Finding an EFT for Nearly Conformal Gauge Theories

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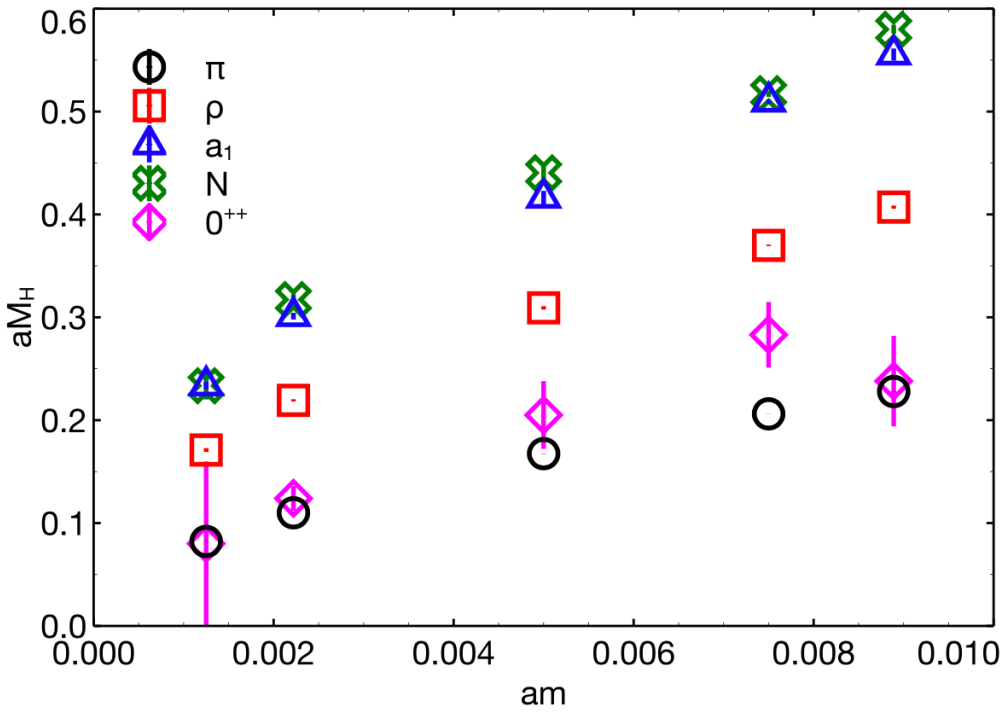
Outline

- Introduction: Fitting data from numerical lattice calculations in nearly conformal gauge theories using EFTs.
- Motivation: Finding UV completion of EW sector
- Dilaton EFT - in collaboration with Thomas Appelquist and Maurizio Piai: 1711.00067
- Linear Sigma EFT – with the LSD collaboration: In preparation.

Our Program

Lattice Simulation Data for $N_c = 3$, $N_f = 8$



1601.04027

$SU_L(N_f) \times SU_R(N_f)$ global symmetry, spontaneously broken to $SU_V(N_f)$ when m=0.

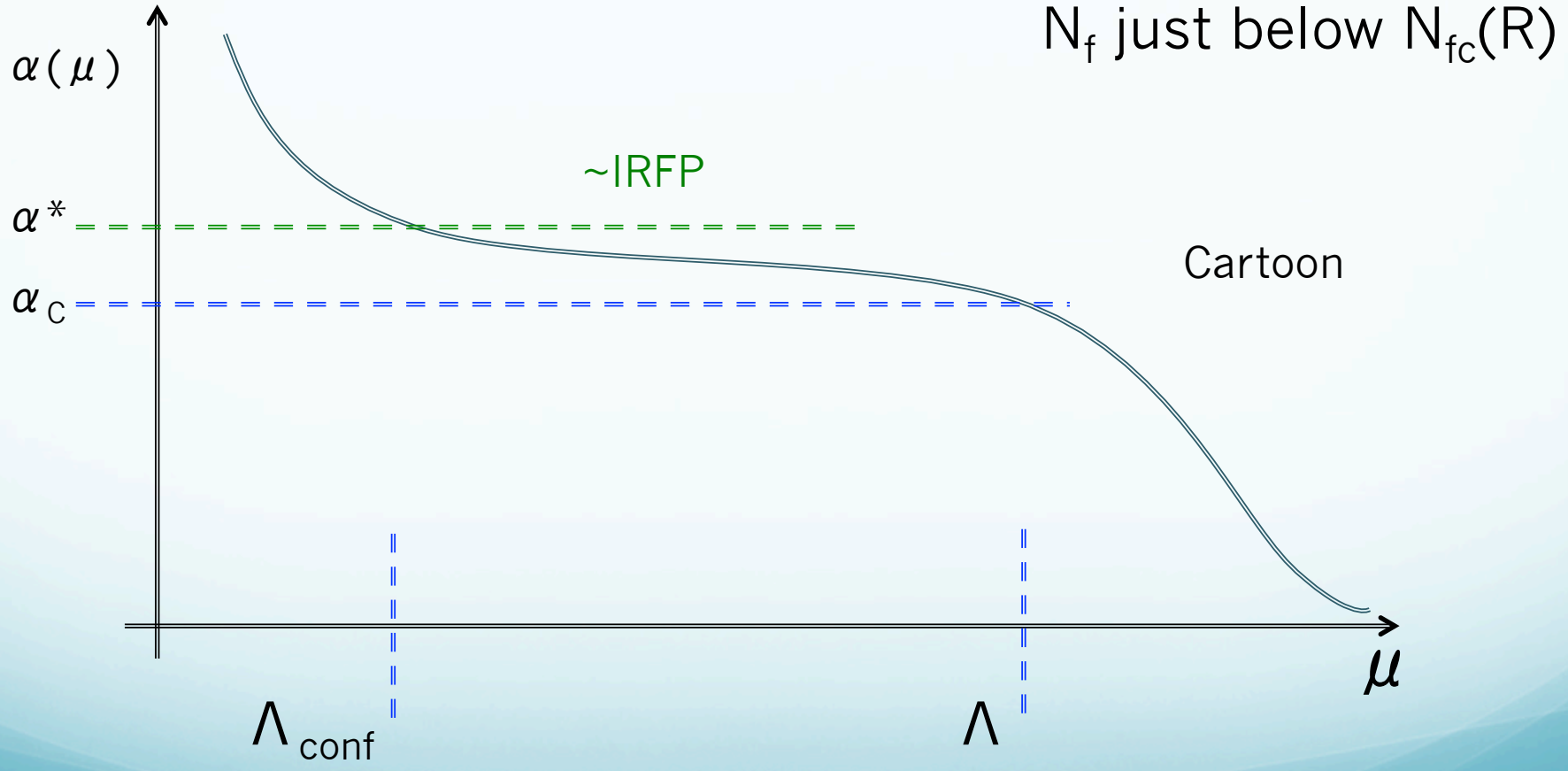
$\mathcal{L} = ?$

$M_{0++} << M_\rho \sim \Lambda_{conf}$?

- Assume light scalar has a weakly coupled EFT description
- Maybe the Higgs boson – connection with EWSB?
- Which properties do scalars from different walking (near conformal) theories share?

Walking Gauge Theories

- Could support a dilaton
- Test by fitting a EFTs to lattice data



The Dilaton EFT

$$L_\pi = \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 Tr\left\{\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right\}$$

$$\Sigma \to L\Sigma R^\dagger \qquad \Sigma = \exp\left\{2i\pi^a T^a / F_\pi\right\}$$

$$L_d = \frac{1}{2}\left(\partial_\mu \chi\right)^2$$

$$\left\langle \chi \right\rangle_{VEV} = F_d$$

$$L_V = -V(\chi)$$

- Potential has a minimum at $\chi = f_d$, and curvature m_d^2
- Let the lattice data determine the form of the potential.

$$L_m = \frac{m_\pi^2 f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y Tr\left\{\Sigma + \Sigma^\dagger\right\}$$

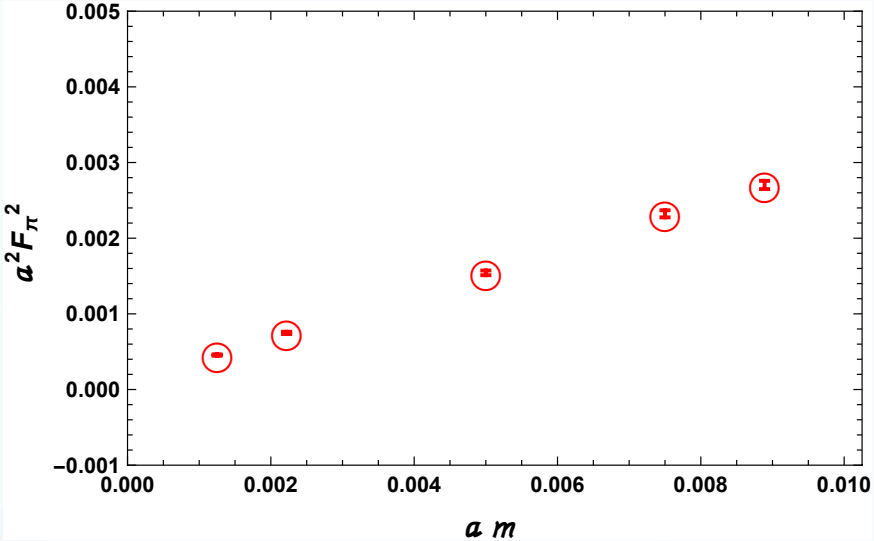
$$m_\pi^2 = 2mB_\pi = \frac{m\left\langle \bar{\psi}\psi \right\rangle}{f_\pi^2}$$

- Quark mass term breaks both scale and chiral symmetry.
- y ~ Scaling dimension of $\bar{\psi}\psi$?
- 5 EFT parameters: f_π, f_d, B_π, m_d, y
- Similar terms have appeared before eg 1603.04575

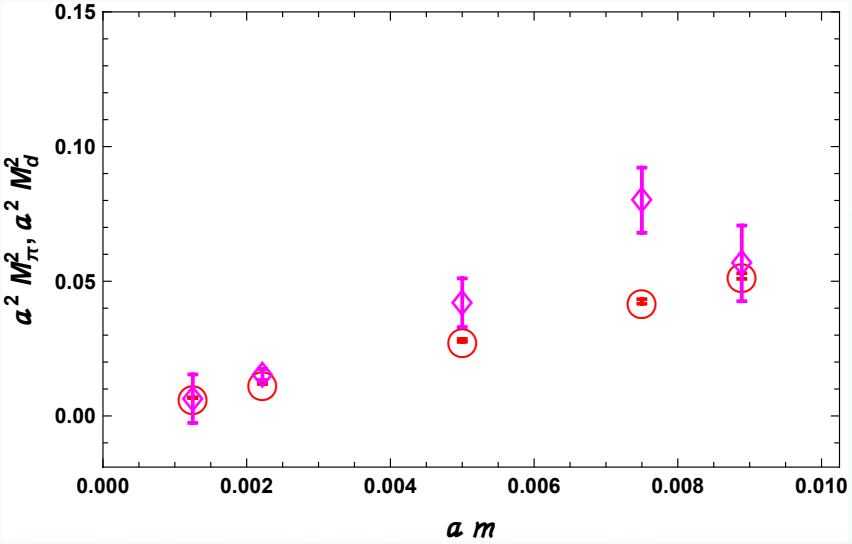
Comparison with LSD Data

$N_c = 3$, $N_f = 8$

F_π squared vs m



NGB and scalar mass squared vs m.



Replotted data from 1601.04027

Lattice data for F_π and M_π only can determine y accurately.

2%

$$M_\pi^2 F_\pi^{2-y} = Cm$$

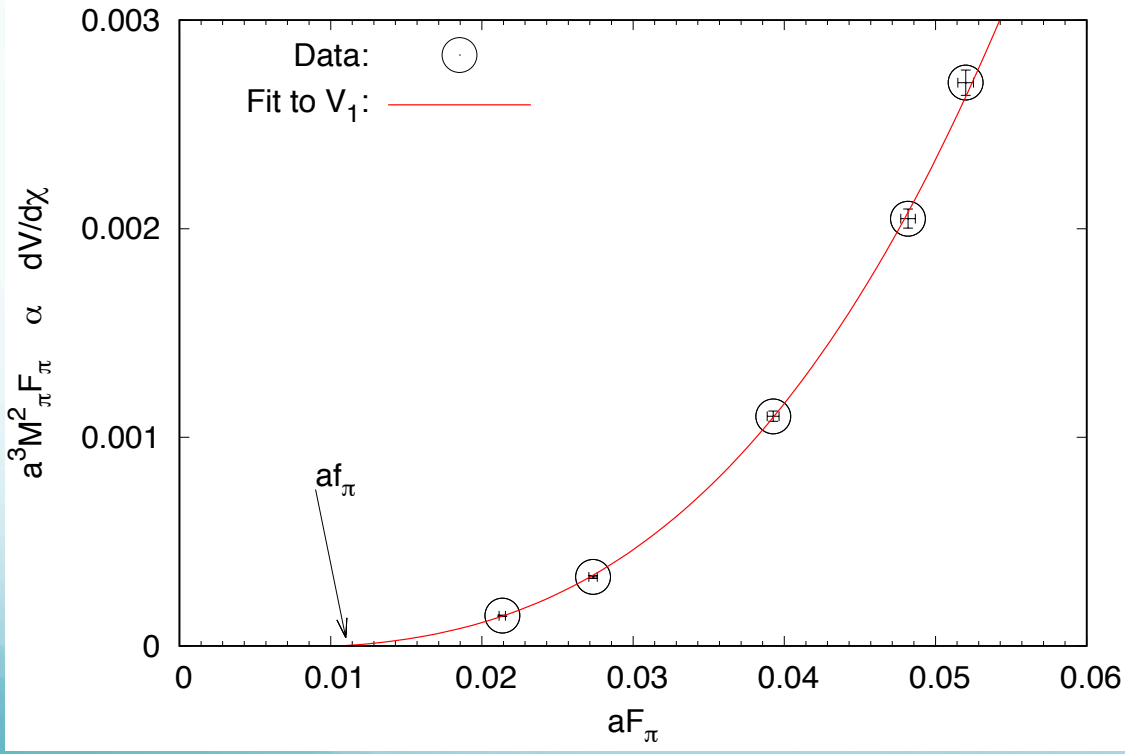
$$y = 2.1 \pm 0.1$$

Determining the form of the potential:

"Inverse scattering problem"

$$\left.\frac{\partial V}{\partial \chi}\right|_{F_d} = \frac{y N_f f_\pi}{f_d} M_\pi^2 F_\pi$$

$\partial V/\partial\chi$ vs χ



$$F_\pi \propto F_d$$

- Both V_1 and V_2 fit the existing data well.
- Need more data especially at smaller m.

$$V_1(\chi) = \frac{m_d^2}{8 f_d^2}\left(\chi^2 - f_d^2\right)^2$$

$$V_2(\chi) = \frac{m_d^2}{16 f_d^2}\chi^4\left(4\ln\frac{\chi}{f_d} - 1\right)$$

Large χ Behavior Of Potential

Use ansatz valid when $F_d >> f_d$

$$V(\chi) = \kappa \chi^p$$

$$M_\pi^2 = B F_\pi^{p-2}$$

Fit to NGB data for $N_f=8$ only:

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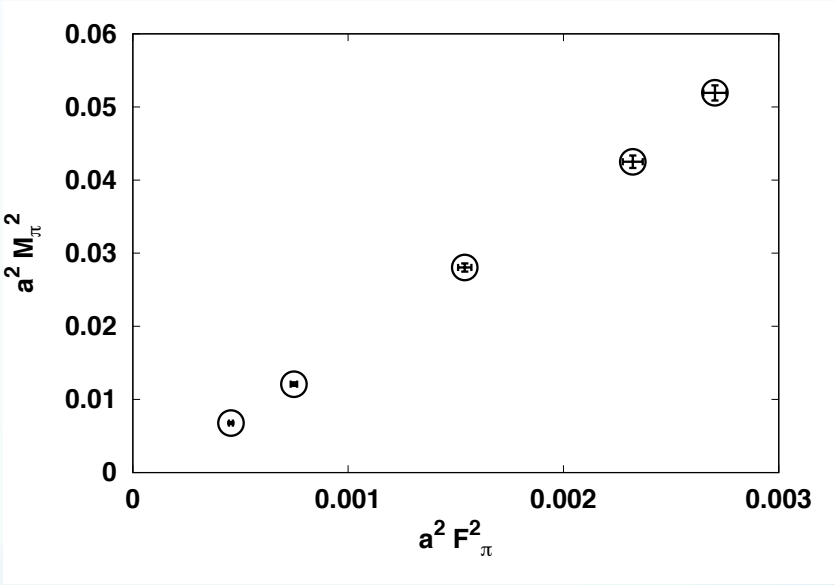
$$p = 4.3 \pm 0.2$$

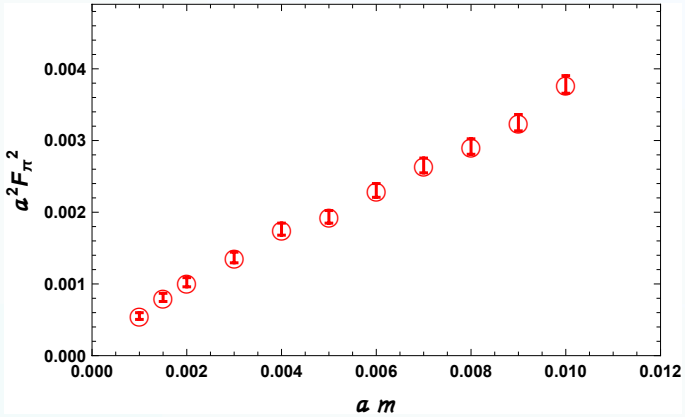


- Ansatz is a good fit.
- Potential grows roughly like χ^4 at large χ.
- A pure χ^4 potential does not break scale symmetry.
- Perhaps some contamination from points near the minimum of the potential? Redo fit without lowest mass points.

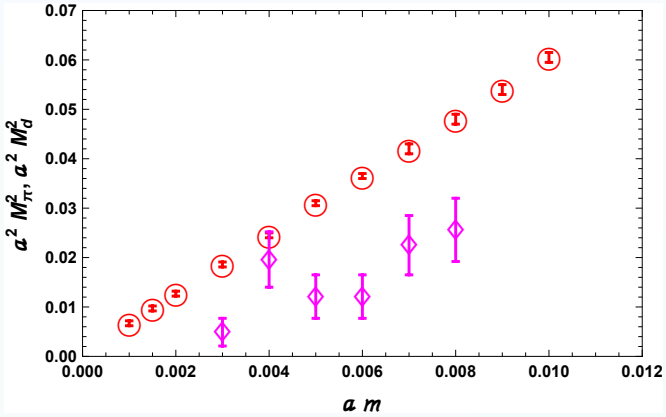
Comparison with LatHC Data

$N_c=3$, $N_f=2$ (sextet)

F_π squared vs m



NGB and scalar mass squared vs m.



Data from 1209.0391, 1502.00028, 1605.08750

$$M_\pi^2 F_\pi^{2-y} = Cm$$

- y = 1.9 ± 0.1 – consistent with LatHC 1712.08594
- p = 4.4 ± 0.3

$$M_\pi^2 = BF_\pi^{p-2}$$

- Very similar values to the $N_f=8$ theory.

The Linear Sigma EFT

Motivation

- Our goal is to fit data to an $SU(N_f)xSU(N_f)$ linear sigma EFT.
- SM Electroweak sector is a gauged SU(2)xSU(2) linear sigma model.
- SMEFT is a linear sigma EFT.
- By testing whether a linear sigma EFT fits lattice data for near conformal gauge theories, you test whether the light states of the gauge theory are SM like.
- The lattice indicates that flavored scalars (analogs of a_0s in QCD) are present in the spectrum which may become light in the chiral limit – 1605.08750. These states appear in the linear sigma model. They are essential for $N_f>2$.

The Model Itself

$SU_L(N_f)xSU_R(N_f)$ global symmetry, spontaneously broken to $SU_V(N_f)$

$$M_a^{\bar{b}} \rightarrow L_a^c M_c^{\bar{d}} R_{\bar{d}}^{\bar{b}}$$

$$\langle M \rangle_{VEV} = \frac{F}{\sqrt{N_f}} \mathrm{I}$$

$$M(x) = \frac{\sigma(x) + i\eta'(x)}{\sqrt{N_f}} + \left(a_j + i\pi_j\right) T_j$$

$2N_f^2$ real degrees of freedom total. Transform linearly under $SU_L(N_f)xSU_R(N_f)$.

$$L_{LO} = \tfrac{1}{2} Tr\left[\partial_\mu M \partial^\mu M^\dagger\right] - V_0\left(M\right) - V_{SB}\left(M\right)$$

Schechter '71

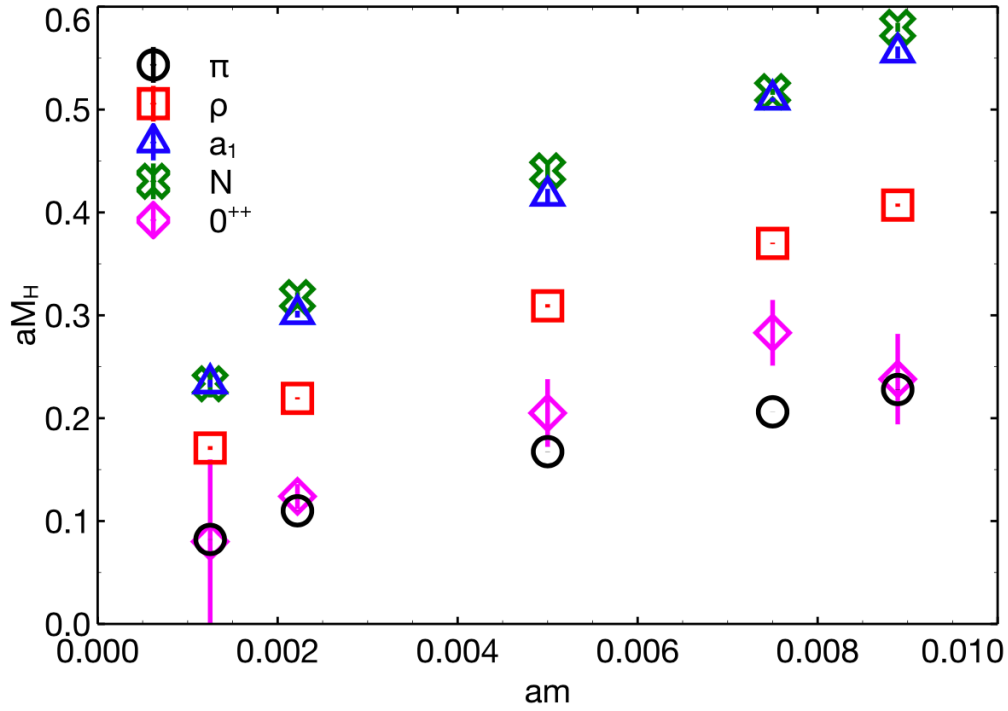
$$V_0\left(M\right) = -\tfrac{m_\sigma^2}{4} Tr\left[M^\dagger M\right] + \tfrac{m_\sigma^2 - m_a^2}{8f^2}\left(Tr\left[M^\dagger M\right]\right)^2 + \tfrac{N_f m_a^2}{8f^2} Tr\left[M^\dagger M M^\dagger M\right]$$

There will also be irrelevant ops in L beyond leading order in EFT expansion.

An Inequality

Using traditional breaking term: $V_{SB}\left(M\right) = -c_1 m\, Tr\left[M + M^{\dagger}\right]$

$N_c=3$, $N_f=8$



Analysis of the tree level theory leads to:

$$M_\sigma^2 \geq 3M_\pi^2$$

1601.04027

Solution

- Is this the only breaking term possible?
- Treating quark mass as a spurion, write down the most general breaking term consistent with power counting and symmetries.

Define spurion: $\chi \equiv Am\,\mathrm{I}$

Choose a power counting, applicable when m is somewhat large:

$$\frac{\partial}{\Lambda} \sim \frac{M}{\Lambda} \sim \frac{\chi}{\Lambda}$$

We end up with a more general leading order breaking term, for which there is no inequality!

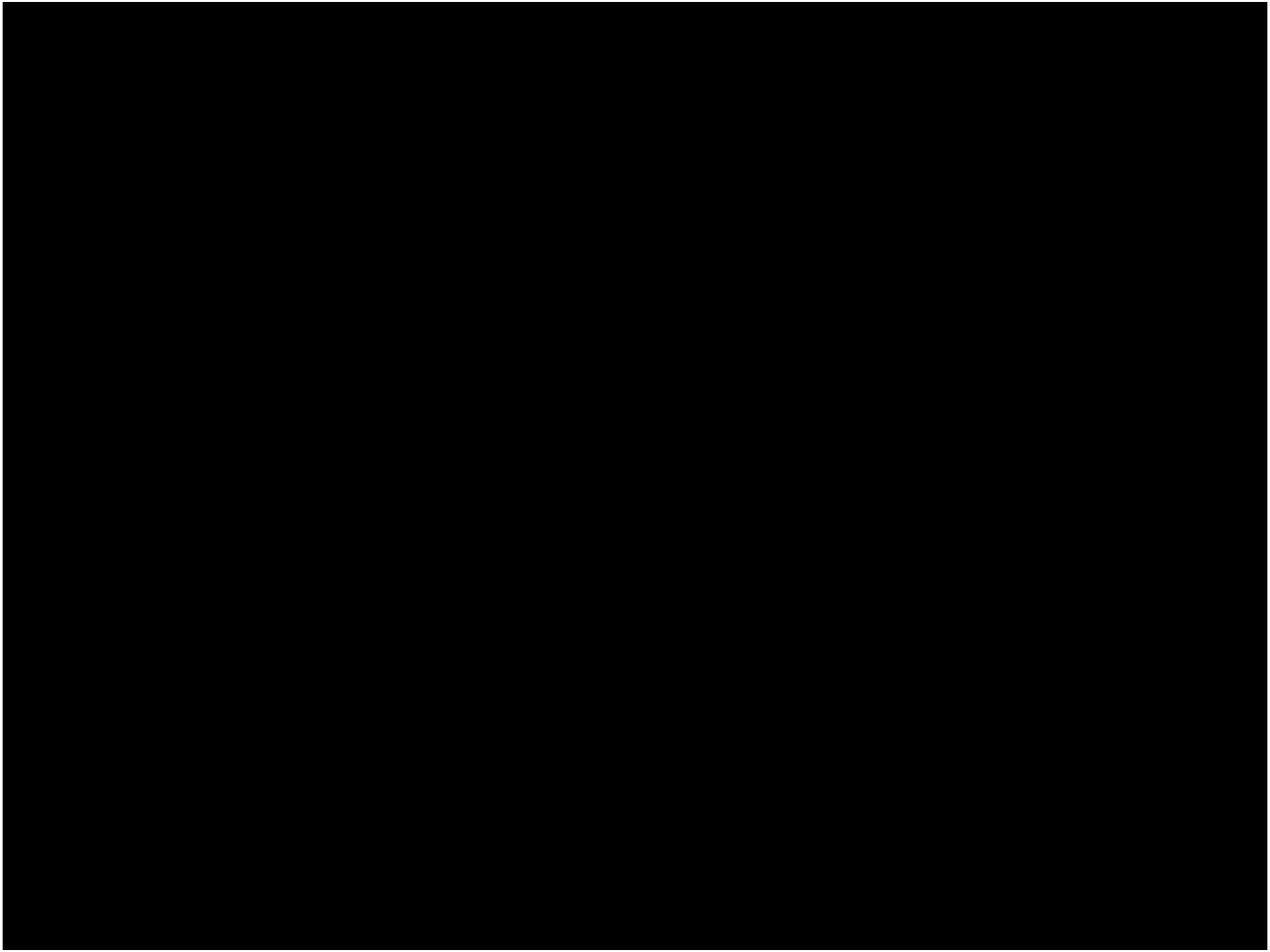
$$V_{SB} = -\sum_{i=1}^{9} c_i O_i$$

Symbol	Operator
O_1	$\langle \chi^\dagger M + M^\dagger \chi \rangle$
O_2	$\langle M^\dagger M \rangle \langle \chi^\dagger M + M^\dagger \chi \rangle$
O_3	$\langle (M^\dagger M)(\chi^\dagger M + M^\dagger \chi) \rangle$
O_4	$\langle \chi^\dagger M + M^\dagger \chi \rangle^2$
O_5	$\langle \chi^\dagger \chi M^\dagger M \rangle$
O_6	$\langle \chi^\dagger \chi \rangle \langle M^\dagger M \rangle$
O_7	$\langle \chi^\dagger M \chi^\dagger M + M^\dagger \chi M^\dagger \chi \rangle$
O_8	$\langle \chi^\dagger \chi \rangle \langle \chi^\dagger M + M^\dagger \chi \rangle$
O_9	$\langle (\chi^\dagger \chi)(\chi^\dagger M + M^\dagger \chi) \rangle$

Angle brackets denote traces here.

Summary

- We've developed EFT frameworks to test whether the light scalar in walking theories is a dilaton, or whether it transforms along with the pions in a linear representation of the chiral symmetry, without needing negligible quark mass.
- Dilaton EFT fits lattice data for two different gauge theories well.
- $y \sim 2$ for both.
- $p \gtrsim 4$ for both.
- The linear sigma EFT could potentially fit the data well, if the correct power counting is chosen.



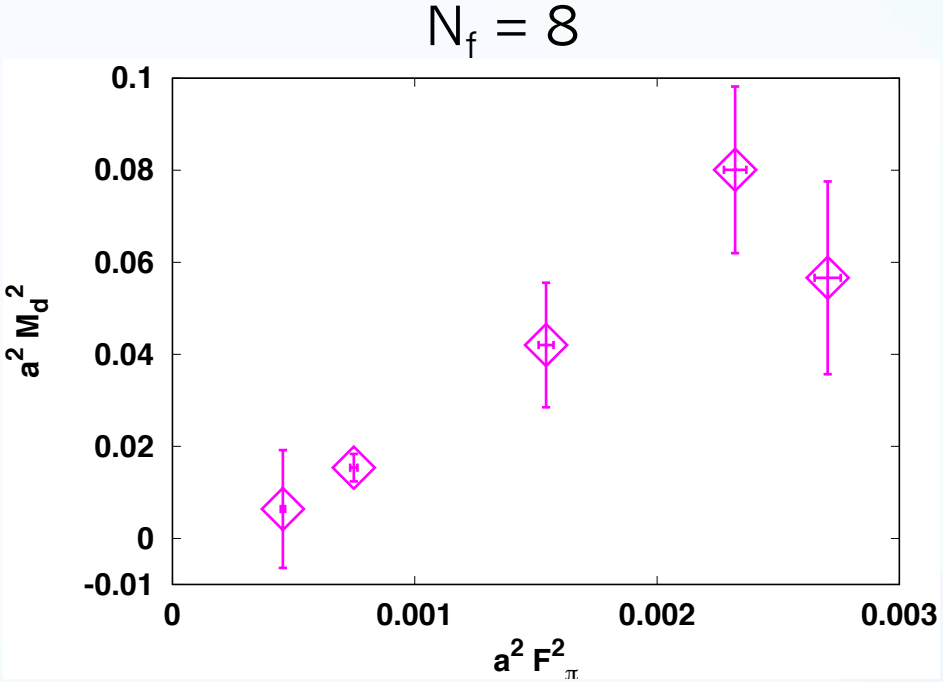
Bounds On Model

- Lattice calculations suggest that S parameter is suppressed relative to QCD - see 1602.00796, 1405.4752, 1009.5967.
- Direct detection of heavy spin 1 states challenging due to their large width.
- Coupling of SM fermions to these walking models is model dependent.

Including Scalar Mass Data

$$M_d^2 = \left.\frac{\partial^2 V}{\partial \chi^2}\right|_{F_d} - \frac{y(y-1)N_f f_\pi^2}{2f_d^2} M_\pi^2$$

- Errors too large to add much precision to determination of potential.
- Use large χ ansatz.



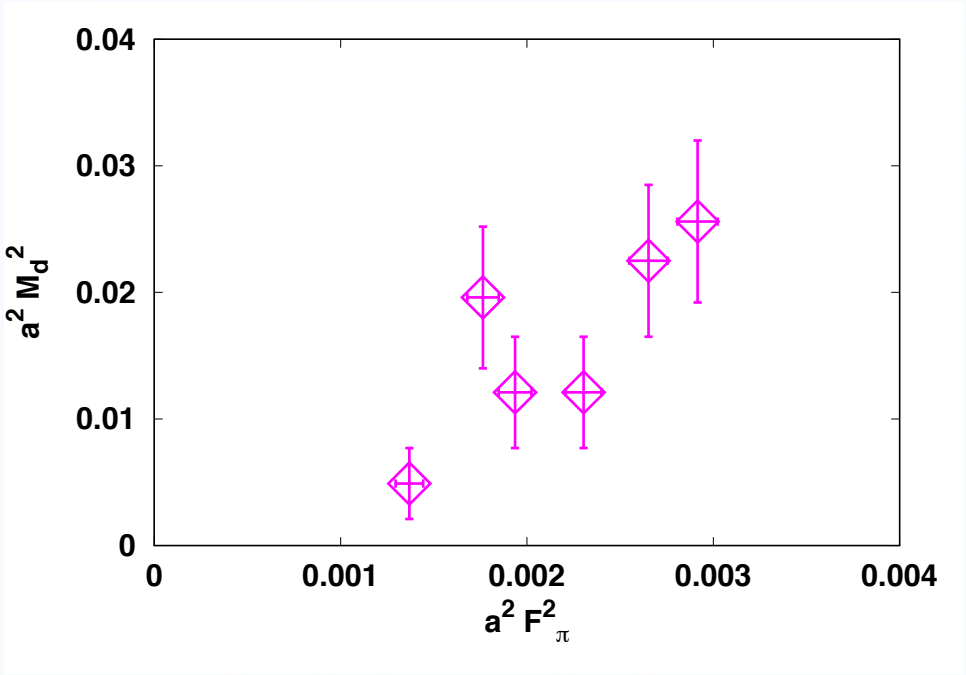
$$M_d^2 = \frac{yN_f f_\pi^2}{2f_d^2}(p-y)BF_\pi^{p-2}$$

2%

$f^2_\pi/f^2_d = 0.08 \pm 0.04$

- Any potential ansatz that fits the data well gives a similar bound.
- The ratio f^2_π/f^2_d is related to the h-WW coupling once the fermions are electroweak gauged.

$N_f = 2$ Sextet



$$M_d^2 = \frac{yN_f f_\pi^2}{2f_d^2}(p-y)BF_\pi^{p-2}$$

$f^2_\pi / f^2_d = 0.09 \pm 0.06$