Quasi-fixed points from scalar sequestering and the little hierarchy problem in supersymmetry

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The **little hierarchy problem** in SUSY:

$m_Z^2 = -2(|\mu|^2 + m_{H_u}^2) + \mathcal{O}(1/\tan^2\beta) +$ loop corrections.

Radiative corrections enhanced by large logarithms make $m_{H_u}^2$ sensitive to gluino and top-squarks with order 1 coefficients. Naively, suggests a worse than 1% level fine-tuning cancellation between μ^2 and $m_{H_u}^2$.

However, this conclusion should be examined critically.

All we really need is that the particular combination:

$$\widehat{m}^2_{H_u} \quad \equiv \quad m^2_{H_u} + |\mu|^2$$

is small, even if $|\mu|^2$ and $m^2_{H_u}$ are individually large. Can renormalization group running do this?

If Q is the renormalization scale, then near a conformal fixed point, could have power-law renormalization group running:

$$\widehat{m}^2_{H_u}(Q) \quad = \quad \left(\frac{Q}{M_*}\right)^{\Gamma} \widehat{m}^2_{H_u}(M_*),$$

where M_* is some very large input scale (perhaps the GUT or Planck scale).

We want a scaling dimension Γ that is positive and large.

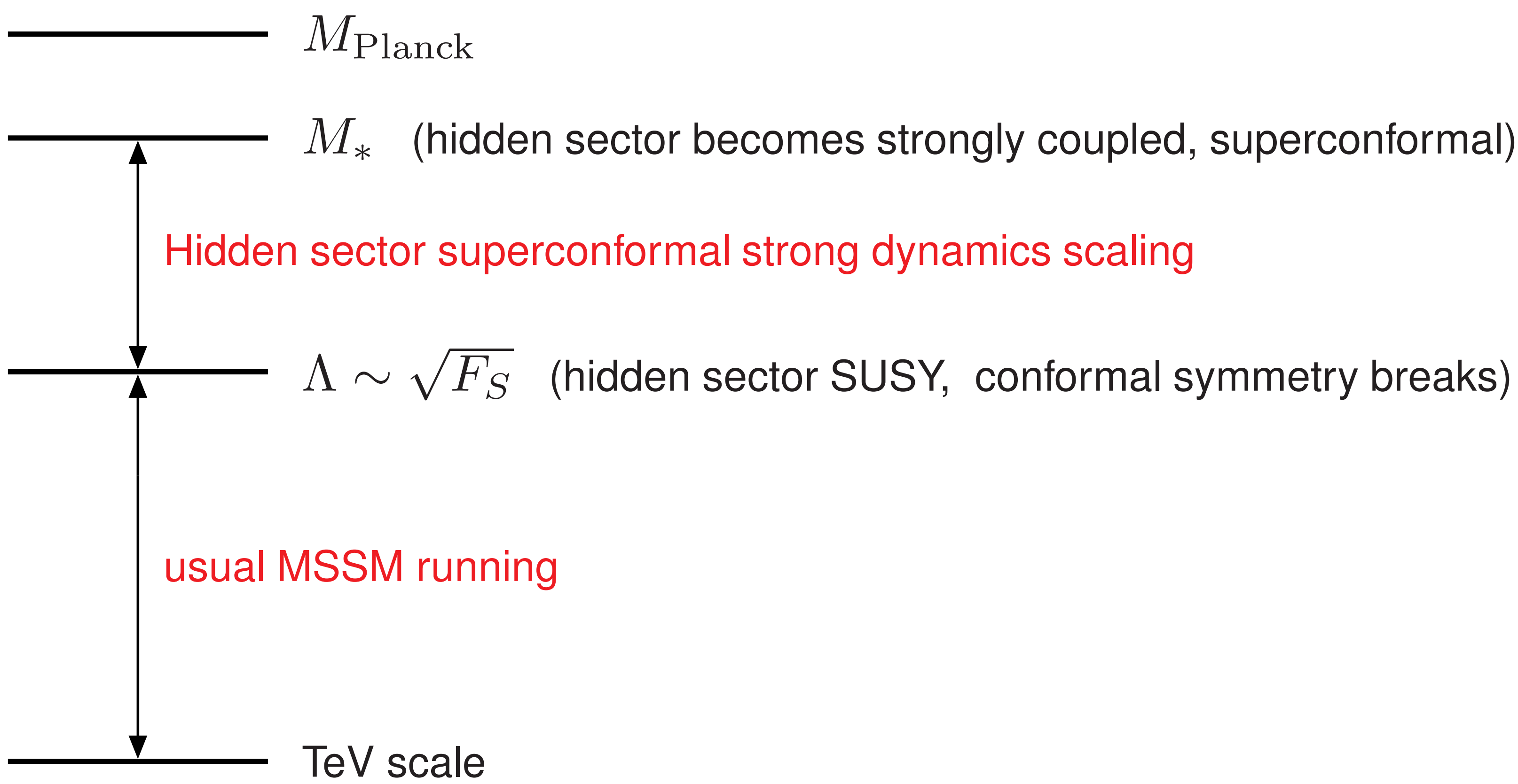
The setup:

- SUSY is broken in a hidden sector, parameterized by F_S ,
- The chiral superfield S that contains F_S is part of a strongly coupled theory ,
- SUSY breaking is communicated to the MSSM (visible) sector by non-renormalizable Lagrangian terms suppressed by a scale M_* ,
- Above a scale $\Lambda \sim \sqrt{F_S}$, which is supposed to be much less than M_*, the strongly coupled theory is approximately conformal, so there is power-law renormalization group running ,
- Scalar squared masses are driven towards 0 by renormalization group running.

This is **scalar sequestering**.

Roy and Schmaltz 0708.3593; Murayama, Nomura, Poland 0709.0775;
Perez, Roy, Schmaltz 0811.3206, . . .

The Big Picture: scales and running



Naively, expect relative suppression factor $(\Lambda/M_*)^\Gamma$ for scalar squared masses.

An important subtlety from Murayama, Nomura, Poland, 0709.0775 and Perez, Roy, Schmaltz, 0811.3206:

The Higgs squared masses that have hidden-sector superconformal scaling are the **combined** SUSY-breaking and SUSY-preserving ones:

$$\begin{aligned} \widehat{m}^2_{H_u} &\equiv m^2_{H_u} + |\mu|^2, \\ \widehat{m}^2_{H_d} &\equiv m^2_{H_d} + |\mu|^2 \end{aligned}$$

This seems like just what we want to cure the SUSY little hierarchy problem!

Generic notatations M_A and m_i^2 for parameters of mass dimensions 1 and 2:

$$\begin{aligned} M_A &= \text{gaugino masses, } a \text{ terms, and the } \mu \text{ term,} \\ m_i^2 &= \text{squark and slepton squared masses, } \widehat{m}^2_{H_u}, \widehat{m}^2_{H_d}, \text{ and } b, \end{aligned}$$

Then renormalization group equations above scale Λ are:

$$\begin{aligned} \frac{d}{dt} M_A &= \beta^{\text{MSSM}}_{M_A}, \qquad \text{(run as usual!)} \\ \frac{d}{dt} m_i^2 &= \Gamma m_i^2 + \beta^{\text{MSSM}}_{m_i^2}, \end{aligned}$$

where

$$t \equiv \ln(Q/Q_0).$$

We now know Γ can't be too large:

$$\Gamma \lesssim 0.3$$

from conformal bootstrap, Poland, Simmons-Duffin, Vichi, 1109.5176; Poland and Stergiou, 1509.06368.

Classic (2008) version of scalar sequestering

At the scale $Q = \Lambda$, boundary conditions from power-law suppression:

$$\widehat{m}^2_{H_u},\ \widehat{m}^2_{H_d},\ b,\ m^2_{\rm squarks},\ m^2_{\rm sleptons}\ \approx\ 0.$$

Prediction: light scalars including all Higgs bosons; heavy gauginos, heavy Higgsinos.

Unfortunately, the classic prediction is somewhat too naive. Some issues that limit the power-law suppression:

- Γ cannot be very large (now know $\lesssim\ 0.3$),
- The range of scales over which the superconformal scaling takes place is limited to $Q > \Lambda \sim \sqrt{F_S}\ \gtrsim\ 10^{10}$ GeV.
- **Need to include visible sector running as well.**

Instead of power-law running to 0 in the infrared, dimension-2 terms will run towards quasi-fixed trajectories where the beta functions vanish:

$$m^2_{i,\,\text{quasi-fixed}} \approx -\beta^{\text{MSSM}}_{m_i^2}/\Gamma.$$

These quasi-fixed points are moving targets, in reality may not be reached as one runs down to Λ.

Below the scale Λ, the hidden sector superconformal scaling is broken, and the running continues with $\Gamma = 0$ and the usual $\beta^{\text{MSSM}}_{m_i^2}$

Fortunately, MSSM scalar squared mass beta functions are negative, and dominated by gaugino masses. Reduces flavor violation.

For squarks, including only gluino contribution for simplicity:

$$m_{\tilde{q},\text{quasi-fixed}} \quad \approx \quad \sqrt{\frac{2}{3}} \frac{g_3 M_{\tilde{g}}}{\pi\sqrt{\Gamma}} = 0.365 \left(\frac{g_3}{0.77}\right) \left(\frac{0.3}{\Gamma}\right)^{1/2} M_{\tilde{g}}.$$

This quasi-fixed point is often reached, but running below the scale Λ increases the squark masses substantially.

Still, $M_{\rm squark} < M_{\rm gluino}$ is a fairly robust prediction.
(See numerical examples below.)

More importantly, what about quasi-fixed point for Higgs squared mass?

$$\widehat{m}^2_{H_u,\text{quasi-fixed}} \approx \frac{3}{8\pi^2\Gamma}\Big[g_2^2(M_2^2+\mu^2)+\frac{g_1^2}{5}(M_1^2+\mu^2) \\ -a_t^2-\mu^2(y_b^2+2y_\tau^2)-y_t^2(m^2_{Q_3}+m^2_{u_3})\Big].$$

For two reasons, I don't view this as a complete solution to the SUSY little hierarchy problem:

- Prefactor $\frac{3}{8\pi^2\Gamma}$ is no smaller than about 0.12
- Running below scale Λ is also significant

However, it has some helpful features:

- Terms of both signs, so cancellation can occur
- Predictive! Correlations between different parameters

Numerical examples

Input parameters at scale $M_* = M_{\rm GUT} = 2.5 \times 10^{16}$ GeV:

- Gaugino masses M_1, M_2, M_3,
- Higgsino mass μ,
- Common scalar[3] parameter A_0
- Common scalar squared mass m_0^2 (dependence on scalar squared masses is weak, due to quasi-fixed point behavior, but not negligible)

Require $M_Z = 91.2$ GeV and $\tan\beta$ fixed: in practice, this allows us to solve for μ and A_0.

Also demand $123\,{\rm GeV} < M_h < 127\,{\rm GeV}$; very roughly fixes M_3.

Example Model Line: non-unified gaugino masses

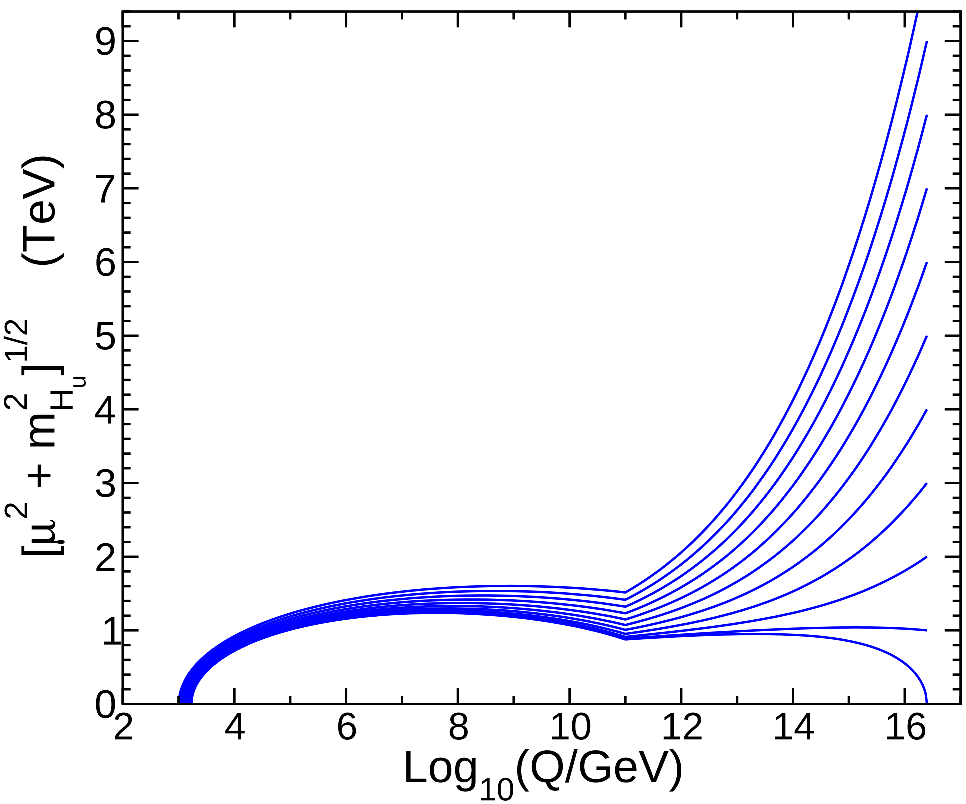
Assume fixed $\tan\beta = 15$ and at the unification scale:

$$\begin{aligned} M_3 &= \text{1200 GeV}, \\ M_2 &= \text{4100 GeV}, \\ M_1 &= \text{2400 GeV}. \end{aligned}$$

Take m_0 variable, and solve for μ and A_0.

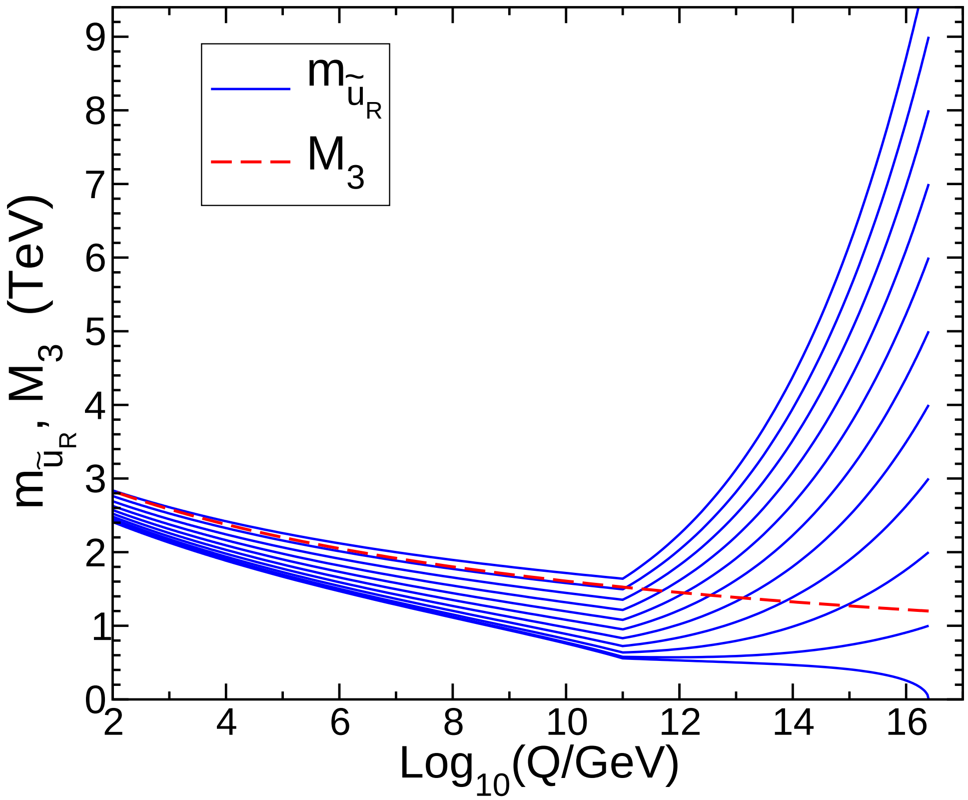
In this case, the solved-for A_0 is negative and large in magnitude, so get large top-squark mixing. This in turn allows $M_h \approx 125$ GeV with relatively light top squarks. That's why M_3 can be so much smaller.

Renormalization group running of $\widehat{m}^2_{H_u} = \mu^2 + m^2_{H_u}$:



I wouldn't claim a complete solution to the SUSY little hierarchy problem, but subjectively, the smaller $m^2_{H_u} + \mu^2$ at the quasi-fixed point suggests less "tuning" than in traditional models.

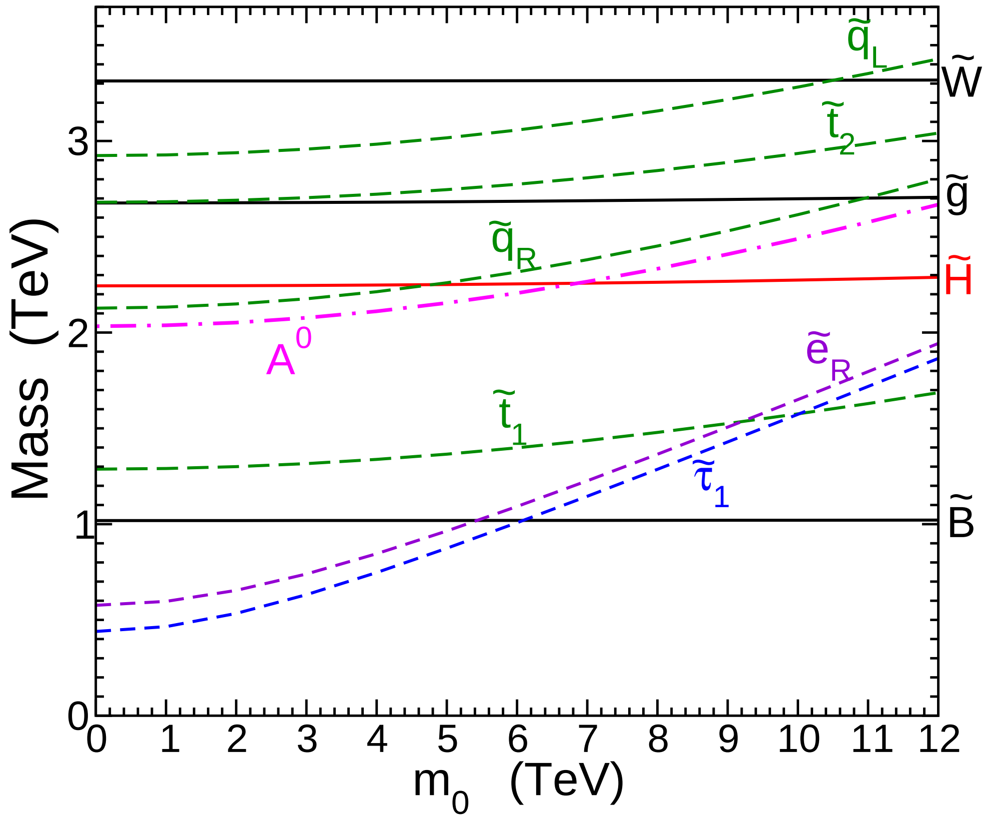
Renormalization group running of squark, gluino masses:



Squarks and gluino below 3 TeV, consistent with $M_h = 125$ GeV.

Within striking distance of the LHC!

Features of the superpartner mass spectrum with non-unified gaugino masses:



$M_h \approx 125$ GeV, nearly independent of high-scale m_0.

Higgsino still very heavy, Winos could be the heaviest superpartners.

Model not excluded by the LHC, but not hopeless for eventual LHC discovery.

Conclusion:

- Interplay between visible sector renormalization and hidden sector superconformal scaling: quasi-fixed point behavior with predictive power
- According to my subjective standards, some improvement in the SUSY little hierarchy problem, but not a completely satisfying "solution".
- Results are more optimistic with non-unified gaugino masses, in particular $M_2 > M_3$.
- Hope for SUSY discovery at LHC.

"We are, I think, in the right Road of Improvement, for we are making Experiments."

– Benjamin Franklin

BACKUP

For sleptons:

$$m_{\tilde{e}_R,\text{quasi-fixed}} \approx \sqrt{\frac{3}{10}\frac{g_1 M_1}{\pi\sqrt{\Gamma}}} = 0.18\left(\frac{g_1}{0.57}\right)\left(\frac{0.3}{\Gamma}\right)^{1/2} M_1,$$

where M_1 = bino mass parameter.

Running below the scale Λ increases the selectron mass, but naively the LSP (Lightest SUSY Particle) is a charged slepton. To avoid disaster in cosmology from charged stable particle:

- R-parity violation allows slepton LSP to decay
- Quasi-fixed point not quite reached, and LSP is neutralino (see numerical examples soon. . .)

How small can the scale Λ be? (Knapen and Shih, 1311.7107)

Gaugino mass estimate at the scale Λ is

$$M_{\rm gaugino} \;=\; c_a \left(\frac{F_S}{M_*}\right)\left(\frac{\Lambda}{M_*}\right)^{\gamma_S}.$$

So, using $\Lambda \gtrsim \sqrt{F_S}$, and taking c_a of order unity, and requiring $M_{\rm gaugino} \gtrsim 1000\ {\rm GeV}$, we need:

$$\Lambda \gtrsim [(1000\ {\rm GeV}) M_*^{1+\gamma_S}]^{1/(2+\gamma_S)}.$$

Using the indications from the conformal bootstrap for $\gamma_S = 3/7$, and taking $M_* = M_{\rm GUT} = 2.5 \times 10^{16}\ {\rm GeV}$, we need:

$$\Lambda \gtrsim \sqrt{F_S} \gtrsim 8 \times 10^{10}\ {\rm GeV}$$

In the following, for numerical examples I will optimistically take:

$$\Gamma = 0.3, \qquad M_* = M_{\rm GUT}, \qquad \Lambda = 10^{11}\ {\rm GeV}.$$

Communication of supersymmetry breaking to the MSSM sector:

$$\mathcal{L}_{\text{gaugino masses}} = -\frac{c_a}{2M_*}\int d^2\theta\, S\,\mathcal{W}^{a\alpha}\mathcal{W}^a_\alpha + \text{c.c.}$$

$$\mathcal{L}_{a\text{ terms}} = -\frac{c^{ijk}}{6M_*}\int d^2\theta\, S\phi_i\phi_j\phi_k + \text{c.c.}$$

$$\mathcal{L}_{\mu\text{ term}} = \frac{c_\mu}{M_*}\int d^4\theta\, S^* H_u H_d + \text{c.c.}$$

$$\mathcal{L}_{b\text{ term}} = \frac{c_b}{M_*^2} Z_{S^*S}\int d^4\theta\, S^*S H_u H_d + \text{c.c.}$$

$$\mathcal{L}_{m^2\text{ terms}} = -\frac{c_i^j}{M_*^2} Z_{S^*S}\int d^4\theta\, S^*S\phi^{*i}\phi_j,$$

Key feature: the last two terms are **non-holomorphic** in S, so they have an additional scaling factor $Z_{S^*S} \sim (Q/Q_0)^\Gamma$.

Dimension-2 terms (scalar squared masses) have extra power-law suppression compared to dimension-1 (gaugino masses, scalar cubic couplings, μ term).

To realize this, need a positive exponent from scaling dimensions:

$$\Gamma = \Delta_{S^*S} - 2\Delta_S,$$

in which

- Δ_{S^*S} is the scaling dimension for the operator S^*S, and
- $\Delta_S = 1 + \gamma_S$ is the scaling dimension for S.

Does such a superconformal theory exist?

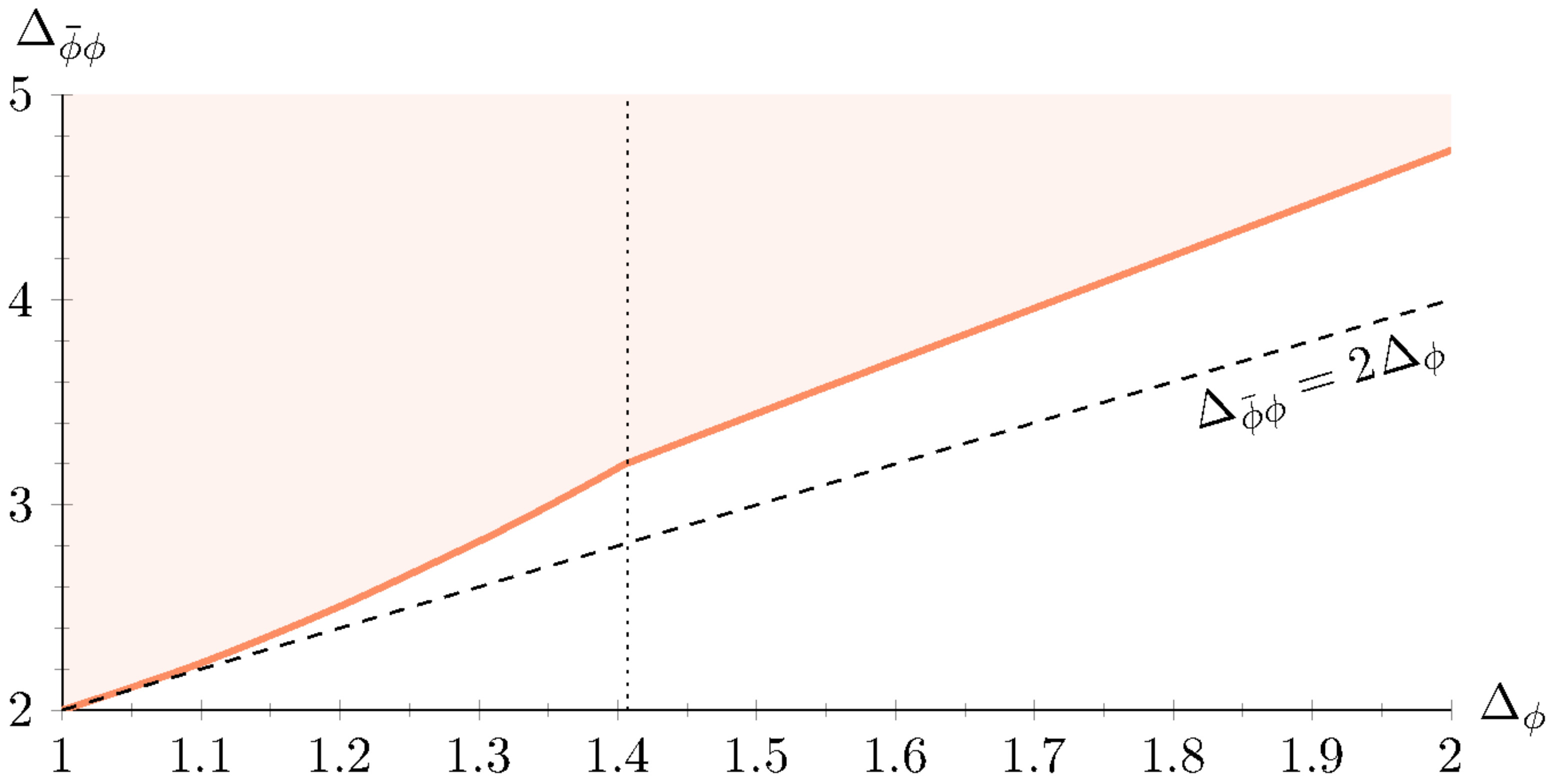
If so, what can one say about Γ and Δ_S?

No actual models with positive Γ are known, but. . .

There are now strong constraints and hints from the conformal bootstrap:

Poland, Simmons-Duffin, Vichi, 1109.5176; Poland and Stergiou, 1509.06368.

From Poland and Stergiou, 1509.06368, shaded is excluded:



- $\Gamma = \Delta_{S*S} - 2\Delta_S$ can be positive, but is bounded from above
- “Kink” near $\Delta_S = 10/7$, circumstantial evidence a theory exists near there?
- For $\Delta_S = 10/7$, find that $\Gamma \lesssim 0.3$
- For smaller Δ_S, Γ is constrainted to be (much) smaller

Example Model Line 1: unified gaugino masses

Assume $M_1 = M_2 = M_3 \equiv m_{1/2}$.

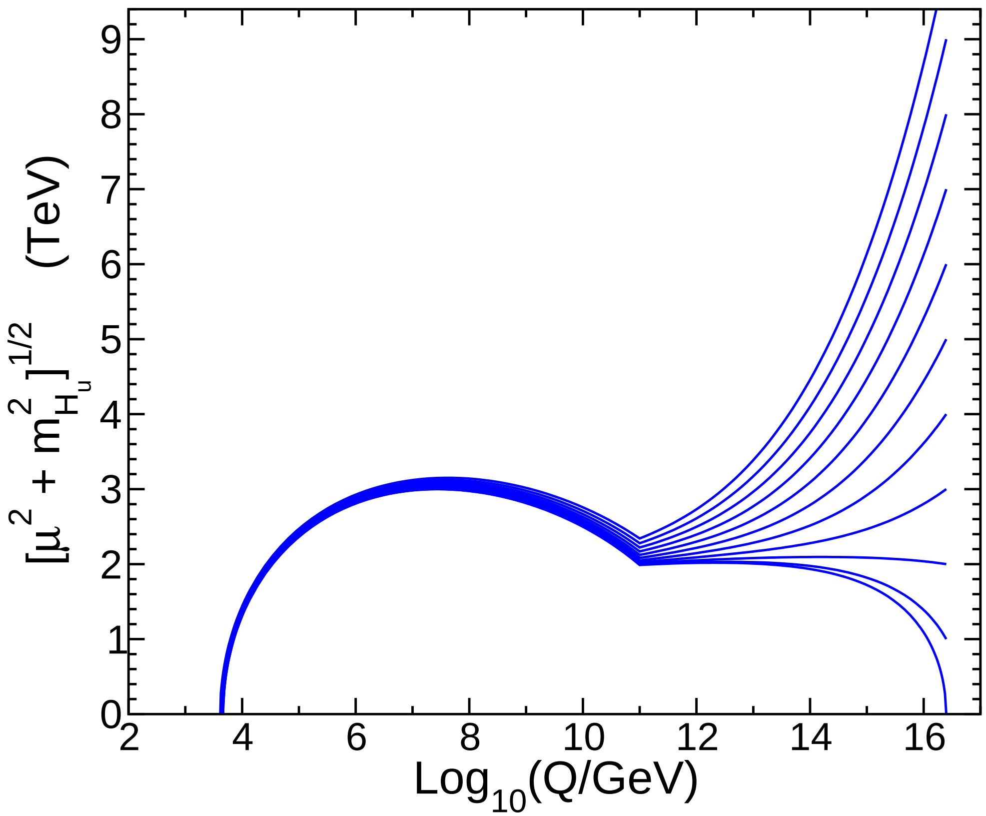
- Free parameters: $m_{1/2},\ m_0,\ \tan\beta$
- Solved for using electroweak symmetry breaking: μ, A_0

It turns out that one can only get the correct $M_Z = 91.2$ GeV with small positive A_0, so that top-squark mixing is moderate.

This in turn requires that $m_{1/2}$ is large, to give heavy top squarks, to allow $M_h = 125$ GeV.

A typical range of allowed values is 2.7 TeV $\lesssim\ m_{1/2}\ \lesssim\ 8.5$ TeV.

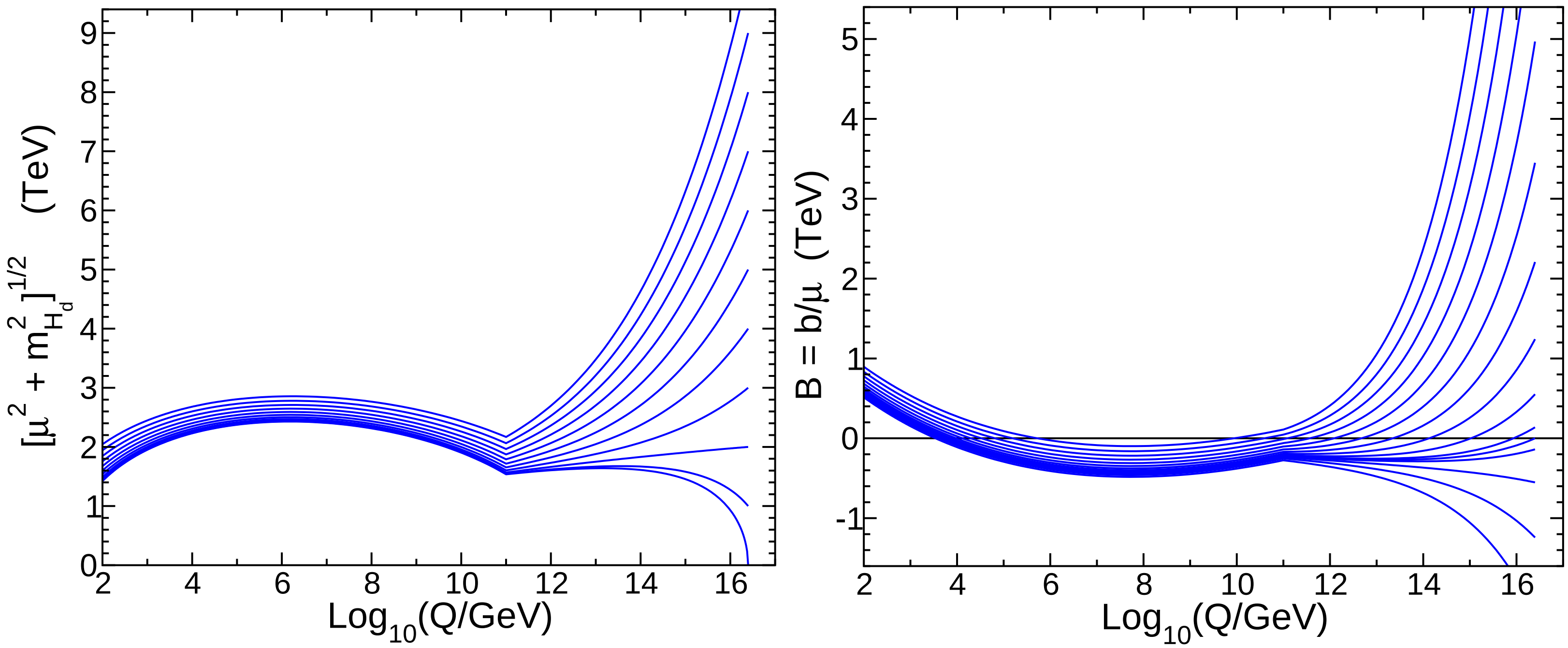
Renormalization group running of $\widehat{m}^2_{H_u}$, for $m_{1/2} = 4.5$ TeV, $\tan\beta = 15$:



Lines = different m_0 input values at the high scale $M_* = M_{\mathrm{GUT}}$.

Quasi-fixed point focusing behavior near 2 TeV, and further focusing behavior below scale $\Lambda = 10^{11}$ GeV. Still needs some “tuning”.

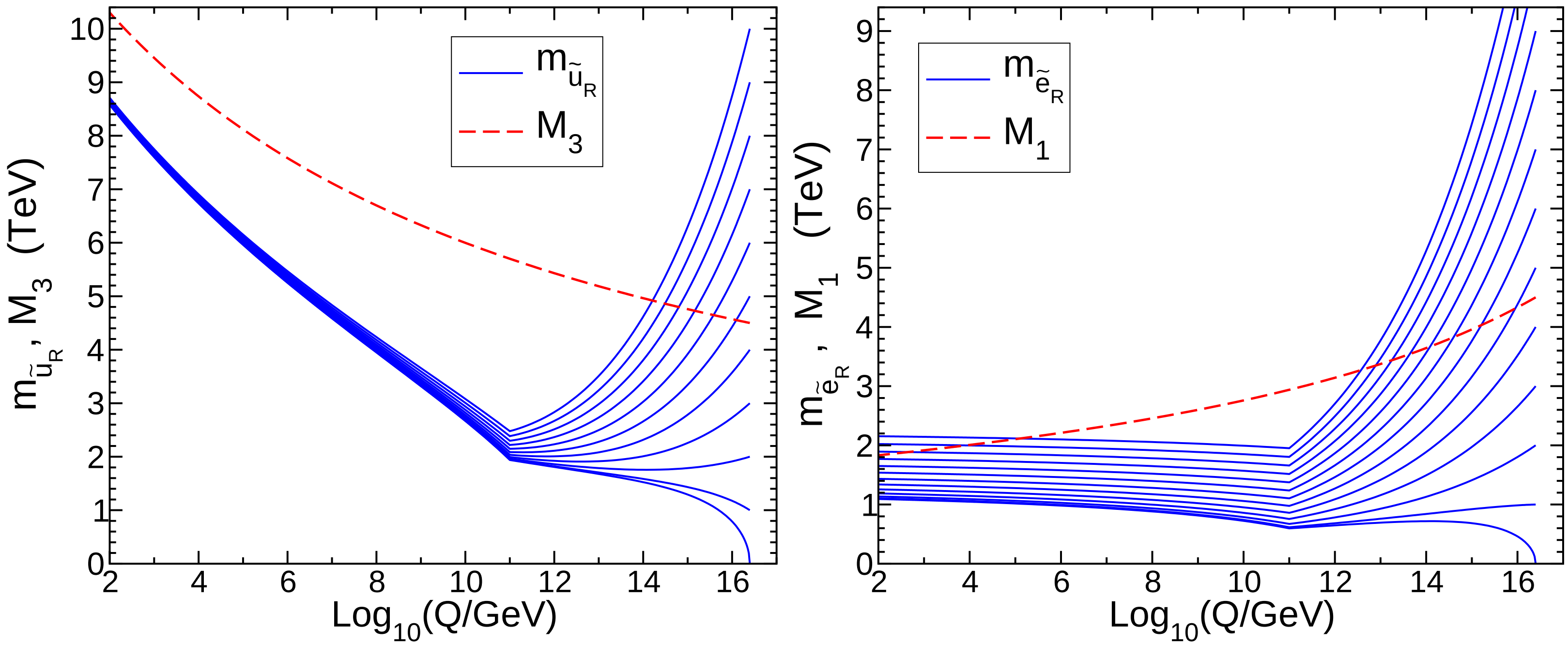
Renormalization group running of $\widehat{m}^2_{H_d}$, B, for $m_{1/2} = 4.5$ TeV:



Quasi-fixed point trajectory is somewhat less robustly attractive.

Small B is easy to achieve; one of the classic motivations for scalar sequestering.

Renormalization group running of squark, slepton masses, for $m_{1/2} = 4.5$ TeV:

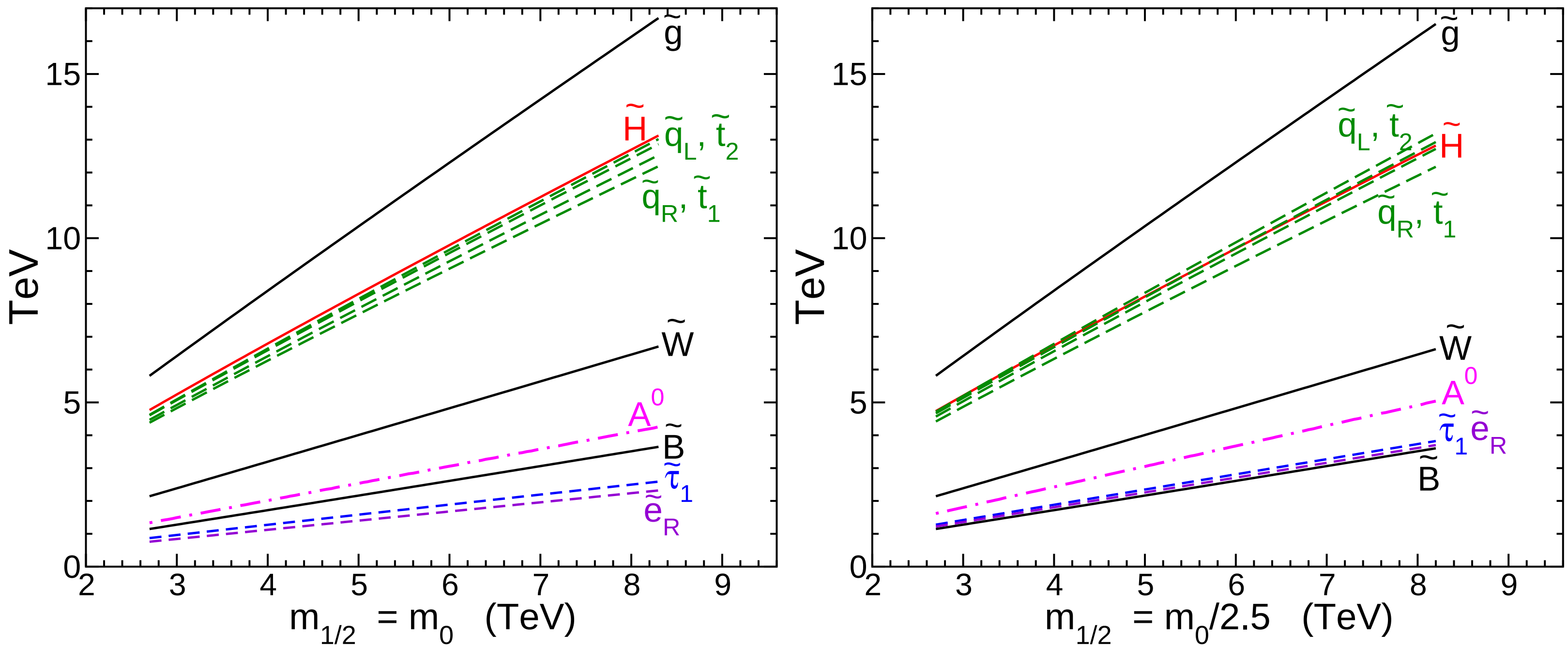


The squarks are lighter than gluino; quasi-fixed point not so important for squarks, because SUSYQCD running below Λ dominates.

Slepton masses less strongly attracted to quasi-fixed point, running below Λ is weak.

If $m_0 \lesssim 2.5 m_{1/2}$, then LSP is a charged slepton.
If $m_0 \gtrsim 2.5 m_{1/2}$, then LSP is a bino-like neutralino.

Sample mass spectra for $m_{1/2} = 4.5$ TeV, $\tan\beta = 15$, and two different assumptions for m_0:



Horizontal range shown corresponds to 123 GeV $< M_h <$ 127 GeV.

New particles safely out of reach of present LHC and future upgrades.