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The NNLO QCD soft function for 1-jettiness

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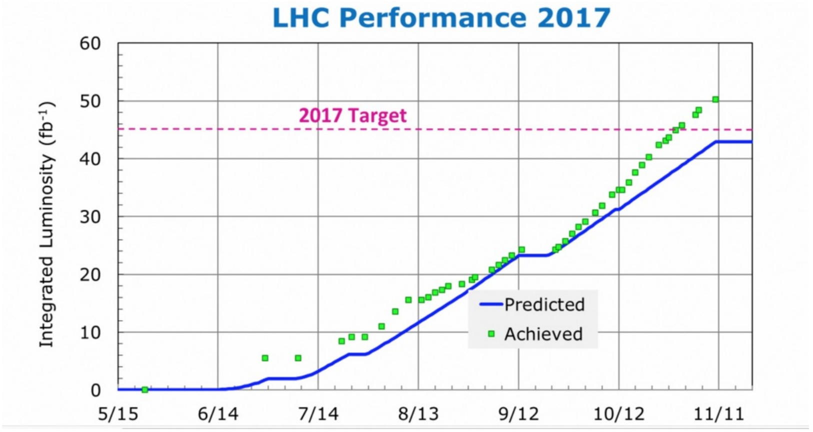
Introduction and motivation

The continued successful operation of the Large Hadron Collider (LHC) has led to the accumulation of a very large data set with which to study the Standard Model (SM) in unprecedented detail



The ever-increasing precision of the experimental analyses mandates a similar increase in the precision of the corresponding theoretical predictions

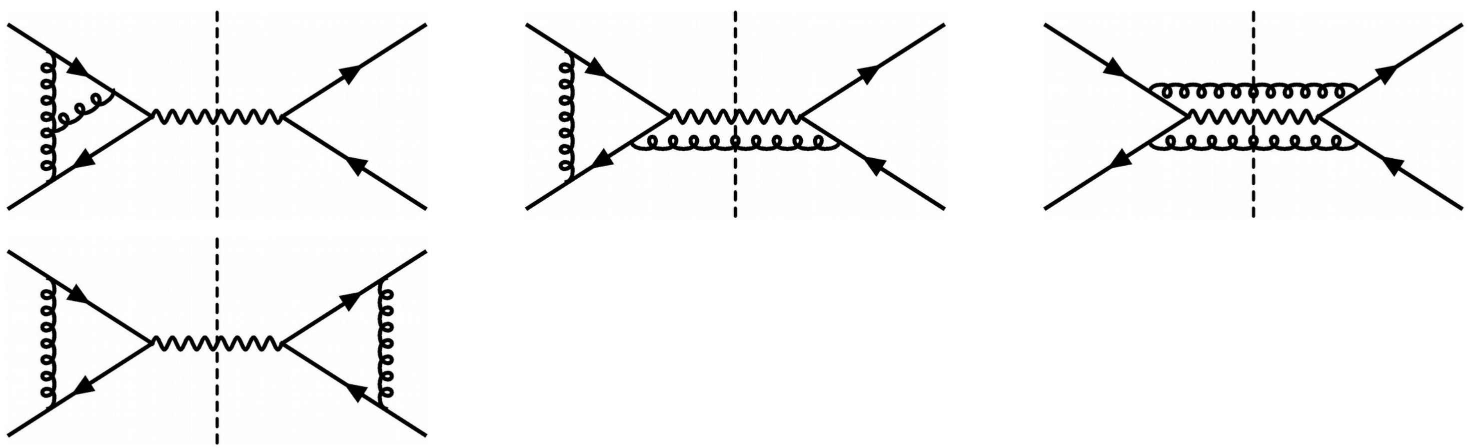
We need predictions accurate to at least **next-to-next-to-leading order** (NNLO) in QCD (possibly also NLO EW, NNLO QCDxEW, etc) to successfully compare theory and experiment

Over the last few years a concerted effort has been made in the theoretical community to provide predictions accurate to NNLO QCD

Next-to-next-to-leading order calculations

At NNLO the calculation of a cross section involves phase-space integrals of differing dimensionality:

$$\sigma_{\mathrm{NNLO}} = \int |\mathcal{M}_{VV}|^2 \, \mathrm{d}\Phi_n + \int |\mathcal{M}_{RV}|^2 \, \mathrm{d}\Phi_{n+1} + \int |\mathcal{M}_{RR}|^2 \, \mathrm{d}\Phi_{n+2}$$



Each term is **divergent**. Three types of singularities occur:

- **Ultraviolet**: in loop integrals. Removed through renormalization
- **Infrared soft**: radiation momentum vanishes
- **Infrared collinear**: angle between massless particles vanishes

Phase-space slicing

Since singularities cancel only when combined in suitably-inclusive observables, we need a manageable way to handle them in component pieces of the NNLO calculation. One possible approach: **Phase-space slicing** [Giele, Glover]

Use a global parameter to divide the phase space into (at least) **two** regions:

region that includes all of the doubly-unresolved emissions

use factorization theorems to compute the cross section in this region

region that has at most one singly-unresolved parton.

this is a NLO calculation (use existing NLO technology)

q_T subtraction [Catani, Grazzini] (for color-singlet final states only)

N-jettiness subtraction [Boughezal, Focke, Liu, Petriello; Gaunt, Stahlhofen, Tackmann, Walsh]

N-jettiness

Event-shape variable N-jettiness [Stewart, Tackmann, Waalewijn 1004.2489]

$$\mathcal{T}_N = \sum_m \min_r \{2\hat{p}_r \cdot q_m\}$$

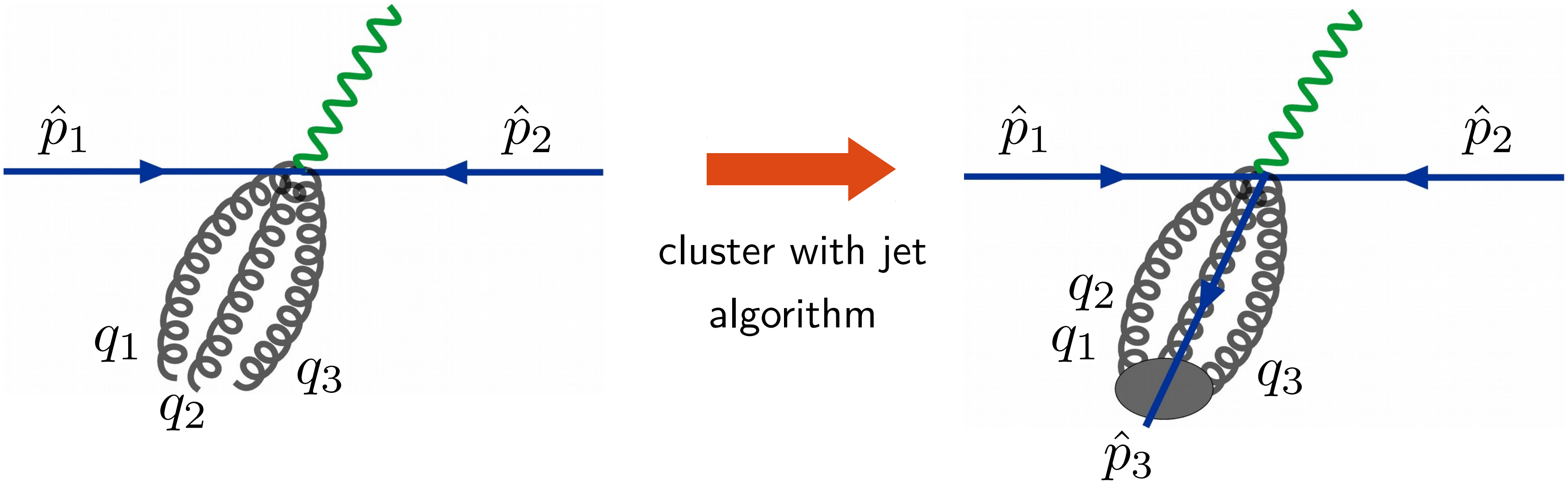
- N = number of final-state jets in the scattering event at Born level

- $\hat{p}_r$ are the directions of the initial-state partons and final-state jets at Born level (N=0 and r=1,2 for LHC color-singlet production, N=1 and r=1,2,3 for LHC processes with one final-state jet)

- q_m denote the momenta of final-state radiation (m=1 for NLO real and NNLO RV corrections, m=1,2 for NNLO double-real corrections)

N-jettiness

e.g. $X + j$ at NNLO



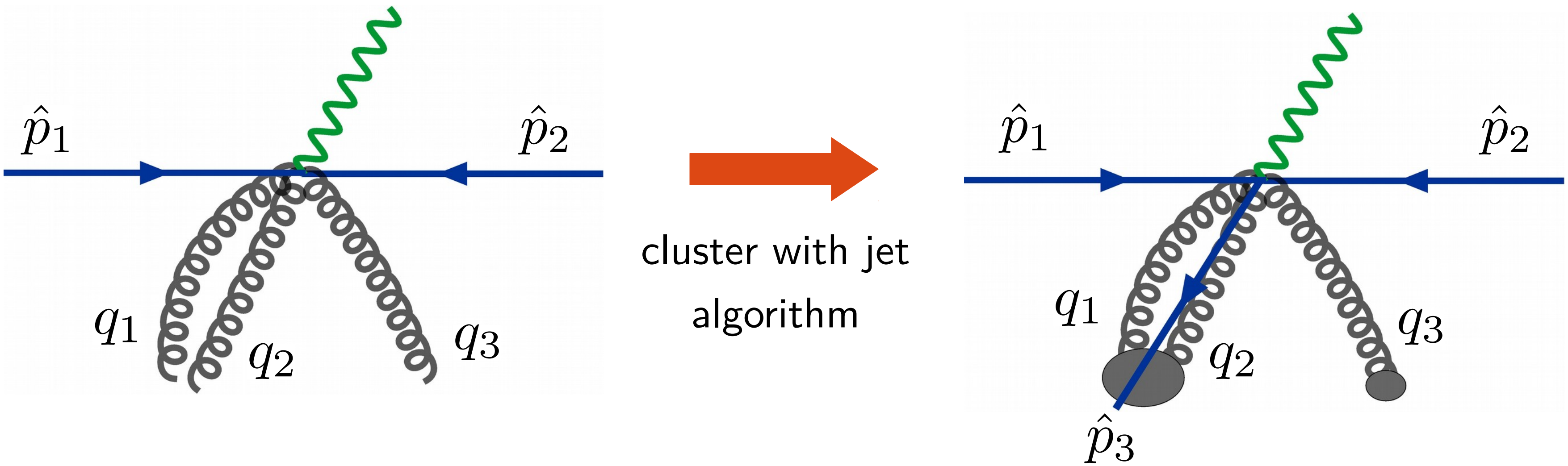
$$\mathcal{T}_N = \sum_m \min_{r=1,2,3} \{2\hat{p}_r \cdot q_m\} \approx 0$$

Doubly-unresolved region

All radiation is either soft or collinear

N-jettiness

e.g. $X + j$ at NNLO



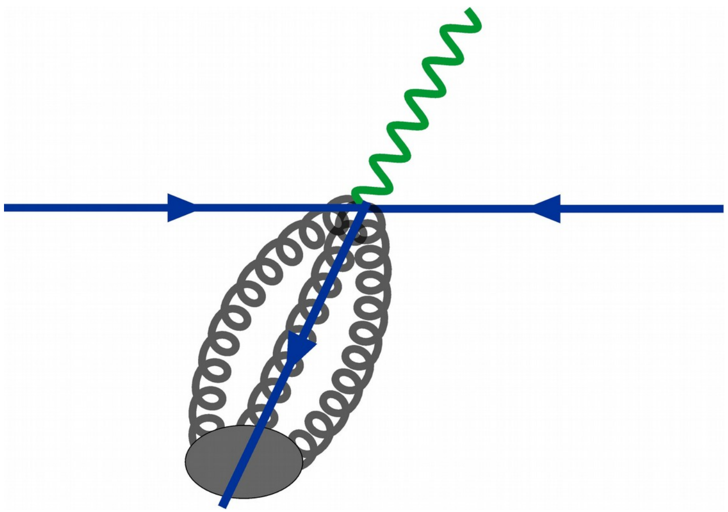
$$\mathcal{T}_N = \sum_m \min_{r=1,2,3} \{2\hat{p}_r \cdot q_m\} > 0$$

Singly-unresolved region

At least one parton is resolved

N-jettiness

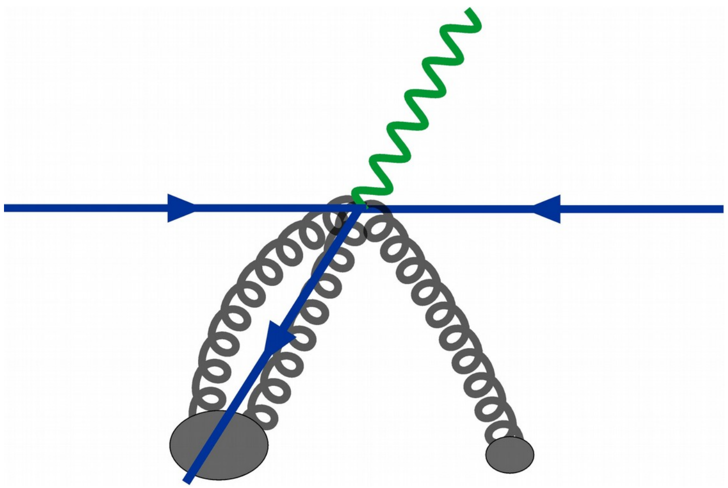
$$\text{e.g. } X + j \text{ at NNLO}$$



Doubly-unresolved region $\mathcal{T}_N \approx 0$

All radiation is either soft or collinear

Introduce $\mathcal{T}_N^{\mathrm{cut}}$ and use factorization theorem for region $\mathcal{T}_N < \mathcal{T}_N^{\mathrm{cut}}$



Singly-unresolved region $\mathcal{T}_N > 0$

At least one parton is resolved

NLO calculation of $X + 2j$ in the region $\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}$

N-jettiness: factorization theorem

$$\sigma_{\rm NNLO} = \sigma_{\rm NNLO}\left[\theta(\mathcal{T}_N^{\rm cut} - \mathcal{T}_N) + \theta(\mathcal{T}_N - \mathcal{T}_N^{\rm cut})\right]$$
$$= \sigma_{\rm NNLO}(\mathcal{T}_N < \mathcal{T}_N^{\rm cut}) + \sigma_{\rm NNLO}(\mathcal{T}_N > \mathcal{T}_N^{\rm cut})$$

$\sigma_{\rm NNLO}(\mathcal{T}_N > \mathcal{T}_N^{\rm cut})$ can be computed as a NLO calculation using any available NLO code

Factorization theorem based on SCET for the cross section in the region of small $\mathcal{T}_N$ [Stewart, Tackmann, Waalewijn 0910.0467]:

$$\sigma(\mathcal{T}_N < \mathcal{T}_N^{\rm cut}) = \int B \otimes B \otimes S \otimes H \otimes \prod_{i=1}^{N} J_i + \mathcal{O}(\mathcal{T}_N^{\rm cut})$$

- ***B*** denotes the **beam function**, which describes initial-state collinear radiation
- ***J*** is the **jet function**, which describes final-state collinear radiation
- ***H*** represents the **hard function**, which is process-specific and finite
- ***S*** is the **soft function**, which describes soft radiation

N-jettiness factorization theorem

$$\sigma(\mathcal{T}_N < \mathcal{T}_N^{\text{cut}}) = \int B \otimes B \otimes S \otimes H \otimes \prod_{i=1}^{N} J_i + \mathcal{O}(\mathcal{T}_N^{\text{cut}})$$

In order to obtain the cross section at NNLO, we need to expand the formula to $\mathcal{O}(\alpha_s^2)$ and therefore we need each function at NNLO:

- ***B*** at NNLO: [Gaunt, Stahlhofen, Tackmann (2014)]
- ***J*** at NNLO: [Becher, Neubert (2006), Becher, Bell (2011)]
- ***H*** at NNLO: e.g. for Z/W+j [Gehrmann, Tancredi (2012)], H+j [Gehrmann, Jaquier, Glover, Koukoutsakis (2012)]
- ***S*** at NNLO: 1-jettiness [Boughezal, Liu, Petriello (2015)]

Provide an independent calculation of the 1-jettiness soft function at NNLO in a form that can be implemented and used in Monte Carlo codes.

Soft function

Perturbative expansion of the N-jettiness soft function:

$$S(\mathcal{T}_N) = S^{(0)}(\mathcal{T}_N) + \left[\frac{\alpha_s}{2\pi}\right] S^{(1)}(\mathcal{T}_N) + \left[\frac{\alpha_s}{2\pi}\right]^2 S^{(2)}(\mathcal{T}_N) + \mathcal{O}(\alpha_s^3)$$

Virtual corrections vanish in DR because of scaling arguments, so only the real corrections contribute. At NNLO:

$$S^{(2)}(\mathcal{T}_N) = -\frac{\beta_0}{2\epsilon} S^{(1)}(\mathcal{T}_N) + S^{(2)}_{RV}(\mathcal{T}_N) + S^{(2)}_{gg,ab}(\mathcal{T}_N) + S^{(2)}_{gg,nab}(\mathcal{T}_N) + S^{(2)}_{q\bar{q}}(\mathcal{T}_N)$$

The Abelian term can be determined from the NLO result, so we focus on the **non-Abelian** contribution

Setup of the calculation (I)

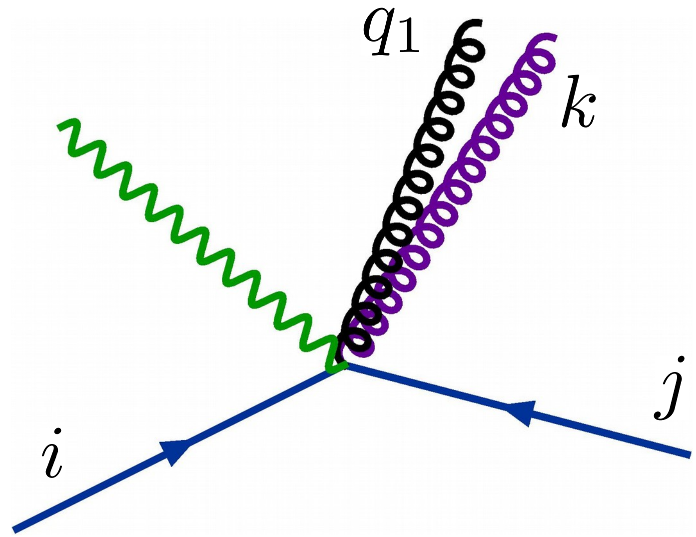
Single emission: $\mathcal{T}_N = \min_r \{q_1^r\}$ $\qquad q^x = 2\hat{p}_x \cdot q$

For the case of **1-jettiness** we have three hard directions: r = i, j, k.

Partition of single-emission phase space:

$$F = F_i + F_j + F_k$$

$$F_k = \delta(\mathcal{T}_N - q_1^k)\,\theta(q_1^i - q_1^k)\theta(q_1^j - q_1^k)$$



Setup of the calculation (II)

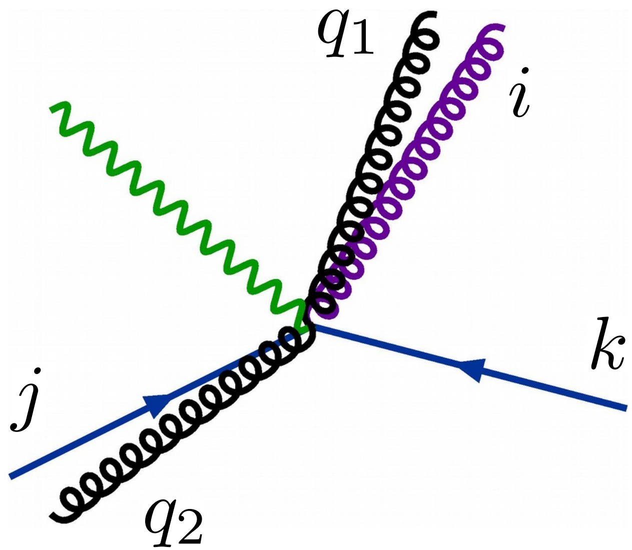
Double emission:

$$\mathcal{T}_N = \min_r \{q_1^r\} + \min_r \{q_2^r\}$$

For the case of **1-jettiness** we have three hard directions: i, j and k.

Partition of double-emission phase space:

$$\begin{aligned} F &= F_{ii} + F_{jj} \\ &+ F_{ij} + F_{ji} \\ &+ F_{ik} + F_{jk} + F_{ki} + F_{kj} \\ &+ F_{kk} \end{aligned}$$



$$F_{ij} = \delta(\mathcal{T}_N - q_1^i - q_2^j)\,\theta(q_1^j - q_1^i)\theta(q_1^k - q_1^i)\,\theta(q_2^i - q_2^j)\theta(q_2^k - q_2^j)$$

Calculation of 1-jettiness soft function at NNLO: overview

We focus on the non-Abelian contribution to the soft function:

$$S^{(2)}_{nab}(\mathcal{T}_N) = -\frac{\beta_0}{2\epsilon}\, S^{(1)}(\mathcal{T}_N) + S^{(2)}_{RV}(\mathcal{T}_N) + S^{(2)}_{gg,nab}(\mathcal{T}_N) + S^{(2)}_{q\bar{q}}(\mathcal{T}_N)$$

- Contribution from **renormalization**: derived from NLO result

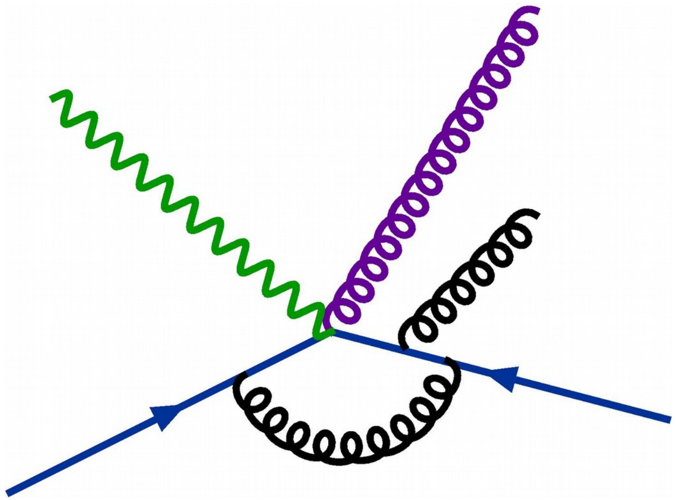
- **Real-virtual** contribution:

$$S^{(2)}_{RV} \propto \sum_{i,j} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \, |\mathcal{M}^{(RV)}_{ij}|^2 \left\{ PS^{(1)} \left[F^{ij}_i + F^{ij}_j + F^{ij}_k \right] \right\}$$

Soft-gluon squared matrix element:

$$|\mathcal{M}^{(RV)}_{ij}|^2 \propto \left(\frac{y_{ij}}{q^i q^j} \right)^{1+\epsilon}$$

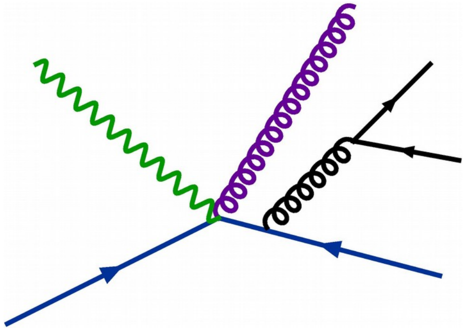
$$y_{ij} = 2\hat{p}_i \cdot \hat{p}_j$$



Calculation of 1-jettiness soft function at NNLO: overview

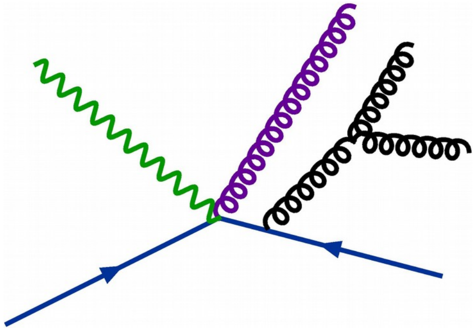
- **Double-real** contribution: emission of soft quark-antiquark pair

$$S^{(2)}_{q\bar{q}} \propto \sum_{i,j} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \, \mathcal{U}_{ij} \; PS^{(2)} \; F^{ij}$$



- **Double-real** contribution: non-Abelian emission of two soft gluons

$$S^{(2)}_{gg,nab} \propto \sum_{i,j} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \, \mathcal{T}_{ij} \; PS^{(2)} \; F^{ij}$$



Calculation of 1-jettiness soft function at NNLO: overview

- Rewrite squared matrix element, phase space, and measurement function using Sudakov decomposition for radiation momenta
- Perform change of variables to map all singularities to unit hypercube
- Introduce additional phase-space partitions to resolve overlapping singularities (sector-decomposition approach)

Singular terms of the type $x^{-1-k\epsilon}$ are expanded in terms of delta and plus distributions:

$$x^{-1-k\epsilon} = -\frac{\delta(x)}{k\,\epsilon} + \sum_{n=0}^{\infty} \frac{(-k\,\epsilon)^n}{n!} \left[\frac{\ln^n(x)}{x}\right]_+$$

where

$$\int_0^1 \mathrm{d}x \left[\frac{\ln^n(x)}{x}\right]_+ f(x) = \int_0^1 \mathrm{d}x\, \frac{\ln^n(x)}{x} \left[f(x) - f(0)\right]$$

Calculation of 1-jettiness soft function at NNLO: overview

The measurement function contains θ-functions of non-trivial arguments. Therefore after expanding all the integrals as Laurent series in ϵ we resort to numerical integrations.

The final expression for the soft function is obtained by expanding the overall factor:

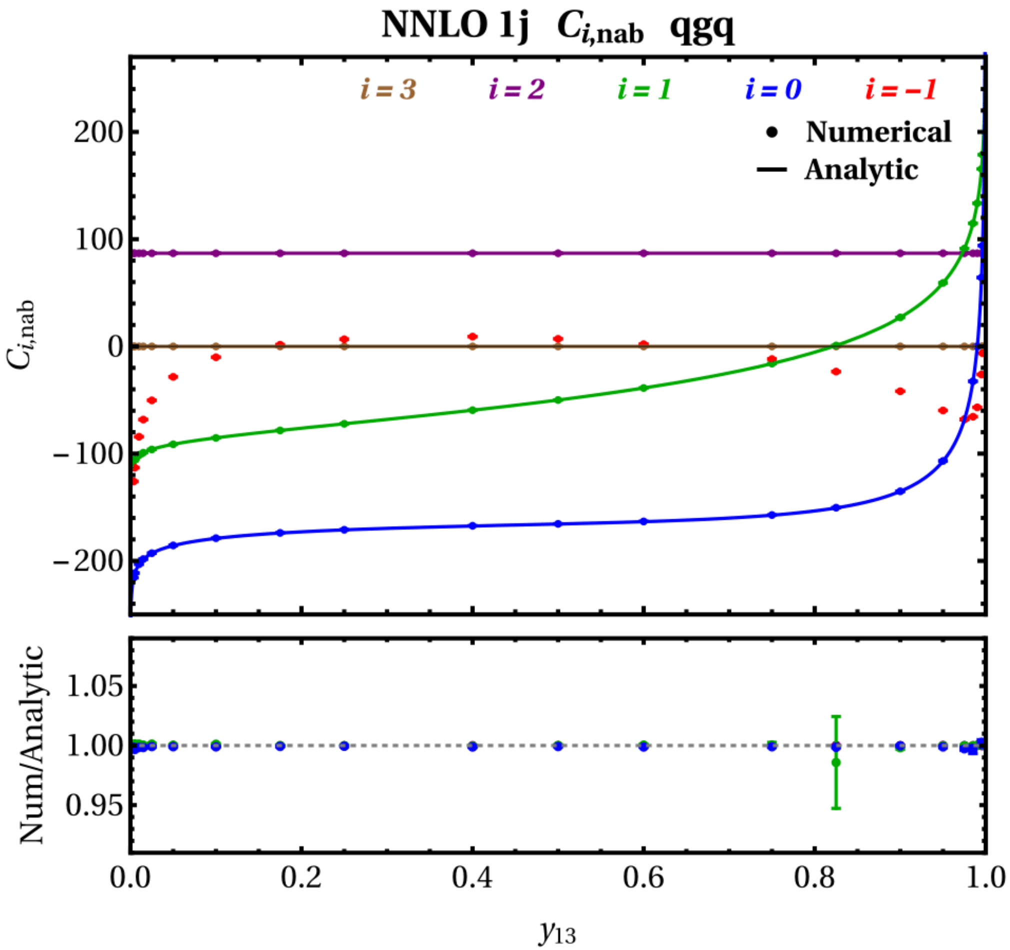
$$\mathcal{T}_1^{-1-4\epsilon} = -\frac{\delta(\mathcal{T}_1)}{4\,\epsilon} + \sum_{n=0}^{\infty} \frac{(-4\,\epsilon)^n}{n!} \mathcal{L}_n(\mathcal{T}_1)$$

The result is then given as

$$S_{nab}^{(2)}(\mathcal{T}_1) = C_{-1}\,\delta(\mathcal{T}_1) + \sum_{i=0}^{3} C_i\,\mathcal{L}_i(\mathcal{T}_1) \qquad\qquad \mathcal{L}_i(\mathcal{T}_1) = \left(\frac{\ln^i(\mathcal{T}_1)}{\mathcal{T}_1}\right)_+$$

Results for NNLO 1-jettiness

$$S^{(2)}_{nab}(\mathcal{T}_1) = C_{-1}\,\delta(\mathcal{T}_1) + \sum_{i=0}^{3} C_i\,\mathcal{L}_i(\mathcal{T}_1)$$



Three partonic configurations:

$$gg \to g \qquad q\bar{q} \to g$$
$$qg \to q$$

LHC kinematics: $\quad y_{ij} = 2\hat{p}_i \cdot \hat{p}_j$

$$y_{12} = 1$$
$$y_{23} = 1 - y_{13}$$

$C_0 - C_3$ are known analytically from renormalization group constraints [Jouttenus et al. (2011), Gaunt et al. (2015)]

Agreement at the per-mille level

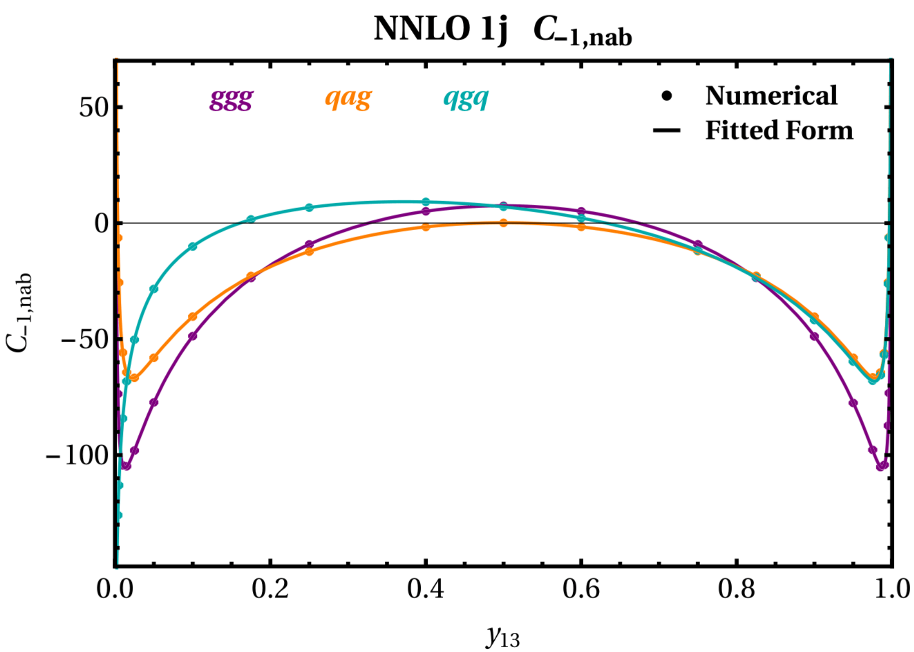
Results for the 1-jettiness soft function at NNLO

$$S^{(2)}_{nab}(\mathcal{T}_1) = {\color{red}C_{-1}}\,\delta(\mathcal{T}_1) + \sum_{n=0}^{3} C_n\,\mathcal{L}_n(\mathcal{T}_1)$$

We have performed **numerical fits** to our results:

$$C_{-1,\text{fit}}(y_{13}) = \sum_{m,n=0}^{3} c_{(m,n)}\,[\ln(y_{13})]^m\,[\ln(1-y_{13})]^n$$

We have tested the fits using randomly-generated phase-space points for comparison: every event agrees with fit prediction within MC uncertainties.



	$gg \to g$	$q\bar{q} \to g$	$qg \to q$
$c_{(0,0)}$	63.187 ± 0.903	42.357 ± 0.786	39.101 ± 0.698
$c_{(1,0)}$	33.599 ± 0.779	25.158 ± 0.678	13.726 ± 0.615
$c_{(2,0)}$	-11.056 ± 0.227	-9.100 ± 0.197	-2.737 ± 0.186
$c_{(3,0)}$	-2.273 ± 0.021	-2.158 ± 0.019	0.016 ± 0.018
$c_{(0,1)}$	33.599 ± 0.779	25.158 ± 0.678	25.591 ± 0.602
$c_{(0,2)}$	-11.056 ± 0.227	-9.100 ± 0.197	-8.749 ± 0.177
$c_{(0,3)}$	-2.273 ± 0.021	-2.158 ± 0.019	-2.126 ± 0.017

Summary

- The LHC is operating very well. In order to compare theory and experiment in Run II, NNLO corrections are needed for most processes.

- NNLO calculations can be performed using phase-space slicing to handle the IR singularities and in particular the N-jettiness variable for processes containing jets.

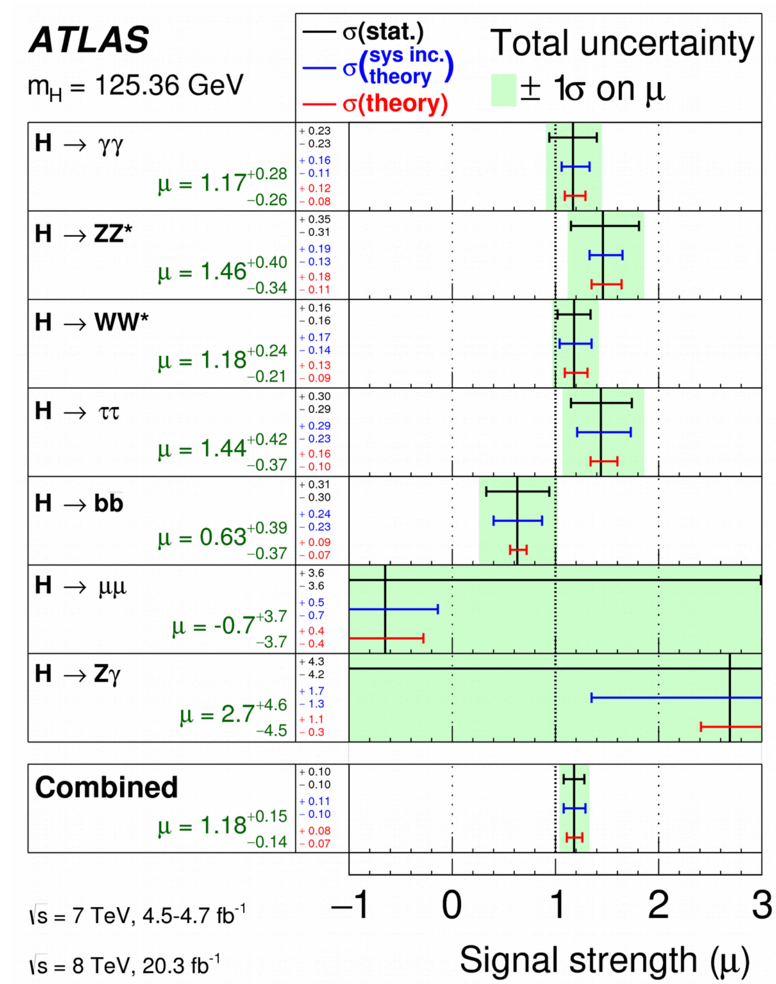
- One of the needed ingredients for the N-jettiness subtraction scheme is the soft function at NNLO, which we have computed numerically for the 1-jettiness case. We have performed several checks on our results and produced numerical fits for the endpoint contribution $\delta(\mathcal{T}_1)$.

Extra slides

Introduction and motivation

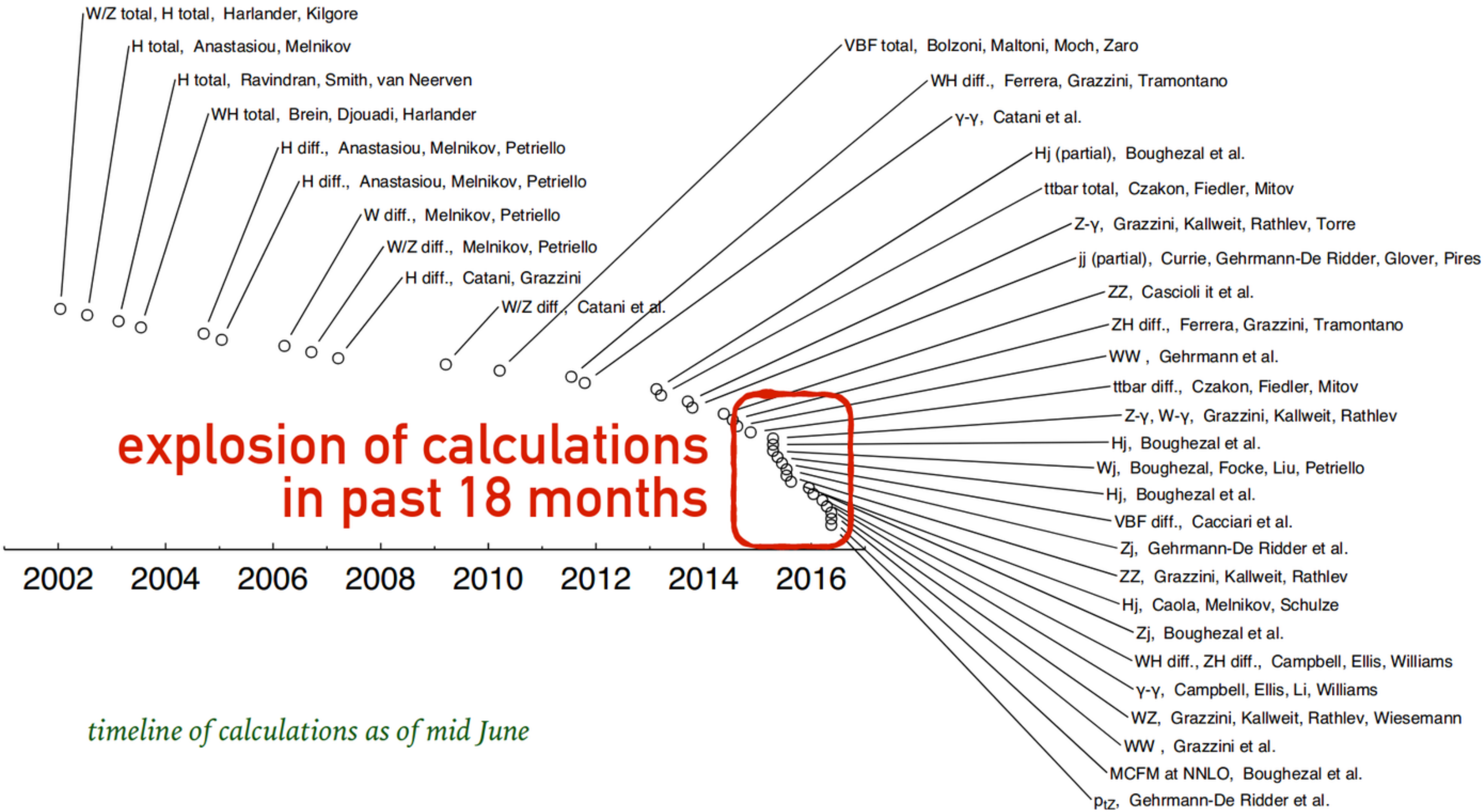
Already in Run I for some processes the theoretical uncertainties dominate the systematic uncertainty.

Theory will become the main source of uncertainty when Run II data are included and statistical uncertainties are reduced



Introduction and motivation

From G. Salam (2016)



Setup of the calculation (I)

Sudakov decomposition of the radiation momenta in terms of pair of directions:

$$q^\mu = q^j \, \frac{\hat{p}_i^\mu}{y_{ij}} + q^i \, \frac{\hat{p}_j^\mu}{y_{ij}} + q^\mu_{ij\perp}$$

where

$$q^x = 2\hat{p}_x \cdot q \qquad\qquad y_{ij} = 2\hat{p}_i \cdot \hat{p}_j$$

In terms of the projected momenta, the definition of the N-jettiness is

$$\mathcal{T}_N = \sum_m \min_r \{q_m^r\}$$

The minimum over *r* provides a natural division of the phase space for extra emission into regions where each projection is smallest.

Setup of the calculation (IV)

We can rewrite the single- and double-emission phase spaces in terms of the Sudakov variables q^i and q^j:

$$PS^{(1)}(i,j) \propto \frac{1}{y_{ij}^{1-\epsilon}} \int \mathrm{d}q^i \, \mathrm{d}q^j \, \left[q^i q^j\right]^{-\epsilon} \int_0^\pi \frac{d\phi}{N_\phi} \sin^{-2\epsilon}\phi$$

$$PS^{(2)}(i,j) \propto \frac{1}{y_{ij}^{2-2\epsilon}} \int \mathrm{d}q_1^i \mathrm{d}q_1^j \mathrm{d}q_2^i \mathrm{d}q_2^j \, \left[q_1^i q_1^j q_2^i q_2^j\right]^{-\epsilon}$$
$$\times \int_0^\pi \frac{d\phi_1}{N_{\phi_1}} \sin^{-2\epsilon}\phi_1 \int_0^\pi \frac{d\phi_2}{N_{\phi_2}} \sin^{-2\epsilon}\phi_2 \int_0^\pi \frac{d\beta}{N_\beta} \sin^{-1-2\epsilon}\beta$$

Example: 1-jettiness soft function at NLO

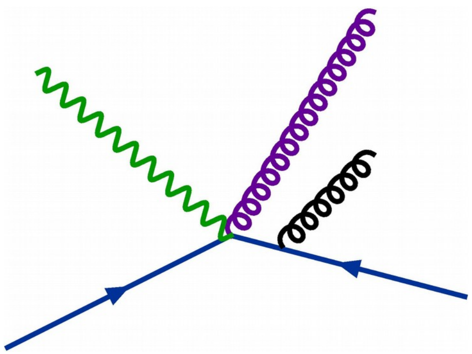
Only the real corrections contribute. The soft-gluon approximation for the squared matrix element is (factoring out the LO amplitude):

$$|M^{(1)}|^2 = -4g^2\mu^{2\epsilon}S_\epsilon \sum_{i<j} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \, \mathcal{S}_{ij}(q) \equiv \sum_{i<j} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \, |M_{ij}^{(1)}|^2$$

where

$$\mathcal{S}_{ij}(q) = \frac{p_i \cdot p_j}{2(p_i \cdot q)\,(p_j \cdot q)} = \frac{y_{ij}}{q^i q^j} \qquad S_\epsilon = \left(\frac{e^{\gamma_E}}{4\pi}\right)^\epsilon$$

The products $\boldsymbol{T}_i \cdot \boldsymbol{T}_j$ represent the color correlations and are expressed in terms of sums of Casimirs $\boldsymbol{T}_i^2 = C_A, C_F$



The **total result for the soft function at NLO** reads:

$$\left[\frac{\alpha_s}{2\pi}\right]\tilde{S}^{(1)} = \sum_{i<j} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \, |M_{ij}^{(1)}|^2 \; PS^{(1)} \; F$$

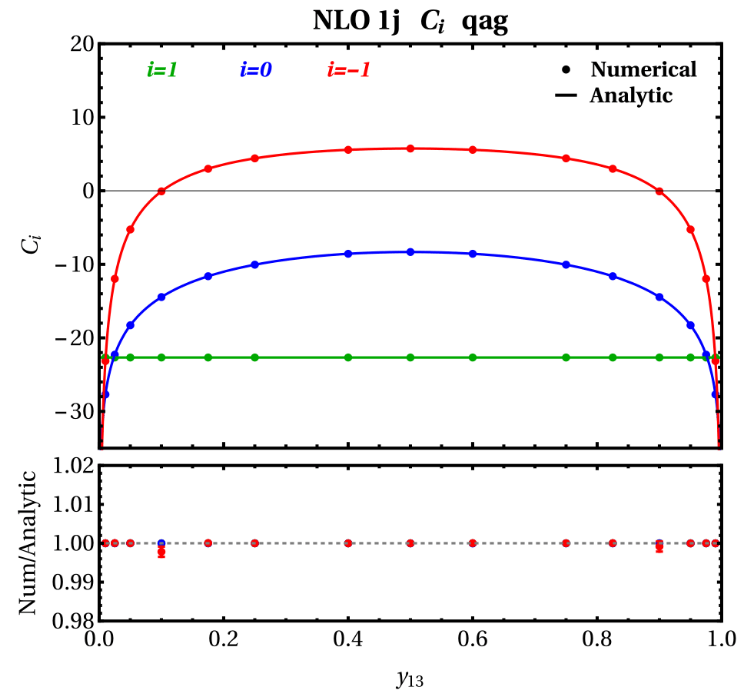
$$= \sum_{i<j} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \, |M_{ij}^{(1)}|^2 \left\{ PS^{(1)}(i,j) \left[F_i^{ij} + F_j^{ij}\right] + PS^{(1)}(k,i) \left[F_k^{ij}\right] \right\}$$

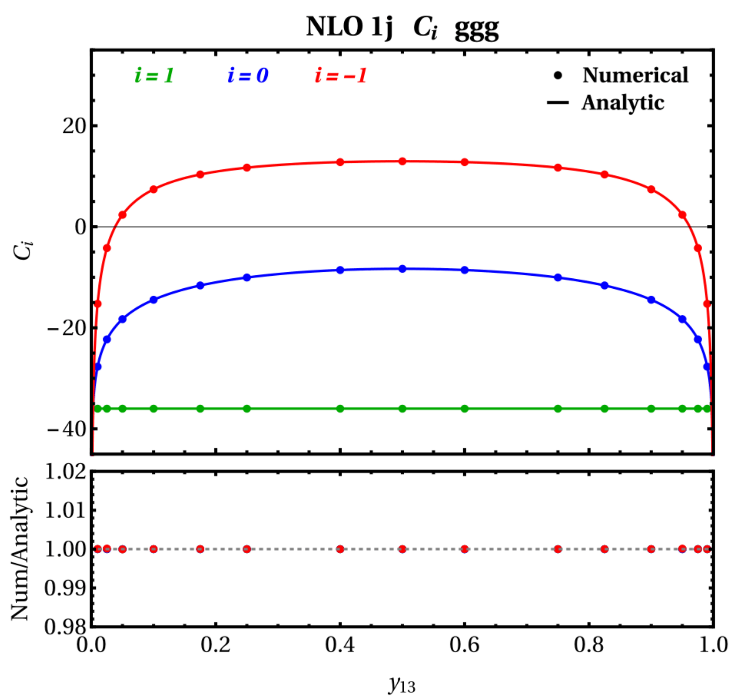
1-jettiness soft function at NLO

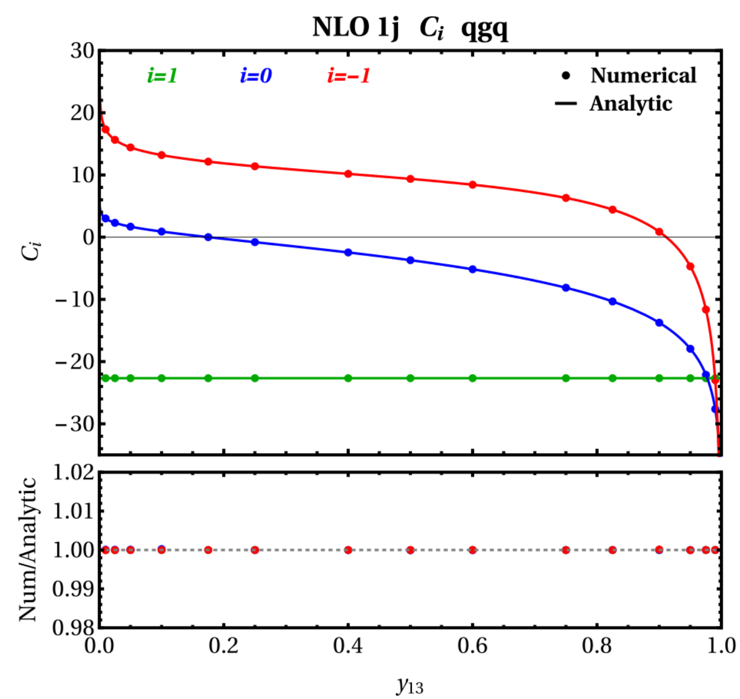
For each partonic channel we compute

$$S^{(1)}(\mathcal{T}_1) = C_{-1}\,\delta(\mathcal{T}_1) + C_0\,\mathcal{L}_0(\mathcal{T}_1) + C_1\,\mathcal{L}_1(\mathcal{T}_1)$$

Agreement between numerical and analytic results [Jouttenus et al. 1102.4344] is at the level of 0.2% or better.





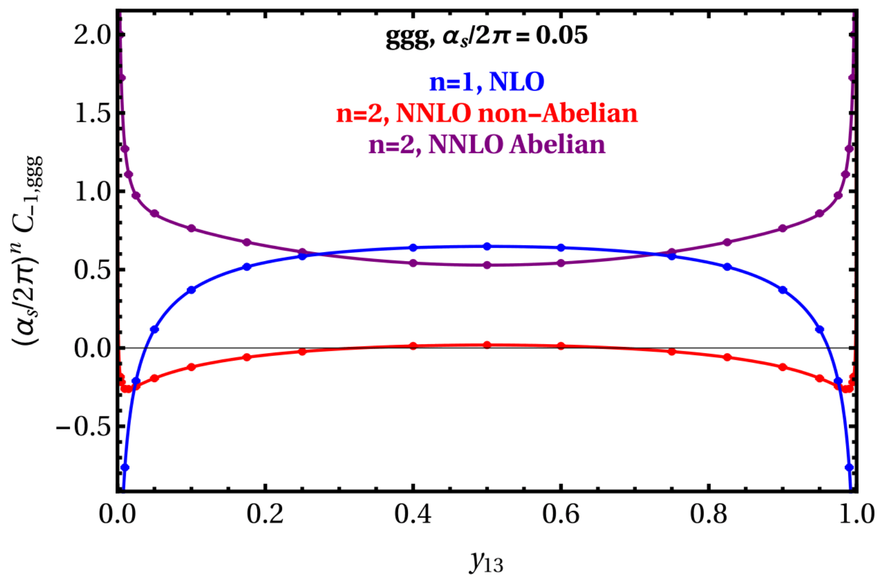


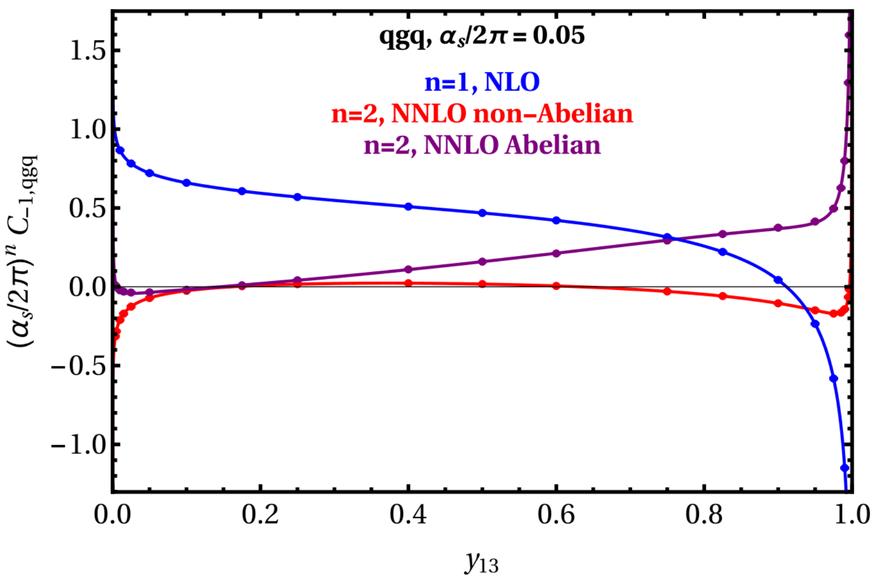
NLO and NNLO 1-jettiness soft functions

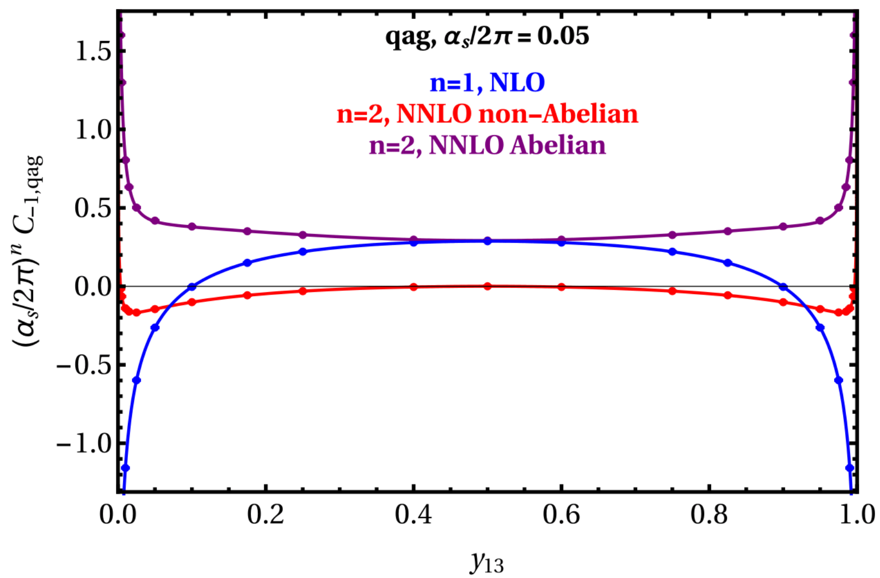
The Abelian term is significantly larger than the non-Abelian piece because it is enhanced by convolutions of the form:

$$(\mathcal{L}_m \otimes \mathcal{L}_n)(\mathcal{T}) \equiv \int d\mathcal{T}' \mathcal{L}_m(\mathcal{T}' - \mathcal{T}) \mathcal{L}_n(\mathcal{T}')$$

$$= V_{-1}^{mn} \delta(\mathcal{T}) + \sum_{k=0}^{m+n+1} V_k^{mn} \mathcal{L}_k(\mathcal{T})$$







1-jettiness soft function at NNLO for generic kinematics

We compute the non-Abelian part of the NNLO 1-jettiness soft function with generic kinematics (scattering process has three colored partons at Born level):

$$\hat{p}_1 = \frac{1}{2}(1,0,0,1) \qquad y_{12} = \frac{1}{2}(1-\cos\theta_2) \qquad \theta_2, \theta_3 \in [0,\pi]$$

$$\hat{p}_2 = \frac{1}{2}(1,0,\sin\theta_2,\cos\theta_2) \qquad y_{13} = \frac{1}{2}(1-\cos\theta_3) \qquad \phi \in [0,2\pi]$$

$$\hat{p}_3 = \frac{1}{2}(1,\sin\phi\sin\theta_3,\cos\phi\sin\theta_3,\cos\theta_3) \qquad y_{23} = \frac{1}{2}(1-\cos\phi\sin\theta_2\sin\theta_3-\cos\theta_2\cos\theta_3)$$

The LHC limit is recovered by setting $\phi = 0$, $\theta = \pi$

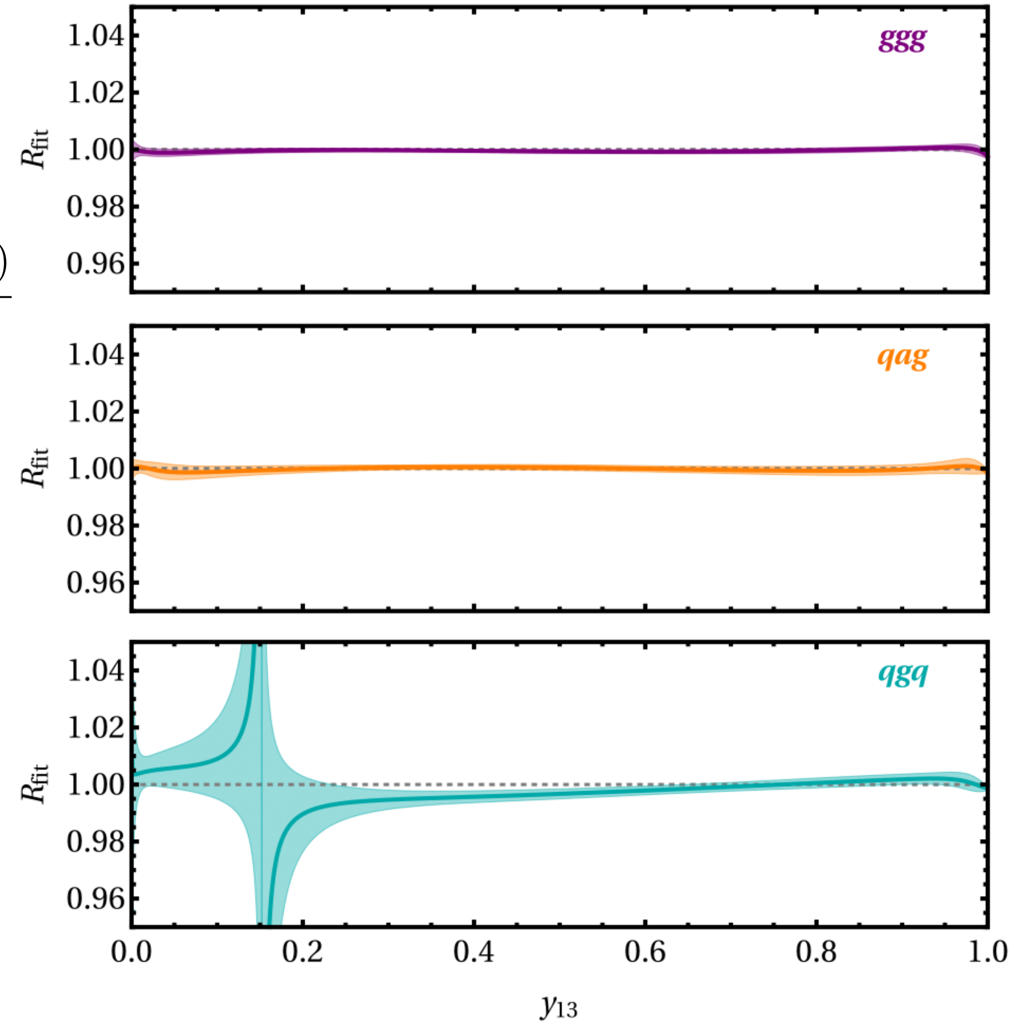
The coefficients C_n of the soft function are computed by choosing 200 random values for the angles. We then perform numerical fits to our results for the coefficient C_{-1}:

$$C^{\text{gen}}_{-1,\text{fit}}(y_{12},y_{13},y_{23}) = \sum_{k,m,n=0}^{3} c_{(k,m,n)} \left[\ln\left(y_{12}\right)\right]^k \left[\ln\left(y_{13}\right)\right]^m \left[\ln\left(y_{23}\right)\right]^n$$

1-jettiness soft function at NNLO for generic kinematics

We can check that the fits correctly reproduce the dedicated LHC fits when choosing $y_{12}=1$ and $y_{23}=1-y_{13}$

$$R_{\rm fit}(y_{13}) = \frac{C^{\rm ab}_{-1}(y_{13}) + C^{\rm gen}_{-1,\rm fit}(1, y_{13}, 1-y_{13})}{C^{\rm ab}_{-1}(y_{13}) + C_{-1,\rm fit}(y_{13})}$$



Agreement better than 1% for **ggg** and **qag** channels. For **qgq**, agreement within 1% away from region where both Abelian and non-Abelian pieces vanish.