Soft Collinear Effective Theory for Gravity

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Why another effective theory for gravity?

Many surprising properties of gravity scattering amplitudes, e.g.

1. ***Soft graviton theorem***

   $$A(1,2,3,4,s) = \left(S_0(s) + S_{sub}(s)\right) A(1,2,3,4)$$

2. ***Decoupling of collinear gravitons***

   No collinear graviton couplings at leading order

3. ***Asymptotic symmetries***

   Infinite dimensional group of symmetries at conformal infinity

4. $\boldsymbol{gravity = gauge^2}$

5. **…**

All these are invisible at Lagrangian level or from Feynman rules!

- Restrict to a smaller region of phase space
- integrating out unnecessary modes

→

- Capture some of these properties at Lagrangian level!
- discover new properties of gravity amplitudes

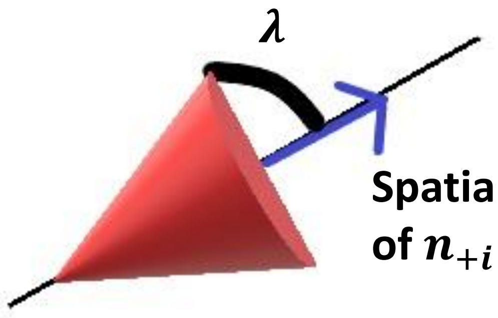
We have developed Soft-Collinear Effective Theory (SCET) for gravity at leading and next-to-leading powers of a small parameter (λ).

SCET for gravity also simplifies calculations greatly.
A three graviton vertex in full theory has 171 terms, each with 8 pairs of contracted indices!

Phase space of SCET for Gravity

of collinear sectors (N)
energy scale of collinear particles (Q)
Small parameter (λ)

→ Defines the target phase space



λ is the small parameter used for power-counting.

Light-cone coordinates:

$$n_{+i}.n_{+i} = n_{-i}.n_{-i} = 0$$
$$n_{+i}.n_{-i} = 1$$

Scaling of i-th collinear momentum in sector i-th coordinates

$$(p^{+i}, p^{-i}, p^{\perp i}) \sim Q(1, \lambda^2, \lambda)$$

Scaling of soft momenta expanded in sector i-th coordinates

$$(p^{+s}, p^{-s}, p^{\perp s}) \sim Q(\lambda^2, \lambda^2, \lambda^2)$$

$N + 1$ different momentum modes in target phase space → Define $N + 1$ independent field for each particle species

- So any field $\Phi(x)$ in full theory, becomes $\Phi_1(x), \Phi_2(x), \dots, \Phi_N(x), \Phi_s(x)$ in SCET.
  - This applies to gauge fields as well (including graviton field).
    - Gauge symmetry of full theory also factorizes into $N + 1$ subgroups in SCET.

Gauge symmetry of gravity:

$$G_{gravity} = (diffeomorphism) \times (local\ Lorentz\ group)$$

In SCET for gravity

$$G_{gravity} \longrightarrow G_s \ltimes G_1 \times G_2 \times \cdots \times G_N$$

Soft Wilson line

- Any collinear field $\Phi_i(x)$ transforms under soft gauge symmetry.
- To achieve true mode separation we redefine $\Phi_i(x)$ such that it is invariant under soft gauge transformation:

$$\Phi_i(x) \to Y_i(x)\Phi_i(x)$$

- Soft Wilson line, $Y_i(x)$, is given by

$$exp\left[\int_{-\infty}^{0} ds\, h^{s}_{+i+i}\,(x + sn_{+i})\partial_{-i}\right]$$

Note the spin independence.

1. **No interaction between soft gravitons and other soft fields.**

   Each soft graviton interaction comes with a positive power of $\lambda^2 Q$ at leading order, so in the limit of $\lambda \to 0$ vanishes.

2. **The only interaction of soft gravitons and collinear particles is through soft Wilson line $Y_i(x)$.**

Soft graviton theorem at LP is obvious in our Lagrangian!

3. **There is no collinear graviton interaction at each collinear sector.**

   Each collinear graviton interaction comes with a positive power of λQ at leading order, so in the limit of $\lambda \to 0$ vanishes.

Collinear graviton theorem is trivial in our LP Lagrangian.

Gravity SCET at next-to-leading power (NLP)

- We have a simple recipe for writing down gravity SCET Lagrangian and effective operators at NLP.
- Interesting interactions between collinear gravitons and other collinear fields (including gravitons).
- Collinear fields transform under collinear gauge transformations at NLP

Factorized effective gauge symmetry → make each collinear field invariant

Gravitational collinear Wilson line for local Lorentz groups:

$$W_r^{(i)}(x) \equiv exp\left[-\frac{1}{2}\int ds\, \gamma^{(i)}_{-i\alpha\beta}(x+sn_{-i})\Sigma_r^{\alpha\beta}\right]$$

Gravitational collinear Wilson line for collinear diff groups:

$$V^{(i)}(x) \equiv 1 - \int_{-\infty}^{0} dt \int_{-\infty}^{t} ds\, \Gamma^{\mu}_{-i-i}\,(x+sn_{-i})\partial_\mu$$

Summary

1. Simple recipe for writing down effective operators including gravitons at LP and NLP
2. Expansion of soft and collinear Lagrangians at LP and NLP
3. Interesting proofs of soft and collinear theorems
4. Many pages of calculations in full theory become very short
5. Interesting properties observed in scattering amplitudes including gravitons

Thank you.
Any questions?

As a side note:

- Do not fly Delta airlines!
- Use Atlanta only as a connection hub!