Bayesian analysis and naturalness of (Next-to-)Minimal SUSY Models

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Based on: P. Athron, C. Balázs, B. Farmer, A. Fowlie, D. Harries, and D. Kim, JHEP **10**, 160 (2017) [arXiv:1709.07895]

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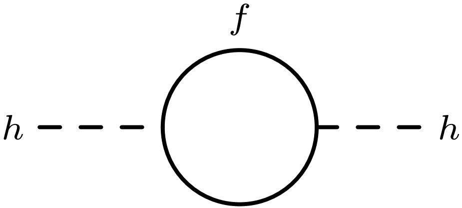
Naturalness Problems

- Common motivation for SUSY is the hierarchy problem,



$$m_h^2 \ll \Lambda_{NP}^2?$$

- Example of a "naturalness problem"
  - See also: strong CP problem, cosmological constant problem, flatness problem, . . .

- Possible characterisation: propensity of model to reproduce data

- Related to fine-tuning, e.g., if observations require a priori unjustified tuning of parameters
  $\Rightarrow$ model unnatural

- Prefer models in which fine-tuning not required/reduced?

- E.g., "little hierarchy problem" in MSSM (at tree-level)

$$m_{h_1}^2 \leq m_Z^2 \cos^2 2\beta \lesssim (91 \text{ GeV})^2$$

$\Rightarrow$ non-minimal SUSY models?

Quantifying Fine-tuning

Typical Motivation

<model name> raises the Higgs mass at tree-level, and therefore is more natural than the MSSM, e.g., in the NMSSM

$$m_{h_1}^2 \lesssim m_Z^2 \cos^2 2\beta + \frac{\lambda^2 v^2}{2} \sin^2 2\beta$$

- Justify/check claim that model is more natural than another $\Rightarrow$ quantify naturalness, i.e., fine-tuning
- Traditionally, construct fine-tuning measure, i.e., calculable function of model parameters that is identified with tuning
- Focus on different features of a model's behaviour $\Rightarrow$ different fine-tuning measure
  - Model fine-tuned according to one measure $\Rightarrow$ also fine-tuned under a different measure?
- I.e., depends on your definition of fine-tuning ...

Traditional Tuning Measures

What constitutes "fine-tuning"?
Opinions differ.

- Large cancellations [1]?

$$\Delta_{EW} = \max_i \frac{2|C_i|}{m_Z^2}, \quad C_1 = -\mu^2,\ C_2 = \frac{m_{H_d}^2}{\tan^2\beta - 1}, \ldots$$

- Extreme sensitivities [2]?

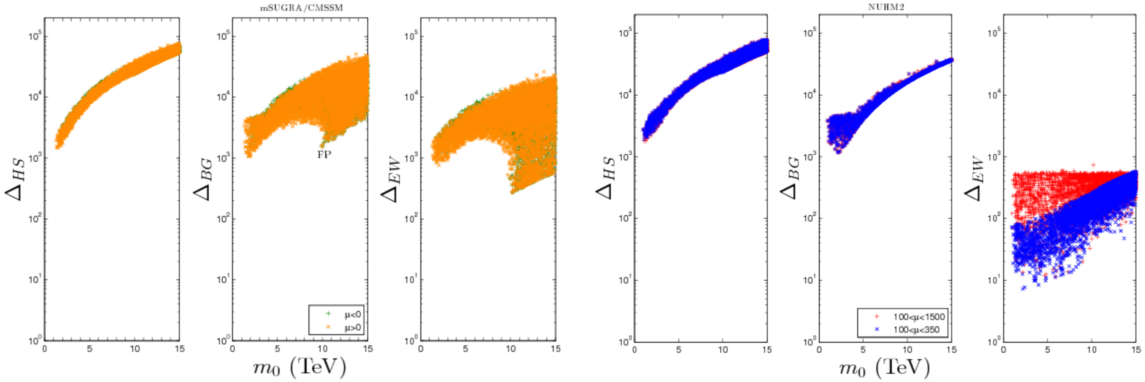
$$\Delta_{BG} = \max_i \left|\frac{\partial \ln m_Z^2}{\partial \ln p_i}\right|, \quad p_i \in \{\text{fundamental parameters}\}$$

- High or low energy contributions (Δ_{EW} or Δ_{HS})? Definition of parameters p_i? Just m_Z?
- Model fine-tuned $\Leftrightarrow \Delta_{EW/HS/BG/\ldots} >$?
- Compare tuning between models?

What does $\Delta_{EW/HS/BC/\ldots} > x$ mean for our degree of belief in model?

[1] H. Baer, V. Barger, P. Huang, A. Mustafayev, and X. Tata, Phys. Rev. Lett. **109** (2012) 161802 [arXiv:1207.3343]
[2] J. R. Ellis, K. Enqvist, D. V. Nanopoulos, and F. Zwirner, Mod. Phys.Lett. **A1** (1986) 57; R. Barbieri and G. F. Giudice, Nucl. Phys. **B306** (1988) 63

Traditional Tuning Measures



[arXiv:1309.2984]

A Bayesian Approach

- Results based on traditional measures depend heavily on choice of measure
- Generally, when we talk about "naturalness" we are really talking about plausibility
  - Unnatural model $\Leftrightarrow$ implausible model
  - Traditional measures do not have an unambiguous interpretation in this sense
- $\Rightarrow$ better questions to ask are,

> *How plausible is a given parameter space point in a model, in light of data?*
>
> *Which model in a given set is the most plausible, in light of data?*

- Rigorous logical framework exists for answering these questions: Bayesian statistics

Naturalness Priors

- Bayesian framework automatically captures intuition about “naturalness”

- Bayes’ theorem applied to model M, parameters $\boldsymbol{x}$, "observables" $\boldsymbol{O}$:

$$p(\boldsymbol{x}|\text{data}, M) = \frac{p(\text{data}|\boldsymbol{x}, M)p(\boldsymbol{x}|M)}{p(\text{data}|M)}, \quad p(M|\text{data}) = \frac{p(\text{data}|M)p(M)}{p(\text{data})}$$

$$p(\text{data}|M) = \int d^n x_i \; p(\text{data}|\boldsymbol{x}, M)p(\boldsymbol{x}|M), \quad p_{\text{eff.}}(x_j, \ldots) = \int dO_i \; p(\boldsymbol{O}, \boldsymbol{x}'|M, O_i^{\text{exp.}})$$

- Reparameterise in favour of $\boldsymbol{O}$ and remaining parameters $\boldsymbol{x}'$, e.g.,

$$p(\boldsymbol{x}|M) = \mathcal{J} p(\boldsymbol{O}, \boldsymbol{x}'|M)$$

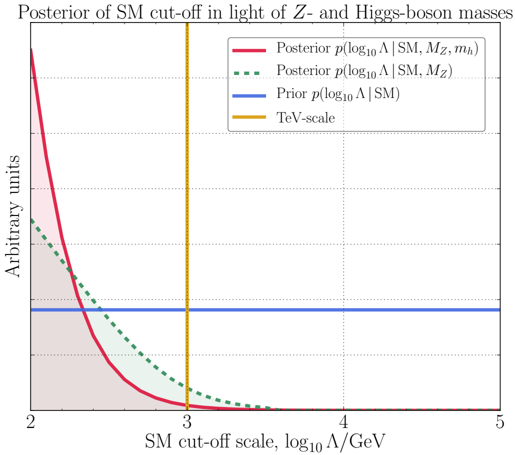
$\Rightarrow$ evidence and effective naturalness priors suppressed by Jacobian $\Delta_J \sim \mathcal{J}$ [3],

$$\Delta_J = \left|\det \frac{\partial \ln O_i}{\partial \ln x_j}\right|$$

[3] D. Kim, P. Athron, C. Balázs, B. Farmer, and E. Hutchison, Phys. Rev. D **90** (2014) 055008 [arXiv:1312.4150]

Example: the SM Hierarchy Problem

Toy model $\Rightarrow M_Z^2 = -\frac{\bar{g}^2}{8\lambda}(\mu^2 + \Lambda_{NP}^2)$



[arXiv:1709.07895]

- Traditional BG measure applied to, e.g., cut-off Λ_{NP}^2,

$$\Delta_{\Lambda_{NP}^2} = \frac{\bar{g}^2}{8\lambda}\frac{\Lambda_{NP}^2}{M_Z^2}$$

$\Rightarrow$ large tuning for $\Lambda_{NP}^2 \gg M_Z^2$ but interpretation unclear

- Compute posterior for Λ_{NP}^2 using

$$p_{\text{eff.}}(\Lambda^2, \lambda) \propto \left|\frac{\partial M_Z}{\partial \mu^2}\right|^{-1}_{\mu=\mu_Z} p(\Lambda^2, \mu_Z^2, \lambda|SM)$$

$\Rightarrow$ captures traditional intuition about tuning

- But also has a well-defined probabilistic interpretation

Models Considered

- CMSSM as reference model with parameters μ_0, $B_0\mu_0$, sign μ, and

$$m_Q^2 = m_{u^c}^2 = \ldots = m_{H_d}^2 = m_{H_u}^2 = m_0^2,$$
$$M_1 = M_2 = M_3 = m_{1/2},$$
$$A_u = A_d = A_e = A_0$$

- Naturalness priors from $\{|\mu_0|, B_0\mu_0\} \to \{M_Z^2, \tan\beta\}$
- Semi-constrained $\mathbb{Z}_3$-NMSSM,

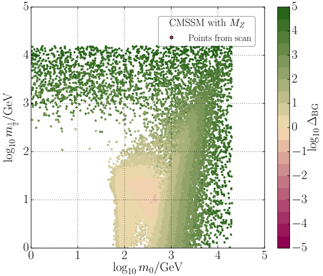
$$\widehat{W}_{\text{NMSSM}} = \widehat{W}_{\text{MSSM}}\Big|_{\mu=0} + \lambda \hat{S}\hat{H}_d \cdot \hat{H}_u + \frac{1}{3}\kappa \hat{S}^3,$$

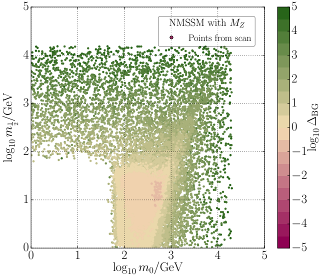
new parameters λ_0, κ_0, $m_{S_0}^2$, sign$(\lambda\langle S\rangle)$, A_λ, A_κ

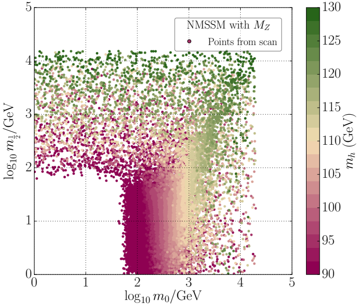
- Trade $\{\lambda_0, \kappa_0, m_{S_0}^2\} \to \{\lambda, M_Z^2, \tan\beta\}$

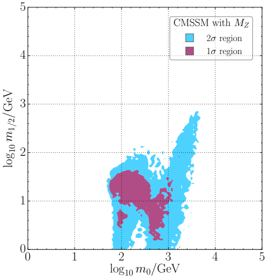
CMSSM	
m_0	Log, 1 GeV – 20 TeV
$m_{1/2}$	Log, 1 GeV – 15 TeV
A_0	Log, 100 GeV $< \lvert A\rvert \leq$ 20 TeV, Flat, $\lvert A\rvert \leq$ 100 GeV
$\lvert\mu_0\rvert$	Log, 100 GeV – 20 TeV
$B_0\mu_0$	Log, $(100\text{ GeV})^2$–$(20\text{ TeV})^2$
sign μ	± 1 with equal probability
NMSSM	
λ_0	Log, 10^{-6} – 1
κ_0	Log for $10^{-10} < \lvert\kappa\rvert < 1$
m_{S_0}	Same as m_0
A_λ	Same as A_0
A_κ	Same as A_0

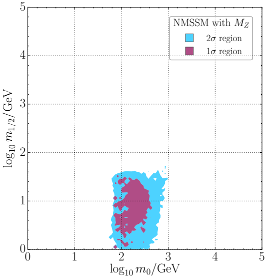
Low Fine-tuning and Credible Regions







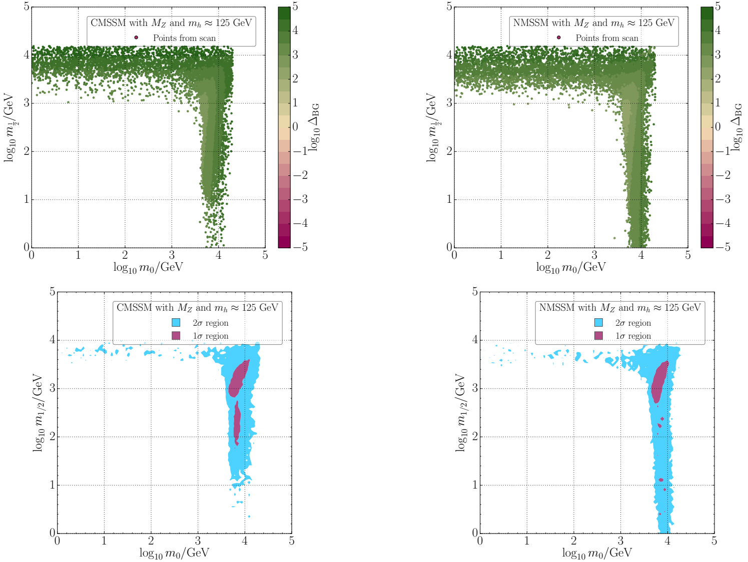




[arXiv:1709.07895]

- High posterior density $\Leftrightarrow$ low fine-tuning (according to Δ_{BG})
- $m_{h_1} \approx 125$ GeV not required $\Rightarrow$ weak-scale soft parameters preferred

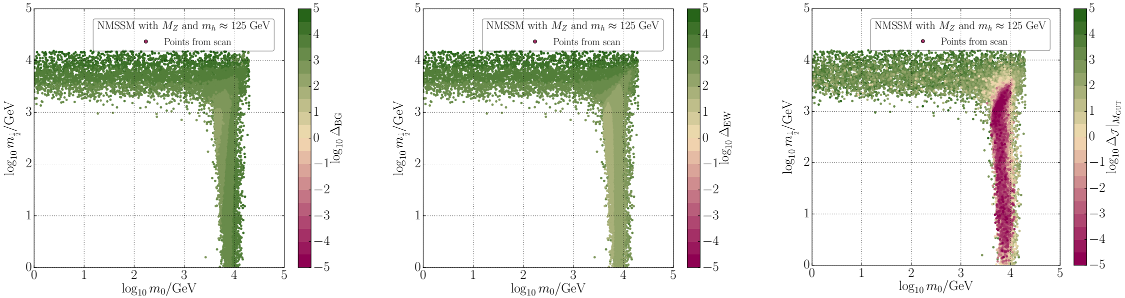
Impact of $m_h \approx 125$ GeV



[arXiv:1709.07895]

- Credible intervals shifted ~ 2 orders of magnitude in m_0, $m_{1/2}$
  - Expected: LHC $\Rightarrow$ SUSY implausible below ~ 1 TeV
- Still find most plausible regions $\Leftrightarrow$ lowest tunings consistent with data
- Both approaches $\Rightarrow$ focus-point region is preferred

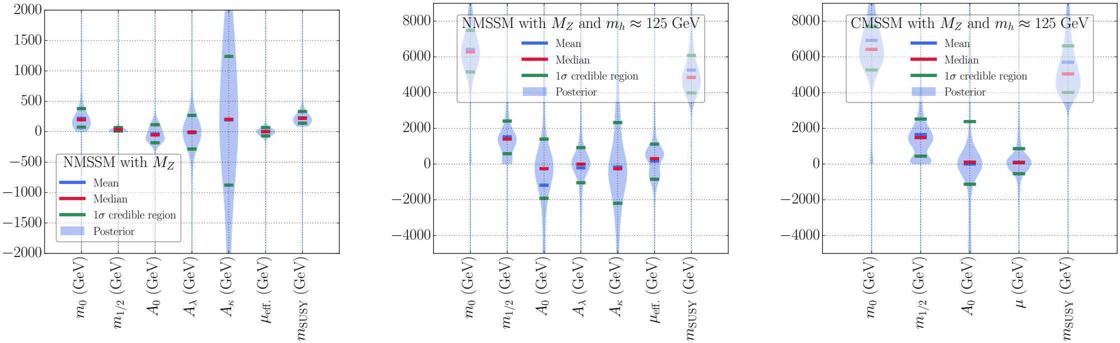
Comparing Fine-tuning Measures



[arXiv:1709.07895]

- Can use Δ_J as a standalone tuning measure
- Qualitative agreement between Δ_{BG}, Δ_{EW}, Δ_J
- But meaningful results ⇒ ideally should compute full posterior densities or evidences
- Some differences, e.g., Δ_J shows much stronger preference for focus point than Δ_{BG}
- Aside: differing definitions of measures ⇒ direct comparison of values not reasonable

Posterior Distributions



[arXiv:1709.07895]

- Posterior densities reflect traditional intuition
- Pre-Higgs densities $\Rightarrow$ low $m_{\mathrm{SUSY}} \lesssim 1$ TeV most plausible (“natural”)
- Inclusion of $m_h \Rightarrow m_{\mathrm{SUSY}} \gtrsim 4$ TeV required, TeV scale soft parameters
- Bayesian approach $\Rightarrow$ systematically update degree of belief given new data
- Ambiguous when using approach based on $\Delta_{EW/BG/\ldots}$

Summary

- Model comparison based on traditional tuning measures is ill-defined
- More relevant to ask which parameter values/models are most plausible given observations $\Rightarrow$ utilitise Bayesian methods
- Bayesian approach automatically incorporates traditional intuitions about fine-tuning
- Effective naturalness priors resemble traditional measures but have a rigorous probabilistic interpretation
- Qualitative agreement between traditional measures and naturalness priors, credible intervals in the CMSSM and semi-constrained NMSSM
- In principle, can go further: have all ingredients to compare model evidences $\Rightarrow$ statistically meanginful model comparisons

Thank you for listening!

Additional Slides

Minimum Tuning Values

	M_Z		M_Z and $m_h \approx 125$ GeV	
	CMSSM	NMSSM	CMSSM	NMSSM
$\Delta_J\big\|_{M_{\mathrm{GUT}}}$	3×10^{-9}	2×10^{-10}	0.004	8×10^{-7}
$\Delta_J\big\|_{m_{\mathrm{SUSY}}}$	6×10^{-7}	2×10^{-10}	0.005	8×10^{-7}
Δ_{EW}	0.3	0.3	48.7	47.4
Δ_{BG}	0.1	0.2	451.9	133.2

- Note: numbers should be compared keeping in mind differing definitions of measures
- $\Delta_J\big|_{m_{\mathrm{SUSY}}} \Rightarrow$ contributions to Jacobian from RG running omitted
- All measures increase once m_h included
- Note Δ_{EW} similar for both models, while Δ_{BG}, Δ_J smaller for NMSSM