Momentum distributions for the $D^0 \bar{D}^0 \pi^0$ and $D^0 \bar{D}^0 \gamma$ decay modes of the X(3872) resonance

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Outline

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- Universal properties of near-threshold S-wave resonance
- Line shapes for $D^0 \bar{D}^0 \pi^0 / D^0 \bar{D}^0 \gamma$
- Momentum distributions for $D^0 \bar{D}^0 \pi^0 / D^0 \bar{D}^0 \gamma$
- Summary
Introduction to the X(3872)

**Discovery**
- **Belle Collaboration (2003)**
  
  \[ B^+ \rightarrow K^+ + X \]
  
  \[ X \rightarrow J/\psi \pi^+ \pi^- \]

**Confirmation**
- **CDF Collaboration**
  
  \[ p\bar{p} \rightarrow X + \text{anything} \]

![Graph showing data for X(3872)](image)

PRL 91, 262001 (2003)

Introduction to the $X(3872)$

- Observation of $D^0 \bar{D}^0 \pi^0$ decay mode
  - Belle Collaboration (2006) [PRL 97, 162002]
Introduction to the $X(3872)$

- **Mass:** $3871.69 \pm 0.17$ MeV (PDG 2017)

$$M_X - (M_{*0} + M_0) = (+0.01 \pm 0.18) \text{ MeV}$$

- **Width:** $< 1.2$ MeV at 90% C.L. [Belle, PRD 84, 052004 (2011)]

- **$J^{PC}$ Quantum numbers:** $J^{PC}=1^{++}$ [LHCb, PRL, 110, 222001(2013)]
Introduction to the \textbf{X(3872)}

- What is the \textbf{X(3872)?}

  Two crucial experimental inputs:
  
  - **Quantum numbers**: $J^{PC}=1^{++}$
    
    \begin{align*}
    \text{S-wave coupling to } & D^0 \bar{D}^0 / \bar{D}^* D^0 \\
    \text{resonant coupling}
    \end{align*}

  - **Mass** is extremely close to $D^* \bar{D}$ threshold

- **Conclusion**: \textbf{X(3872) is a charm meson molecule!}

\[
X = \frac{1}{\sqrt{2}} \left( D^* \bar{D}^0 + D^0 \bar{D}^* \right)
\]
Universal properties

Nonrelativistic Quantum Mechanics:

- Short-range interactions
- S-wave resonance close enough to threshold

  large scattering length $a$ ($\gg$ range)

  universal features depend only on $a$, insensitive to shorter distances

$X(3872)$ close to $D^*0 \bar{D}^0$ threshold

  universal features depend only on large scattering length $a$ or complex inverse scattering length $\gamma(1/a)$ for $D^*0 \bar{D}^0$
The energy distribution summed over all resonant channels:

\[
\frac{dR}{dE} \text{[total]} = R_0 \frac{\text{Re} \left[ \sqrt{2\mu(E + i\Gamma_{*0}/2)} \right] + \text{Im}[\gamma]}{|-\gamma + \kappa(E)|^2}
\]

\( R_0 \): normalization factor depends on production mechanism.

\( \kappa(E) = \sqrt{-2\mu(E + i\Gamma_{*0}/2)} \)

\( E \): energy relative to the \( D^{*0} \bar{D}^0 \) threshold

Inelastic channels:

\( J/\psi \pi^+ \pi^- \), \( J/\psi \pi^+ \pi^- \pi^0 \)

Pole at

\[ E = -E_X - \frac{i\Gamma_X}{2} \]

\[ \gamma = \sqrt{2\mu(E_X + i(\Gamma_X - \Gamma_{*0})/2)} \]
Previous theoretical work on $D^0 \bar{D}^0 \pi^0$ and $D^0 \bar{D}^0 \gamma$

- **Voloshin** [PLB 579, 316 (2004)]:
  - interference between $D^{*0} \bar{D}^0 / \bar{D}^{*0} D^0$:
    - $J^{PC}=1^{++}$ constructive in $D^0 \bar{D}^0 \pi^0$
    - destructive in $D^0 \bar{D}^0 \gamma$

- momentum distributions reveal inner structure of X(3872)

- **Voloshin** [Int.J.Mod.Phys.A21 (2006)]
- **Colangelo, Fazio, Nicotri** [PLB 650, 166 (2007)]
- **Schmidt, Jansen, Hammer** [arXiv: J. C74, 1804.00375]
- ...  

Widths of X and $D^{*0}$ not treated consistently.
Line shapes in $D^0 \bar{D}^0\pi^0 / D^0 \bar{D}^0\gamma$

$\Gamma_X = \Gamma_{*0}$

$\Gamma_X = 3\Gamma_{*0}$

$E$: energy relative to the $D^{*0}\bar{D}^0$ threshold.

Line shape is sensitive to the binding energy $E_0$ and decay width $\Gamma_X$. 
Line shapes in $D^0 \bar{D}^0 \pi^0 / D^0 \bar{D}^0 \gamma$

Gaussian smearing in energy with width 1 MeV:

$\Gamma_x = \Gamma_{*0}$

$\Gamma_x = 3 \Gamma_{*0}$

- Black: $E_x = 0.2$ MeV
- Orange: $E_x = 0$ MeV
- Blue: $E_x = -0.2$ MeV

$E$ (MeV) vs. $dR/dE$
Momentum distributions for $D^0 \bar{D}^0 \pi^0$ decay modes

\[ \frac{dR}{dEdp} = R_0 \frac{1}{| - \gamma + \kappa(E)|^2} \frac{d\Gamma}{dp} \]
\[ d\Gamma = \frac{1}{3} \sum_{spin} |M|^2 d\Pi_3 \]
\[ \gamma = \sqrt{2\mu (E_X + i(\Gamma_X - \Gamma_{*0})/2)} \]
\[ \kappa(E) = \sqrt{-2\mu (E + i\Gamma_{*0}/2)} \]

Kinematic variables in $D^0 \bar{D}^0 \pi^0$ rest frame:

$E = \text{energy of } D^0 \bar{D}^0 \pi^0 \text{ relative to } D^{*0} \bar{D}^0 \text{ threshold}$

$p = \text{momentum of } D^0$

The shape of $D^0$ momentum distribution is insensitive to the binding energy and width of $X(3872)$ at fixed energy
Momentum distributions for $D^0 \bar{D}^0 \pi^0$ decay modes

At $E_x=0$, $\Gamma_x = \Gamma_{*0}$:

- Two peaks
- One wider peak
Momentum distributions

Explanation to double peak structure:

\[ \pi^0 \to D^*^0 \to D^0 \to \pi^0 \]

- \( p \approx 40 \text{ MeV} \)

\[ \bar{D}^*^0 \to \bar{D}^0 \to \pi^0 \to \bar{D}^0 \]

- \( p \approx 0 \)

\[ D^0 \bar{D}^0 \pi^0 : \text{peaks are near } p \approx 0 \text{ and } p \approx 40 \text{ MeV} \]

\[ D^0 \bar{D}^0 \gamma : \text{peaks are near } p \approx 0 \text{ and } p \approx 140 \text{ MeV} \]
Momentum distributions for $D^0 \bar{D}^0 \gamma$ decay modes

At $E_x = 0$, $\Gamma_x = \Gamma_{*0}$:

\[
\frac{dR}{dE dp} \quad \text{E=0} \quad \text{smeared around E=0}
\]
Summary

- **Double peaked structure** in $D^0$ momentum distributions for $D^0\bar{D}^0\pi^0$ and $D^0\bar{D}^0\gamma$ decay modes of the X(3872).

- First consistent theoretical treatment of width of X and $D^{*0}$.

- Position and width of the peaks are insensitive to the binding energy and width of X(3872).

- Smearing from experimental energy resolution:
  * two peaks for $D^0\bar{D}^0\pi^0$ smeared together
  * two peaks for $D^0\bar{D}^0\gamma$ still separated

- The $D^0$ momentum distributions should be measured at Belle II.
Thank you!
Branching ratios:

\[
\frac{\text{Br}[X \to J/\psi \pi^+ \pi^- \pi^0]}{\text{Br}[X \to J/\psi \pi^+ \pi^-]} = \begin{cases} 
1.0 \pm 0.5 & \text{Belle} \\
0.8 \pm 0.3 & \text{BaBar}
\end{cases}
\]

\[
J/\psi \pi^+ \pi^- \pi^0 \approx J/\psi \omega^* \quad \text{isospin 0}
\]

\[
J/\psi \pi^+ \pi^- \approx J/\psi \rho^* \quad \text{isospin 1}
\]

- large violation of isospin symmetry
- stronger coupling to isospin 0

\[
\frac{|G_{X J/\psi \rho}|^2}{|G_{X J/\psi \omega}|^2} \approx 0.08 \pm 0.04
\]

\[
\mathcal{L}_{XD^{*0}D^0} = \bar{g} g_{\mu\nu} X^\mu D^{*0\nu\dagger} D^{0\dagger} + h.c.,
\]

\[
\mathcal{L}_{D^{*0}D^0\gamma} = g_\gamma \epsilon_{\mu\nu\alpha\beta} \partial^\beta D^{*0\nu} D^{0\dagger} \partial^\alpha A^{\mu\dagger} + h.c.
\]

\[
\mathcal{L}_{D^{*0}D^0\pi^0} = \frac{g}{2\sqrt{m_0} f_\pi (M_0 + m_0)} D^{*0} [D^0 (M_0 \vec{\nabla} - m_0 \hat{\nabla}) \pi^0] + h.c.
\]
\[ \Gamma_{*0} : \text{width of } D^{*0}, \text{ obtained by chiral symmetry and isospin symmetry:} \]

\[
\frac{\text{Br}[D^{*0} \rightarrow D^0\pi^0]\Gamma_{*0}}{\text{Br}[D^{*+} \rightarrow D^0\pi^+]\Gamma_{*1}} = \frac{1}{2} \frac{\lambda^{3/2}(M_{*0}^2, M^2_0, m^2_0)/M_{*0}^5}{\lambda^{3/2}(M_{*1}^2, M^2_0, m^2_1)/M_{*1}^5}
\]

\[ \Gamma_{*1} = (83.4 \pm 1.8) \text{ keV} \quad \Gamma_{*0} = (55.4 \pm 1.8) \text{ keV} \]

\[ \frac{dR}{dE} \propto R_0 \frac{\text{Re}[2\mu(E + i\Gamma_{*0}/2)]}{| -\gamma + \kappa(E)|^2} \]

\[ \gamma = \sqrt{2\mu(E_X + i(\Gamma_X - \Gamma_{*0})/2)} \]

\[ dR[D^0 \bar{D}^0\pi^0] = R_0 \frac{1}{| -\gamma + \kappa(E)|^2} dEd\Gamma[D^0 \bar{D}^0\pi^0] \]

**Transition amplitude in the C=+ S-wave channel:**

\[ A(E) = \frac{2\pi/\mu}{-\gamma + \kappa(E)} \]

\[ \kappa(E) = \sqrt{-2\mu(E + i\Gamma_{*0}/2)} \quad E: \text{ energy relative to the } D^{*0}\bar{D}^0 \text{ threshold.} \]
$D^0 \bar{D}^0 \pi^0$

$D^0 \bar{D}^0 \gamma$
Line shape

smearing
Momentum distributions for $D^0 \bar{D}^0 \pi^0$ decay modes

Gaussian smearing in energy with width 1 MeV:

$E = \text{energy of } D^0 \bar{D}^0 \pi^0 \text{ relative to } D^{*0} \bar{D}^0 \text{ threshold}$

$p = \text{momentum of } D^0$

momentum distributions at $E_{x}=0$, $\Gamma_{x} = \Gamma_{*0}$:

\[ \frac{dR}{dE dp} \]
Momentum distributions for $D^0 \bar{D}^0 \gamma$ decay modes

Gaussian smearing in energy with width 1 MeV:

$E$ = energy of $D^0 \bar{D}^0 \pi^0$ relative to $D^{*0} \bar{D}^0$ threshold

$p$ = momentum of $D^0$

momentum distributions at $E_{x}=0$, $\Gamma_x = \Gamma_{*0}$:

- $E=-0.2$ MeV
- $E=+0.2$ MeV