



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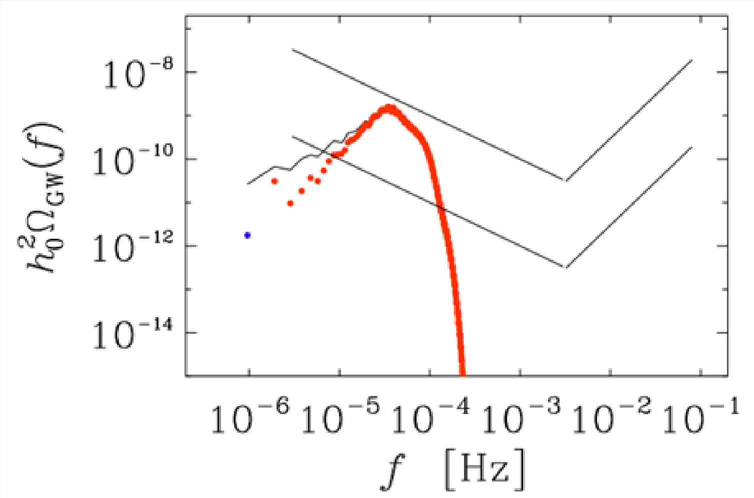
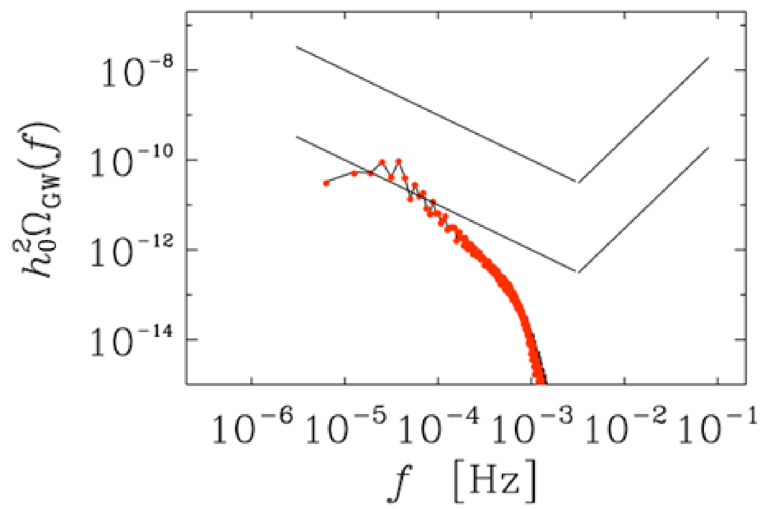


$k_M/k_H = 2$

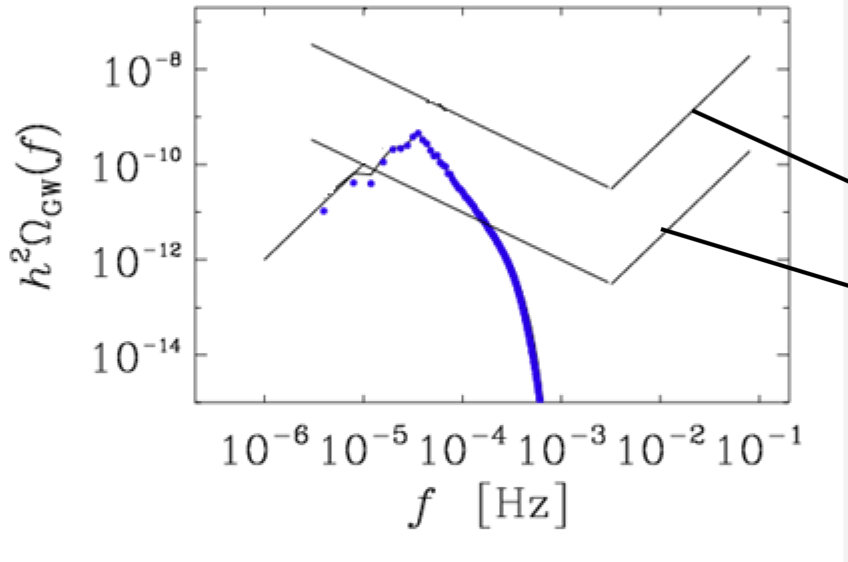


$k_M/k_H = 60$

$$\Omega_{GW} = \frac{\mathcal{E}_{GW}}{\mathcal{E}_{crit}}$$

$$\mathcal{E}_{crit} = \frac{3H_0^2c^2}{8\pi G}$$

$$H_0 = 100\ \mathrm{h}_0\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$$



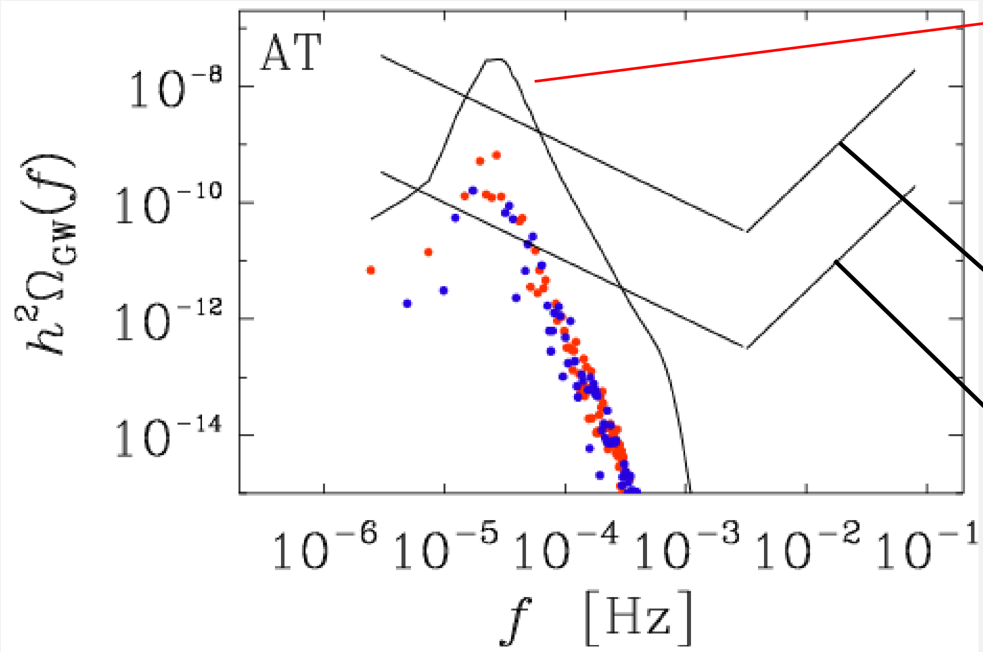
LISA sensitivity range for considered configurations in Caprini *et. al*, JCAP 1604 (2016) after 5 years of observations

$k_M/k_H = 6$

PRELIMINARY RESULTS NOT PUBLISHED

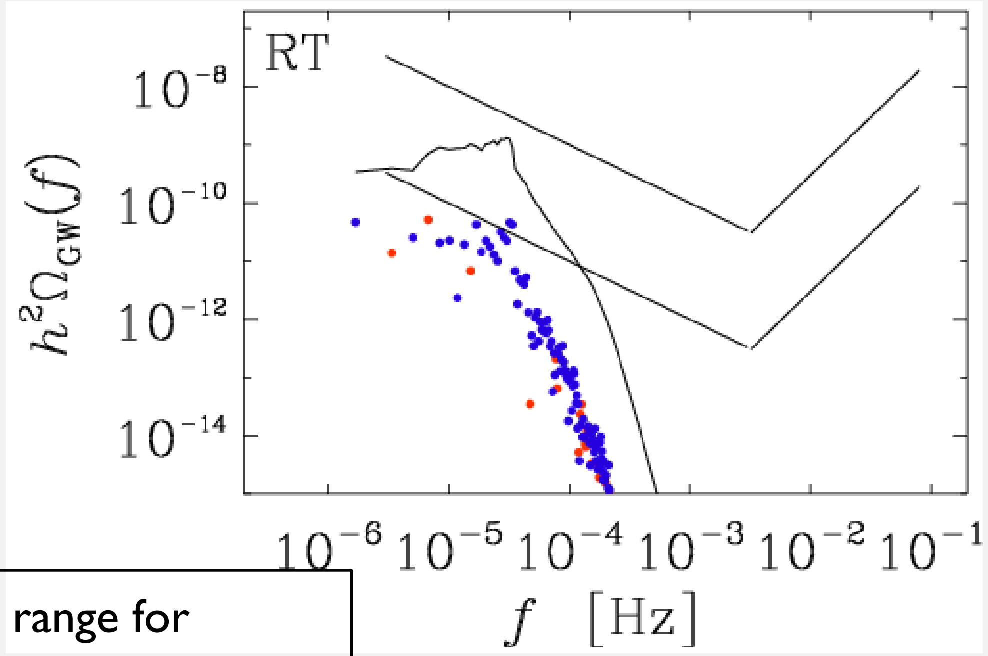
DETECTABILITY OF FORCED HYDRODYNAMIC TURBULENCE WITH LISA

- Acoustic turbulence



STRONGER SIGNAL!

- Rotational turbulence



LISA sensitivity range for considered configurations in Caprini *et. al*, JCAP 1604 (2016) after 5 years of observations

- Blue dots indicate negative polarization
- Red dots indicate positive polarization

CONCLUSIONS

- GW module running in PENCIL CODE: realistic MHD turbulence
- Direct relation between the helicity fraction of B field and polarization degree of GW

  - in agreement with [1]
- Stronger GW energy for acoustic than for vortical turbulence

  - in agreement with [2].
- GW from MHD turbulence above sensitivity limit of LISA

[1] Kahniashvhili, T., Gogoberidze, G., and Ratra, B., Phys. Rev. Lett. 95 (2005)
[2] Hindmarsh, M., *et al.*, Phys. Rev. Lett. 112 (2014)

THANK YOU!! QUESTIONS?