Anomaly Cancellation in Effective Supergravity Theories from the Heterotic String: Two Simple Examples

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Based on work with Mary K. Gaillard

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Background

• Goal was to study anomalies arising from the massless spectrum of the $Z_3$ and $Z_7$ toroidal orbifold compactifications of the $E_8 \times E_8$ Heterotic String

• These are $N=1$ supergravity theories with matter & vector supermultiplets, along with the so-called diagonal Kahler moduli:

$$T_i = G_{i\bar{j}} + iB_{i\bar{j}}$$

• With Kahler potential

$$K = e^{\Phi^a} \bar{\Phi}^a \Phi^a + g(T^i, \bar{T}^i)$$

$$g(T^i, \bar{T}^i) = \sum_{i=1}^{3} g^i(T^i, \bar{T}^i) = -\sum_{i=1}^{3} \ln(T^i + \bar{T}^i)$$

$$g^a(T^i, \bar{T}^i) = \sum_{i} q_i^a g^i(T^i, \bar{T}^i)$$
Background

- Anomalies arise from the SL(2,R) modular transformation of the three diagonal Kahler moduli:

\[ T^i \rightarrow \frac{a_i T_i - i b_i}{i c_i T_i + d_i} \quad a_i b_i - c_i d_i = 1 \]

- Chiral matter supermultiplets must also transform

\[ \Phi^a \rightarrow \exp \left( - \sum_i q_i^a F^i \right) \Phi^a \quad F_i = i c_i T_i + d_i \]

- These field redefinitions in turn force a Kahler transformation:

\[ K \rightarrow K + \sum_i (F^i + \bar{F}^i) = K + F + \bar{F} \]

- Thus the "modular" anomalies are a combination of Kahler + reparametrization anomalies and we get modular/gauge/gauge, modular/gravity/gravity, and pure modular anomalies.

- Our project showed that the anomalies in these supergravity theories has the correct universal form such that they can be cancelled via the 4D Green-Schwarz mechanism.
Motivation

• Modular transformations correspond to T-Duality transformations of the underlying string theory – consistency of the low energy QFT and its relation to string theory relies on the cancellation of anomalies.

• On the string side, showing anomaly cancellation is easy, but one would want to ensure that the cancellation holds up purely from a QFT point of view.

• However, the QFT calculation is subtle. Previous investigations studied only subsets of the total anomaly or found a non-universal total anomaly due to difficulties with calculating the pure modular anomaly.

• Also phenomenological reasons for wanting a preserved symmetry – residual R-parity
Anomaly Cancellation Outline

• Calculate the one-loop effective action

• Introduce Pauli-Villars fields with non-invariant masses to cancel divergences*

• Anomaly arises from variation of terms involving PV mass terms in the effective action that are finite as the masses are taken to infinity

• Cancel the result via the GS mechanism

• Example:

\[ \mathcal{L}_{YM} = -\sqrt{g} \frac{s}{8} \sum_a \left( F^{a}_{\mu\nu} - i \tilde{F}^{a}_{\mu\nu} \right) F^{\mu\nu}_a + \text{h.c.} \]

\[ \Delta \mathcal{L}_{YM1} = -\frac{\sqrt{g}}{64\pi^2} \left( 8\pi^2 b \sum_i F^i \right) \left( F^{a}_{\mu\nu} - i \tilde{F}^{\mu\nu}_a \right) F^{\mu\nu}_a + \text{h.c.} \]

• Cancellation by imposing:

\[ \Delta s = -b \sum_i F^i \]
PV Fields

• A variety of fields are needed to cancel divergences and produce the correct anomaly. Many have Kahler potential & superpotential terms based on the couplings of the matter fields. As an example, a subset of chiral PV fields with non-invariant masses have

\[
W(\Phi^P, \Phi'^P) = \mu_P \Phi^P \Phi'^P
\]

\[
K(\Phi^P, \Phi'^P) = e^{f^P} |\Phi^P| + e^{f'^P} |\Phi'^P|
\]

\[
f^P = \alpha^P K(Z, \bar{Z}) + \beta^P g(T, \bar{T}) + \delta^P k(S, \bar{S}) + \sum_n q_n^C g^n(T^n, \bar{T}^n)
\]
Cancellation of Divergences

• To cancel divergences we require

  • Quadratic Divergences:
    \[ \text{Tr}(\eta \Gamma_\alpha) = 0 \]
    \[ \Gamma^i_{j\alpha} = -\frac{1}{8}(\bar{D}^2 - 8R)D_\alpha Z^k_{jk} \Gamma^i_{jk} \]

  • Logarithmic Divergences:
    \[ \text{Tr}(\eta \Gamma_\alpha \Gamma_\beta) = \text{Tr}(\eta \Gamma_\alpha T^a) = \text{Tr}(\eta (T^a)^2) = 0 \]

  • Linear Divergences:
    \[ \text{Tr}(\eta \phi G \cdot \tilde{G}) = 0 \]

• Where \( G_{\mu\nu} \) is the field strength associated with the fermion connections
  \[ G_{\mu\nu} = [D_\mu, D_\nu] = -\Gamma^C_{D\mu\nu} + iF^{a}_{\mu\nu}(T^a)^C_D + \frac{1}{2}X_{\mu\nu}\delta^C_D \]

• And is the parameter for a chiral rotation corresponding to a T-Duality transformation:
  \[ \phi^C = \left(\frac{1}{2} - \alpha^C - \beta^C\right) F - \sum_i F^i q^C_i \]
Anomaly Result

• After satisfying the regularization conditions & anomaly matching conditions, we find

\[
\delta \mathcal{L}_{\text{anom}} = b \int d^4 \theta E(F + \bar{F}) \Omega \\
= \frac{b \sqrt{g} \text{Im}(F)}{4} \left( F_a \cdot \tilde{F}^a + R \cdot \tilde{R} + \frac{1}{2} \sum_i g^i \cdot \tilde{g}^i + \cdots \right) \\
8\pi^2 b = \frac{1}{24} \left( 2 \sum_p q_n^p - N + N_G - 21 \right) \\
= C_a - C_a^M + 2 \sum_b C_a^b q_n^b \\
= 30
\]
Anomaly Cancellation

- In the chiral superfield description of the dilaton, we have a tree level coupling between the dilaton and the Chern-Simons anomaly superfield of the form

\[ \mathcal{L} = \int d^4 \theta E (S + \bar{S}) \Omega = -\frac{1}{8} \int d^4 \theta \frac{E}{R} S \Phi + \text{h.c.} \]

- So that the anomaly can be cancelled by allowing the dilaton to transform under a modular transformation as

\[ \Delta S = -bF \]
Conclusions & Future Work

• We have shown that, for at least two examples, string-derived supergravities can be regulated in such a way that the anomalies take a universal form and can be eliminated with the GS mechanism.

• Next step is to apply the procedure to more realistic string compactifications to make statements about the phenomenology of string models.
Bibliography