Bayesian analysis and naturalness of (Next-to-)Minimal SUSY Models

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Naturalness Problems

- Common motivation for SUSY is the hierarchy problem,

\[ m_h^2 \ll \Lambda_{NP}^2 ? \]

- Example of a “naturalness problem”
  - See also: strong CP problem, cosmological constant problem, flatness problem, …

- Possible characterisation: propensity of model to reproduce data

- Related to fine-tuning, e.g., if observations require a priori unjustified tuning of parameters \( \Rightarrow \) model unnatural

- Prefer models in which fine-tuning not required/reduced?

- E.g., “little hierarchy problem” in MSSM (at tree-level)
  
  \[ m_{h_1}^2 \leq m_Z^2 \cos^2 2\beta \lesssim (91 \text{ GeV})^2 \]

  \( \Rightarrow \) non-minimal SUSY models?
Quantifying Fine-tuning

Typical Motivation

<model name> raises the Higgs mass at tree-level, and therefore is more natural than the MSSM, e.g., in the NMSSM

\[ m_{h_1}^2 \lesssim m_Z^2 \cos^2 2\beta + \frac{\lambda^2 v^2}{2} \sin^2 2\beta \]

- Justify/check claim that model is more natural than another \(\Rightarrow\) quantify naturalness, i.e., fine-tuning

- Traditionally, construct fine-tuning measure, i.e., calculable function of model parameters that is identified with tuning

- Focus on different features of a model’s behaviour \(\Rightarrow\) different fine-tuning measure
  - Model fine-tuned according to one measure \(\Rightarrow\) also fine-tuned under a different measure?

- I.e., depends on your definition of fine-tuning . . .
Traditional Tuning Measures

What constitutes "fine-tuning"?
Opinions differ.

- Large cancellations [1]?
  \[
  \Delta_{EW} = \max_i \frac{2|C_i|}{m_Z^2}, \quad C_1 = -\mu^2, \quad C_2 = \frac{m_{H_d}^2}{\tan^2 \beta - 1}, \ldots
  \]

- Extreme sensitivities [2]?
  \[
  \Delta_{BG} = \max_i \left| \frac{\partial \ln m_Z^2}{\partial \ln p_i} \right|, \quad p_i \in \{\text{fundamental parameters}\}
  \]

- High or low energy contributions (\(\Delta_{EW}\) or \(\Delta_{HS}\))? Definition of parameters \(p_i\)? Just \(m_Z\)?

- Model fine-tuned \(\Leftrightarrow \Delta_{EW/HS/BG/\ldots} > ?\)

- Compare tuning between models?

  What does \(\Delta_{EW/HS/BC/\ldots} > \times\) mean for our degree of belief in model?

Traditional Tuning Measures

[arXiv:1309.2984]
A Bayesian Approach

- Results based on traditional measures depend heavily on choice of measure

- Generally, when we talk about “naturalness” we are really talking about plausibility
  - Unnatural model ⇔ implausible model
  - Traditional measures do not have an unambiguous interpretation in this sense

- ⇒ better questions to ask are,

  - How plausible is a given parameter space point in a model, in light of data?
  - Which model in a given set is the most plausible, in light of data?

- Rigorous logical framework exists for answering these questions: Bayesian statistics
Naturalness Priors

- Bayesian framework automatically captures intuition about “naturalness”
- Bayes’ theorem applied to model \( M \), parameters \( x \), "observables" \( O \):

\[
p(x | \text{data}, M) = \frac{p(\text{data} | x, M)p(x | M)}{p(\text{data} | M)}, \quad p(M | \text{data}) = \frac{p(\text{data} | M)p(M)}{p(\text{data})}
\]

\[
p(\text{data} | M) = \int d^n x \ p(\text{data} | x, M)p(x | M), \quad p_{\text{eff.}}(x_j, \ldots) = \int dO_i \ p(\mathbf{O}, x' | M, O_i^{\text{exp.}})
\]

- Reparameterise in favour of \( \mathbf{O} \) and remaining parameters \( x' \), e.g.,

\[
p(x | M) = \mathcal{J} p(\mathbf{O}, x' | M)
\]

⇒ evidence and effective naturalness priors suppressed by Jacobian \( \Delta J \sim \mathcal{J} \) [3],

\[
\Delta J = \left| \det \frac{\partial \ln O_i}{\partial \ln x_j} \right|
\]

Example: the SM Hierarchy Problem

Toy model \( \Rightarrow M_Z^2 = -\frac{g^2}{8\lambda}(\mu^2 + \Lambda_{NP}^2) \)

- Traditional BG measure applied to, e.g., cut-off \( \Lambda_{NP}^2 \),
  \[
  \Delta_{\Lambda_{NP}^2} = \frac{g^2}{8\lambda} \frac{\Lambda_{NP}^2}{M_Z^2}
  \]
  \Rightarrow large tuning for \( \Lambda_{NP}^2 \gg M_Z^2 \) but interpretation unclear

- Compute posterior for \( \Lambda_{NP}^2 \) using
  \[
  p_{\text{eff.}}(\Lambda^2, \lambda) \propto \left| \frac{\partial M_Z}{\partial \mu^2} \right|^{-1}_{\mu=\mu_Z} p(\Lambda^2, \mu_Z^2, \lambda|\text{SM})
  \]
  \Rightarrow captures traditional intuition about tuning

- But also has a well-defined probabilistic interpretation

[arXiv:1709.07895]
Models Considered

- CMSSM as reference model with parameters $\mu_0$, $B_0\mu_0$, $\text{sign} \, \mu$, and

$$m_Q = m_{u^c} = \ldots = m_{H_d} = m_{H_u} = m_0^2,$$
$$M_1 = M_2 = M_3 = m_{1/2},$$
$$A_u = A_d = A_e = A_0$$

- Naturalness priors from $\{|\mu_0|, B_0\mu_0\} \rightarrow \{M_Z^2, \tan \beta\}$

- Semi-constrained $\mathbb{Z}_3$-NMSSM,

$$\hat{W}_{\text{NMSSM}} = \hat{W}_{\text{MSSM}} \bigg|_{\mu=0} + \lambda \hat{S} \hat{H}_d \cdot \hat{H}_u + \frac{1}{3} \kappa \hat{S}^3,$$

new parameters $\lambda_0, \kappa_0, m_{S_0}^2, \text{sign}(\lambda \langle S \rangle), A_\lambda, A_\kappa$

- Trade $\{\lambda_0, \kappa_0, m_{S_0}^2\} \rightarrow \{\lambda, M_Z^2, \tan \beta\}$

<table>
<thead>
<tr>
<th>CMSSM</th>
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<tbody>
<tr>
<td>$m_0$</td>
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<tr>
<td>$m_{1/2}$</td>
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<tr>
<td>$A_0$</td>
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<tr>
<td>$B_0\mu_0$</td>
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<tr>
<td>$\text{sign} , \mu$</td>
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<table>
<thead>
<tr>
<th>NMSSM</th>
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<tbody>
<tr>
<td>$\lambda_0$</td>
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<tr>
<td>$\kappa_0$</td>
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<tr>
<td>$m_{S_0}$</td>
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<tr>
<td>$A_\lambda$</td>
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<tr>
<td>$A_\kappa$</td>
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</tbody>
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Low Fine-tuning and Credible Regions

- High posterior density $\Leftrightarrow$ low fine-tuning (according to $\Delta_{BG}$)
- $m_{h_1} \approx 125$ GeV not required $\Rightarrow$ weak-scale soft parameters preferred

[arXiv:1709.07895]
Impact of $m_h \approx 125$ GeV

- Credible intervals shifted $\sim 2$ orders of magnitude in $m_0$, $m_{1/2}$
  - Expected: LHC $\Rightarrow$ SUSY implausible below $\sim 1$ TeV
- Still find most plausible regions $\Leftrightarrow$ lowest tunings consistent with data
- Both approaches $\Rightarrow$ focus-point region is preferred
Comparing Fine-tuning Measures

- Can use $\Delta_J$ as a standalone tuning measure
- Qualitative agreement between $\Delta_{BG}$, $\Delta_{EW}$, $\Delta_J$
- But meaningful results $\Rightarrow$ ideally should compute full posterior densities or evidences
- Some differences, e.g., $\Delta_J$ shows much stronger preference for focus point than $\Delta_{BG}$
- Aside: differing definitions of measures $\Rightarrow$ direct comparison of values not reasonable

[arXiv:1709.07895]
Posterior Distributions

- Posterior densities reflect traditional intuition
- Pre-Higgs densities ⇒ low $m_{\text{SUSY}} \lesssim 1$ TeV most plausible ("natural")
- Inclusion of $m_h \Rightarrow m_{\text{SUSY}} \gtrsim 4$ TeV required, TeV scale soft parameters
- Bayesian approach ⇒ systematically update degree of belief given new data
- Ambiguous when using approach based on $\Delta_{\text{EW}}/BG/…$

[arXiv:1709.07895]
Model comparison based on traditional tuning measures is ill-defined

More relevant to ask which parameter values/models are most plausible given observations ⇒ utilise Bayesian methods

Bayesian approach automatically incorporates traditional intuitions about fine-tuning

Effective naturalness priors resemble traditional measures but have a rigorous probabilistic interpretation

Qualitative agreement between traditional measures and naturalness priors, credible intervals in the CMSSM and semi-constrained NMSSM

In principle, can go further: have all ingredients to compare model evidences ⇒ statistically meaningful model comparisons

Thank you for listening!
Additional Slides
Minimum Tuning Values

<table>
<thead>
<tr>
<th></th>
<th>( M_Z )</th>
<th>( M_Z ) and ( m_h \approx 125 \text{ GeV} )</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>CMSSM</td>
<td>NMSSM</td>
</tr>
<tr>
<td>( \Delta J \rvert_{M_{\text{GUT}}} )</td>
<td>( 3 \times 10^{-9} )</td>
<td>( 2 \times 10^{-10} )</td>
</tr>
<tr>
<td>( \Delta J \rvert_{m_{\text{SUSY}}} )</td>
<td>( 6 \times 10^{-7} )</td>
<td>( 2 \times 10^{-10} )</td>
</tr>
<tr>
<td>( \Delta_{EW} )</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>( \Delta_{BG} )</td>
<td>0.1</td>
<td>0.2</td>
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- Note: numbers should be compared keeping in mind differing definitions of measures.
- \( \Delta J \rvert_{m_{\text{SUSY}}} \Rightarrow \) contributions to Jacobian from RG running omitted.
- All measures increase once \( m_h \) included.
- Note \( \Delta_{EW} \) similar for both models, while \( \Delta_{BG}, \Delta_J \) smaller for NMSSM.