Anomaly Cancellation in Effective Supergravity Theories from the Heterotic String: Two Simple Examples

Jacob Leedom, UC Berkeley

Pheno 2018

Based on work with Mary K. Gaillard

arXiv: 1711.01023 & upcoming work

Background

- Goal was to study anomalies arising from the massless spectrum of the Z_3 and Z_7 toroidal orbifold compactifications of the E_8 x E_8 Heterotic String
- These are N=1 supergravity theories with matter & vector supermultiplets, along with the so-called diagonal Kahler moduli:

$$T_i = G_{i\bar{i}} + iB_{i\bar{i}}$$

- With Kahler potential

$$K = e^{g^a}\bar{\Phi}^a\Phi^a + g(T^i, \bar{T}^i)$$

$$g(T^i, \bar{T}^i) = \sum_{i=1}^{3} g^i(T^i, \bar{T}^i) = -\sum_{i=1}^{3} \ln(T^i + \bar{T}^i)$$

$$g^a(T^i, \bar{T}^i) = \sum_i q_i^a g^i(T^i, \bar{T}^i)$$

Background

- Anomalies arise from the SL(2,R) modular transformation of the three diagonal Kahler moduli:

$$T^i \to \frac{a_i T_i - i b_i}{i c_i T_i + d_i} \qquad a_i b_i - c_i d_i = 1$$

- Chiral matter supermultiplets must also transform

$$\Phi^a \to \exp\left(-\sum_i q_i^a F^i\right)\Phi^a \qquad F_i = i c_i T_i + d_i$$

- These field redefinitions in turn force a Kahler transformation:

$$K \to K + \sum_i (F^i + \bar{F}^i) = K + F + \bar{F}$$

- Thus the ``modular'' anomalies are a combination of Kahler + reparametrization anomalies and we get modular/gauge/gauge, modular/gravity/gravity, and pure modular anomalies.
- Our project showed that the anomalies in these supergravity theories has the correct universal form such that they can be cancelled via the 4D Green-Schwarz mechanism

Motivation

- Modular transformations correspond to T-Duality transformations of the underlying string theory – consistency of the low energy QFT and its relation to string theory relies on the cancellation of anomalies
- On the string side, showing anomaly cancellation is easy, but one would want to ensure that the cancellation holds up purely from a QFT point of view.
- However, the QFT calculation is subtle. Previous investigations studied only subsets of the total anomaly or found a non-universal total anomaly due to difficulties with calculating the pure modular anomaly.
- Also phenomenological reasons for wanting a preserved symmetry – residual R-parity

Anomaly Cancellation Outline

- Calculate the one-loop effective action
- Introduce Pauli-Villars fields with non-invariant masses to cancel divergences*
- Anomaly arises from variation of terms involving PV mass terms in the effective action that are finite as the masses are taken to infinity
- Cancel the result via the GS mechanism
- Example:

$$\mathcal{L}_{YM} = -\sqrt{g}\frac{s}{8}\sum_a \left(F^a_{\mu\nu} - i\tilde{F}^a_{\mu\nu}\right) F^{\mu\nu}_a + \text{h.c.}$$

$$\Delta\mathcal{L}_{YM1} = -\frac{\sqrt{g}}{64\pi^2}\left(8\pi^2 b\sum_i F^i\right)\left(F^a_{\mu\nu} - i\tilde{F}^{\mu\nu}_a\right) F^{\mu\nu}_a + \text{h.c.}$$

- Cancellation by imposing:

$$\Delta s = -b\sum_i F^i$$

PV Fields

- A variety of fields are needed to cancel divergences and produce the correct anomaly. Many have Kahler potential & superpotential terms based on the couplings of the matter fields. As an example, a subset of chiral PV fields with non-invariant masses have

$$W(\Phi^P, \Phi'^P) = \mu_P \Phi^P \Phi'^P$$

$$K(\Phi^P, \Phi'^P) = e^{f^P} |\Phi^P| + e^{f'^P} |\Phi'^P|$$

$$f^P = \alpha^P K(Z, \bar{Z}) + \beta^P g(T, \bar{T}) + \delta^P k(S, \bar{S}) + \sum_n q_n^C g^n(T^n, \bar{T}^n)$$

Cancellation of Divergences

- To cancel divergences we require
  - Quadratic Divergences:

$$\mathrm{Tr}(\eta\Gamma_\alpha) = 0 \qquad \Gamma^i_{j\alpha} = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha Z^k \Gamma^i_{jk}$$

  - Logarithmic Divergences:

$$\mathrm{Tr}(\eta\Gamma_\alpha\Gamma_\beta) = \mathrm{Tr}(\eta\Gamma_\alpha T^a) = \mathrm{Tr}(\eta(T^a)^2) = 0$$

  - Linear Divergences:

$$\mathrm{Tr}(\eta\phi G \cdot \tilde{G}) = 0$$

  - Where $G_{\mu\nu}$ is the field strength associated with the fermion connections

$$G_{\mu\nu} = [D_\mu, D_\nu] = -\Gamma^C_{D\mu\nu} + iF^a_{\mu\nu}(T^a)^C_D + \frac{1}{2}X_{\mu\nu}\delta^C_D$$

  - And is the parameter for a chiral rotation corresponding to a T-Duality transformation:

$$\phi^C = \left(\frac{1}{2} - \alpha^C - \beta^C\right) F - \sum_i F^i q^C_i$$

Anomaly Result

- After satisfying the regularization conditions & anomaly matching conditions, we find

$$\delta\mathcal{L}_{anom} = b\int d^4\theta E(F+\bar{F})\Omega$$
$$= \frac{b\sqrt{g}\mathrm{Im}(F)}{4}\left(F_a\cdot\tilde{F}^a + R\cdot\tilde{R} + \frac{1}{2}\sum_i g^i\cdot\tilde{g}^i + \cdots\right)$$

$$8\pi^2 b = \frac{1}{24}\left(2\sum_p q_n^p - N + N_G - 21\right)$$
$$= C_a - C_a^M + 2\sum_b C_a^b q_n^b$$
$$= 30$$

Anomaly Cancellation

- In the chiral superfield description of the dilaton, we have a tree level coupling between the dilaton and the Chern-Simons anomaly superfield of the form

$$\mathcal{L} = \int d^4\theta E(S + \bar{S})\Omega = -\frac{1}{8}\int d^4\theta \frac{E}{R} S\Phi + \text{h.c.}$$

- So that the anomaly can be cancelled by allowing the dilaton to transform under a modular transformation as

$$\Delta S = -bF$$

Conclusions & Future Work

- We have shown that, for at least two examples, string-derived supergravities can be regulated in such a way that the anomalies take a universal form and can be eliminated with the GS mechanism

- Next step is to apply the procedure to more realistic string compactifications to make statements about the phenomenology of string models

Bibliography

- [1] M.K. Gaillard, J.M. Leedom - arxiv:
- [2]. D. Butter, M.K. Gaillard - arXiv: 1410.6192
- [3]. C. Scruccca, M. Serone - arXiv:hep-th/0012124
- [4] G. Cardoso, B. Ovrut – arxiv: hep-th/9205009
- [5] D. Freedman, B. Kors – arxive: hep-th/0509217
- [6] J.P. Derendinger, S. Ferrara, C. Kounnas, F. Zwirner - Nucl.Phys. B372 (1992) 145-188 (1992)