The Constructive SM

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Feynman Diagrams

# External Gluons	4	5	6	7	8	9	10
# Feynman Diagrams	4	25	220	2,485	34,300	559,405	10,525,900

- Exponential growth in number of diagrams.
- Leading order in perturbation theory.
- Diagrams lose their meaning and utility.
- Complicated gauge cancellations.

Feynman Diagrams

- Part of the problem is that fields and Feynman diagrams contain unphysical degrees of freedom.
- For example, there are only two physical degrees of freedom for the photon, positive and negative helicity.
- Yet, in order to form a manifestly Lorentz invariant theory, we embed the photon in a Lorentz 4-vector, resulting in two unphysical degrees of freedom.
- We are then forced to invent a “gauge symmetry” to cancel the effects of these unphysical degrees of freedom.

Hints of Something Better

- Parke-Taylor formula for n gluons [Phys. Rev. Lett. 56, 2459 (1986)]

$$\mathcal{M}^{1,1,-1,\cdots,-1} = \frac{[12]^4}{[12][23]\cdots[n1]}$$

Maximally-helicity-violating gluon amplitudes can be written with only one mathematical term no matter how many gluons are present.

Clearly, Feynman diagrams are adding a bunch of extraneous garbage that cancels out at the end!

BCFW Recursion Relations

- Britto, Cachazo, Feng and Witten [Phys. Rev. Lett. 94, 181602 (2005)] found recursion relations that gave all gluon amplitudes (at tree level) without Feynman diagrams!

These BCFW diagrams (as opposed to Feynman diagrams):

- do not have unphysical degrees of freedom!
- do not have complicated cancellations between diagrams!
- are always on shell!
- have exponentially fewer diagrams! Sometimes only 1!
- do not have any gauge choice and thus no need for gauge invariance!
- Appear to be minimal!

Constructive Massless Theories

- All 3-point vertices for massless theories can be determined uniquely based purely on the Poincare symmetry.

- For example, the (color stripped) vertex of two helicity +1 particles and one helicity -1 particle must have the structure

$$\mathcal{A}(1,1,-1) = \frac{[12]^3}{[23][31]}$$

Constructive Massive Theories

- Arkani-Hamed, Huang and Huang (arXiv:1709.04891) showed how to generalize this to massive theories.
- The structure of all 3-point vertices for massive theories can be determined based purely on the Poincare symmetry.
- Combine helicity-spinors (for the massless particles) with spin-spinors (for the massive particles) in a way that satisfies the Poincare symmetry for the three point amplitude. For example, for one massless particle and two massive particles of the same mass,

$$M^h{}_{\{\alpha_1\alpha_2\cdots\alpha_{2S_1}\},\{\beta_1\beta_2\cdots\beta_{2S_2}\}} = \sum_{i=|S_1-S_2|}^{(S_1+S_2)} g_i x^{h+i} (\lambda_3^{2i} \varepsilon^{S_1+S_2-i})_{\{\alpha_1\alpha_2\cdots\alpha_{2S_1}\},\{\beta_1\beta_2\cdots\beta_{2S_2}\}}$$

Constructive SM

- NC and B Field (arXiv:1709.04891) specialized this to the SM.

$$x^h \frac{\langle \mathbf{12} \rangle^{2S}}{m^{2S-1}} \qquad\qquad x = \frac{\langle \xi | p_2 | 3]}{m \langle \xi 3 \rangle}$$

- Worked out the complete set of minimal 3-point vertices for the SM.

- Found the high-energy limit of these massive vertices and showed that we recovered the expected purely massless vertices.

Constructive EM

Particles	Coupling	Vertex	High-Energy Limit (Helicity Signature)
$f\bar{f}\gamma^+$	$-ieQ_f$	$x\langle\mathbf{12}\rangle$	$\frac{[23]^2}{[12]}\,(-+), \quad -\frac{[31]^2}{[12]}\,(+-)$
$f\bar{f}\gamma^-$	$-ieQ_f$	$\tilde{x}[\mathbf{12}]$	$\frac{\langle 23\rangle^2}{\langle 12\rangle}\,(+-), \quad -\frac{\langle 31\rangle^2}{\langle 12\rangle}\,(-+)$
$W\bar{W}\gamma^+$	$-ie$	$\frac{x}{M_W}\langle\mathbf{12}\rangle^2$	$\frac{[23]^3}{[12][31]}(-+), \quad 2\frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle}(--), \quad \frac{[31]^3}{[12][23]}(+-), \quad -\frac{1}{2}\frac{[31][23]}{[12]}(00)$
$W\bar{W}\gamma^-$	$-ie$	$\frac{\tilde{x}}{M_W}[\mathbf{12}]^2$	$\frac{\langle 23\rangle^3}{\langle 12\rangle\langle 31\rangle}(+-), \quad 2\frac{[12]^3}{[23][31]}(++), \quad \frac{\langle 31\rangle^3}{\langle 12\rangle\langle 23\rangle}(-+), \quad -\frac{1}{2}\frac{\langle 31\rangle\langle 23\rangle}{\langle 12\rangle}(00)$

$$x = \frac{\langle\xi|p_2|3]}{m\langle\xi 3\rangle} \qquad \tilde{x} = \frac{[\xi|p_2|3\rangle}{m[\xi 3]}$$

Constructive QCD

Particles	Coupling	Vertex	High-Energy Limit (Helicity Signature)
$q\bar{q}g^+$	$ig_s(T^{a_3})_{i_1}^{i_2}$	$x\langle\mathbf{12}\rangle$	$\frac{[23]^2}{[12]}\,(-+),\qquad -\frac{[31]^2}{[12]}\,(+-)$
$q\bar{q}g^-$	$ig_s(T^{a_3})_{i_1}^{i_2}$	$\tilde{x}[\mathbf{12}]$	$\frac{\langle 23\rangle^2}{\langle 12\rangle}\,(+-),\qquad -\frac{\langle 31\rangle^2}{\langle 12\rangle}\,(-+)$
$g^-g^-g^+$	$ig_sf^{a_1a_2a_3}$	$\frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle}$	Already massless
$g^+g^+g^-$	$ig_sf^{a_1a_2a_3}$	$\frac{[12]^3}{[23][31]}$	Already massless

$$x = \frac{\langle\xi|p_2|3]}{m\langle\xi 3\rangle} \qquad \tilde{x} = \frac{[\xi|p_2|3\rangle}{m[\xi 3]}$$

Constructive Gravity

Particles	Coupling	Vertex	High-Energy Limit (Helicity Signature)
hhG^+	$\frac{i}{M_P}$	$x^2 m_h^2$	$\left(\frac{[23][31]}{[12]}\right)^2$
hhG^-	$\frac{i}{M_P}$	$\tilde{x}^2 m_h^2$	$\left(\frac{\langle 23\rangle\langle 31\rangle}{\langle 12\rangle}\right)^2$
$f\bar{f}G^+$	$\frac{i}{M_P}$	$x^2 m_f \langle\mathbf{12}\rangle$	$\frac{[23]^3[31]}{[12]^2}(-+), \quad -\frac{[31]^3[23]}{[12]^2}(+-)$
$f\bar{f}G^-$	$\frac{i}{M_P}$	$\tilde{x}^2 m_f [\mathbf{12}]$	$\frac{\langle 23\rangle^3\langle 31\rangle}{\langle 12\rangle^2}(+-), \quad -\frac{\langle 31\rangle^3\langle 23\rangle}{\langle 12\rangle^2}(-+)$
$V\bar{V}G^+$	$\frac{i}{M_P}$	$x^2\langle\mathbf{12}\rangle^2$	$\frac{[23]^4}{[12]^2}(-+), \quad -\frac{1}{2}\frac{[31]^2[23]^2}{[12]^2}(00), \quad \frac{[31]^4}{[12]^2}(+-)$
$V\bar{V}G^-$	$\frac{i}{M_P}$	$\tilde{x}^2[\mathbf{12}]^2$	$\frac{\langle 23\rangle^4}{\langle 12\rangle^2}(+-), \quad -\frac{1}{2}\frac{\langle 31\rangle^2\langle 23\rangle^2}{\langle 12\rangle^2}(00), \quad \frac{\langle 31\rangle^4}{\langle 12\rangle^2}(-+)$
$\gamma^+\gamma^-G^+$ $g^+g^-G^+$	$\frac{i}{M_P}$	$\frac{[31]^4}{[12]^2}$	Already massless

Constructive Weak

Particles	Coupling	Vertex	High-Energy Limit
$\nu^- \bar{\nu}^+ Z$	$\frac{ie}{\sin 2\theta_w}$	$\frac{\langle \mathbf{3}1\rangle[2\mathbf{3}]}{M_Z}$	$\frac{1}{2}\left(\frac{\langle 31\rangle^2}{\langle 12\rangle} - \frac{[23]^2}{[12]}\right)$
$f\bar{f}Z$	$\frac{ie}{\sin 2\theta_w}$	$\frac{\langle \mathbf{31}\rangle[\mathbf{23}] + [\mathbf{31}]\langle \mathbf{23}\rangle}{M_Z}$	$\frac{1}{2}\left(\frac{[31]^2}{[12]} + \frac{\langle 31\rangle^2}{\langle 12\rangle} - \frac{\langle 23\rangle^2}{\langle 12\rangle} - \frac{[23]^2}{[12]}\right)$, $\pm\frac{m_f}{M_Z}\left([12] + \langle 12\rangle\right)$
$l\bar{\nu}_l^+ W$	$\frac{ie}{2\sqrt{2}\cos\theta_w}$	$\frac{\langle \mathbf{31}\rangle[2\mathbf{3}]}{M_W} + \mathcal{N}_{Wl\nu}[\mathbf{31}][2\mathbf{3}]$	$\frac{1}{2}\left(\frac{\langle 31\rangle^2}{\langle 12\rangle} - \frac{[23]^2}{[12]}\right)$, $\frac{m_l}{M_W}[12]$
$\bar{l}\nu_l^- \bar{W}$	$\frac{ie}{2\sqrt{2}\cos\theta_w}$	$\frac{[\mathbf{31}]\langle 2\mathbf{3}\rangle}{M_W} + \mathcal{N}^*_{Wl\nu}\langle \mathbf{31}\rangle\langle 2\mathbf{3}\rangle$	$\frac{1}{2}\left(\frac{[31]^2}{[12]} - \frac{\langle 23\rangle^2}{\langle 12\rangle}\right)$, $\frac{m_l}{M_W}\langle 12\rangle$
$f_i\bar{f}_j W$	$\frac{ie}{2\sqrt{2}\cos\theta_w}$	$\frac{\langle \mathbf{31}\rangle[\mathbf{23}] + [\mathbf{31}]\langle \mathbf{23}\rangle}{M_W}$ $+\mathcal{N}_{Wff}\left(\langle \mathbf{31}\rangle\langle \mathbf{23}\rangle + [\mathbf{31}][\mathbf{23}]\right)$	$\frac{1}{2}\left(\frac{[31]^2}{[12]} + \frac{\langle 31\rangle^2}{\langle 12\rangle} - \frac{\langle 23\rangle^2}{\langle 12\rangle} - \frac{[23]^2}{[12]}\right)$, $+\frac{m_i}{M_W}\left([12] + \langle 12\rangle\right)$, $-\frac{m_j}{M_W}\left([12] + \langle 12\rangle\right)$
			$-\frac{1}{4}\left(\frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle} + \frac{[12]^3}{[23][31]}\right)$ $+\frac{1}{4}\frac{M_W}{M_Z}\left(\frac{\langle 12\rangle\langle 31\rangle}{\langle 23\rangle} + \frac{[12][31]}{[23]} + \frac{\langle 12\rangle\langle 23\rangle}{\langle 31\rangle} + \frac{[12][23]}{[31]}\right)$,

Conclusions

- We have constructed the complete set of 3-point amplitudes for the SM.
  - We have only used the Poincare and little group symmetries.
  - We have done this without the use of fields, Feynman rules or gauges.
  - We have found the high-energy behavior of these massive vertices and showed that they agreed with the well-known massless vertices.
- The massive constructive theory is still not complete. There is still much to do. For example:
  - Work out all the 4-point "contact" terms of the SM.
  - Work out a complete consistent BCFW-like recursion relation for any n-point amplitude.
  - Work out a complete set of rules giving any loop contribution in terms of the 3-point amplitudes.